8.16 1) $\det(A) = \begin{vmatrix} 3 & -1 \\ 2 & 4 \end{vmatrix} = 3 \cdot 4 - 2 \cdot (-1) = 14 \neq 0$

$A^{-1} = \frac{1}{14}\, {}^t\!\begin{pmatrix} 4 & -2 \\ -(-1) & 3 \end{pmatrix} = \frac{1}{14} \begin{pmatrix} 4 & 1 \\ -2 & 3 \end{pmatrix}$

2) $\det(A) = \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = 0 \cdot 1 - 1 \cdot 1 = -1 \neq 0$

$A^{-1} = \frac{1}{-1}\, {}^t\!\begin{pmatrix} 1 & -1 \\ -1 & 0 \end{pmatrix} = -\begin{pmatrix} 1 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix}$

3) $\det(A) = \begin{vmatrix} 8 & 12 \\ 2 & 3 \end{vmatrix} = 4 \begin{vmatrix} 2 & 3 \\ 2 & 3 \end{vmatrix} = 4 \cdot 0 = 0$: A n'est donc pas inversible

4) $\det(A) = \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = 2 \cdot 3 - 1 \cdot 1 = 5 \neq 0$

$A^{-1} = \frac{1}{5}\, {}^t\!\begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix} = \frac{1}{5} \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix}$

5) $\det(A) = \begin{vmatrix} \cos^2(\alpha) & \sin^2(\alpha) \\ \sin^2(\alpha) & \cos^2(\alpha) \end{vmatrix} = \cos^2(\alpha) \cdot \cos^2(\alpha) - \sin^2(\alpha) \cdot \sin^2(\alpha)$

$= \cos^4(\alpha) - \sin^4(\alpha) = \underbrace{\big(\cos^2(\alpha) - \sin^2(\alpha)\big)}_{\cos(2\,\alpha)}\ \underbrace{\big(\cos^2(\alpha) + \sin^2(\alpha)\big)}_{1}$

$= \cos(2\,\alpha)$

$\det(A) = 0$ si $\cos(2\,\alpha) = 0$, c'est-à-dire si $2\,\alpha = \frac{\pi}{2} + k\,\pi$ ou encore si $\alpha = \frac{\pi}{4} + k\,\frac{\pi}{2}$ $(k \in \mathbb{Z})$. Dans ce cas, A n'est pas inversible.

Dans le cas où $\alpha \neq \frac{\pi}{4} + k\,\frac{\pi}{2}$ $(k \in \mathbb{Z})$, A est inversible et son inverse vaut :

$A^{-1} = \frac{1}{\cos(2\,\alpha)}\, {}^t\!\begin{pmatrix} \cos^2(\alpha) & -\sin^2(\alpha) \\ -\sin^2(\alpha) & \cos^2(\alpha) \end{pmatrix} = \frac{1}{\cos(2\,\alpha)} \begin{pmatrix} \cos^2(\alpha) & -\sin^2(\alpha) \\ -\sin^2(\alpha) & \cos^2(\alpha) \end{pmatrix}$