**2.13** 1) 
$$\left(\frac{16}{625}\right)^{-\frac{1}{4}} = \left(\frac{2^4}{5^4}\right)^{-\frac{1}{4}} = \frac{(2^4)^{-\frac{1}{4}}}{(5^4)^{-\frac{1}{4}}} = \frac{2^{4 \cdot (-\frac{1}{4})}}{5^{4 \cdot (-\frac{1}{4})}} = \frac{2^{-1}}{5^{-1}} = \frac{\frac{1}{2^1}}{\frac{1}{5^1}} = \frac{\frac{1}{2}}{\frac{1}{5}} = \frac{5}{2}$$

2) 
$$49^{\frac{1}{6}} \cdot 49^{\frac{1}{3}} = (7^2)^{\frac{1}{6}} \cdot (7^2)^{\frac{1}{3}} = 7^{2 \cdot \frac{1}{6}} \cdot 7^{2 \cdot \frac{1}{3}} = 7^{\frac{1}{3}} \cdot 7^{\frac{2}{3}} = 7^{\frac{1}{3} + \frac{2}{3}} = 7^1 = 7$$

3) 
$$\left(8^{\frac{3}{2}}\right)^{\frac{4}{9}} = \left((2^3)^{\frac{3}{2}}\right)^{\frac{4}{9}} = 2^{3 \cdot \frac{3}{2} \cdot \frac{4}{9}} = 2^2 = 4$$

4) 
$$9^{-0,3} \cdot 9^{0,7} \cdot 9^{1,1} \cdot 9 = 9^{-\frac{3}{10}} \cdot 9^{\frac{7}{10}} \cdot 9^{\frac{11}{10}} \cdot 9 = 9^{-\frac{3}{10} + \frac{7}{10} + \frac{11}{10} + 1} = 9^{\frac{5}{2}} = (3^2)^{\frac{5}{2}} = 3^{\frac{5}{2}} = 3^5 = 243$$

5) 
$$5^{-\frac{1}{5}} \cdot 5^{\frac{4}{5}} \cdot 5^{\frac{9}{10}} = 5^{-\frac{1}{5} + \frac{4}{5} + \frac{9}{10}} = 5^{\frac{3}{2}} = \sqrt[2]{5^3} = \sqrt{5^{2+1}} = \sqrt{5^2 \cdot 5} = 5\sqrt{5}$$

6) 
$$(100^{0.4})^{1.25} = ((10^2)^{\frac{2}{5}})^{\frac{5}{4}} = 10^{2 \cdot \frac{2}{5} \cdot \frac{5}{4}} = 10^1 = 10$$

7) 
$$\left(\frac{1}{216}\right)^{\frac{1}{3}} = \left(\frac{1}{6^3}\right)^{\frac{1}{3}} = (6^{-3})^{\frac{1}{3}} = 6^{-3 \cdot \frac{1}{3}} = 6^{-1} = \frac{1}{6^1} = \frac{1}{6}$$

8) 
$$\frac{6^{\frac{1}{4}}}{6^{\frac{1}{3}}} = 6^{\frac{1}{4} - \frac{1}{3}} = 6^{-\frac{1}{12}} = \frac{1}{6^{\frac{1}{12}}} = \frac{1}{\sqrt[12]{6}}$$

9) 
$$\left(\frac{3^{-4} \cdot 5^{3}}{3^{11} \cdot 5^{-2}}\right)^{\frac{1}{5}} = \left(\frac{3^{-4}}{3^{11}} \cdot \frac{5^{3}}{5^{-2}}\right)^{\frac{1}{5}} = (3^{-4-11} \cdot 5^{3-(-2)})^{\frac{1}{5}} = (3^{-15} \cdot 5^{5})^{\frac{1}{5}} = (3^{-15} \cdot 5^{5})^{\frac{1}{5}} = (3^{-15} \cdot 5^{5})^{\frac{1}{5}} = 3^{-15 \cdot \frac{1}{5}} \cdot 5^{5 \cdot \frac{1}{5}} = 3^{-3} \cdot 5^{1} = \frac{1}{3^{3}} \cdot 5 = \frac{1}{27} \cdot 5 = \frac{5}{27}$$

10) 
$$a^{\frac{2}{3}} : (a^{\frac{2}{3}} : a^{\frac{3}{2}}) = a^{\frac{2}{3}} : a^{\frac{2}{3} - \frac{3}{2}} = a^{\frac{2}{3}} : a^{-\frac{5}{6}} = a^{\frac{2}{3} - (-\frac{5}{6})} = a^{\frac{3}{2}} = \sqrt[2]{a^3} = \sqrt[2]{a^3} = \sqrt[2]{a^2 \cdot a} = a\sqrt[2]{a}$$

11) 
$$\left(a^{-\frac{3}{4}} \cdot a\right)^{-2} = \left(a^{-\frac{3}{4}+1}\right)^{-2} = \left(a^{\frac{1}{4}}\right)^{-2} = a^{\frac{1}{4}\cdot(-2)} = a^{-\frac{1}{2}} = \frac{1}{a^{\frac{1}{2}}} = \frac{1}{\sqrt{a}} = \frac{\sqrt{a}}{a}$$

12) 
$$\left(a^{\frac{1}{5}}:a^{\frac{1}{10}}\right) \cdot a^{\frac{2}{5}} = a^{\frac{1}{5} - \frac{1}{10}} \cdot a^{\frac{2}{5}} = a^{\frac{1}{10}} \cdot a^{\frac{2}{5}} = a^{\frac{1}{10} + \frac{2}{5}} = a^{\frac{1}{2}} = \sqrt{a}$$

Algèbre : puissances