

3.15

1) Désignons par $C(t)$ le capital après t années.

On pose $C(0) = C$.

Chaque année, le capital augmente de 5 % :

$$C(t+1) = C(t) + \frac{5}{100} C(t) = \left(1 + \frac{5}{100}\right) C(t) = 1,05 \cdot C(t)$$

On obtient ainsi :

$$C(1) = 1,05 \cdot C$$

$$C(2) = 1,05 \cdot C(1) = 1,05 \cdot 1,05 \cdot C = 1,05^2 \cdot C$$

$$C(3) = 1,05 \cdot C(2) = 1,05 \cdot 1,05^2 \cdot C = 1,05^3 \cdot C$$

...

$$C(t) = 1,05 \cdot C(t-1) = 1,05 \cdot 1,05^{t-1} \cdot C = 1,05^t \cdot C$$

2) $C(t) = 2 \cdot C$

$$1,05^t \cdot C = 2 \cdot C$$

$$1,05^t = 2$$

$$t = \log_{1,05}(2) = \frac{\log(2)}{\log(1,05)} \approx 14,21$$