

7.1

1)

x	reste r	n	$3^{2^n} \bmod 19$	contribution (si $r = 1$)
100	0	0	3	
50	0	1	$3^2 \equiv 9$	
25	1	2	$9^2 \equiv 5$	5
12	0	3	$5^2 \equiv 6$	
6	0	4	$6^2 \equiv -2$	
3	1	5	$(-2)^2 \equiv 4$	4
1	1	6	$4^2 \equiv 16$	16

$$3^{100} \equiv \underbrace{5 \cdot 4}_{\equiv 1} \cdot 16 \equiv 16 \bmod 19$$

2)

x	reste r	n	$12^{2^n} \bmod 34$	contribution (si $r = 1$)
364	0	0	12	
182	0	1	$12^2 \equiv 8$	
91	1	2	$8^2 \equiv -4$	-4
45	1	3	$(-4)^2 \equiv 16$	16
22	0	4	$16^2 \equiv 18$	
11	1	5	$18^2 \equiv 18$	18
5	1	6	$18^2 \equiv 18$	18
2	0	7	$18^2 \equiv 18$	
1	1	8	$18^2 \equiv 18$	18

$$12^{364} \equiv \underbrace{-4 \cdot 16}_{\equiv 4} \cdot \underbrace{18 \cdot 18 \cdot 18}_{\equiv 18} \equiv 4 \cdot 18 \equiv 4 \bmod 34$$

3)

x	reste r	n	$5^{2^n} \bmod 97$	contribution (si $r = 1$)
51	1	0	5	5
25	1	1	$5^2 \equiv 25$	25
12	0	2	$25^2 \equiv 43$	
6	0	3	$43^2 \equiv 6$	
3	1	4	$6^2 \equiv 36$	36
1	1	5	$36^2 \equiv 35$	35

$$5^{51} \equiv \underbrace{5 \cdot 25}_{\equiv 28} \cdot \underbrace{36 \cdot 35}_{\equiv -1} \equiv -28 \equiv 69 \bmod 97$$

4)

x	reste r	n	$9^{2^n} \bmod 113$	contribution (si $r = 1$)
71	1	0	9	9
35	1	1	$9^2 \equiv -32$	-32
17	1	2	$(-32)^2 \equiv 7$	7
8	0	3	$7^2 \equiv 49$	
4	0	4	$49^2 \equiv 28$	
2	0	5	$28^2 \equiv -7$	
1	1	6	$(-7)^2 \equiv 49$	49

$$9^{71} \equiv \underbrace{9 \cdot (-32)}_{\equiv 51} \cdot \underbrace{7 \cdot 49}_{\equiv 4} \equiv 51 \cdot 4 \equiv 91 \pmod{113}$$