

5.10

- 1) $|1 - i| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$
 $1 - i = \sqrt{2} \left(\frac{1}{\sqrt{2}} - i \frac{1}{\sqrt{2}} \right) = \sqrt{2} \left(\frac{\sqrt{2}}{2} + i \left(-\frac{\sqrt{2}}{2} \right) \right) = \sqrt{2} \left(\cos\left(\frac{7\pi}{4}\right) + i \sin\left(\frac{7\pi}{4}\right) \right)$
 $-3i = 3 \left(0 + i \cdot (-1) \right) = 3 \left(\cos\left(\frac{3\pi}{2}\right) + i \sin\left(\frac{3\pi}{2}\right) \right)$
 $|(1 - i)(-3i)| = \sqrt{2} \cdot 3 = 3\sqrt{2}$
 $\arg((1 - i)(-3i)) = \frac{7\pi}{4} + \frac{3\pi}{2} = \frac{13\pi}{4} = \frac{5\pi}{4} + 2\pi$
- 2) $-2i = 2 \left(0 + i \cdot (-1) \right) = 2 \left(\cos\left(\frac{3\pi}{2}\right) + i \sin\left(\frac{3\pi}{2}\right) \right)$
 $|(-2i)^{10}| = 2^{10} = 1024$
 $\arg((-2i)^{10}) = 10 \cdot \frac{3\pi}{2} = 15\pi = \pi + 7 \cdot 2\pi$
- 3) $|1 + \sqrt{3}i| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1 + 3} = \sqrt{4} = 2$
 $1 + \sqrt{3}i = 2 \left(\frac{1}{2} + i \frac{\sqrt{3}}{2} \right) = 2 \left(\cos\left(\frac{\pi}{3}\right) + i \sin\left(\frac{\pi}{3}\right) \right)$
 $|(1 + \sqrt{3}i)^2| = 2^2 = 4$
 $\arg((1 + \sqrt{3}i)^2) = 2 \cdot \frac{\pi}{3} = \frac{2\pi}{3}$
- 4) $|-1 + i| = \sqrt{(-1)^2 + 1^2} = \sqrt{2}$
 $-1 + i = \sqrt{2} \left(-\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right) = \sqrt{2} \left(-\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right) = \sqrt{2} \left(\cos\left(\frac{3\pi}{4}\right) + i \sin\left(\frac{3\pi}{4}\right) \right)$
 $|2 + 2i| = |2(1 + i)| = |2| |1 + i| = 2 \sqrt{1^2 + 1^2} = 2\sqrt{2}$
 $2 + 2i = 2\sqrt{2} \left(\frac{2}{2\sqrt{2}} + i \frac{2}{2\sqrt{2}} \right) = 2\sqrt{2} \left(\frac{1}{\sqrt{2}} + i \frac{1}{\sqrt{2}} \right) = 2\sqrt{2} \left(\frac{\sqrt{2}}{2} + i \frac{\sqrt{2}}{2} \right) =$
 $2\sqrt{2} \left(\cos\left(\frac{\pi}{4}\right) + i \sin\left(\frac{\pi}{4}\right) \right)$
 $|(-1 + i)^5 (2 + 2i)^4| = (\sqrt{2})^5 \cdot (2\sqrt{2})^4 = 4\sqrt{2} \cdot 16 \cdot 4 = 256\sqrt{2}$
 $\arg((-1 + i)^5 (2 + 2i)^4) = 5 \cdot \frac{3\pi}{4} + 4 \cdot \frac{\pi}{4} = \frac{15\pi}{4} + \pi = \frac{19\pi}{4} = \frac{3\pi}{4} + 2 \cdot 2\pi$
- 5) $|\sqrt{3} - i| = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{3 + 1} = \sqrt{4} = 2$
 $\sqrt{3} - i = 2 \left(\frac{\sqrt{3}}{2} + i \left(-\frac{1}{2} \right) \right) = 2 \left(\cos\left(\frac{5\pi}{6}\right) + i \sin\left(\frac{5\pi}{6}\right) \right)$
 $|\sqrt{3} + i| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3 + 1} = \sqrt{4} = 2$
 $\sqrt{3} + i = 2 \left(\frac{\sqrt{3}}{2} + i \frac{1}{2} \right) = 2 \left(\cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right) \right)$
 $\left| \left(\frac{\sqrt{3} - i}{\sqrt{3} + i} \right)^{30} \right| = \left(\frac{2}{2} \right)^{30} = 1^{30} = 1$
 $\arg \left(\left(\frac{\sqrt{3} - i}{\sqrt{3} + i} \right)^{30} \right) = 30 \cdot \left(\frac{5\pi}{6} - \frac{\pi}{6} \right) = 30 \cdot \frac{2\pi}{3} = 20\pi = 0 + 10 \cdot 2\pi$

$$6) \quad |1 - \sqrt{3}i| = \sqrt{1^2 + (-\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$$

$$1 - \sqrt{3}i = 2 \left(\frac{1}{2} + i \left(-\frac{\sqrt{3}}{2} \right) \right) = 2 \left(\cos\left(\frac{5\pi}{3}\right) + i \sin\left(\frac{5\pi}{3}\right) \right)$$

$$\text{En 5), on a déjà établi } \sqrt{3} + i = 2 \left(\cos\left(\frac{\pi}{6}\right) + i \sin\left(\frac{\pi}{6}\right) \right)$$

$$\left| \left(\frac{1 - \sqrt{3}i}{\sqrt{3} + i} \right)^{17} \right| = \left(\frac{2}{2} \right)^{17} = 1^{17} = 1$$

$$\arg \left(\left(\frac{1 - \sqrt{3}i}{\sqrt{3} + i} \right)^{17} \right) = 17 \cdot \left(\frac{5\pi}{3} - \frac{\pi}{6} \right) = 17 \cdot \frac{3\pi}{2} = \frac{51\pi}{2} = \frac{3\pi}{2} + 12 \cdot 2\pi$$