

4.21

$$\begin{aligned} 1) \quad & (1-r)(u_1 + u_2 + u_3 + \dots + u_n) = \\ & (1-r)u_1 + (1-r)u_2 + (1-r)u_3 + \dots + (1-r)u_n = \\ & u_1 - u_1r + u_2 - u_2r + u_3 - u_3r + \dots + u_n - u_nr = \\ & u_1 - u_2 + u_2 - u_3 + u_3 - u_4 + \dots + u_n - u_{n+1} = \\ & u_1 - u_{n+1} \end{aligned}$$

$$\begin{aligned} 2) \quad & (1-r)(u_1 + u_2 + u_3 + \dots + u_n) = u_1 - u_{n+1} = u_1 - u_1 \cdot r^{(n+1)-1} \\ & = u_1 - u_1 \cdot r^n = u_1(1 - r^n) \end{aligned}$$

Comme $r \neq 1$, on a $1-r \neq 0$; en divisant l'équation précédente par $1-r$, on obtient la formule

$$u_1 + u_2 + u_3 + \dots + u_n = u_1 \cdot \frac{1 - r^n}{1 - r}$$