5.14

1) 
$$2(\sqrt{k+1} - \sqrt{k}) = 2 \cdot \frac{(\sqrt{k+1} - \sqrt{k})(\sqrt{k+1} + \sqrt{k})}{\sqrt{k+1} + \sqrt{k}} = 2 \cdot \frac{(k+1) - k}{\sqrt{k+1} + \sqrt{k}}$$

$$= 2 \cdot \frac{1}{\sqrt{k+1} + \sqrt{k}} < 2 \cdot \frac{1}{\sqrt{k} + \sqrt{k}} = 2 \cdot \frac{1}{2\sqrt{k}} = \frac{1}{\sqrt{k}}$$
2)  $\sum_{k=1}^{n} \frac{1}{\sqrt{k}} > \sum_{k=1}^{n} 2(\sqrt{k+1} - \sqrt{k}) = 2\sum_{k=1}^{n} \sqrt{k+1} - \sqrt{k}$ 

2) 
$$\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}} > \sum_{k=1}^{\infty} 2\left(\sqrt{k+1} - \sqrt{k}\right) = 2\sum_{k=1}^{\infty} \sqrt{k+1} - \sqrt{k}$$
$$= 2\left(\underbrace{\sqrt{2} - \sqrt{1}}_{k=1} + \underbrace{\sqrt{3} - \sqrt{2}}_{k=2} + \underbrace{\sqrt{4} - \sqrt{3}}_{k=3} + \dots + \underbrace{\sqrt{n+1} - \sqrt{n}}_{k=n}\right)$$
$$= 2\left(-\sqrt{1} + \sqrt{n+1}\right) = 2\left(\sqrt{n+1} - 1\right)$$

3) Puisque la suite des sommes partielles est non bornée, elle diverge.

Analyse : séries Corrigé 5.14