

5.5

$$1) \quad 1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + 4 \cdot 5 + \dots + n(n+1) = \sum_{k=1}^n k(k+1)$$

$$\begin{aligned} 2) \quad \sum_{k=1}^n k(k+1) &= \sum_{k=1}^n k^2 + k = \sum_{k=1}^n k^2 + \sum_{k=1}^n k = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} \\ &= \frac{n(n+1)(2n+1) + 3n(n+1)}{6} = \frac{n(n+1)((2n+1)+3)}{6} \\ &= \frac{n(n+1)\overbrace{(2n+4)}^{2(n+2)}}{6} = \frac{2n(n+1)(n+2)}{6} = \frac{n(n+1)(n+2)}{3} \end{aligned}$$