

5.15

$$1) \begin{array}{c|c} 100 & 2 \\ 50 & 2 \\ 25 & 5 \\ 5 & 5 \\ 1 & \end{array}$$

$$\varphi(100) = \varphi(2^2 \cdot 5^2) = 100 \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{5}\right) = 100 \cdot \frac{1}{2} \cdot \frac{4}{5} = 40$$

$$2) \begin{array}{c|c} 720 & 2 \\ 360 & 2 \\ 180 & 2 \\ 90 & 2 \\ 45 & 3 \\ 15 & 3 \\ 5 & 5 \\ 1 & \end{array}$$

$$\varphi(720) = \varphi(2^4 \cdot 3^2 \cdot 5) = 720 \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{5}\right) = 720 \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{4}{5} = 192$$

$$3) \begin{array}{c|c} 1001 & 7 \\ 143 & 11 \\ 13 & 13 \\ 1 & \end{array}$$

$$\begin{aligned} \varphi(1001) &= \varphi(7 \cdot 11 \cdot 13) = 1001 \left(1 - \frac{1}{7}\right) \left(1 - \frac{1}{11}\right) \left(1 - \frac{1}{13}\right) = 1001 \cdot \frac{6}{7} \cdot \frac{10}{11} \cdot \frac{12}{13} \\ &= 720 \end{aligned}$$

$$\begin{aligned} 4) \quad 10! &= 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \\ &= 2 \cdot 5 \cdot 3^2 \cdot 2^3 \cdot 7 \cdot 2 \cdot 3 \cdot 5 \cdot 2^2 \cdot 3 \cdot 2 \\ &= 2^8 \cdot 3^4 \cdot 5^2 \cdot 7 \end{aligned}$$

$$\begin{aligned} \varphi(10!) &= \varphi(2^8 \cdot 3^4 \cdot 5^2 \cdot 7) = 10! \cdot \left(1 - \frac{1}{2}\right) \left(1 - \frac{1}{3}\right) \left(1 - \frac{1}{5}\right) \left(1 - \frac{1}{7}\right) \\ &= 10! \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{5} \cdot \frac{1}{7} = 829440 \end{aligned}$$