

$$\begin{aligned}
8.9 \quad (\tan(x))' &= \left( \frac{\sin(x)}{\cos(x)} \right)' = \frac{(\sin(x))' \cos(x) - \sin(x) (\cos(x))'}{\cos^2(x)} \\
&= \frac{\cos^2(x) - (-\sin^2(x))}{\cos^2(x)} = \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)} \\
&= \frac{\cos^2(x)}{\cos^2(x)} + \frac{\sin^2(x)}{\cos^2(x)} = 1 + \left( \frac{\sin(x)}{\cos(x)} \right)^2 = 1 + \tan^2(x)
\end{aligned}$$