A New Class of Gradient Adaptive Step-Size LMS Algorithms

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***Abstract*—The gradient adaptive step-size least-mean-square (LMS) algorithms [an important family of variable step-size LMS (VSLMS) algorithms] are revisited. We propose a simplification to a class of the studied algorithms and show that this leads to a new class of VSLMS algorithms with reduced complexity but with no observable loss in performance.**

***Index Terms*—Adaptive filters, adaptive signal processing, channel estimation, time-varying environments, tracking, variable step-size LMS algorithm, VSLMS algorithm.**

I. INTRODUCTION

IN a nonstationary environment, the optimum value of the step-size parameter μ of the least-mean-square (LMS) algorithm strikes a balance between the amount of *lag* noise and *gradient* noise [8]. However, in practice, the optimum value of the step-size parameter μ cannot be determined *a priori* due to unknown channel parameters. In addition, a fixed μ may not respond to time-variant channel parameters, resulting in poor performance. To deal with this problem, a number of variable step-size LMS (VSLMS) algorithms have been developed in the last two decades. Two earlier reported classes of these algorithms, which employ the gradient descent technique to adjust the step-size parameter, are Mathews' algorithm[4] and Benveniste's algorithm [1], [7], focusing on a common step size.

One advantage of Mathews' algorithm over Benveniste's algorithm is its simplicity. However, this causes degradation in performance when the measurement noise is high relative to the variation noise at the filter output due to the plant's time variations. In this case, the adaptation parameter used to update the LMS algorithm step-size parameter(s) needs to be relatively small for the mean square error (MSE) to converge close to its optimum value. Consequently, the speed of convergence of the step-size parameter(s) is slowed down. On the other hand, Benveniste's algorithm outperforms Mathews' algorithm by smoothing the gradient vector estimates that are used to update the LMS algorithm step-size parameter(s). However, it is computationally more involved than Mathews' algorithm.

There are other variations of the basic algorithm of Mathews and Benveniste. One of them uses multiple μ's to update the different filter tap weights, resulting in the multiple step-size VSLMS algorithm [4], [5]. This achieves better tracking behavior compared with using a common step-size parameter when identifying a two-path communications channel [5], [8], and it also overcomes the problem of compensating nonexistent tap weights of the communications channel [6]. Another area where the multiple step-size algorithm is effective is in acoustic echo cancellation [3]. Yet another variation of the VSLMS algorithm is when the linear update recursion of μ's is replaced by the multiplicative update recursion [5]. This outperforms its linear counterpart in general (see Table I for differences in linear updates and multiplicative updates). It is also noted in [5] that it is useful to normalize the parameter ρ (which is used to update μ) with respect to an estimate of the gradient power $E[|e(n)x^*(n-i)|^2|]$. This is done by replacing ρ with $\rho_i(n) = \rho_o/\phi_i(n) \quad i = 0 \cdots N-1$, where ρ_o is a common constant and $\phi_i(n)$, which is an estimate of the gradient power, may be obtained by using the recursion $\phi_i(n) = \beta\phi_i(n-1) + (1-\beta)|e(n)x^*(n-i)|^2$. Here, β is a constant smaller than but close to one.

Throughout this paper, $\mathbf{w}_o(n)$ and $\mathbf{w}(n)$ denote the length-N plant and adaptive filter weight vectors, respectively, $\mathbf{x}(n) = [x(n)x(n-1)\cdots x(n-N+1)]^T$ is the filter tap-input vector, $d(n)$ is the desired output of the filter, and $e(n) = d(n) - \mathbf{w}^H(n)\mathbf{x}(n)$. The superscripts $*$, T, and H denote conjugate, transpose, and Hermitian, respectively, and $\Re\{x\}$ denotes real part of x.

II. NEW CLASS OF VSLMS ALGORITHMS

In this section, we propose a simplification to Benveniste's algorithm, which leads to a new class of VSLMS algorithms with comparable performance to the original Benveniste's algorithm but with less complexity. The proposed simplification will be best explained by starting with a formulation of the multiple step-size Benveniste's algorithm, although the original Benveniste's VSLMS algorithm uses a common step-size parameter for all the filter taps. We thus begin with this formulation of Benveniste's algorithm.

A. Benveniste's Multiple Step-Size Algorithm

Starting with the multiple step-size parameters update equation and following the approach of Benveniste *et al.*, we arrive at the following recursions:

$$\mu_i(n) = \mu_i(n-1) + \rho\Re\{e(n)x^*(n-i)\psi_i(n)\} \quad \text{for } i = 0, 1, \cdots, N-1 \tag{1}$$

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$$\psi_i(n) \triangleq \frac{\partial w_i(n)}{\partial \mu_i(n-1)} = [1-\mu_i(n-1)|x(n-1-i)|^2] \cdot \psi_i(n-1) + e^*(n-1)x(n-1-i). \quad (2)$$

B. Proposed Simplification

In (2), if we define $\delta_i(n-1) = \mu_i(n-1)|x(n-1-i)|^2$ and substitute for $\psi_i(n-1)$ backward $n-1$ steps, we obtain

$$\psi_i(n) = \prod_{k=0}^{n-1}(1-\delta_i(k))\psi_i(0) + \sum_{j=0}^{n-2}\prod_{k=0}^{n-2-j}(1-\delta_i(n-1-k))x(j-i) \cdot e^*(j) + e^*(n-1)x(n-1-i). \quad (3)$$

Noting that $\delta_i(k)$s are small numbers since μ_is are small, we may write $\prod_{k=0}^{n-1}(1-\delta_i(k)) \approx 1 - \sum_{k=0}^{n-1}\delta_i(k) \approx 1 - n\delta_{i,ave} \approx (1-\delta_{i,ave})^n$, where $\delta_{i,ave}$ is the average value of $\delta_i(k)$s. Similarly, $\prod_{k=0}^{n-2-j}(1-\delta_i(k)) \approx (1-\delta_{i,ave})^{n-1-j}$. Thus, we obtain

$$\begin{aligned}\psi_i(n) &= (1-\delta_{i,ave})^n\psi_i(0) + \sum_{j=0}^{n-1}(1-\delta_{i,ave})^{n-1-j}x(j-i)e^*(j) \\ &= (1-\delta_{i,ave})\psi_i(n-1) + x(n-i-1)e^*(n-1). \quad (4)\end{aligned}$$

Next, we may replace $1-\delta_{i,ave}$ by a constant α smaller than but close to one. This leads to the recursion

$$\psi_i(n) = \alpha\psi_i(n-1) + x(n-i-1)e^*(n-1) \quad (5)$$

which we propose as a replacement for (2). This clearly simplifies the update equation (2) of Benveniste's algorithm. Mathews' algorithm is obtained when α is set equal to zero. This, of course, is the simplest algorithm among the three.

C. Gradient Filtering View of the Proposed and Benveniste's Algorithms

The recursive equation (5) may be viewed as follows. The process $\psi_i(n)$ is obtained by passing the noisy gradient term $x(n-i-1)e^*(n-1)$ through a filter with the transfer function $B(z) = (1/1-\alpha z^{-1})$. For α close to but smaller than one, this corresponds to a lowpass filtering operation that effectively takes a weighted average of the present and past observation. This, to a great extent, reduces the noise content of the past gradient vector, resulting in a more stable (less noisy) adaptation of the step-size parameters when compared with Mathews' algorithm, for which $\alpha = 0$.

In Benveniste's algorithm, the operation of lowpass filtering is also carried out in the same manner, with the difference that the constant coefficient α is replaced by the input dependent variable coefficient $1-\mu_i(n-1)|x(n-1-i)|^2$.

D. Common Step-Size Case

In vector form, (5) is written as

$$\boldsymbol{\psi}(n) = \alpha\boldsymbol{\psi}(n-1) + e^*(n-1)\mathbf{x}(n-1). \quad (6)$$

The update equation for the common step-size parameter $\mu(n)$ is then

$$\mu(n) = \mu(n-1) + \rho\Re\{e(n)\mathbf{x}^H(n)\boldsymbol{\psi}(n)\}. \quad (7)$$

III. Classification of the Algorithms and Computational Complexity

Table I summarizes the various algorithms investigated, including a count of the number of multiplication and addition operations needed. Two categories of algorithms [common step-size algorithms (named with prefix "c") and multiple step-size algorithms (named with prefix "m")] are presented. The table also highlights the two different forms of step-size update recursions mentioned: linear (named with suffix "L") and multiplicative updates (named with the suffix "M"). The meanings of linear and multiplicative updates are understood from the context of Table I. The common step-size and multiple step-size LMS algorithms using the optimum step-size parameter(s) reported in [8] are named c-OVSLMS and m-OVSLMS, respectively.

IV. Simulation Results

This section presents a thorough relative performance study of the new class of algorithms with the existing Mathews' and Benveniste's algorithms in the context of plant modeling (Section IV-A) and multipath channel identification[1] (Section IV-B), showing that the proposed algorithms outperform their previous counterparts. Only the results for multiplicative updates are presented, having noted that they outperform their linear counterparts in general [5]. The detailed comparison of Mathews' and Benveniste's algorithms presented here has not been given in any previous report. The results obtained using c-OVSLMS and m-OVSLMS are used as a baseline for evaluating the performance of the various versions of the VSLMS algorithm.

A. Tracking a Time-Varying Plant Using c-VSLMS Algorithms

The input $x(n)$ to the plant and adaptive filter is a zero-mean white Gaussian process of unit variance. The desired plant output $d(n)$ is corrupted with a zero-mean white Gaussian noise $e_o(n)$ with variance of 0.01. The plant is modeled as a transversal filter whose tap-weight vector $\mathbf{w}_o(n)$ is a multivariate random-walk process, where $\mathbf{w}_o(n) = \mathbf{w}_o(n-1) + \boldsymbol{\epsilon}_o(n)$, and $\sigma^2_{\epsilon_o} = 10^{-6}$. Without loss of generality, the initial value of $\mathbf{w}_o(n)$ is set to a length-N vector of zeros. Both the plant and adaptive filter length are chosen to be 16. The parameter α used in the proposed algorithm (c-VSLMS-III-M) is set equal to 0.95. The step-size parameter is checked at the end of every iteration of the algorithm and limited to stay within a range that satisfies[2] $\mu < 1/\mathrm{tr}[\mathbf{R}]$, [8]

[1]The performance of the algorithm in other applications, e.g., echo cancellation, is not investigated by the authors in this paper and may be different.

[2]This follows the approach in [4] since the conditions on ρ guaranteeing the convergence of the LMS algorithm is difficult to derive. tr[$\cdot$] denotes trace of $\mathbf{R} = \sum_i E[|x(n-i)|^2] = \sum_i \sigma^2_{x_i}$. $\sigma^2_{x_i}$ can be estimated by using the recursion $\hat{\sigma}^2_{x_i}(n) = \lambda\hat{\sigma}^2_{x_i}(n-1) + (1-\lambda)|x(n-i)|^2$, where $0 < \lambda < 1$. Note that here, the limit is for the complex-valued LMS algorithm [8].

TABLE I
CLASSIFICATION OF VSLMS ALGORITHMS AND COMPLEXITY

Algorithm	Name	Step-Size Update Equations	No. of complex multiply(add)‡
Mathews' common step-size	*c-VSLMS-I-L*	$\mu(n)=\mu(n-1)+$ $\rho\Re\{e(n)\mathbf{x}^H(n)e^*(n-1)\mathbf{x}(n-1)\}$	$3N(2N)$
	c-VSLMS-I-M	$\mu(n)=\mu(n-1)\times$ $[1+\rho\Re\{e(n)\mathbf{x}^H(n)e^*(n-1)\mathbf{x}(n-1)\}]$	$3N(2N)$
Benveniste's common step-size	*c-VSLMS-II-L*	$\mu(n)=\mu(n-1)+\rho\Re\{e(n)\mathbf{x}^H(n)\boldsymbol{\psi}(n)\}$ $\boldsymbol{\psi}(n)=\left[\mathbf{I}-\mu(n-1)\mathbf{x}(n-1)\mathbf{x}^H(n-1)\right]\times$ $\boldsymbol{\psi}(n-1)+e^*(n-1)\mathbf{x}(n-1)$	$5.5N(4.5N)$
	c-VSLMS-II-M	$\mu(n)=\mu(n-1)\times$ $[1+\rho\Re\{e(n)\mathbf{x}^H(n)\boldsymbol{\psi}(n)\}]$ $\boldsymbol{\psi}(n)=\left[\mathbf{I}-\mu(n-1)\mathbf{x}(n-1)\mathbf{x}^H(n-1)\right]\times$ $\boldsymbol{\psi}(n-1)+e^*(n-1)\mathbf{x}(n-1)$	$5.5N(4.5N)$
Proposed simplified Benveniste's common step-size	*c-VSLMS-III-L*	$\mu(n)=\mu(n-1)+\rho\Re\{e(n)\mathbf{x}^H(n)\boldsymbol{\psi}(n)\}$ $\boldsymbol{\psi}(n)=\alpha\boldsymbol{\psi}(n-1)+e^*(n-1)\mathbf{x}(n-1)$	$3.5N(2.5N)$
	c-VSLMS-III-M	$\mu(n)=\mu(n-1)[1+\rho\Re\{e(n)\mathbf{x}^H(n)\boldsymbol{\psi}(n)\}]$ $\boldsymbol{\psi}(n)=\alpha\boldsymbol{\psi}(n-1)+e^*(n-1)\mathbf{x}(n-1)$	$3.5N(2.5N)$
Mathews' multiple step-size	*m-VSLMS-I-L*	$\mu_i(n)=\mu_i(n-1)+$ $\rho_i\Re\{e(n)x^*(n-i)x(n-1-i)e^*(n-1)\}$	$5N(1.5N)$
	m-VSLMS-I-M	$\mu_i(n)=\mu_i(n-1)\times$ $[1+\rho_i\Re\{e(n)x^*(n-i)x(n-1-i)e^*(n-1)\}]$	$5N(1.5N)$
Benveniste's multiple step-size	*m-VSLMS-II-L*	$\mu_i(n)=\mu_i(n-1)+\rho_i\Re\{e(n)x^*(n-i)\psi_i(n)\}$ $\psi_i(n)=[1-\mu_i(n-1)\lvert x(n-1-i)\rvert^2]\times$ $\psi_i(n-1)+x(n-1-i)e^*(n-1)$	$6N(2.5N)$
	m-VSLMS-II-M	$\mu_i(n)=[1+\rho_i\Re\{e(n)x^*(n-i)\psi_i(n)\}]\mu_i(n-1)$ $\psi_i(n)=[1-\mu_i(n-1)\lvert x(n-1-i)\rvert^2]\times$ $\psi_i(n-1)+x(n-1-i)e^*(n-1)$	$6N(2.5N)$
Proposed simplified Benveniste's multiple step-size	*m-VSLMS-III-L*	$\mu_i(n)=\mu_i(n-1)+\rho_i\Re\{e(n)x^*(n-i)\psi_i(n)\}$ $\psi_i(n)=\alpha\psi_i(n-1)+x(n-1-i)e^*(n-1)$	$5.5N(2N)$
	m-VSLMS-III-M	$\mu_i(n)=[1+\rho_i\Re\{e(n)x^*(n-i)\psi_i(n)\}]\mu_i(n-1)$ $\psi_i(n)=\alpha\psi_i(n-1)+x(n-1-i)e^*(n-1)$	$5.5N(2N)$

‡ The operations include computation of the filter output, the tap weights and step-size parameter(s) updates, and the updates of the normalizing factor $\phi_i(n)$ for ρ.

to ensure the stability of the LMS algorithm. When this is not satisfied, the step-size parameter is hard limited to $1/\mathrm{tr}[\mathbf{R}]$. The parameter ρ is chosen such that the excess MSEs and the temporal root-mean-squared (rms) excess MSE [2] obtained for all algorithms are about the same. This ensures a fair comparison of the convergence speed and the steady-state behavior of various algorithms—a larger ρ compared with a smaller ρ achieves faster convergence and sometimes lower (average) excess MSE but causes large fluctuations of the temporal excess MSE about its average value. The initial convergence of the adaptive filter tap weights is bypassed by starting each simulation run with $\mathbf{w}(0)=\mathbf{w}_o(0)$. The subsequent changes of $\mathbf{w}_o(n)$ are then tracked by $\mathbf{w}(n)$.

In Fig. 1, we show results obtained from ensemble averaging of 50 independent runs for $\mu(n)$ and the mean squared norm of the coefficient error vector $\mathrm{tr}[\mathbf{K}]$ as a function of time. Here, $\mathrm{tr}[\mathbf{K}]=E[\mathbf{v}^H(n)\mathbf{v}(n)]$, and $\mathbf{v}(n)=\mathbf{w}(n)-\mathbf{w}_o(n)$. All three algorithms converge toward the optimum step-size parameter [8] predicted by the theory, with Benveniste's algorithm, as well as the proposed algorithm, converging much faster than Mathews' algorithm.

Fig. 2 studied the excess MSEs dependence on ρ, showing that the proposed and Benveniste's algorithms depend weakly on ρ for a large range of values compared with Mathews' algorithm. This is believed to be due to the gradient filtering/smoothing property of the algorithms.

B. Tracking a Multipath Channel Using m-VSLMS Algorithms

The simulation setup is shown in Fig. 3. The data symbols $x(n)$ are QPSK symbols with unity variance, at a rate of $1/T$. The combined transmitter and receiver filters result in a Nyquist raised cosine filter with 50% rolloff. The multipath channel is a two-path Rayleigh fading channel with impulse response $c(t)=a_1(t)\delta(t)+a_2(t)\delta(t-\tau(t))$, where $\tau(t)$ is the delay between the first and second path. $a_1(t)$ and $a_2(t)$ are

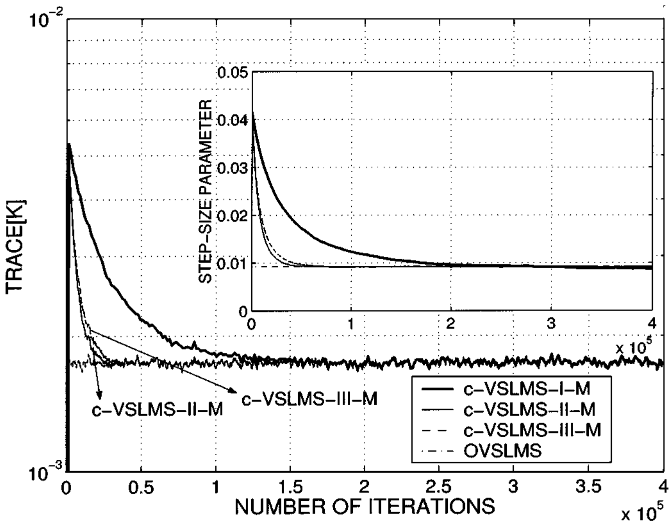

Fig. 1. Tracking performance of c-VSLMS algorithms using multiplicative update recursion. The plant and adaptive filter length is equal to 16. Variance of plant input, $\sigma_x^2 = 1.0$, plant measurement noise, $\sigma_{e_o}^2 = 0.01$, process noise variance of plant tap weights, $\sigma_{c_O}^2 = 10^{-6}$, $\rho = 2 \times 10^{-4}$ is used in all the algorithms, and $\alpha = 0.95$ is used in c-VSLMS-III-M.

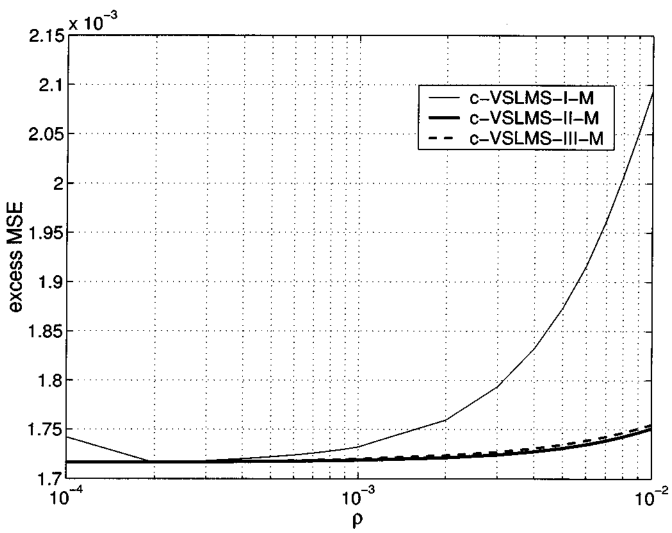

Fig. 2. Variation of excess MSE obtained as a function of ρ. This compares the sensitivity of various c-VSLMS algorithms with the parameter ρ. The simulation parameters are the same as those used in Fig. 1.

the complex path gains assumed to be uncorrelated with one another. Thus, the combined continuous-time impulse response consisting of the transmitter and receiver filter and the multipath channel is given as $h_t(t_o) = a_1(t_o)p(t) + a_2(t_o)p(t - \tau(t_o))$, where t_o is the time the channel response is measured, and $p(t)$ is the raised cosine pulse with 50% roll-off factor. The discrete-time combined channel model to be tracked by the adaptive filter is $W_o(z) = \sum_{i=0}^{N-1} w_{o,i}(n)z^{-i}$, and it is related to the continuous-time combined channel response $h_t(t_o)$ by $w_{o,i}(n) = h_{iT}(nT)$, for $i = 0, 1, \cdots, N-1$ and nT, is the time at which the discrete-time combined channel is measured. $a_1(nT)$ and $a_2(nT)$ are generated independently by passing independent complex-valued unity variance white Gaussian sequences through a single-pole lowpass filter given by $H(z) = (\sqrt{1-\gamma^2}/1 - \gamma z^{-1})$. The parameter γ is equal

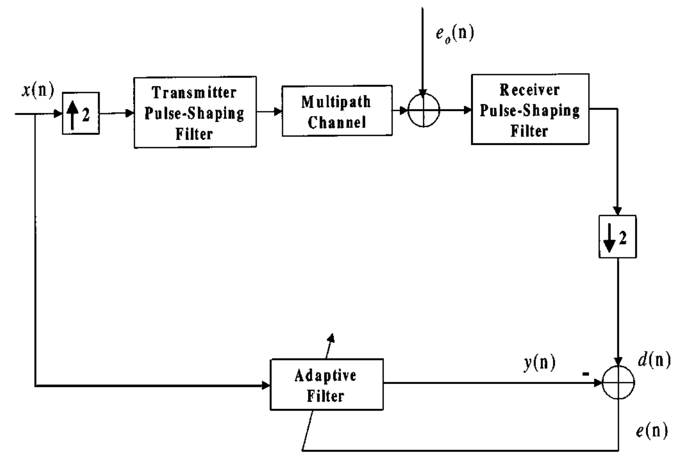

Fig. 3. Simulation setup for investigating the tracking performance of the various m-VSLMS-M algorithms.

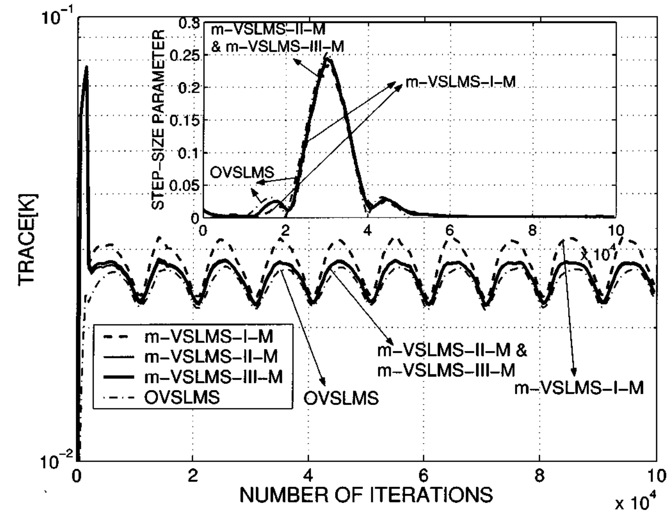

Fig. 4. Tracking performance of m-VSLMS algorithms using multiplicative update recursion under high SNR and slow channel variations. SNR $= 20$ dB, $f_d = 1$ Hz, and $\beta = 0.95$ are used in all simulations. $\rho_o = 2 \times 10^{-2}$ is used in m-VSLMS-I-M, $\rho_o = 1 \times 10^{-2}$ is used in both m-VSLMS-II-M and m-VSLMS-III-M. $\alpha = 0.95$ is used in m-VSLMS-III-M.

to $1 - (\pi f_d/2400)$, where f_d is proportional to the multipath channel fading rate in Hertz, and 2400 is the data baud rate. The multipath channel variations can be controlled by varying the rate the second path moves away from the first path and/or the value of f_d. The parameter ρ is normalized with respect to an estimate of $E[|e(n)x_i^*(n)\psi_i(n)|]$, for $i = 0 \cdots N-1$, similar to that proposed in [8]. It is determined by using the recursive equations $\phi_i(n) = \beta\phi_i(n-1) + (1-\beta)|e(n)x^*(n-i)\psi_i(n)|$, where β is a forgetting factor close to but less than one, and $\beta = 0.95$ in all simulations. The parameter ρ_o is chosen to achieve the best excess MSE for each of the algorithms. Hence, it may differ among the algorithms. These values are listed in the captions of the associated figures. In each experiment, the results are ensemble averaged over 40 independent runs. For each algorithm, one of the step-size parameters and the mean squared norm of the coefficient error vector, tr[$\mathbf{K}$] (tr[$\mathbf{K}$] $= E[\mathbf{v}^H(n)\mathbf{v}(n)]$, where $\mathbf{v}(n) = \mathbf{w}(n) - \mathbf{w}_o(n)$), as a function of time, are plotted. The parameter α used in the algorithm m-VSLMS-III-M is set equal to 0.95. The step-size

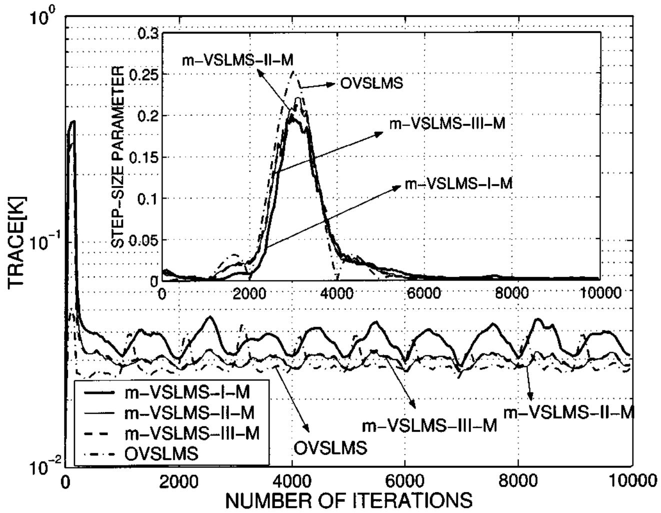

Fig. 5. Tracking performance of m-VSLMS algorithms using multiplicative update recursion under high SNR and fast channel variations. The simulation conditions are the same as in Fig. 4, except that the rate of separation of the two multipath is increased ten times and $\rho_o = 0.1$ in all algorithms.

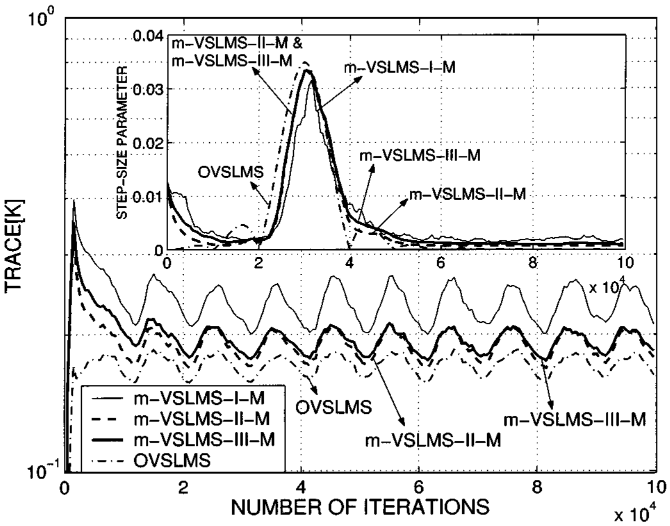

Fig. 6. Tracking performance of m-VSLMS algorithms using multiplicative update recursion under low SNR (SNR $= 0$ dB) and slow channel variations. The rest of the simulation conditions are the same as in Fig. 4.

parameters are checked at the end of every iteration of the algorithm and limited to stay within a range that satisfies tr$[\boldsymbol{\mu}\mathbf{R}] < 1$ [8] to ensure stability of the LMS algorithm. When this is not satisfied, all the step-size parameters are scaled down by the same factor such that tr$[\boldsymbol{\mu}\mathbf{R}]$ reduces to its upper bound 1. The step-size parameters are also hard-limited to the minimum value of $\mu_{\min} = 0.01/\mathrm{tr}[\mathbf{R}]$.

In Fig. 4, under high average signal-to-noise ratio (SNR) of 20 dB at the channel output and slow channel variations where $\tau(nT) = 2T + nT/10\,000$ and $f_d = 1$ Hz, Benveniste's (m-VSLMS-II-M) and the proposed (m-VSLMS-III-M) algorithm have about the same performance, tracking very closely to the optimum multiple step-size VSLMS algorithm (m-OVSLMS). On the other hand, the Mathews algorithm (m-VSLMS-I-M) has slightly poorer performance.

When the speed of separation of the second path from the first path is increased ten times, i.e., $\tau(nT) = 2T + nT/1000$, keeping $f_d = 1$ Hz and the average SNR at 20 dB at the channel output, m-VSLMS-II-M and m-VSLMS-III-M clearly respond faster and track the faster channel variations better than m-VSLMS-I-M (Fig. 5).

In Fig. 6, the performance of the algorithms is compared under a very low SNR of 0 dB, whereas the rest of the conditions are the same as in Fig. 4. Here, m-VSLMS-II-M and m-VSLMS-III-M again outperform m-VSLMS-I-M. The step-size parameters of m-VSLMS-I-M respond more slowly to the channel variations due to the low SNR.

V. Conclusions

In this paper, we proposed a new class of VSLMS algorithms. The proposed algorithms may be thought as a simplified version of their counterparts in the class of algorithms proposed by Benveniste *et al.* [1]. The VSLMS algorithm proposed by Mathews and Xie [4] was shown to be a special case of the proposed algorithm. The use of multiple step-size parameters and multiplicative update equations were also emphasized. Extensive computer simulations showed that the proposed algorithm, as well as Benveniste's, had very similar performance and that they outperformed Mathews' algorithm.

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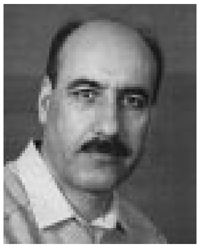

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