3.1 1) Posons 
$$y = \log_2(8)$$
.

$$2^{y} = 8$$

$$2^y = 2^3$$

$$y = 3$$

$$\log_2(8) = 3$$

2) Posons 
$$y = \log_3(1)$$
.

$$3^{y} = 1$$

$$3^y = 3^0$$

$$y = 0$$

$$\log_3(1) = 0$$

3) Posons 
$$y = \log_2(1024)$$
.

$$2^y = 1024$$

$$2^y = 2^{10}$$

$$y = 10$$

$$\log_2(1024) = 10$$

4) Posons 
$$y = \log_2(1)$$
.

$$2^{y} = 1$$

$$2^y = 2^0$$

$$y = 0$$

$$\log_2(1) = 0$$

5) Posons 
$$y = \log_2(512)$$
.

$$2^y = 512$$

$$2^y = 2^9$$

$$y = 9$$

$$\log_2(512) = 9$$

6) Posons 
$$y = \log_3\left(\sqrt[5]{3^2}\right)$$
.

$$3^y = \sqrt[5]{3^2}$$

$$3^y = 3^{\frac{2}{5}}$$

$$y = \frac{2}{5}$$

$$\log_3\left(\sqrt[5]{3^2}\right) = \frac{2}{5}$$

7) Posons 
$$y = \log_3(3)$$
.

$$3^{y} = 3$$

$$3^y = 3^1$$

$$y = 1$$

$$\log_3(3) = 1$$

8) Posons 
$$y = \log_2(\sqrt{2})$$
.  

$$2^y = \sqrt{2}$$

$$2^y = 2^{\frac{1}{2}}$$

$$y = \frac{1}{2}$$

$$\log_2(\sqrt{2}) = \frac{1}{2}$$

9) Posons 
$$y = \log_4(\sqrt[5]{64})$$
.  
 $4^y = \sqrt[5]{64}$   
 $4^y = \sqrt[5]{4^3}$   
 $4^y = 4^{\frac{3}{5}}$   
 $y = \frac{3}{5}$   
 $\log_4(\sqrt[5]{64}) = \frac{3}{5}$ 

10) Posons 
$$y = \log_3(\frac{1}{243})$$
.  

$$3^y = \frac{1}{243}$$

$$3^y = \frac{1}{3^5}$$

$$3^y = 3^{-5}$$

$$y = -5$$

$$\log_3(\frac{1}{243}) = -5$$

11) Posons 
$$y = \log_3(\sqrt[4]{27})$$
.  
 $3^y = \sqrt[4]{27}$   
 $3^y = \sqrt[4]{3^3}$   
 $3^y = 3^{\frac{3}{4}}$   
 $y = \frac{3}{4}$   
 $\log_3(\sqrt[4]{27}) = \frac{3}{4}$ 

12) Posons 
$$y = \log_4\left(\frac{1}{\sqrt[3]{16}}\right)$$
.  

$$4^y = \frac{1}{\sqrt[3]{16}}$$

$$4^y = \frac{1}{\sqrt[3]{4^2}}$$

$$4^y = \frac{1}{4^{\frac{2}{3}}}$$

$$4^y = 4^{-\frac{2}{3}}$$

$$y = -\frac{2}{3}$$

$$\log_4\left(\frac{1}{\sqrt[3]{16}}\right) = -\frac{2}{3}$$

13) Posons 
$$y = \log_3(27)$$
.  
 $3^y = 27$   
 $3^y = 3^3$   
 $y = 3$   
 $\log_3(27) = 3$ 

14) Posons 
$$y = \log_3(\frac{1}{81})$$
.  

$$3^y = \frac{1}{81}$$

$$3^y = \frac{1}{3^4}$$

$$3^y = 3^{-4}$$

$$y = -4$$

$$\log_3(\frac{1}{81}) = -4$$

15) Posons 
$$y = \log_5(0,04)$$
.  

$$5^y = 0,04$$

$$5^y = \frac{4}{100}$$

$$5^y = \frac{1}{25}$$

$$5^y = \frac{1}{5^2}$$

$$5^y = 5^{-2}$$

$$y = -2$$

$$\log_5(0,04) = -2$$

16) Posons 
$$y = \log_{\frac{1}{8}}(64)$$
.  

$$(\frac{1}{8})^y = 64$$

$$(8^{-1})^y = 8^2$$

$$8^{-y} = 8^2$$

$$-y = 2$$

$$y = -2$$

$$\log_{\frac{1}{8}}(64) = -2$$

17) Posons 
$$y = \log_{27}(3)$$
.  
 $27^y = 3$   
 $(3^3)^y = 3^1$   
 $3^3y = 3^1$   
 $3y = 1$   
 $y = \frac{1}{3}$   
 $\log_{27}(3) = \frac{1}{3}$ 

18) Posons 
$$y = \log(100)$$
.  
 $10^y = 100$   
 $10^y = 10^2$   
 $y = 2$   
 $\log(100) = 2$ 

19) Posons 
$$y = \log_4(\sqrt{2})$$
.  
 $4^y = \sqrt{2}$   
 $(2^2)^y = 2^{\frac{1}{2}}$   
 $2^{2y} = 2^{\frac{1}{2}}$ 

$$2y = \frac{1}{2}$$

$$y = \frac{1}{4}$$

$$\log_4(\sqrt{2}) = \frac{1}{4}$$

20) Posons 
$$y = \log_{49}(\sqrt[3]{7})$$
.  
 $49^y = \sqrt[3]{7}$   
 $(7^2)^y = \sqrt[3]{7^1}$   
 $7^{2y} = 7^{\frac{1}{3}}$   
 $2y = \frac{1}{3}$   
 $y = \frac{1}{6}$   
 $\log_{49}(\sqrt[3]{7}) = \frac{1}{6}$ 

21) Posons 
$$y = \log_9(\sqrt[4]{81})$$
.  
 $9^y = \sqrt[4]{81}$   
 $9^y = \sqrt[4]{9^2}$   
 $9^y = 9^{\frac{2}{4}}$   
 $y = \frac{2}{4} = \frac{1}{2}$   
 $\log_9(\sqrt[4]{81}) = \frac{1}{2}$ 

22) Posons 
$$y = \log_{0,25}(\sqrt{8})$$
.  
 $0.25^y = \sqrt{8}$   
 $(\frac{1}{4})^y = \sqrt{2^3}$   
 $(2^{-2})^y = 2^{\frac{3}{2}}$   
 $2^{-2y} = 2^{\frac{3}{2}}$   
 $-2y = \frac{3}{2}$   
 $y = -\frac{3}{4}$   
 $\log_{0,25}(\sqrt{8}) = -\frac{3}{4}$ 

23) Posons 
$$y = \ln(e) = \log_e(e)$$
.  
 $e^y = e$   
 $e^y = e^1$   
 $y = 1$   
 $\ln(e) = 1$ 

24) Posons 
$$y = \log_{0,1}(0,000\ 01)$$
.  
 $0,1^y = 0,000\ 01$   
 $\left(\frac{1}{10}\right)^y = \frac{1}{100000}$   
 $(10^{-1})^y = \frac{1}{10^5}$   
 $10^{-y} = 10^{-5}$   
 $-y = -5$   
 $y = 5$   
 $\log_{0,1}(0,000\ 01) = 5$ 

25) Posons 
$$y = \log_a(1)$$
.  

$$a^y = 1$$

$$a^y = a^0$$

$$y = 0$$

$$\log_a(1) = 0$$

26) Posons 
$$y = \log_a(a)$$
. 
$$a^y = a$$
 
$$a^y = a^1$$
 
$$y = 1$$
 
$$\log_a(a) = 1$$

27) Posons 
$$y = \log_a(a^3)$$
.  
 $a^y = a^3$   
 $y = 3$   
 $\log_a(a^3) = 3$ 

28) Posons 
$$y = \log_a(\frac{1}{a})$$
.  

$$a^y = \frac{1}{a}$$

$$a^y = a^{-1}$$

$$y = -1$$

$$\log_a(\frac{1}{a}) = -1$$
.

29) Posons 
$$y = \log_a(\sqrt{a})$$
.  
 $a^y = \sqrt{a}$   
 $a^y = a^{\frac{1}{2}}$   
 $y = \frac{1}{2}$   
 $\log_a(\sqrt{a}) = \frac{1}{2}$ 

30) Posons 
$$y = \log_a(\sqrt[3]{a^5})$$
.  

$$a^y = \sqrt[3]{a^5}$$

$$a^y = a^{\frac{5}{3}}$$

$$y = \frac{5}{3}$$

$$\log_a(\sqrt[3]{a^5}) = \frac{5}{3}$$

31) Posons 
$$y = \log_a\left(\frac{1}{\sqrt{a}}\right)$$
.
$$a^y = \frac{1}{\sqrt{a}}$$

$$a^y = \frac{1}{a^{\frac{1}{2}}}$$

$$a^y = a^{-\frac{1}{2}}$$

$$y = -\frac{1}{2}$$

$$\log_a\left(\frac{1}{\sqrt{a}}\right) = -\frac{1}{2}$$

32) Posons 
$$y = \log_a \left( a^{-2} \cdot \sqrt[3]{a} \right)$$
.  
 $a^y = a^{-2} \cdot \sqrt[3]{a}$   
 $a^y = a^{-2} \cdot a^{\frac{1}{3}}$   
 $a^y = a^{-2 + \frac{1}{3}} = a^{-\frac{5}{3}}$   
 $y = -\frac{5}{3}$   
 $\log_a \left( a^{-2} \cdot \sqrt[3]{a} \right) = -\frac{5}{3}$ 

Algèbre : logarithmes Corrigé 3.1