Chamblandes 2008 — Problème 6

a) $f(x) = \dfrac{x^3 - 2\,x^2}{x^2 + 2\,x + 1} = \dfrac{x^2\,(x-2)}{(x+1)^2}$

		-1		0		2	
x^2	$+$	‖	$+$	0	$+$	|	$+$
$x-2$	$-$	‖	$-$	|	$-$	0	$+$
$(x+1)^2$	$+$	‖	$+$	|	$+$	|	$+$
f	$-$	‖	$-$	0	$-$	0	$+$

b) $\mathrm{D}_f = \mathbb{R} - \{-1\}$

$$\lim_{x \to -1} \frac{x^3 - 2\,x^2}{x^2 + 2\,x + 1} = \frac{(-1)^3 - 2\cdot(-1)^2}{(-1)^2 + 2\cdot(-1) + 1} = \text{« } \frac{-3}{0} \text{ »} = -\infty$$

$x = -1$ est asymptote verticale

$$\begin{array}{rrrr|l}
x^3 & -\,2\,x^2 & & & x^2 + 2\,x + 1 \\ \cline{5-5}
-x^3 & -\,2\,x^2 & -\;\;x & & x - 4 \\ \cline{1-4}
 & -\,4\,x^2 & -\;\;x & & \\
 & 4\,x^2 & +\,8\,x & +\,4 & \\ \cline{2-4}
 & & 7\,x & +\,4 &
\end{array}$$

$y = x - 4$ est asymptote oblique

$\delta(x) = \dfrac{7\,x + 4}{x^2 + 2\,x + 1} = \dfrac{7\,x + 4}{(x+1)^2}$

		-1		$-\frac{4}{7}$	
$7\,x+4$	$-$	‖	$-$	0	$+$
$(x+1)^2$	$+$	‖	$+$	|	$+$
δ	$-$	‖	$-$	0	$+$

c) $f'(x) = \dfrac{x^3 + 3\,x^2 - 4\,x}{(x+1)^3} = \dfrac{x\,(x^2 + 3\,x - 4)}{(x+1)^3} = \dfrac{x\,(x+4)\,(x-1)}{(x+1)^3}$

		-4		-1		0		1	
x	$-$	|	$-$	‖	$-$	0	$+$	|	$+$
$x+4$	$-$	0	$+$	‖	$+$	|	$+$	|	$+$
$x-1$	$-$	|	$-$	‖	$-$	|	$-$	0	$+$
$(x+1)^3$	$-$	|	$-$	‖	$+$	|	$+$	|	$+$
f'	$+$	0	$-$	‖	$+$	0	$-$	0	$+$
f	$\nearrow$	max	$\searrow$	‖	$\nearrow$	max	$\searrow$	min	$\nearrow$

$f(-4) = \dfrac{(-4)^3 - 2\cdot(-4)^2}{(-4)^2 + 2\cdot(-4) + 1} = \dfrac{-96}{9} = -\dfrac{32}{3}$

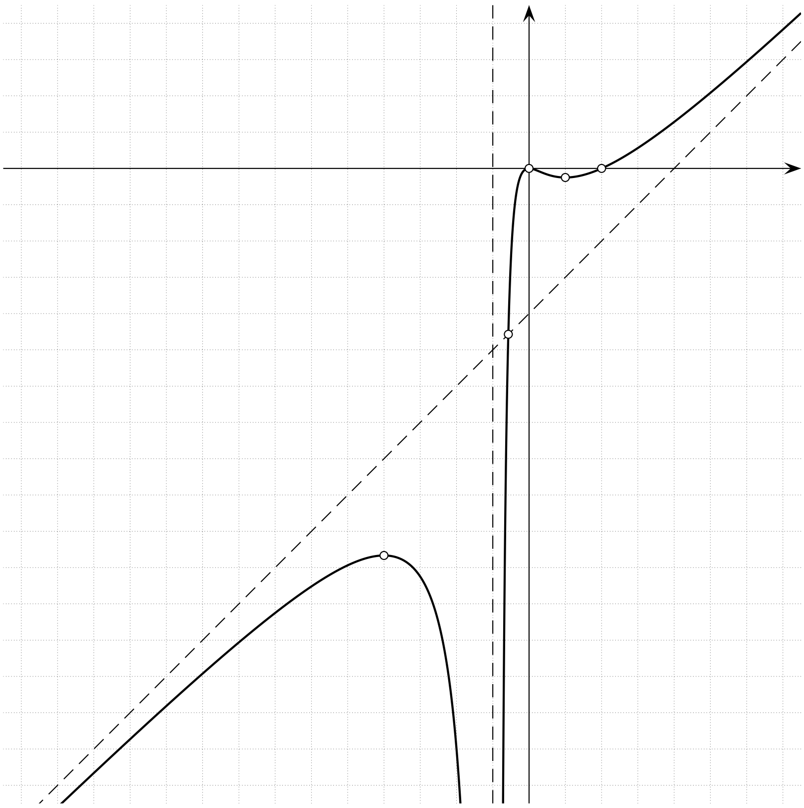
Le point $\left(-4\,;-\frac{32}{3}\right)$ est un maximum.

$$f(0) = \frac{0^3 - 2\cdot 0^2}{0^2 + 2\cdot 0 + 1} = \frac{0}{1} = 0$$

Le point $(0\,;0)$ est un maximum.

$$f(1) = \frac{1^3 - 2\cdot 1^2}{1^2 + 2\cdot 1 + 1} = \frac{-1}{4}$$

Le point $\left(1\,;-\frac{1}{4}\right)$ est un minimum.

d) 

e) $$f'(x) = \left(\frac{x^3 - 2\,x^2}{(x+1)^2}\right)' = \frac{(x^3 - 2\,x^2)'\,(x+1)^2 - (x^3 - 2\,x^2)\,\big((x+1)^2\big)'}{\big((x+1)^2\big)^2}$$

$$= \frac{(3\,x^2 - 4\,x)\,(x+1)^2 - (x^3 - 2\,x^2)\,2\,(x+1)\,\overbrace{(x+1)'}^{1}}{(x+1)^4}$$

$$= \frac{(3\,x^2 - 4\,x)\,(x+1)^2 - 2\,(x^3 - 2\,x^2)\,(x+1)}{(x+1)^4}$$

$$= \frac{(x+1)\,\big((3\,x^2 - 4\,x)\,(x+1) - 2\,(x^3 - 2\,x^2)\big)}{(x+1)^4}$$

$$= \frac{(x+1)\,(3\,x^3 + 3\,x^2 - 4\,x^2 - 4\,x - 2\,x^3 + 4\,x^2)}{(x+1)^4} = \frac{(x+1)\,(x^3 + 3\,x^2 - 4\,x)}{(x+1)^4}$$

$$= \frac{x^3 + 3\,x^2 - 4\,x}{(x+1)^3}$$