

**1.10**

$$1) (n+1)^3 = \underbrace{n^3 + 3n^2}_{n^2(n+3)} + 3n + 1 = n^2(n+3) + 3n + 1$$

2) Le reste de la division de  $(n+1)^3$  par  $n^2$  vaut  $3n+1$  uniquement si  $0 \leq 3n+1 < n^2$ .

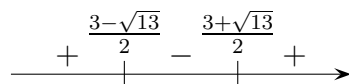
$$(a) 0 \leq 3n+1 \iff -1 \leq 3n \iff -\frac{1}{3} \leq n$$

$$(b) 3n+1 < n^2 \iff 0 < n^2 - 3n - 1$$

Étudions le signe de  $n^2 - 3n - 1$ .

$$\Delta = (-3)^2 - 4 \cdot 1 \cdot (-1) = 13$$

$$n_1 = \frac{3-\sqrt{13}}{2} \approx -0,30 \quad \text{et} \quad n_2 = \frac{3+\sqrt{13}}{2} \approx 3,30$$



Toutes ces conditions sont satisfaites si  $n \geq 4$ .