3.3 Posons $\alpha = \log_a(x)$ et $\beta = \log_a(y)$.

Par définition, on a $x = a^{\alpha}$ et $y = a^{\beta}$.

1)
$$\log_a(x y) = \log_a(a^{\alpha} \cdot a^{\beta}) = \log_a(a^{\alpha+\beta}) = \alpha + \beta = \log_a(x) + \log_a(y)$$

2)
$$\log_a(\frac{1}{y}) = \log_a(\frac{1}{a^{\beta}}) = \log_a(a^{-\beta}) = -\beta = -\log_a(y)$$

3)
$$\log_a(\frac{x}{y}) = \log_a(\frac{a^{\alpha}}{a^{\beta}}) = \log_a(a^{\alpha-\beta}) = \alpha - \beta = \log_a(x) - \log_a(y)$$

4)
$$\log_a(x^p) = \log_a((a^\alpha)^p) = \log_a(a^{p\alpha}) = p\alpha = p\log_a(x)$$

Algèbre : logarithmes Corrigé 3.3