

10.2

$$1) \quad \|x\|^2 = \left(\sqrt{x_1^2 + \dots + x_n^2}\right)^2 = x_1^2 + \dots + x_n^2 = x \cdot x$$

$$2) \quad \|x\| = 0 \iff \|x\|^2 = 0 \iff x \cdot x = 0 \iff x = 0$$

$$\begin{aligned} 3) \quad \|\alpha x\| &= \|\alpha(x_1; \dots; x_n)\| \\ &= \|(\alpha x_1; \dots; \alpha x_n)\| \\ &= \sqrt{(\alpha x_1)^2 + \dots + (\alpha x_n)^2} \\ &= \sqrt{\alpha^2 x_1^2 + \dots + \alpha^2 x_n^2} \\ &= \sqrt{\alpha^2 (x_1^2 + \dots + x_n^2)} \\ &= \sqrt{\alpha^2} \sqrt{x_1^2 + \dots + x_n^2} \\ &= |\alpha| \|x\| \end{aligned}$$