3.9 Posons $\mathrm{F} = \{(x\,;y\,;z\,;t) \in \mathbb{R}^4 : x = 2\,y \text{ et } t = 5\,x\}$.

Soient $u = (x\,;y\,;z\,;t) \in \mathrm{F}$ et $v = (x'\,;y'\,;z'\,;t') \in \mathrm{F}$ et $\alpha \in \mathbb{R}$.

Puisque $u \in \mathrm{F}$, on a $x = 2\,y$ et $t = 5\,x$.
De même, $x' = 2\,y'$ et $t' = 5\,x'$, car $v \in \mathrm{F}$.

1) Posons $w = u + v = (x\,;y\,;z\,;t) + (x'\,;y'\,;z'\,;t')$
$$= (\underbrace{x + x'}_{x''}\,;\underbrace{y + y'}_{y''}\,;\underbrace{z + z'}_{z''}\,;\underbrace{t + t'}_{t''})\,.$$

Montrons que $w = (x''\,;y''\,;z''\,;t'') \in \mathrm{F}$:

(a) $x'' = x + x' = 2\,y + 2\,y' = 2\,(y + y') = 2\,y''$

(b) $t'' = t + t' = 5\,x + 5\,x' = 5\,(x + x') = 5\,t''$

2) Posons $w = \alpha \cdot u = \alpha \cdot (x\,;y\,;z\,;t) = (\underbrace{\alpha\,x}_{x'''}\,;\underbrace{\alpha\,y}_{y'''}\,;\underbrace{\alpha\,z}_{z'''}\,;\underbrace{\alpha\,t}_{t'''})$.

Vérifions que $w = (x'''\,;y'''\,;z'''\,;t''') \in \mathrm{F}$:

(a) $x''' = \alpha\,x = \alpha\,(2\,y) = 2\,(\alpha\,y) = 2\,y'''$

(b) $t''' = \alpha\,t = \alpha\,(5\,x) = 5\,(\alpha\,x) = 5\,x'''$