3.3 Soient $(u_n)_{n\in\mathbb{N}}, (v_n)_{n\in\mathbb{N}}$ et $(w_n)_{n\in\mathbb{N}}$ des suites réelles. Soient $\alpha, \beta \in \mathbb{R}$.

1) (a) $\big((u_n)_{n\in\mathbb{N}} + (v_n)_{n\in\mathbb{N}}\big) + (w_n)_{n\in\mathbb{N}} =$
$(u_n + v_n)_{n\in\mathbb{N}} + (w_n)_{n\in\mathbb{N}} =$
$\big((u_n + v_n) + w_n\big)_{n\in\mathbb{N}} =$
$\big(u_n + (v_n + w_n)\big)_{n\in\mathbb{N}} =$
$(u_n)_{n\in\mathbb{N}} + (v_n + w_n)_{n\in\mathbb{N}} =$
$(u_n)_{n\in\mathbb{N}} + \big((v_n)_{n\in\mathbb{N}} + (w_n)_{n\in\mathbb{N}}\big)$

(b) Considérons la suite $(z_n)_{n\in\mathbb{N}}$ définie par $z_n = 0$ pour tout $n \in \mathbb{N}$.
$(u_n)_{n\in\mathbb{N}} + (z_n)_{n\in\mathbb{N}} = (u_n + z_n)_{n\in\mathbb{N}} = (u_n + 0)_{n\in\mathbb{N}} = (u_n)_{n\in\mathbb{N}}$
$(z_n)_{n\in\mathbb{N}} + (u_n)_{n\in\mathbb{N}} = (z_n + u_n)_{n\in\mathbb{N}} = (0 + u_n)_{n\in\mathbb{N}} = (u_n)_{n\in\mathbb{N}}$

(c) $(u_n)_{n\in\mathbb{N}} + (-u_n)_{n\in\mathbb{N}} = (u_n + (-u_n))_{n\in\mathbb{N}} = (0)_{n\in\mathbb{N}} = (z_n)_{n\in\mathbb{N}}$

(d) $(u_n)_{n\in\mathbb{N}} + (v_n)_{n\in\mathbb{N}} = (u_n + v_n)_{n\in\mathbb{N}} = (v_n + u_n)_{n\in\mathbb{N}} = (v_n)_{n\in\mathbb{N}} + (u_n)_{n\in\mathbb{N}}$

2) (a) $\alpha \cdot \big(\beta \cdot (u_n)_{n\in\mathbb{N}}\big) = \alpha \cdot (\beta\, u_n)_{n\in\mathbb{N}} = \big(\alpha\,(\beta\, u_n)\big)_{n\in\mathbb{N}} = \big((\alpha\,\beta)\, u_n\big)_{n\in\mathbb{N}} =$
$(\alpha\,\beta) \cdot (u_n)_{n\in\mathbb{N}}$

(b) $(\alpha + \beta) \cdot (u_n)_{n\in\mathbb{N}} = \big((\alpha + \beta)\, u_n\big)_{n\in\mathbb{N}} = (\alpha\, u_n + \beta\, u_n)_{n\in\mathbb{N}} =$
$(\alpha\, u_n)_{n\in\mathbb{N}} + (\beta\, u_n)_{n\in\mathbb{N}} = \alpha \cdot (u_n)_{n\in\mathbb{N}} + \beta \cdot (u_n)_{n\in\mathbb{N}}$

(c) $\alpha \cdot \big((u_n)_{n\in\mathbb{N}} + (v_n)_{n\in\mathbb{N}}\big) = \alpha \cdot (u_n + v_n)_{n\in\mathbb{N}} = \big(\alpha\,(u_n + v_n)\big)_{n\in\mathbb{N}} =$
$(\alpha\, u_n + \alpha\, v_n)_{n\in\mathbb{N}} = (\alpha\, u_n)_{n\in\mathbb{N}} + (\alpha\, v_n)_{n\in\mathbb{N}} = \alpha \cdot (u_n)_{n\in\mathbb{N}} + \alpha \cdot (v_n)_{n\in\mathbb{N}}$

(d) $1 \cdot (u_n)_{n\in\mathbb{N}} = (1 \cdot u_n)_{n\in\mathbb{N}} = (u_n)_{n\in\mathbb{N}}$