

4.1

1) 1^{re} méthode

$$\begin{aligned}\cos(\varphi) &= \frac{\begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}}{\left\| \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} \right\| \left\| \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \right\|} = \frac{1 \cdot 3 + (-2) \cdot 1 + 5 \cdot 1}{\sqrt{1^2 + (-2)^2 + 5^2} \sqrt{3^2 + 1^2 + 1^2}} = \frac{6}{\sqrt{30} \sqrt{11}} \\ &= \frac{6 \sqrt{330}}{330} = \frac{\sqrt{330}}{55} \\ \varphi &= \arccos \left(\frac{\sqrt{330}}{55} \right) \approx 70,71^\circ\end{aligned}$$

2^e méthode

$$\begin{aligned}\sin(\varphi) &= \frac{\left\| \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} \times \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \right\|}{\left\| \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} \right\| \left\| \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \right\|} = \frac{\left\| \begin{pmatrix} -7 \\ 14 \\ 7 \end{pmatrix} \right\|}{\left\| \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} \right\| \left\| \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \right\|} = \frac{7 \left\| \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix} \right\|}{\left\| \begin{pmatrix} 1 \\ -2 \\ 5 \end{pmatrix} \right\| \left\| \begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix} \right\|} \\ &= \frac{7 \sqrt{6}}{\sqrt{30} \sqrt{11}} = \frac{7 \sqrt{6 \cdot 30 \cdot 11}}{330} = \frac{7 \cdot 6 \sqrt{5 \cdot 11}}{330} = \frac{7 \sqrt{55}}{55} \\ \varphi &= \arcsin \left(\frac{7 \sqrt{55}}{55} \right) \approx 70,71^\circ\end{aligned}$$

2) 1^{re} méthode

$$\begin{aligned}\cos(\varphi) &= \frac{\begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}}{\left\| \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \right\| \left\| \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right\|} = \frac{1}{\sqrt{14} \sqrt{6}} = \frac{1}{2 \sqrt{21}} = \frac{\sqrt{21}}{2 \cdot 21} = \frac{\sqrt{21}}{42} \\ \varphi &= \arccos \left(\frac{\sqrt{21}}{42} \right) \approx 83,74^\circ\end{aligned}$$

2^e méthode

$$\sin(\varphi) = \frac{\left\| \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \times \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right\|}{\left\| \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \right\| \left\| \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right\|} = \frac{\left\| \begin{pmatrix} -7 \\ -5 \\ -3 \end{pmatrix} \right\|}{\left\| \begin{pmatrix} 2 \\ -1 \\ -3 \end{pmatrix} \right\| \left\| \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} \right\|} = \frac{\sqrt{83}}{2 \sqrt{21}}$$

$$= \frac{\sqrt{83 \cdot 21}}{2 \cdot 21} = \frac{\sqrt{1743}}{42}$$

$$\varphi = \arcsin \left(\frac{\sqrt{1743}}{42} \right) \approx 83,74^\circ$$