3.15 Soient $w_1 \in \mathrm{F}$, $w_2 \in \mathrm{F}$ et $\alpha \in \mathbb{R}$.

Vu que $w_1 \in \mathrm{F}$, il existe $\lambda_1, \mu_1 \in \mathbb{R}$ tels que $w_1 = \lambda_1 \cdot u + \mu_1 \cdot v$.
Comme $w_2 \in \mathrm{F}$, il existe $\lambda_2, \mu_2 \in \mathbb{R}$ tels que $w_2 = \lambda_2 \cdot u + \mu_2 \cdot v$.

1) $$\begin{aligned} w_1 + w_2 &= (\lambda_1 \cdot u + \mu_1 \cdot v) + (\lambda_2 \cdot u + \mu_2 \cdot v) = \lambda_1 \cdot u + \lambda_2 \cdot u + \mu_1 \cdot v + \mu_2 \cdot v \\ &= (\underbrace{\lambda_1 + \lambda_2}_{\lambda}) \cdot u + (\underbrace{\mu_1 + \mu_2}_{\mu}) \cdot v = \lambda \cdot u + \mu \cdot v \in \mathrm{F} \end{aligned}$$

2) $$\begin{aligned} \alpha \cdot w_1 &= \alpha \cdot (\lambda_1 \cdot u + \mu_1 \cdot v) = \alpha \cdot (\lambda_1 \cdot u) + \alpha \cdot (\mu_1 \cdot v) = (\underbrace{\alpha\, \lambda_1}_{\lambda}) \cdot u + (\underbrace{\alpha\, \mu_1}_{\mu}) \cdot v \\ &= \lambda \cdot u + \mu \cdot v \in \mathrm{F} \end{aligned}$$