3.2 Soient $u=\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}, v=\begin{pmatrix}v_1\\v_2\\\vdots\\v_n\end{pmatrix}$ et $w=\begin{pmatrix}w_1\\w_2\\\vdots\\w_n\end{pmatrix}\in\mathbb{R}^n$. Soient $\alpha,\beta\in\mathbb{R}$.

1) (a)
$$\begin{aligned}(u+v)+w &= \left(\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}+\begin{pmatrix}v_1\\v_2\\\vdots\\v_n\end{pmatrix}\right)+\begin{pmatrix}w_1\\w_2\\\vdots\\w_n\end{pmatrix}=\begin{pmatrix}u_1+v_1\\u_2+v_2\\\vdots\\u_n+v_n\end{pmatrix}+\begin{pmatrix}w_1\\w_2\\\vdots\\w_n\end{pmatrix}\\ &=\begin{pmatrix}(u_1+v_1)+w_1\\(u_2+v_2)+w_2\\\vdots\\(u_n+v_n)+w_n\end{pmatrix}=\begin{pmatrix}u_1+(v_1+w_1)\\u_2+(v_2+w_2)\\\vdots\\u_n+(v_n+w_n)\end{pmatrix}\\ &=\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}+\begin{pmatrix}v_1+w_1\\v_2+w_2\\\vdots\\v_n+w_n\end{pmatrix}=\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}+\left(\begin{pmatrix}v_1\\v_2\\\vdots\\v_n\end{pmatrix}+\begin{pmatrix}w_1\\w_2\\\vdots\\w_n\end{pmatrix}\right)\\ &=u+(v+w)\end{aligned}$$

(b) Posons $0=\begin{pmatrix}0\\0\\\vdots\\0\end{pmatrix}$.

$$u+0=\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}+\begin{pmatrix}0\\0\\\vdots\\0\end{pmatrix}=\begin{pmatrix}u_1+0\\u_2+0\\\vdots\\u_n+0\end{pmatrix}=\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}=u$$

$$0+u=\begin{pmatrix}0\\0\\\vdots\\0\end{pmatrix}+\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}=\begin{pmatrix}0+u_1\\0+u_2\\\vdots\\0+u_n\end{pmatrix}=\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}=u$$

(c) Posons $-u=\begin{pmatrix}-u_1\\-u_2\\\vdots\\-u_n\end{pmatrix}$.

$$u+(-u)=\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}+\begin{pmatrix}-u_1\\-u_2\\\vdots\\-u_n\end{pmatrix}=\begin{pmatrix}u_1+(-u_1)\\u_2+(-u_2)\\\vdots\\u_n+(-u_n)\end{pmatrix}=\begin{pmatrix}0\\0\\\vdots\\0\end{pmatrix}=0$$

(d) $$u+v=\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}+\begin{pmatrix}v_1\\v_2\\\vdots\\v_n\end{pmatrix}=\begin{pmatrix}u_1+v_1\\u_2+v_2\\\vdots\\u_n+v_n\end{pmatrix}=\begin{pmatrix}v_1+u_1\\v_2+u_2\\\vdots\\v_n+u_n\end{pmatrix}$$

$$=\begin{pmatrix}v_1\\v_2\\\vdots\\v_n\end{pmatrix}+\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}=v+u$$

2) (a) $$\alpha\cdot(\beta\cdot u)=\alpha\cdot\left(\beta\cdot\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}\right)=\alpha\cdot\begin{pmatrix}\beta\,u_1\\\beta\,u_2\\\vdots\\\beta\,u_n\end{pmatrix}=\begin{pmatrix}\alpha\,(\beta\,u_1)\\\alpha\,(\beta\,u_2)\\\vdots\\\alpha\,(\beta\,u_n)\end{pmatrix}$$

$$=\begin{pmatrix}(\alpha\,\beta)\,u_1\\(\alpha\,\beta)\,u_2\\\vdots\\(\alpha\,\beta)\,u_n\end{pmatrix}=(\alpha\,\beta)\cdot\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}=(\alpha\,\beta)\cdot u$$

(b) $$(\alpha+\beta)\cdot u=(\alpha+\beta)\cdot\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}=\begin{pmatrix}(\alpha+\beta)\,u_1\\(\alpha+\beta)\,u_2\\\vdots\\(\alpha+\beta)\,u_n\end{pmatrix}=\begin{pmatrix}\alpha\,u_1+\beta\,u_1\\\alpha\,u_2+\beta\,u_2\\\vdots\\\alpha\,u_n+\beta\,u_n\end{pmatrix}$$

$$=\begin{pmatrix}\alpha\,u_1\\\alpha\,u_2\\\vdots\\\alpha\,u_n\end{pmatrix}+\begin{pmatrix}\beta\,u_1\\\beta\,u_1\\\vdots\\\beta\,u_n\end{pmatrix}=\alpha\cdot\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}+\beta\cdot\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}=\alpha\cdot u+\beta\cdot u$$

(c) $$\alpha\cdot(u+v)=\alpha\cdot\left(\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}+\begin{pmatrix}v_1\\v_2\\\vdots\\v_n\end{pmatrix}\right)=\alpha\cdot\begin{pmatrix}u_1+v_1\\u_2+v_2\\\vdots\\u_n+v_n\end{pmatrix}=\begin{pmatrix}\alpha\,(u_1+v_1)\\\alpha\,(u_2+v_2)\\\vdots\\\alpha\,(u_n+v_n)\end{pmatrix}$$

$$=\begin{pmatrix}\alpha\,u_1+\alpha\,v_1\\\alpha\,u_2+\alpha\,v_2\\\vdots\\\alpha\,u_n+\alpha\,v_n\end{pmatrix}=\begin{pmatrix}\alpha\,u_1\\\alpha\,u_2\\\vdots\\\alpha\,u_n\end{pmatrix}+\begin{pmatrix}\alpha\,v_1\\\alpha\,v_2\\\vdots\\\alpha\,v_n\end{pmatrix}$$

$$=\alpha\cdot\begin{pmatrix}u_1\\u_2\\\vdots\\u_n\end{pmatrix}+\alpha\cdot\begin{pmatrix}v_1\\v_2\\\vdots\\v_n\end{pmatrix}=\alpha\cdot u+\alpha\cdot v$$

(d) $1 \cdot u = 1 \cdot \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} = \begin{pmatrix} 1 \cdot u_1 \\ 1 \cdot u_2 \\ \vdots \\ 1 \cdot u_n \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{pmatrix} = u$