4.5 1) (a) i.
$$\lim_{x \to -\infty} \frac{\sqrt{4x^2 + 9}}{x} = \lim_{x \to -\infty} \frac{\sqrt{4x^2}}{x} = \lim_{x \to -\infty} \frac{|2x|}{x} = \lim_{x \to -\infty} \frac{-2x}{x} = \lim_{x \to -\infty} \frac{-2x}{x} = \lim_{x \to -\infty} \frac{-2x}{x} = \lim_{x \to -\infty} \frac{|2x|}{x} = \lim_{x \to -\infty} \frac{-2x}{x} = \lim_{x \to -\infty} \frac{$$

ii.
$$\lim_{\substack{x\to -\infty\\\text{indéterminé}}} \sqrt{4\,x^2+9} - (-2\,x) = \lim_{\substack{x\to -\infty\\\text{indéterminé}}} \sqrt{4\,x^2+9} + 2\,x = +\infty - \infty :$$

$$\lim_{x \to -\infty} \sqrt{4\,x^2 + 9} + 2\,x = \lim_{x \to -\infty} \frac{\left(\sqrt{4\,x^2 + 9} + 2\,x\right)\left(\sqrt{4\,x^2 + 9} - 2\,x\right)}{\sqrt{4\,x^2 + 9} - 2\,x} = \frac{1}{2}$$

$$\lim_{x \to -\infty} \frac{(4x^2 + 9) - (2x)^2}{\sqrt{4x^2} - 2x} = \lim_{x \to -\infty} \frac{9}{|2x| - 2x} = \lim_{x \to -\infty} \frac{9}{-2x - 2x} =$$

$$\lim_{x \to -\infty} \frac{9}{-4x} = 0$$

y = -2x est une asymptote oblique à gauche de f.

(b) i.
$$\lim_{x \to +\infty} \frac{\sqrt{4x^2 + 9}}{x} = \lim_{x \to +\infty} \frac{\sqrt{4x^2}}{x} = \lim_{x \to +\infty} \frac{|2x|}{x} = \lim_{x \to +\infty} \frac{2x}{x} = \lim_{x \to +\infty} \frac{2x}{x}$$

ii.
$$\lim_{x \to +\infty} \sqrt{4x^2 + 9} - 2x = +\infty - \infty$$
: indéterminé

$$\lim_{x \to +\infty} \sqrt{4 \, x^2 + 9} - 2 \, x = \lim_{x \to +\infty} \frac{(\sqrt{4 \, x^2 + 9} - 2 \, x) \, (\sqrt{4 \, x^2 + 9} + 2 \, x)}{\sqrt{4 \, x^2 + 9} + 2 \, x} = 0$$

$$\lim_{x \to +\infty} \frac{(4x^2 + 9) - (2x)^2}{\sqrt{4x^2} + 2x} = \lim_{x \to +\infty} \frac{9}{|2x| + 2x} = \lim_{x \to +\infty} \frac{9}{2x + 2x} = \lim_{x \to +\infty} \frac{9}{4x} = 0$$

y = 2x est une asymptote oblique à droite de f.

2) (a) i.
$$\lim_{x \to -\infty} \frac{1 + \sqrt{3}x^2 + 2}{x} = \lim_{x \to -\infty} \frac{1 + \sqrt{3}x^2}{x} = \lim_{x \to -\infty} \frac{1 + |\sqrt{3}x|}{x} = \lim_{x \to -\infty} \frac{1 - \sqrt{3}x}{x} = \lim_{x \to -\infty} \frac{-\sqrt{3}x}{x} = \lim_{x \to -\infty} -\sqrt{3} = -\sqrt{3}$$

ii.
$$\lim_{\substack{x\to-\infty\\x\to-\infty}}1+\sqrt{3\,x^2+2}-(-\sqrt{3}\,x)=\lim_{\substack{x\to-\infty\\1+\infty}}1+\sqrt{3\,x^2+2}+\sqrt{3}\,x=1+\infty$$
 1 + \infty - \infty: indéterminé

$$\lim_{x \to -\infty} 1 + \sqrt{3x^2 + 2} + \sqrt{3x} =$$

$$\lim_{x \to -\infty} 1 + \frac{(\sqrt{3}x^2 + 2 + \sqrt{3}x)(\sqrt{3}x^2 + 2 - \sqrt{3}x)}{\sqrt{3}x^2 + 2 - \sqrt{3}x} =$$

$$\lim_{x \to -\infty} 1 + \frac{(3x^2 + 2) - (\sqrt{3}x)^2}{\sqrt{3x^2} - \sqrt{3x}} = \lim_{x \to -\infty} 1 + \frac{2}{|\sqrt{3}x| - \sqrt{3}x} = \frac{1}{|\sqrt{3}x|} = \frac{1}{|\sqrt{3$$

$$\lim_{x \to -\infty} 1 + \frac{2}{-\sqrt{3}x - \sqrt{3}x} = \lim_{x \to -\infty} 1 + \frac{2}{-2\sqrt{3}x} = \frac{1}{2}$$

$$\lim_{x \to -\infty} 1 - \frac{1}{\sqrt{3}x} = 1 - 0 = 1$$

 $y = -\sqrt{3}x + 1$ est une asymptote oblique à gauche de f.

(b) i.
$$\lim_{x \to +\infty} \frac{1 + \sqrt{3}x^2 + 2}{x} = \lim_{x \to +\infty} \frac{1 + \sqrt{3}x^2}{x} = \lim_{x \to +\infty} \frac{1 + |\sqrt{3}x|}{x} = \lim_{x \to +\infty} \frac{1 + \sqrt{3}x}{x} = \lim_{x \to +\infty} \frac{\sqrt{3}x}{x} = \lim_{x \to +\infty} \sqrt{3} = \sqrt{3}$$

ii.
$$\lim_{x\to+\infty}1+\sqrt{3\,x^2+2}-\sqrt{3}\,x=1+\infty-\infty: \text{ indéterminé}$$

$$\lim_{x\to+\infty}1+\sqrt{3\,x^2+2}-\sqrt{3}\,x=$$

$$\lim_{x \to +\infty} 1 + \frac{\left(\sqrt{3} x^2 + 2 - \sqrt{3} x\right) \left(\sqrt{3} x^2 + 2 + \sqrt{3} x\right)}{\sqrt{3} x^2 + 2 + \sqrt{3} x} = \lim_{x \to +\infty} 1 + \frac{\left(3 x^2 + 2\right) - \left(\sqrt{3} x\right)^2}{\sqrt{3} x^2 + \sqrt{3} x} = \lim_{x \to +\infty} 1 + \frac{2}{|\sqrt{3} x| + \sqrt{3} x} = \lim_{x \to +\infty} 1 + \frac{2}{\sqrt{3} x + \sqrt{3} x} = \lim_{x \to +\infty} 1 + \frac{2}{2\sqrt{3} x} = \lim_{x \to +\infty} 1 + \frac{1}{\sqrt{3} x} = 1 + 0 = 1$$

 $y = \sqrt{3}x + 1$ est une asymptote oblique à droite de f.

3) (a) i.
$$\lim_{x \to -\infty} \frac{x - \sqrt{x^2 - 1}}{x} = \lim_{x \to -\infty} \frac{x - \sqrt{x^2}}{x} = \lim_{x \to -\infty} \frac{x - |x|}{x} = \lim_{x \to -\infty} \frac{x - (-x)}{x} = \lim_{x \to -\infty} \frac{2x}{x} = \lim_{x \to -\infty} 2 = 2$$

ii.
$$\lim_{\substack{x\to-\infty\\\text{indétermin\'e}}} x-\sqrt{x^2-1}-2\,x=\lim_{\substack{x\to-\infty\\}} -x-\sqrt{x^2-1}=+\infty-\infty:$$

$$\lim_{x \to -\infty} -x - \sqrt{x^2 - 1} = \lim_{x \to -\infty} \frac{(-x - \sqrt{x^2 - 1})(-x + \sqrt{x^2 - 1})}{-x + \sqrt{x^2 - 1}} = \lim_{x \to -\infty} \frac{x^2 - (x^2 - 1)}{-x + \sqrt{x^2}} = \lim_{x \to -\infty} \frac{1}{-x + |x|} = \lim_{x \to -\infty} \frac{1}{-2x} = 0$$

y = 2x est une asymptote oblique à gauche de f.

(b) i.
$$\lim_{x \to +\infty} \frac{x - \sqrt{x^2 - 1}}{x} = \lim_{x \to +\infty} \frac{x - \sqrt{x^2}}{x} = \lim_{x \to +\infty} \frac{x - |x|}{x} = \lim_{x \to +\infty} \frac{x - x}{x} = \lim_{x \to +\infty} \frac{0}{x} = 0$$

ii.
$$\lim_{x\to +\infty}x-\sqrt{x^2-1}-0$$
 $x=\lim_{x\to +\infty}x-\sqrt{x^2-1}=+\infty-\infty$: indéterminé

$$\lim_{x \to +\infty} x - \sqrt{x^2 - 1} = \lim_{x \to +\infty} \frac{(x - \sqrt{x^2 - 1})(x + \sqrt{x^2 - 1})}{x + \sqrt{x^2 - 1}} = \lim_{x \to +\infty} \frac{x^2 - (x^2 - 1)}{x} = \lim_{x \to +\infty} \frac{1}{x + \sqrt{x^2 - 1}} = 0$$

$$\lim_{x \to +\infty} \frac{x^2 - (x^2 - 1)}{x + \sqrt{x^2}} = \lim_{x \to +\infty} \frac{1}{x + |x|} = \lim_{x \to +\infty} \frac{1}{2x} = 0$$

y=0 est une asymptote horizontale à droite de f.

$$\begin{array}{lll} \text{4)} & \text{i.} & \lim_{x \to -\infty} \frac{2\,x - \sqrt{4\,x^2 + 2\,x + 1}}{x} = \lim_{x \to -\infty} \frac{2\,x - \sqrt{4\,x^2}}{x} = \lim_{x \to -\infty} \frac{2\,x - |2\,x|}{x} = \\ & \lim_{x \to -\infty} \frac{2\,x - (-2\,x)}{x} = \lim_{x \to -\infty} \frac{4\,x}{x} = \lim_{x \to -\infty} 4 = 4 \\ & \text{ii.} & \lim_{x \to -\infty} 2\,x - \sqrt{4\,x^2 + 2\,x + 1} - 4\,x = \lim_{x \to -\infty} -2\,x - \sqrt{4\,x^2 + 2\,x + 1} = \\ & +\infty - \infty : & \text{indéterminé} \\ & \lim_{x \to -\infty} -2\,x - \sqrt{4\,x^2 + 2\,x + 1} = \\ & \lim_{x \to -\infty} \frac{(-2\,x - \sqrt{4\,x^2 + 2\,x + 1})\,(-2\,x + \sqrt{4\,x^2 + 2\,x + 1})}{-2\,x + \sqrt{4\,x^2 + 2\,x + 1}} = \\ & \lim_{x \to -\infty} \frac{4\,x^2 - (4\,x^2 + 2\,x + 1)}{-2\,x + \sqrt{4\,x^2}} = \lim_{x \to -\infty} \frac{-2\,x - 1}{-2\,x + |2\,x|} = \lim_{x \to -\infty} \frac{-2\,x}{-2\,x - 2\,x} = \\ & \lim_{x \to -\infty} \frac{-2\,x}{-4\,x} = \lim_{x \to -\infty} \frac{1}{2} = \frac{1}{2} \\ & y = 4\,x + \frac{1}{2} \text{ est une asymptote oblique à gauche de } f. \end{array}$$

$$\begin{array}{ll} \text{(b)} & \text{i. } \lim_{x \to +\infty} \frac{2\,x - \sqrt{4\,x^2 + 2\,x + 1}}{x} = \lim_{x \to +\infty} \frac{2\,x - \sqrt{4\,x^2}}{x} = \lim_{x \to +\infty} \frac{2\,x - |2\,x|}{x} = \\ & \lim_{x \to +\infty} \frac{2\,x - 2\,x}{x} = \lim_{x \to +\infty} \frac{0}{x} = 0 \\ & \text{ii. } \lim_{x \to +\infty} 2\,x - \sqrt{4\,x^2 + 2\,x + 1} - 0\,x = \lim_{x \to +\infty} 2\,x - \sqrt{4\,x^2 + 2\,x + 1} = \\ & \infty - \infty : \text{ indéterminé} \\ & \lim_{x \to +\infty} 2\,x - \sqrt{4\,x^2 + 2\,x + 1} = \\ & \lim_{x \to +\infty} \frac{2\,x - \sqrt{4\,x^2 + 2\,x + 1} + 1}{2\,x - \sqrt{4\,x^2 + 2\,x + 1}} = \\ & \lim_{x \to +\infty} \frac{(2\,x - \sqrt{4\,x^2 + 2\,x + 1})\,(2\,x + \sqrt{4\,x^2 + 2\,x + 1})}{2\,x + \sqrt{4\,x^2 + 2\,x + 1}} = \\ & \lim_{x \to +\infty} \frac{4\,x^2 - (4\,x^2 + 2\,x + 1)}{2\,x + \sqrt{4\,x^2}} = \lim_{x \to +\infty} \frac{-2\,x - 1}{2\,x + |2\,x|} = \lim_{x \to +\infty} \frac{-2\,x}{2\,x + 2\,x} = \\ & \lim_{x \to +\infty} \frac{-2\,x}{4\,x} = \lim_{x \to +\infty} \frac{-1}{2} = -\frac{1}{2} \end{array}$$

 $y = -\frac{1}{2}$ est une asymptote horizontale à droite de f.

Analyse: asymptotes