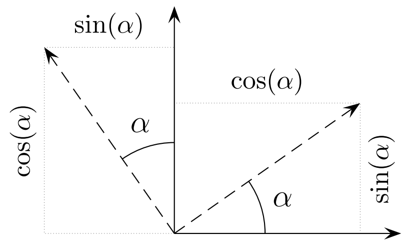
11.5 1)



(a) $r(e_1) = \cos(\alpha) \cdot e_1 + \sin(\alpha) \cdot e_2 = \begin{pmatrix} \cos(\alpha) \\ \sin(\alpha) \end{pmatrix}$

(b) $r(e_2) = -\sin(\alpha) \cdot e_1 + \cos(\alpha) \cdot e_2 = \begin{pmatrix} -\sin(\alpha) \\ \cos(\alpha) \end{pmatrix}$

(c) $\mathrm{A} = \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) \\ \underbrace{\sin(\alpha)}_{r(e_1)} & \underbrace{\cos(\alpha)}_{r(e_2)} \end{pmatrix}$

2) (a) $^t\mathrm{A}\mathrm{A} = \begin{pmatrix} \cos(\alpha) & \sin(\alpha) \\ -\sin(\alpha) & \cos(\alpha) \end{pmatrix} \begin{pmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{pmatrix}$

$= \begin{pmatrix} \cos^2(\alpha) + \sin^2(\alpha) & -\cos(\alpha)\,\sin(\alpha) + \cos(\alpha)\,\sin(\alpha) \\ -\cos(\alpha)\,\sin(\alpha) + \cos(\alpha)\,\sin(\alpha) & \cos^2(\alpha) + \sin^2(\alpha) \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathrm{I}$

(b) $\det(\mathrm{A}) = \begin{vmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{vmatrix} = \cos^2(\alpha) + \sin^2(\alpha) = 1$

3) $\begin{vmatrix} \cos(\alpha) - \lambda & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) - \lambda \end{vmatrix} = \big(\cos(\alpha) - \lambda\big)^2 + \sin^2(\alpha) =$

$\lambda^2 - 2\,\cos(\alpha)\,\lambda + \cos^2(\alpha) + \sin^2(\alpha) = \lambda^2 - 2\,\cos(\alpha)\,\lambda + 1$

$\Delta = \big(-2\,\cos(\alpha)\big)^2 - 4 \cdot 1 \cdot 1 = 4\,\cos^2(\alpha) - 4 = 4\,\big(\cos^2(\alpha) - 1\big)$

Puisque $-1 \leqslant \cos(\alpha) \leqslant 1$, on en déduit que :

(a) $\Delta = 0$ si $\alpha = k\,\pi$ avec $k \in \mathbb{Z}$;

(b) $\Delta < 0$ sinon.

Si $\alpha = 2\,k\,\pi$ avec $k \in \mathbb{Z}$, alors $\mathrm{A} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \mathrm{I}$.

A admet la valeur propre 1 et est l'identité.

Si $\alpha = (2\,k + 1)\,\pi$ avec $k \in \mathbb{Z}$, alors $\mathrm{A} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -\mathrm{I}$.

A admet la valeur propre -1 et constitue une symétrie centrale.