

5.1

$$1) \quad f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h) - x}{h} = \lim_{h \rightarrow 0} \frac{h}{h} = \lim_{h \rightarrow 0} 1 = 1$$

$$2) \quad f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \rightarrow 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\ = \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} = \lim_{h \rightarrow 0} \frac{h(2x + h)}{h} = \lim_{h \rightarrow 0} 2x + h = 2x + 0 = 2x$$

$$3) \quad f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^3 - x^3}{h} \\ = \lim_{h \rightarrow 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h} = \lim_{h \rightarrow 0} \frac{3x^2h + 3xh^2 + h^3}{h} \\ = \lim_{h \rightarrow 0} \frac{h(3x^2 + 3xh + h^2)}{h} = \lim_{h \rightarrow 0} 3x^2 + 3xh + h^2 = 3x^2 + 3x \cdot 0 + 0 \\ = 3x^2$$

$$4) \quad f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{(x+h)^4 - x^4}{h} \\ = \lim_{h \rightarrow 0} \frac{x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4 - x^4}{h} \\ = \lim_{h \rightarrow 0} \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h} = \lim_{h \rightarrow 0} \frac{h(4x^3 + 6x^2h + 4xh^2 + h^3)}{h} \\ = \lim_{h \rightarrow 0} 4x^3 + 6x^2h + 4xh^2 + h^3 = 4x^3 + 6x^2 \cdot 0 + 4x \cdot 0 + 0 = 4x^3$$

$$5) \quad f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \lim_{h \rightarrow 0} \frac{\frac{x - (x+h)}{x(x+h)}}{h} \\ = \lim_{h \rightarrow 0} \frac{\frac{-h}{x(x+h)}}{h} = \lim_{h \rightarrow 0} \frac{-1}{x(x+h)} = \frac{-1}{x(x+0)} = -\frac{1}{x^2}$$

$$6) \quad f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \\ = \lim_{h \rightarrow 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{(x+h) - x}{h(\sqrt{x+h} + \sqrt{x})} \\ = \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h} + \sqrt{x})} = \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x+0} + \sqrt{x}} \\ = \frac{1}{2\sqrt{x}}$$