3.7 Soient $f, g, h \in \mathcal{F}_{[a\,;b]}$ et $\alpha, \beta \in \mathbb{R}$.

1) (a) $\big((f+g)+h\big)(x) = (f+g)(x)+h(x) = \big(f(x)+g(x)\big)+h(x) = f(x)+\big(g(x)+h(x)\big) = f(x)+(g+h)(x) = \big(f+(g+h)\big)(x)$

(b) Posons $o(x) = 0$ pour tout $x \in [a\,;b]$.
$(f+o)(x) = f(x)+o(x) = f(x)+0 = f(x)$
$(o+f)(x) = o(x)+f(x) = 0+f(x) = f(x)$

(c) Posons $(-f)(x) = -f(x)$.
$\big(f+(-f)\big)(x) = f(x)+(-f)(x) = f(x)+\big(-f(x)\big) = f(x)-f(x) = 0 = o(x)$

(d) $(f+g)(x) = f(x)+g(x) = g(x)+f(x) = (g+f)(x)$

2) (a) $\big(\alpha\cdot(\beta\cdot f)\big)(x) = \alpha\,\big((\beta\cdot f)(x)\big) = \alpha\,\big(\beta\,f(x)\big) = (\alpha\,\beta)\,f(x) = \big((\alpha\,\beta)\cdot f\big)(x)$

(b) $\big((\alpha+\beta)\cdot f\big)(x) = (\alpha+\beta)\,f(x) = \alpha\,f(x)+\beta\,f(x) = (\alpha\cdot f)(x)+(\beta\cdot f)(x) = (\alpha\cdot f+\beta\cdot f)(x)$

(c) $\big(\alpha\cdot(f+g)\big)(x) = \alpha\,(f+g)(x) = \alpha\,\big(f(x)+g(x)\big) = \alpha\,f(x)+\alpha\,g(x) = (\alpha\cdot f)(x)+(\alpha\cdot g)(x) = (\alpha\cdot f+\alpha\cdot g)(x)$

(d) $(1\cdot f)(x) = 1\cdot f(x) = f(x)$