**4.6**

$$1) \frac{5+3i}{2+4i} = \frac{(5+3i)(2-4i)}{(2+4i)(2-4i)} = \frac{10-20i+6i-12i^2}{4-8i+8i-16i^2} = \frac{10-14i+12}{4+16} = \frac{22-14i}{20} = \frac{22}{20} - \frac{14}{20}i = \frac{11}{10} - \frac{7}{10}i$$

2) 
$$\frac{63+16i}{4+3i} = \frac{(63+16i)(4-3i)}{(4+3i)(4-3i)} = \frac{252-189i+64i-48i^2}{16-12i+12i-9i^2} = \frac{252-125i+48}{16+9} = \frac{300-125i}{25} = \frac{300}{25} - \frac{125}{25}i = 12-5i$$

3) 
$$\frac{56+33i}{12-5i} = \frac{(56+33i)(12+5i)}{(12-5i)(12+5i)} = \frac{672+280i+396i+165i^2}{144+60i-60i-25i^2} = \frac{672+676i-165}{144+25} = \frac{507+676i}{169} = \frac{507}{169} + \frac{676}{169}i = 3+4i$$

4) 
$$\frac{13-5i}{1-i} = \frac{(13-5i)(1+i)}{(1-i)(1+i)} = \frac{13+13i-5i-5i^2}{1+i-i-i^2} = \frac{13+8i+5}{1+1} = \frac{18+8i}{2} = \frac{18}{2} + \frac{8}{2}i = 9+4i$$

5) 
$$\frac{2i}{1+3i} = \frac{2i(1-3i)}{(1+3i)(1-3i)} = \frac{2i-6i^2}{1-3i+3i-9i^2} = \frac{2i+6}{1+9} = \frac{6+2i}{10} = \frac{6}{10} + \frac{2}{10}i = \frac{3}{5} + \frac{1}{5}i$$

6) 
$$\frac{i}{2-3i} = \frac{i(2+3i)}{(2-3i)(2+3i)} = \frac{2i+3i^2}{4+6i-6i-9i^2} = \frac{2i-3}{4+9} = \frac{-3+2i}{13} = \frac{-3+2i}{13} = \frac{3}{13} + \frac{2}{13}i$$

7) 
$$\frac{7+i}{3-2i} = \frac{(7+i)(3+2i)}{(3-2i)(3+2i)} = \frac{21+14i+3i+2i^2}{9+6i-6i-4i^2} = \frac{21+17i-2}{9+4} = \frac{19+17i}{13} = \frac{19}{13} + \frac{17}{13}i$$

8) 
$$\frac{-3}{(1+i)(2-i)} = \frac{-3}{2-i+2i-i^2} = \frac{-3}{2+i+1} = \frac{-3}{3+i} = \frac{-3(3-i)}{(3+i)(3-i)} = \frac{-9+3i}{9-3i+3i-i^2} = \frac{-9+3i}{9+1} = \frac{-9+3i}{10} = -\frac{9}{10} + \frac{3}{10}i$$