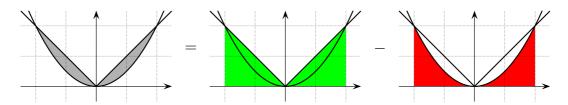
11.9



Les fonctions f et g sont paires :

$$f(-x) = \frac{1}{2}(-x)^2 = \frac{1}{2}x^2 = f(x)$$
 et $g(-x) = |-x| = |x| = g(x)$

C'est pourquoi, aire
$$\left\{ \left\{ (x;y) \in \mathbb{R}^2 : f(x) \leqslant y \leqslant g(x) \right\} \right\} =$$

$$2 \cdot \operatorname{aire} \Big(\big\{ (x; y) \in \mathbb{R}^2 : x \geqslant 0 \text{ et } f(x) \leqslant y \leqslant g(x) \big\} \Big).$$

Déterminons les abscisses des points d'intersection des graphes de f et de g lorsque $x\geqslant 0$:

$$\frac{1}{2}x^2 = |x|$$

$$\frac{1}{2}x^2 = x$$

$$0 = x - \frac{1}{2}x^2 = \frac{1}{2}x(2 - x)$$

$$x = 0$$
 ou $x = 2$

$$2 \cdot \left(\int_0^2 x \, dx - \int_0^2 \frac{1}{2} \, x^2 \, dx \right) = 2 \cdot \left(\int_0^2 \left(x - \frac{1}{2} \, x^2 \right) \, dx \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{6} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{2} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{2} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^2 - \frac{1}{2} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^3 - \frac{1}{2} \, x^3 \, \Big|_0^2 \right) = 2 \cdot \left(\frac{1}{2} \, x^3 - \frac{1}{2} \, x^3 + \frac{$$

$$2 \cdot \left(\left(\frac{1}{2} \cdot 2^2 - \frac{1}{6} \cdot 2^3 \right) - \left(\frac{1}{2} \cdot 0^2 - \frac{1}{6} \cdot 0^3 \right) \right) = 2 \cdot \left(\left(2 - \frac{4}{3} \right) - \left(0 - 0 \right) \right) = 2 \cdot \left(\frac{2}{3} - 0 \right) = 2 \cdot \frac{2}{3} = \frac{4}{3}$$

Analyse : intégrales Corrigé 11.9