

## 5.21

$$\begin{aligned}
1) \quad f'(x) &= (\sqrt{5x^2 - 2x + 1})' = ((5x^2 - 2x + 1)^{\frac{1}{2}})' \\
&= \frac{1}{2} (5x^2 - 2x + 1)^{-\frac{1}{2}} (5x^2 - 2x + 1)' \\
&= \frac{1}{2} \cdot \frac{1}{(5x^2 - 2x + 1)^{\frac{1}{2}}} (10x - 2) \\
&= \frac{1}{2} \cdot \frac{1}{\sqrt{5x^2 - 2x + 1}} \cdot 2(5x - 1) \\
&= \frac{5x - 1}{\sqrt{5x^2 - 2x + 1}}
\end{aligned}$$

$$\begin{aligned}
2) \quad f'(x) &= (\sqrt{(3x^2 + 1)^3})' = ((3x^2 + 1)^{\frac{3}{2}})' \\
&= \frac{3}{2} (3x^2 + 1)^{\frac{1}{2}} (3x^2 + 1)' \\
&= \frac{3}{2} \sqrt{3x^2 + 1} \cdot 6x \\
&= 9x \sqrt{3x^2 + 1}
\end{aligned}$$

$$\begin{aligned}
3) \quad f'(x) &= (\sqrt{(x+1)(2-3x)})' = \left( ((x+1)(2-3x))^{\frac{1}{2}} \right)' \\
&= \frac{1}{2} ((x+1)(2-3x))^{-\frac{1}{2}} ((x+1)(2-3x))' \\
&= \frac{1}{2} \cdot \frac{1}{((x+1)(2-3x))^{\frac{1}{2}}} \left( \underbrace{(x+1)'}_1 (2-3x) + (x+1) \underbrace{(2-3x)'}_{-3} \right) \\
&= \frac{1}{2} \cdot \frac{1}{\sqrt{(x+1)(2-3x)}} (2-3x-3x-3) \\
&= \frac{-6x-1}{2\sqrt{(x+1)(2-3x)}}
\end{aligned}$$

$$\begin{aligned}
4) \quad f'(x) &= ((1-x)\sqrt{1-x^2})' \\
&= (1-x)' \sqrt{1-x^2} + (1-x) (\sqrt{1-x^2})' \\
&= -1 \sqrt{1-x^2} + (1-x) ((1-x^2)^{\frac{1}{2}})' \\
&= -\sqrt{1-x^2} + (1-x) \frac{1}{2} (1-x^2)^{-\frac{1}{2}} (1-x^2)' \\
&= -\sqrt{1-x^2} + (1-x) \frac{1}{2} \cdot \frac{1}{(1-x^2)^{\frac{1}{2}}} (-2x) \\
&= -\sqrt{1-x^2} + (1-x) \frac{1}{\sqrt{1-x^2}} (-x) \\
&= -\sqrt{1-x^2} + \frac{x(x-1)}{\sqrt{1-x^2}} \\
&= \frac{-(1-x^2) + x(x-1)}{\sqrt{1-x^2}} \\
&= \frac{(x-1)(x+1) + x(x-1)}{\sqrt{1-x^2}}
\end{aligned}$$

$$\begin{aligned}
&= \frac{(x-1)((x+1)+x)}{\sqrt{1-x^2}} \\
&= \frac{(x-1)(2x+1)}{\sqrt{1-x^2}}
\end{aligned}$$

$$\begin{aligned}
5) \quad f'(x) &= \left( \frac{x}{\sqrt{1+x^2}} \right)' \\
&= \frac{(x)' \sqrt{1+x^2} - x (\sqrt{1+x^2})'}{(\sqrt{1+x^2})^2} \\
&= \frac{1 \sqrt{1+x^2} - x ((1+x^2)^{\frac{1}{2}})'}{1+x^2} \\
&= \frac{\sqrt{1+x^2} - x \frac{1}{2} (1+x^2)^{-\frac{1}{2}} (1+x^2)'}{1+x^2} \\
&= \frac{\sqrt{1+x^2} - x \frac{1}{2} \frac{1}{(1+x^2)^{\frac{1}{2}}} 2x}{1+x^2} \\
&= \frac{\sqrt{1+x^2} - \frac{x^2}{\sqrt{1+x^2}}}{1+x^2} \\
&= \frac{\frac{(1+x^2) - x^2}{\sqrt{1+x^2}}}{1+x^2} \\
&= \frac{1}{\sqrt{1+x^2} (1+x^2)}
\end{aligned}$$

$$\begin{aligned}
6) \quad f'(x) &= \left( \sqrt{\frac{1-2x}{3x+2}} \right)' \\
&= \left( \left( \frac{1-2x}{3x+2} \right)^{\frac{1}{2}} \right)' \\
&= \frac{1}{2} \left( \frac{1-2x}{3x+2} \right)^{-\frac{1}{2}} \left( \frac{1-2x}{3x+2} \right)' \\
&= \frac{1}{2} \left( \left( \frac{1-2x}{3x+2} \right)^{-1} \right)^{\frac{1}{2}} \frac{(1-2x)'(3x+2) - (1-2x)(3x+2)'}{(3x+2)^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2} \left( \frac{1}{\frac{1-2x}{3x+2}} \right)^{\frac{1}{2}} \frac{-2(3x+2) - (1-2x)3}{(3x+2)^2} \\
&= \frac{1}{2} \left( \frac{3x+2}{1-2x} \right)^{\frac{1}{2}} \frac{-6x-4-3+6x}{(3x+2)^2} \\
&= \frac{1}{2} \sqrt{\frac{3x+2}{1-2x}} \frac{-7}{(3x+2)^2} \\
&= \frac{-7}{2(3x+2)^2} \sqrt{\frac{3x+2}{1-2x}}
\end{aligned}$$

$$\begin{aligned}
7) \quad f'(x) &= \left( \frac{1}{x + \sqrt{1+x^2}} \right)' \\
&= \frac{-(x + \sqrt{1+x^2})'}{(x + \sqrt{1+x^2})^2} \\
&= \frac{-(x)' - (\sqrt{1+x^2})'}{(x + \sqrt{1+x^2})^2} \\
&= \frac{-1 - ((1+x^2)^{\frac{1}{2}})'}{(x + \sqrt{1+x^2})^2} \\
&= \frac{-1 - \frac{1}{2}(1+x^2)^{-\frac{1}{2}}(1+x^2)'}{(x + \sqrt{1+x^2})^2} \\
&= \frac{-1 - \frac{1}{2} \frac{1}{(1+x^2)^{\frac{1}{2}}} 2x}{(x + \sqrt{1+x^2})^2} \\
&= \frac{-1 - \frac{x}{\sqrt{1+x^2}}}{(x + \sqrt{1+x^2})^2} \\
&= \frac{\frac{-\sqrt{1+x^2} - x}{\sqrt{1+x^2}}}{(x + \sqrt{1+x^2})^2} \\
&= \frac{-1(x + \sqrt{1+x^2})}{\sqrt{1+x^2}(x + \sqrt{1+x^2})^2} \\
&= \frac{-1}{\sqrt{1+x^2}(x + \sqrt{1+x^2})}
\end{aligned}$$

$$\begin{aligned}
8) \quad f'(x) &= \left( \sqrt{x + \sqrt{x}} \right)' \\
&= (x + \sqrt{x})^{\frac{1}{2}} \\
&= \frac{1}{2} (x + \sqrt{x})^{-\frac{1}{2}} (x + \sqrt{x})' \\
&= \frac{1}{2} \frac{1}{(x + \sqrt{x})^{\frac{1}{2}}} ((x)' + (\sqrt{x})') \\
&= \frac{1}{2} \frac{1}{\sqrt{x + \sqrt{x}}} (1 + (x^{\frac{1}{2}})') \\
&= \frac{1}{2} \frac{1}{\sqrt{x + \sqrt{x}}} (1 + \frac{1}{2} x^{-\frac{1}{2}} (x)') \\
&= \frac{1}{2} \frac{1}{\sqrt{x + \sqrt{x}}} \left( 1 + \frac{1}{2\sqrt{x}} \right) \\
&= \frac{1}{2} \frac{1}{\sqrt{x + \sqrt{x}}} \frac{2\sqrt{x} + 1}{2\sqrt{x}} \\
&= \frac{2\sqrt{x} + 1}{4\sqrt{x} \sqrt{x + \sqrt{x}}}
\end{aligned}$$