10.12   1)
$$
\begin{array}{r|l}
x^2 - \ x & x+1 \\
-x^2 - \ x & \overline{x-2} \\
\hline
\ \ - 2\,x & \\
\ \ \ \ 2\,x + 2 & \\
\hline
\ \ \ \ \ \ \ \ 2 &
\end{array}
$$

$$x^2 - x = (x+1)(x-2) + 2 \quad \text{implique} \quad \frac{x^2 - x}{x+1} = x - 2 + \frac{2}{x+1}$$

$$\int \frac{x^2 - x}{x+1}\, dx = \int \Big(x - 2 + \frac{2}{x+1}\Big)\, dx = \int (x-2)\, dx + \int \frac{2}{x+1}\, dx$$

$$= \int (x-2)\, dx + 2\int \frac{1}{x+1}\, dx = \tfrac{1}{2}\,x^2 - 2\,x + 2\,\ln\big(|x+1|\big) + c$$

2)
$$
\begin{array}{r|l}
6\,x^2 - \ 4\,x + \ 2 & 3\,x+4 \\
-6\,x^2 - \ 8\,x & \overline{2\,x-4} \\
\hline
\ \ \ - 12\,x + \ 2 & \\
\ \ \ \ \ \ 12\,x + 16 & \\
\hline
\ \ \ \ \ \ \ \ \ \ 18 &
\end{array}
$$

De $6\,x^2 - 4\,x + 2 = (3\,x + 4)(2\,x - 4) + 18$ on tire que

$$\frac{6\,x^2 - 4\,x + 2}{3\,x + 4} = 2\,x - 4 + \frac{18}{3\,x + 4}$$

$$\int \frac{6\,x^2 - 4\,x + 2}{3\,x + 4}\, dx = \int \Big(2\,x - 4 + \frac{18}{3\,x + 4}\Big)\, dx = \int (2\,x - 4)\, dx + \int \frac{18}{3\,x + 4}\, dx$$

$$= \int (2\,x - 4)\, dx + 6\int \frac{1}{3\,x + 4}\cdot 3\, dx$$

$$= x^2 - 4\,x + 6\,\ln\big(|3\,x + 4|\big) + c$$

3)
$$
\begin{array}{r|l}
x^2 + 2\,x + 1 & x-1 \\
-x^2 + \ \ x & \overline{x+3} \\
\hline
\ \ \ \ 3\,x + 1 & \\
\ \ - 3\,x + 3 & \\
\hline
\ \ \ \ \ \ \ \ 4 &
\end{array}
$$

$(x+1)^2 = x^2 + 2\,x + 1 = (x-1)(x+3) + 4$ fournit

$$\frac{(x+1)^2}{x-1} = x + 3 + \frac{4}{x-1}$$

$$\int \frac{(x+1)^2}{x-1}\, dx = \int \Big(x + 3 + \frac{4}{x-1}\Big)\, dx = \int (x+3)\, dx + \int \frac{4}{x-1}\, dx$$

$$= \int (x+3)\, dx + 4\int \frac{1}{x-1}\, dx = \tfrac{1}{2}\,x^2 + 3\,x + 4\,\ln\big(|x-1|\big) + c$$

4)

$$\begin{array}{r|l} x^3 - 9\,x & x^2 - 1 \\ \underline{-x^3 + x} & x \\ -8\,x \end{array}$$

$$x\,(x^2 - 9) = x^3 - 9\,x = (x^2 - 1)\,x - 8\,x \quad \text{donne} \quad \frac{x\,(x^2 - 9)}{x^2 - 1} = x - \frac{8\,x}{x^2 - 1}$$

$$\int \frac{x\,(x^2 - 9)}{x^2 - 1}\,dx = \int \left(x - \frac{8\,x}{x^2 - 1}\right)dx = \int x\,dx - \int \frac{8\,x}{x^2 - 1}\,dx$$

$$= \int x\,dx - 4\int \frac{1}{x^2 - 1}\cdot 2\,x\,dx = \tfrac{1}{2}\,x^2 - 4\,\ln\big(|x^2 - 1|\big) + c$$