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Interference Simulation using Matlab

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Contents

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Abstract

The double-slit experiment has been of great interest to philosophers, because the quantum mechanical behavior it reveals has forced them to reevaluate their ideas about classical concepts such as particles, waves, location, and movement from one place to another.

Thomas Young first demonstrated the interference of light in 1801. His experiment gave strong support to the wave theory of light. This experiment shows interference fringes created when a coherent light source is shone through double slits. The interference is observable since each slit acts as coherent sources of light as they are derived from a single source. The interference can be either constructive, when the net intensity is greater than the individual intensities, or destructive, when the net intensity is less than the individual intensities.

The aim of this project was to describe the interference of waves using the Fast Fourier Transformation FFT, MATLAB command. The result of the double-slit, and multy-slits experiment shows the same tendency as that of theoretical.

The simulation of double-slit experiment shows the intensity of the central fringe is larger than the other on both sides. While the progression to a larger number of slits shows a pattern of narrowing the high intensity peaks and a relative increase in their peak intensity.

The result of this project shows that the FFT is a powerful technique to studies the interference of the wave.

Chapter One
Introduction

1.1 Introduction

Light, or visible light, is electromagnetic radiation of a wavelength that is visible to the human eye (about 400–700 nm), or up to 380–750 nm. In the broader field of physics, *light* is sometimes used to refer to electromagnetic radiation of all wavelengths, whether visible or not.

Until the middle of the 1800's, the generally accepted theory of light was the particle picture. In this viewpoint, advocated by Newton, light was considered to be a stream of tiny particles. However, in the late 1800's, the particle picture was replaced by the wave theory of light. This was because certain phenomena associated with light, namely refraction, diffraction and interference could only be explained using the wave picture.

In the early 20th century, experiments revealed that there were some phenomena associated with light that could only be explained by a particle picture. Thus, light as it is now understood, has attributes of both particles and waves. In this Chapter we will deal mainly with the wave attributes of light. The particle-like behavior of light is described by the modern theory of quantum mechanics.

1.2 Waves and Wavefronts:

The electric field vector due to an electromagnetic field at a point in space is composed of an amplitude and a phase

$$E(x, y, z) = A(x, y, z)e^{i\phi(x,y,z,t)} \tag{1.1}$$

or

$$E(r,t) = A(r,t)e^{i\phi(r,t)} \tag{1.2}$$

where r is the position vector and both the amplitude A and phase $\emptyset$ are functions of the spatial coordinate and time. The polarization state of the field is contained in the temporal variations in the amplitude vector.

This expression can be simplified if a linearly polarized monochromatic wave is assumed:

$$E(x,y,z) = A(x,y,z)e^{i(wt-\phi(x,y,z))} \tag{1.3}$$

Where w is the angular frequency in radians per second and is related to the frequency v by

$$w = 2\pi v \tag{1.4}$$

Some typical values for the optical frequency are $5\times10^{14}\,Hz$ for the visible, $10^{13}\,HZ$ for the infrared, and $10^{16}\,Hz$ for the ultraviolet.

1.2.1 Plane Wave:

The simplest example of an electromagnetic wave is the plane wave. The plane wave is produced by a monochromatic point source at infinity and is approximated by a collimated light source.

The complex amplitude of a linearly polarized plane wave is:

$$E(x,y,z,t) = E(r,t) = Ae^{i(wt-kr)} \tag{1.5}$$

where k is the wave vector. The wave vector points in the direction of propagation, and its magnitude is the wave number K related to the temporal frequency by the speed of light v in the medium. The wavelength is

$$\lambda = \gamma / \nu = 2\pi * \nu / w = c / n\nu = 2\pi c / nw \tag{1.6}$$

where n is the index of refraction, and c is the speed of light in a vacuum. The amplitude A of a plane wave is a constant over all space, and the plane wave is clearly an idealization. If the direction of propagation is parallel to the z axis, the expression for the complex amplitude of the plane wave simplifies to

$$E(x,y,z,t) = Ae^{i(wt-kz)} \tag{1.7}$$

We see that the plane wave is periodic in both space and time. The spatial period equals the wavelength in the medium, and the temporal period equals $1/v$.

1.2.2 Spherical Wave:

The second special case of an electromagnetic wave is the spherical wave which radiates from an isotropic point source. If the source is located at the origin , the complex amplitude is

$$E(r,t) = (A/r)e^{i(wt-kr)} \tag{1.8}$$

where $r = (x^2 + y^2 + z^2)$. The field is spherically symmetric and varies harmonically with time and the radial distance. The radial period is the wavelength in the medium. The amplitude of the field decreases as *1/r* for energy conservation. At a large distance from the source, the spherical wave can be approximated by a plane wave, show Figure (1.1).

1.2.3 Aberrated Plane wave:

When an aberrated or irregularly shaped wavefront is interfered with a reference wavefront, an irregularly shaped fringe pattern is produced. However, the rules for analyzing this pattern are the same as with any two wavefronts. A given fringe represents a contour of constant OPD or phase difference between the two wavefronts. Adjacent fringes differ in OPD by one wavelength or equivalently correspond to a phase difference.

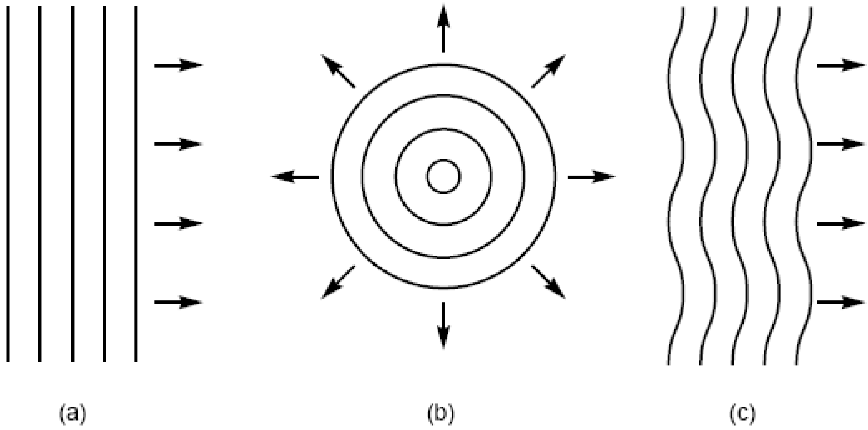


Figure (1.1): *Examples of wave fronts: (a) plane wave; (b) spherical wave; and (c) aberrated Plane wave.*

1.3 Electromagnetic Spectrum:

Visible light is only a tiny fraction of the entire range of electro-magnetic radiation. The electromagnetic spectrum is arranged by the frequency of its waves, from the longest, lowest energy waves to the shortest, high-energy waves. Show Figure (1.2).

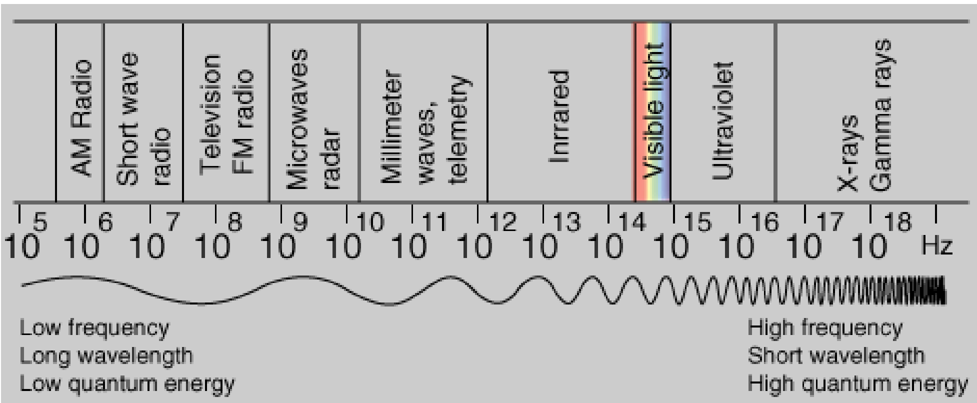


Figure (1.2): *The Electromagnetic Spectrum*

Radio: We use the radio band of the spectrum for a wide range of uses, including wireless communication, television and radio broadcasting, navigation, radar and even cooking.

Infrared: Just below the range of human vision, infrared gives off heat. About 75% of the radiation emitted by a light bulb is infrared.

Visible light: The range of frequencies that can be seen with the naked eye.

Ultraviolet: Dangerous to living organisms, about 9% of the energy radiated from the sun is ultraviolet light. Ultraviolet radiation is often used to sterilize medical instruments because it kills bacteria and viruses.

X-Rays: An invisible form of light produced in the cosmos by gas heated to millions of degrees. X-rays are absorbed depending on the atomic weight of the matter they penetrate. Since x-rays affect photographic emulsion in the same way visible light does, we can use them to take pictures of the insides of things.

Gamma Rays: The product of radioactive decay, nuclear explosions and violent cosmic phenomena such as supernovae. Earth's atmosphere shields us from the cosmic rays.

The different types of radiation are distinguished by their wavelength, or frequency, as shown in Table (1.1).

Table (1.1): *The Electromagnetic Spectrum.*

Region	**Wavelength (Angstroms)**	**Wavelength (centimeters)**
Radio	$> 10^9$	> 10
Microwave	$10^9 - 10^6$	$10 - 0.01$
Infrared	$10^6 - 7000$	$0.01 - 7 \times 10^{-5}$
Visible	$7000 - 4000$	$7 \times 10^{-5} - 4 \times 10^{-5}$
Ultraviolet	$4000 - 10$	$4 \times 10^{-5} - 10^{-7}$
X-Rays	$10 - 0.1$	$10^{-7} - 10^{-9}$
Gamma Rays	< 0.1	$< 10^{-9}$

1.4 Electromagnetic theory of light:

James clack Maxwell, a brilliants scientists of the middle 19'th century, showed by constructing an oscillating electrical circuit that electromagnetic waves could moves through empty space.

Current light theory says that, light is made up of very small packets of electromagnetic energy called photons (the smallest unit of electromagnetic energy). The electromagnetic energy of light is a form of electromagnetic radiation, which are made up of moving electric and magnetic force and move as waves, as shown in Figure (1.3).

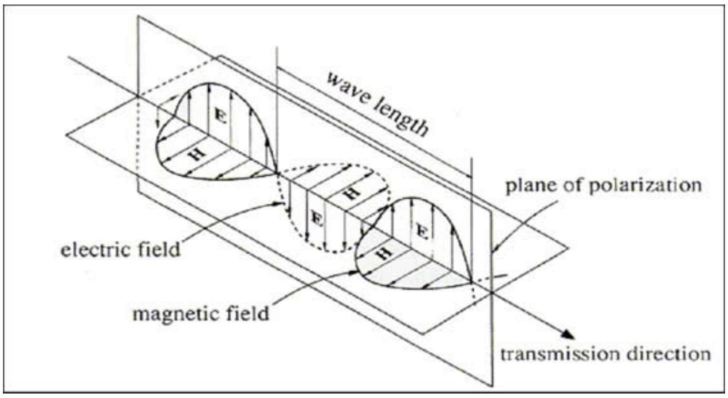


Figure (1.3): *Showing the two oscillating components of light; an electric field and a magnetic field perpendicular to each other and to the direction of motion (a transverse wave).*

According to Maxwells electromagnetic theory the energy $\boldsymbol{E}$ and momentum $\boldsymbol{P}$ of an electromagnetic wave are related by the expression:

$$E = cp \qquad (1.9)$$

Alternatively the energy and momentum of a particle of rest mass are related by way of the formula:

$$c = E(m_o{}^2c^2 + p^2)^{1/2} \qquad (1.10)$$

whose origins are in the special theory of relativity. Inasmuch as the photon is a creature of both these disciplines we can expect either equation to be equally applicable indeed they must be identical. It follows that *the rest mass of a photon is equal to zero.* The photons total energy as with any particle is given by the relativistic expression $E = mc^2$ where

$$m = \frac{m_o}{\sqrt{1 - v^2 / c^2}} \tag{1.11}$$

Thus, since it has a finite relativistic mass m and since $m_o = 0$, it follows that a photon can only exist at a speed *c:* the energy *E* is purely kinetic.

The fact that the photon possesses inertial mass leads to some rather interesting results e.g. the gravitational red shift, and the deflection of starlight by the sun .The red shift was actually observed under laboratory conditions in 1960 by R. V. Pound and G. A. Rebka Jr. at Harvard University. In brief if a particle of *mass m* moves upward height *d* in the earth gravitational field it will do work in overcoming the field and thus decrease in energy by an amount *mgd* Therefore if the photons initial energy is *hv* its final energy after traveling a vertical distance *d* will be given by:

$$hv_f = hv_i - hgd \tag{1.12}$$

$v_f < v_i$ and so

Pound and ribka using gamma-ray photos were able to confirm that quanta of the electromagnetic field behave as if they had a mass $m = E/c^2$

Form Eq.(1) the momentum of a photon can be written as

$$p = E / c = hv / c \tag{1.13}$$

or

$$p = h / v \tag{1.14}$$

If we had a perfectly monochromatic beam of light of wave length λ each constituent photon would possess a momentum of h/λ , equivalently

$$p = hk \tag{1.15}$$

We can arrive at this some end by way of a some what different route. Momentum quite generally is the product of mass and speed thus

$$p = mc = E / c \tag{1.16}$$

The momentum relationship ($p = h/\lambda$)for photon was confirmed in 1923 by Arthur holly Compton (1892-1962). In a classic experiment he irradiated electrons with x-ray quanta and studied the frequency of the scattered photon. By applying the laws of conservation of momentum and energy relativistically as if the collisions were between particles Compton was able to account for an otherwise inexplicable decrease in the frequency of the scattered radiant energy.

A few years later in Francw Louis Victor Prince De broglie (b.1891) in his doctoral thesis drew a marvelous analogy between photons and matter particles. He proposed that every particle and not just the photon should have an associated wave nature. Thus since $p = h/\lambda$ the wavelength of a particle having a momentum m v would then be:

$$\lambda = h/mv \tag{1.17}$$

1.5 The wave properties of light

Light are a very complex phenomenon, but in many situations its behavior can be understood with a simple model based on rays and wave fronts. A ray is a thin beam of light that travels in a straight line. A wave front is the line (not necessarily straight) or surface connecting all the light that left a source at the same time. For a source like the Sun, rays radiate out in all directions; the wave fronts are spheres centered on the Sun. If the source is a long way away, the wave fronts can be treated as parallel lines.

Rays and wave fronts can generally be used to represent light when the light is interacting with objects that are much larger than the wavelength of light, which is about 500 nm.

1.5.1 Reflection:

The first property of light we consider is reflection from a surface, such as that of a mirror. This is illustrated in Figure (1.4).

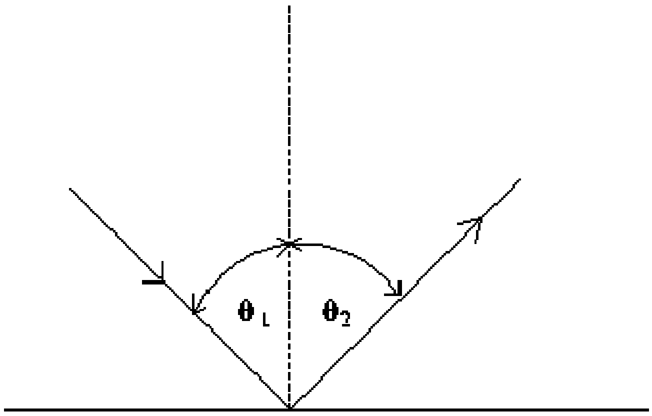


Figure (1.4): *Law of reflection*

When light is reflected off any surface, the angle of incidence θ_1 is always equal to the angle of reflection θ_2. The angles are always measured with respect to the normal to the surface.

The law of reflection is also consistent with the particle picture of light. Table (1.2) shows the variation of refraction index of glass.

Table (1.2): *Variations of Index of Refraction in Glass:*

Color	Wavelength	Index of Refraction
blue	434 nm	1.528
yellow	550 nm	1.517
red	700 nm	1.510

1.5.2 Refraction:

Refraction is the bending of light as it passes between materials of different optical density. While the index of Refraction of a material is the ratio of the speed of light in vacuum to the speed of light in that material:

$$n = \frac{c}{v} \tag{1.18}$$

where v is the speed of light in the material.

The more dense the material, the slower the speed of light in that material. Thus $n > 1$ for all materials, and increases with increasing density. $n = 1$ in vacuum.

The frequency of light does not change when it passes from one medium to another. According to the formula $v = \lambda f$, the wavelength must change. The index of refraction can therefore be written in terms of wavelengths as:

$$n = \frac{\lambda_o}{\lambda} \tag{1.19}$$

Where λ_o is the wavelength of the light in the vacuum and λ is the wavelength of the light in the medium.

The change in speed and wavelength at the boundary between two materials causes light to change direction. Think of a car approaching a patch of mud at a sharp angle from a well paved road. The tire that hits the mud first will slow down, while the other tire is still going fast on the good road. This will cause the car to turn, until both tires are in the mud and going at the same speed. If θ_1 is the angle of the ray relative to the normal to the surface in medium 1, and θ_2 is the angle relative to the normal in medium 2, then:

$$\frac{sin\theta_1}{sin\theta_2} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2} = \frac{n_2}{n_1} \tag{1.20}$$

where v_1 and λ_1 are the speed and wavelength in medium 1, etc. This is illustrated in Figure (1.5).

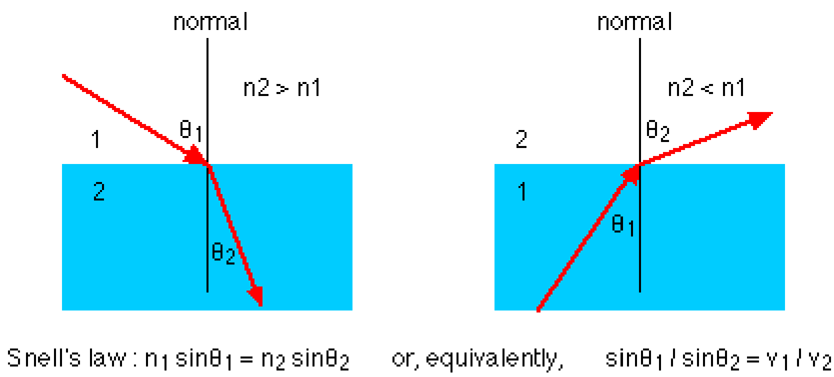


***Figure (1.5):** Law of refraction.*

This relationship between the angles is called Snell's Law. The relation between the two angles is the same whether the ray is moving from medium 1 to 2 (so that θ_1 is the angle of incidence and θ_2 is the angle of refraction) or whether the ray moves from medium 2 to medium 1, so that θ_2 is the angle of incidence and θ_1 is the angle of refraction.

Total Internal Reflection

For a light ray passing from a more dense to a less dense material, there is a critical angle of incidence θ_c for which the angle of refraction is 90^{o}. For greater angles of incidence, the light cannot pass through the boundary between the materials, and is reflected within the more dense material. For a light ray trying to pass from medium 2 to medium 1, the critical angle is given by:

$$sin\theta_c = \frac{n_1}{n_2} sin90 = \frac{n_1}{n_2} \tag{1.21}$$

Where n_1 is the index of refraction of the less dense material, and n_2 is the index of refraction of the more dense material.

Notes the formula for the critical angle shows that n_2 must be greater than n_1 for there to be total internal reflection. That is, medium 2 must be denser than medium 1, otherwise $\boldsymbol{sin\theta_c} > 1$, which is not possible dispersion.

The velocity of light in a material, and hence its index of refraction, depends on the wavelength of the light. In general, n varies inversely with wavelength: it is greater for shorter wavelengths. This causes light inside materials to be refracted by different amounts according to the wavelength (or color). This gives rise to the colors seen through a prism. Rainbows are caused by a combination of dispersion inside the raindrop and total internal reflection of light from the back of raindrops. The following is a chart giving the index of refraction for various wavelengths of light in glass.

Table (1.3): *Variations of Index of Refraction in Glass.*

Color	Wavelength	Index of Refraction
blue	434 nm	1.528
yellow	550 nm	1.517
red	700 nm	1.510

In general shorter Wavelengths (i.e. light towards the blue end of the spectrum) have higher indices of refraction and get bent more than light with longer wavelengths (towards the red end).

1.5.3 Diffraction:

Huygens' Principle tells us that a "new" wavefront of a traveling wave may be constructed at a later time by the envelope of many wavelets generated at the "old" wave front. One assumes that a primary wave generates fictitious spherical waves at each point of the "old" wavefront. The fictitious spherical wave is called Huygens' wavelet and the superposition of all these wavelets results in the "new" wavefront. This is schematically shown in Figure (1.6). The distance between the generating source points is infinitely small and therefore, integration has to be applied for their superposition.

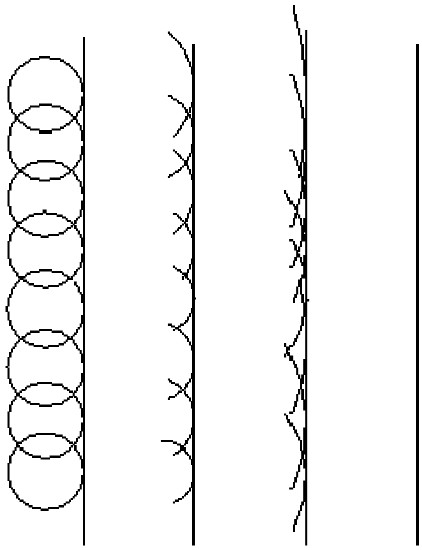


Figure (1.6): *Schematic of wave front construction using Huygens' Principle.*

Applying this division process to an open aperture, the incident wave generates new waves in the plane of the aperture, and these newly generated waves have fixed phase relations with the incident wave and with one another. We assume that all waves generated by the incident wave propagate only in the forward direction, and not backward to the source of light. Let us consider the diffraction on a slit (Figure 1.7). The observed pattern depends on the wavelength and the size of the opening. A slit of a width of several orders of magnitude larger than the wavelength of the incident light will give us almost the geometrical shadow (Figure 1.7-a). A slit of width of an order or two larger than the wavelength will bend the light and fringes will occur; see Figure (1.7-b). A slit smaller than the wavelength will shown intensity pattern with no fringes and decreasing intensity for larger angles; see Figure (1.7-c). All openings will show small deformations of the wavefront close to the edges of the slit.

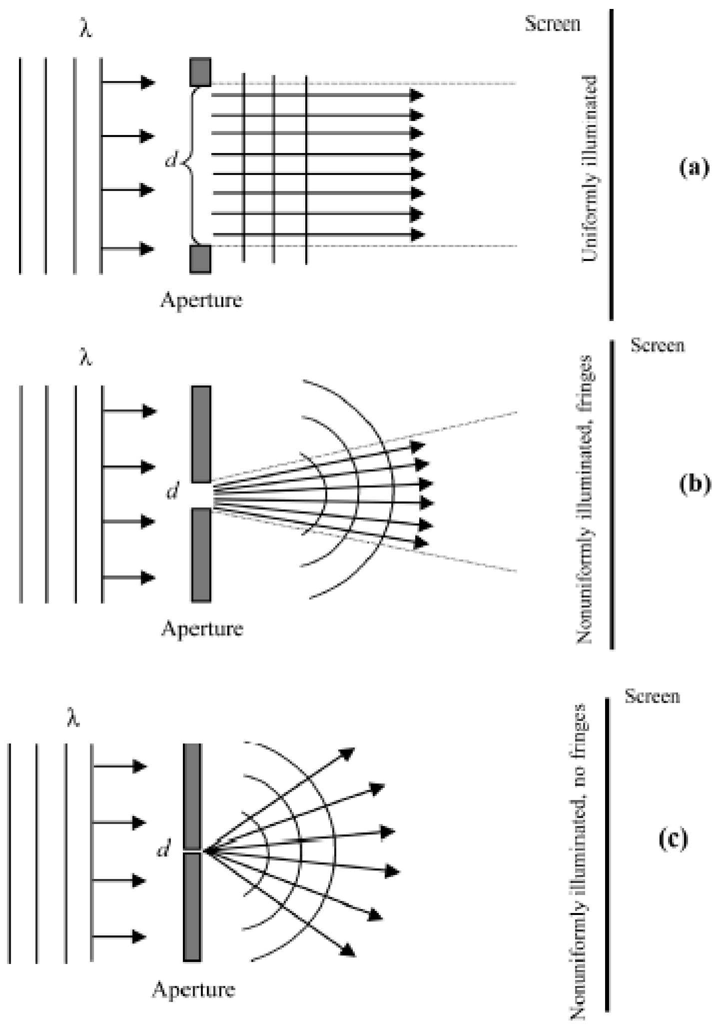


Figure (1.7): *Conditions for diffraction on a single slit:*
(a) d >> λ, no appreciable diffraction;
(b) d of the same order of magnitude of λ, diffraction is observed (fringes)
(c) d << λ, nonuniformly illuminated observation screen, but no fringes

Diffraction limits the resolving power of microscopes and other magnifying devices. If the object being viewed is smaller than the wavelength of light used, then the light diffracts around the object, and severely distorts the image. Thus microscopes using visible light have a resolving power of only about 600 nm $\approx 10^{-6}$m, but X-rays, whose wavelength is about 0.1 nm (10^{-10} m) have a resolving power four orders of magnitude smaller.

1.5.4 Polarization:

Corresponding to the electromagnetic theory of light it is compound from electric field and magnetic field, they vibrating in the plane perpendicular to each other and perpendicular to the direction of light wave propagation, and in the natural light the electric field will vibrate in all perpendicular direction of the light. When the natural light is incident on a polarizer like Nicole prism or same type crystals then the transmitted light will be polarized partially or complete or the polarizer will allow only to the electric field compound that vibration parallel to the polarizer axis ,and when the polarized light passed through another polarizer (analyzer) the intensity of outer light will depend on the angle between the transmission direction of polarizer and analyzer (θ) and the amplitude of the transmitted light (A) will give by:

$$A = A_o \, cos\theta \qquad (1.22)$$

then the intensity of the light from the second polarizer (analyzer) will give by

$$I = I_o \, cos^2\theta \qquad (1.23)$$

and this is the maul's law

A retarder can be made from any birefringent material, that is, any material whose refractive index depends on direction. As an example, let us take the uniaxial crystal characterized by refractive indices ne and no. The orthogonal linearly polarized component waves are the e-wave and the o-wave. It is further assumed that the front and back surfaces of the retarder are parallel to the optic axis of the crystal, and the propagation direction of the incident light is normal to the front surface of the retarder. In this situation, the directions of the component e-wave and o-wave do not separate as they propagate through the retarder; rather, they emerge together. Depending on which is smaller, ne or no, one of the component waves moves through the retarder faster than the other, as shown in Figure (1.8).

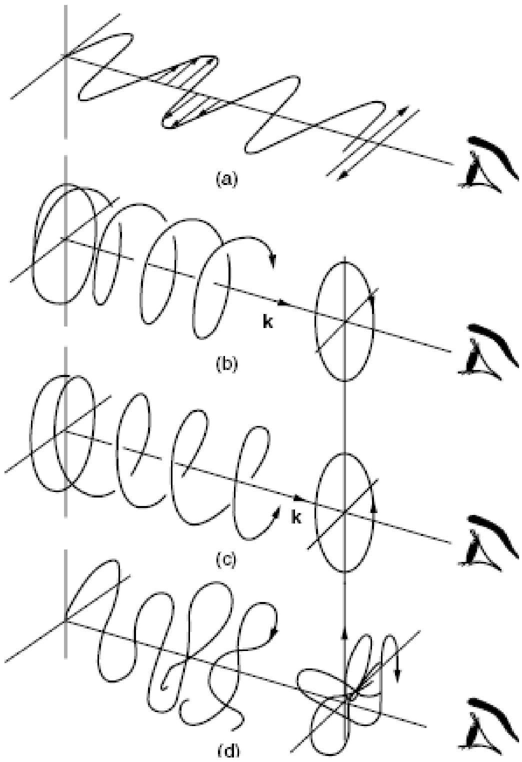


Figure (1.8): *Various states of polarization (SOP). (a) Linearly (horizontally) polarized. (b) Right-handed circularly polarized. (c) Left-handed circularly polarized. (d) Depolarized.*

The relative phase difference is the retardance Δ. The polarization direction of the faster component wave is called the fast axis of the retarder, and the polarization direction of the slower component wave is called the slow axis. The emergent state of polarization is the superposition of the two component waves and will depend on the relative amplitudes of the two component waves, as well as the retardance.

A circle diagram will be used to find the state of polarization as the incident linearly polarized light transmits through the retarder. Figure (1.9) shows the configuration. A 55^{o}. is incident onto a retarder with retardance delta=60

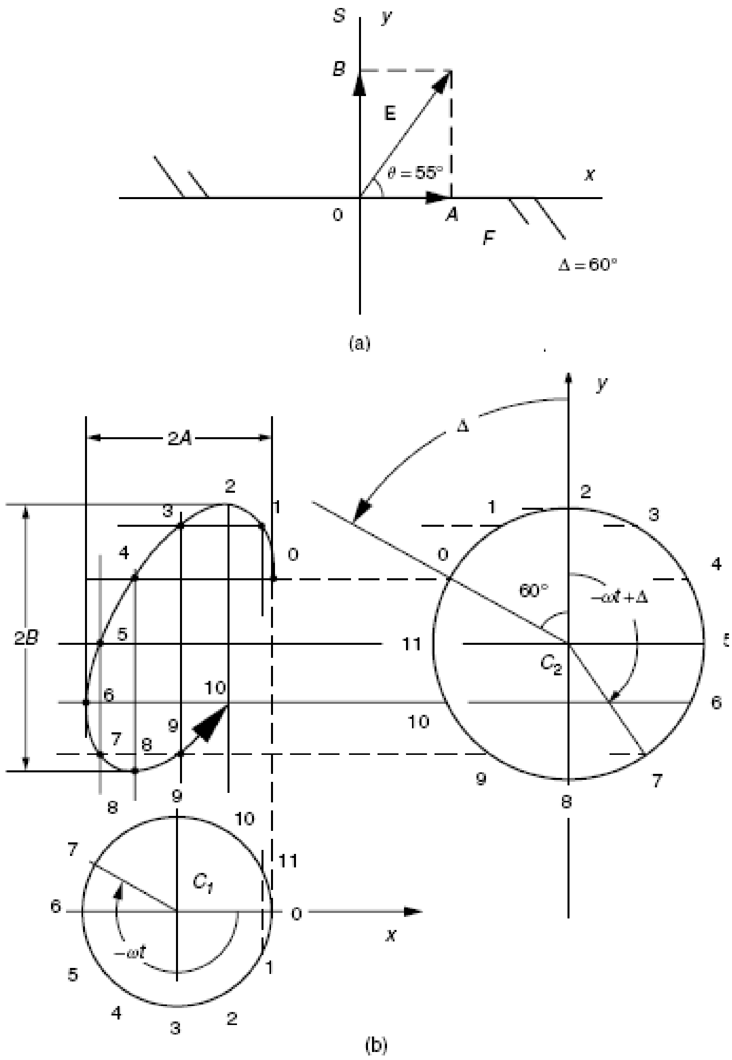


Figure (1.9): *Graphical solution. (a) Geometry. (b) Circle diagram.*

The direction of the fast axis of the retarder is designated by an elongated *F* and in this case is oriented in the *x* direction. The direction of the slow axis is perpendicular to that of the fast axis and is taken as the *y* direction. The *z* direction is the direction of propagation.

The incident light *E* is decomposed into the directions of the fast and slow axes, that is, in the *x* and *y* directions. In complex notation, the component waves are:

$$E_x = Ae^{i(-wt-\beta z)} \tag{1.24}$$

$$E_y = Be^{i(-wt-\beta z+\Delta)} \tag{1.25}$$

With

$$E = E_x \,\hat{\imath} + E_y \,\hat{\jmath} \tag{1.26}$$

$A = |E| cos\, 55^o$

$B = |E| sin\, 55^o$

and the corresponding real expressions are

$$E_x = A\cos(-wt + \beta z) \quad (1.27)$$

$$E_y = B\cos(-wt + \beta z + \Delta) \quad (1.28)$$

The phasor circle $\boldsymbol{C_1}$ in Figure (1.9) represents Eq. (1.27) and $\boldsymbol{C_2}$ represents Eq. (1.28). As time progresses, both phasors rotate at the same angular velocity as (*exp jωt*) (for now a fixed *z*), or clockwise as indicated by *0, 1, 2, 3, . . . , 11*. The phase of however, lags by . because of the retarder. The projection from the circumference of circle $\boldsymbol{C_1}$ onto the *x* axis represents $\boldsymbol{E_x}$, and that from the $\boldsymbol{C_2}$ circle onto the *y* axis represents $\boldsymbol{E_y}$. It should be noted that the phase angle *ωt* in $\boldsymbol{C_1}$ is with respect to the horizontal axis and $\omega t + \Delta$ in $\boldsymbol{C_2}$ is with respect to the vertical axis.

By connecting the cross points of the projections from *0, 1, 2, 3, . . . , 11* on each phasor circle, the desired vectorial sum of $\boldsymbol{E_x}$ and $\boldsymbol{E_y}$ is obtained. The emergent light is elliptically polarized with left-handed or counterclockwise rotation. Next, the case when the fast axis is not necessarily along the *x* axis will be treated. For this example, a retarder with $\Delta = 90^o$. will be used.

1.5.5 Superposition and Interference of wave:

Superposition of two waves depending on space and time coordinates; the description of the interference of two waves in a simple way, using the superposition of two harmonic waves u_1 and u_2. Both waves will propagate in the *x* direction and vibrate in the *y* direction, as shown in Figure (1.10).

$$u_1 = A\cos[2\pi(x/\lambda - t/T)] \quad (1.29)$$

$$u_2 = A\cos[2\pi((x - \delta)/\lambda - t/T)] \quad (1.30)$$

Assuming that the two waves have an optical path difference *δ*. At time instance *t = 0*, the wave u_1 has its first maximum at *x = 0*, and u_2 at *x = δ*

Adding u_1 and u_2 we have

$U = u_1 + u_2$

$U = A\cos[2\pi(x/\lambda - t/T)] + u_2 = A\cos[2\pi((x-\delta)/\lambda - t/T)]$ (1.30)

Using:

$\cos(\alpha) + \cos)\beta) = 2\cos[(\alpha-\beta)/2]\cos\ [(\alpha+\beta)/2]$ (1.31)

we get

$U = \{2A\cos[2\pi(\delta/2)/\lambda]\}\{\cos[2\pi(x/\lambda - t/T) - 2\pi(\delta/2)/\lambda]\}$ (1.32)

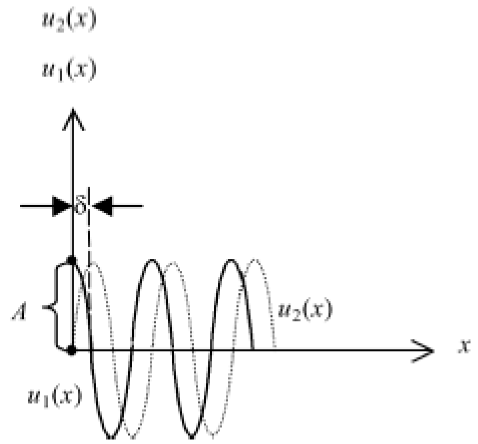


Figure (1.10): *Two waves with magnitude A and wavelength λ.* $u_1 = A$ *for x= 0 and* $u_2 = A$ *for x= δ.*

By discussing the two factors from equation (1.32).The first factor

$$2A\cos\{2\pi(\delta/2)/\lambda\}$$

depends on *δ* and *λ*, but not on *x* and *t* . One obtains for *δ* equal to *0* or a multipleinteger of the wavelength

$$[2A\cos\{2\pi(\delta/2)/\lambda\}]2 \quad is \quad 4A2$$

and for *δ* equal to a multiple of half a wavelength

$$[2A\cos\{2\pi(\delta/2)/\lambda\}]2 \quad is \quad 0$$

The first factor in equation (1.32) may be called the amplitude factor and is used for characterization of the interference maxima and minima.

One has

Maxima for *δ* = *mλ*, where *m* is *0* or an integer

Minima for *δ* = *mλ*, where m is *1/2* plus an integer

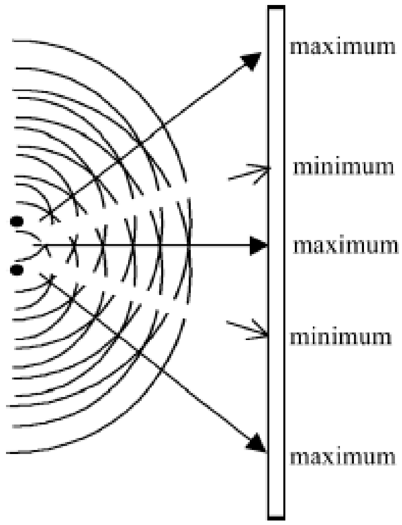
and *m* is called the order of interference.

The second factor is a time-dependent cosine wave with a phase constant depending on δ and λ. For the description of the interference pattern this factor is averaged over time and results in a constant, which may be factored out and included in the normalization constant.

Figure (1.11) shows schematically the interference of two water waves with a fixed phase relation. When the interference factor is zero one has minima, indicated by white strips. They do not depend on time.

At the crossing of the lines, the amplitudes of the waves of both sources are the same and adding. Taking the time dependence into account, the magnitude changes between maximum and minimum.

These are the maxima when considering light. Between the maxima we indicate the two lines corresponding to the minima. Along these lines the amplitude of the two waves compensates opposed to each other; their sum is zero for all times.



Figure(1.11): *Schematic of the interference pattern produced by two sources vibrating in phase.*

Interference results from the superposition of two or more electromagnetic waves. From a classical optics perspective, interference is the mechanism by which light interacts with light. Other phenomena, such as refraction, scattering, and diffraction, describe how light interacts with its physical environment. Historically, interference was instrumental in establishing the wave nature of light. The earliest observations were of colored fringe patterns in thin films. Using the wavelength of light as a scale, interference continues to be of great practical importance in areas such as spectroscopy and metrology.

An example of interference is Michelson interferometer; a schematic of the Michelson interferometer is shown in the figure (1.12). The input light is split at the beam splitter and directed to two mirrors. These mirrors reflect the light back to the beam splitter where the two waves combine. The phase difference between these waves depends on the optical path length difference between the two arms of the interferometer. The resulting interference can be observed on a screen placed at the output. When the optical path length of the two arms are exactly equal or differ by an integral multiple of the wave length λ_0, the interfering waves have the same phase and the output is bright. When the path lengths differ from an integral multiple of λ_0 by $\lambda_0 / 2$, the output is dark.

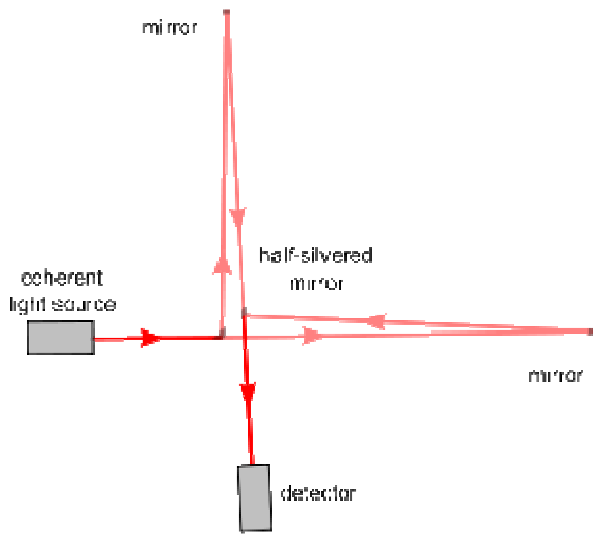


Figure (1.12): *A* schematic of the Michelson interferometer.

The phase curvatures (wave fronts) of the two interfering waves may depend on the optical path length of the interferometer arms. For example when the input field is a diverging Gaussian beam, different path lengths result in different phase curvatures for the interfering waves. Hence, the interference pattern assumes a circular pattern. In other words, the phase difference becomes a function of the radial distance on the transverse plane.

Chapter Two

Young double slit Experiment

2.1 Introduction:

In 1801, an English physicist named Thomas Young performed an experiment that strongly inferred the wave-like nature of light. Because he believed that light was composed of waves, Young reasoned that some type of interaction would occur when two light waves met. This interactive tutorial explores how coherent light waves interact when passed through two closely spaced slits.

The tutorial initializes with rays from the sun being passed through a single slit in a screen to produce coherent light. This light is then projected onto another screen that has twin (or double) slits, which again diffracts the incident illumination as it passes through. The results of interference between the diffracted light beams can be visualized as light intensity distributions on the dark film, as shown in Figure (2.1). The slider labeled distance between slits can be utilized to vary the distance between the slits and produce corresponding variations in the interference intensity distribution patterns.

Young's experiment was based on the hypothesis that if light were wave-like in nature, then it should behave in a manner similar to ripples or waves on a pond of water. Where two opposing water waves meet, they should react in a specific manner to either reinforce or destroy each other. If the two waves are in step (the crests meet), then they should combine to make a larger wave. In contrast, when two waves meet that are out of step (the crest of one meets the trough of another) the waves should cancel and produce a flat surface in that area.

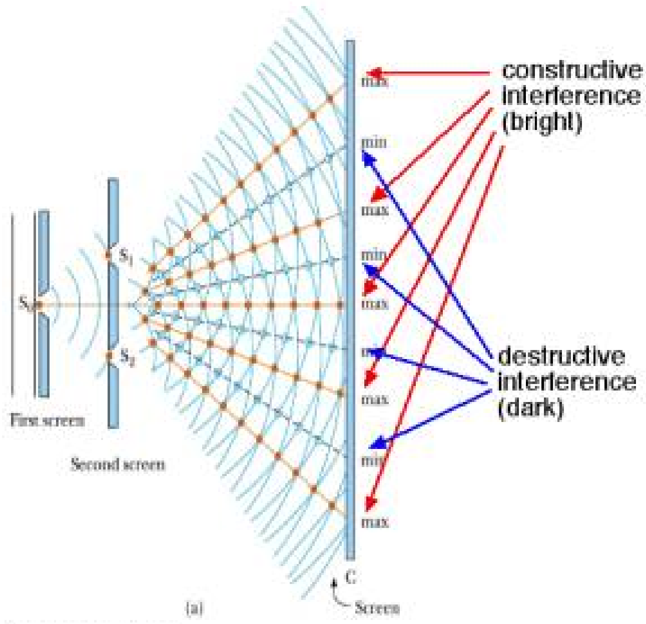


Figure (2.1): *Schematic of Interference from double slits.*

2.2 Interference Conditions:

Wave can be added together either constructively or destructively. The result of adding two waves of the same frequency depends on the value of the phase of the wave at the point in which the waves are added. Electromagnetic waves are subject to interference.

For sustained interference between two sources of light to be observed, there are some conditions, which must be met:

1. The sources must be coherent; to produce coherent source currently it must more common to use a laser as a coherent source, because laser produces an intense, coherent, monochromatic beam over a width of several millimeters, and can be used to illuminate multiple slits directly.
2. They must maintain a constant phase with respect to each other.
3. The waves must have an identical wavelength.

2.3 Young's Double Slit Experiment:

This is a classic example of interference effects in light waves. Two light rays pass through two slits, separated by a distance d and strike a screen a distance L, from the slits, as in Figure (2.2).

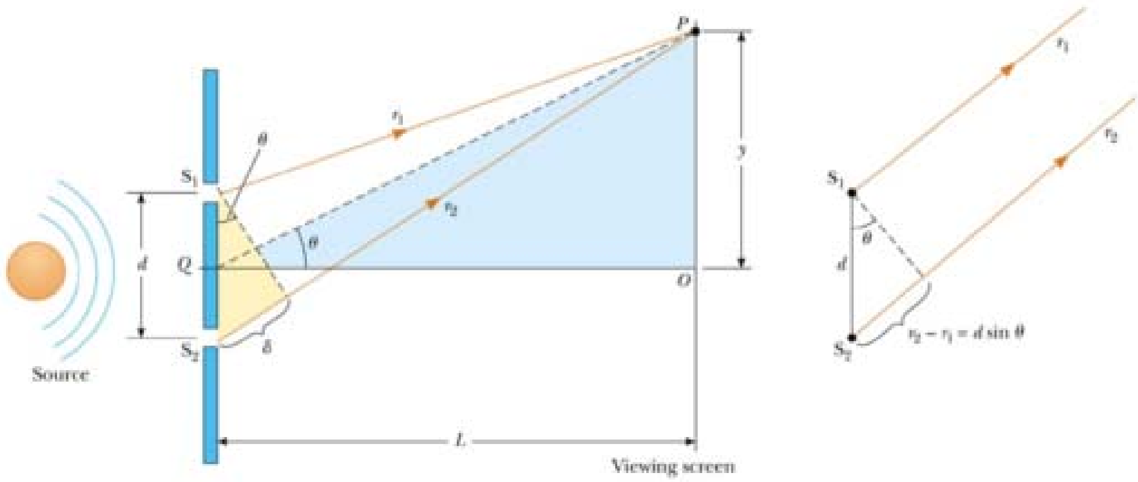


Figure (2.2): *Double slit diffraction.*

If $d << L$ then the difference in path length $r_1 - r_2$ traveled by the two rays is approximately:

$$r_1 - r_2 = d\, sin\theta \tag{2.1}$$

Where θ is approximately equal to the angle that the rays make relative to a perpendicular line joining the slits to the screen.

If the rays were in phase when they passed through the slits, then the condition for constructive interference at the screen is:

$$d\, sin\theta = m\lambda\,, \qquad m = \pm 1, \pm 2, \pm 3, \dots \dots \tag{2.2}$$

whereas the condition for destructive interference at the screen is:

$$d\, sin\theta = (m + \tfrac{1}{2})\lambda\,, \qquad m = \pm 1, \pm 2, \pm 3, \dots \dots \tag{2.3}$$

The points of constructive interference will appear as bright bands on the screen and the points of destructive interference will appear as dark bands. These dark and bright spots are called interference fringes.

In the case that y (the distance from the interference fringe to the point of the screen opposite the center of the slits) is much less than L ($y << L$), one can use the approximate formula:

$$sin\theta \approx y/L \tag{2.4}$$

so that the formulas specifying the y - coordinates of the bright and dark spots, respectively are:

$$y_m^B = \frac{m\lambda L}{d} \qquad (\textit{Bright spots}) \tag{2.5}$$

$$y_m^D = \frac{(m+\frac{1}{2})\lambda L}{d} \qquad (\textit{Dark spots}) \tag{2.6}$$

The spacing between the dark spots is:

$$\Delta y = \frac{\lambda L}{d} \tag{2.7}$$

If $d << L$ then the spacing between the interference can be large even when the wavelength of the light is very small (as in the case of visible light). This gives a method for (indirectly) measuring the wavelength of light.

The above formulas assume that the slit width is very small compared to the wavelength of light, so that the slits behave essentially like point sources of light.

Finally, the uses of Young's double slit experiment are:

1. Young's double slit experiment provides a method for measuring wavelength of light.
2. This experiment gave the wave model of light a great deal of credibility.
3. It is inconceivable that particles of light could cancel each other.

2.4 Intensity in double slit interference:

By illuminating two narrow slits with the same monochromatic, coherent light source, but we now expect to see a different pattern. First of all, more light is now going to reach the screen, and so expect that overall pattern to be brighter (more intense). But more interestingly, and now expecting to see an interference pattern due to the fact that the light from the two slits will travel different distances to arrive at the same point on the screen. consider two very narrow slits separated by a small distance *d*, the diffraction of the light from each slit will cause the light to spread out essentially uniformly over a broad central region and we would see a pattern such as the one depicted in Figure (2.3).

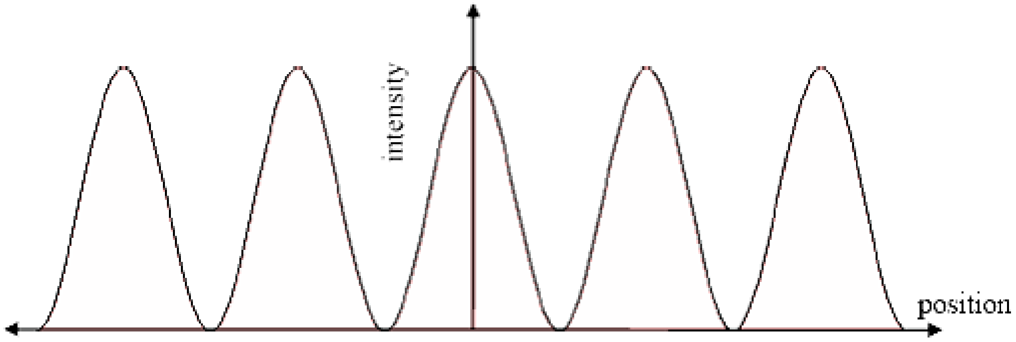


Figure (2.3): *Idealized double-slit intensity pattern as a function of position. This is the pattern we would see if the size of the individual slits is relatively small.*

If the individual slits are somewhat larger, so that the diffraction patterns are not so spread out, we would expect to see a somewhat more complicated pattern, which shows both the diffraction pattern and double-slit interference pattern simultaneously, as depicted in Figure (2.4). In both cases, the maxima and minima will appear where the conditions for constructive and destructive interference are satisfied, respectively

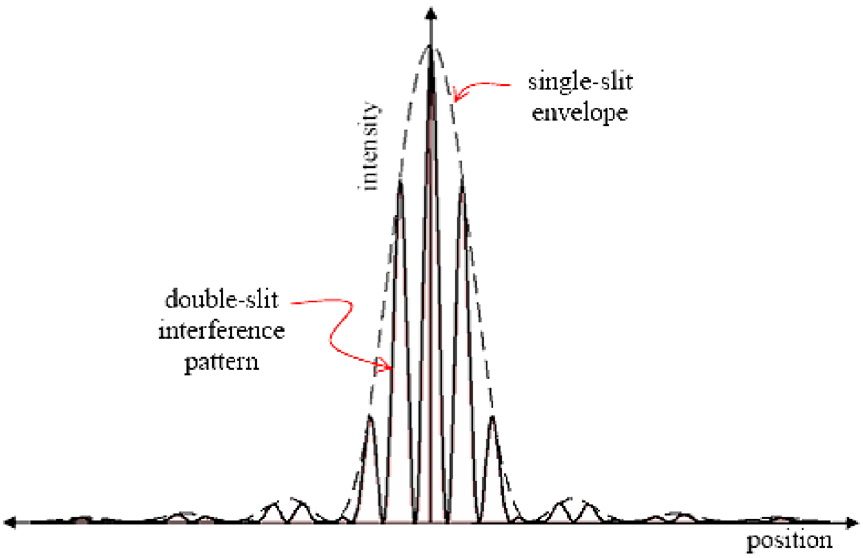


Figure (2.4): *Actual (non-ideal) double-slit interference pattern.*

The maxima and minima appear where constructive and destructive interference occur, respectively, due to the path length difference between the waves propagating from each slit to the observation point (screen). This pattern is attenuated by the single-slit "envelope".

For the double-slit interference pattern, intensity *maxima* will be located at an angle θ relative to the central maximum, where θ will obey the relation

$$dsin\theta = m\lambda \qquad \text{m=0,1,2,3…….} \qquad (2.8)$$

The intensity *minima*, on the other hand, due to the double-slit interference will occur at an angle θ relative to the central maximum given by

$$dsin\theta = \frac{1}{2}(m + \lambda) \qquad \text{m=0,1,2,3…….} \qquad (2.9)$$

2.5 Effect of Slit Width:

The light used to produce the interference pattern is diffracted by the pinholes or slits. Interference is possible only if light is directed in that direction. The overall interference intensity pattern is therefore modulated by the single-slit diffraction pattern λ (assuming slit apertures):

$$I(x) = I_o sinc^2(\frac{DX}{\lambda L})\left[1 + \gamma(x)\cos\left(\frac{2\pi xd}{\lambda L}\right)\right] \qquad (2.10)$$

where *D* is the slit width, and a one-dimensional expression is shown. The definition of a sinc function is

$$\mathrm{sinc}(\theta) = \frac{\sin\pi\theta}{\theta\pi} \qquad (2.11)$$

where the zeros of the function occur when the argument $\propto$ is an integer. The intensity variation in the *y* direction is due to diffraction only and is not shown. Since the two slits are assumed to be illuminated by a single source, there are no coherence effects introduced by using a pinhole or slit of finite size. The term $\gamma(x)$ is included in Eq. (2.10) to account for variations in the fringe visibility. These could be due to unequal illumination of the two slits, a phase difference of the light reaching the slits, or a lack of temporal or spatial coherence of the source S_o.

2.6 Repetition:

By replacing a double slit with a triple slit, Figure (2.5-a). We can think of this as a third *repetition* of the structures that were present in the double slit, as can be shown in Figures (2.5-b) and (2.5-c). For ease of visualization, we have violated our usual rule of only considering points very far from the diffracting object. The scale of the drawing is such that a wavelengths is one cm. In (2.5-b), all three waves travel an integer number of wavelengths to reach the same point, so there is a bright central spot, as one would expect from the experience with the double slit. In Figure (2.5-c), it shows the path lengths to a new point. This point is farther from slit **A** by a quarter of a wavelength, and correspondingly closer to slit **C**. The distance from slit **B** has hardly changed at all. Because the paths lengths traveled from slits **A** and **C** differ from half a wavelength, there will be perfect destructive interference between these two waves. There is still some un canceled wave intensity because of slit **B**, but the amplitude will be three times less than in Figure (2.5-b), resulting in a factor of 9 decrease in brightness.

Thus, by moving off to the right a little, one can gone from the bright central maximum to a point that is quite dark.

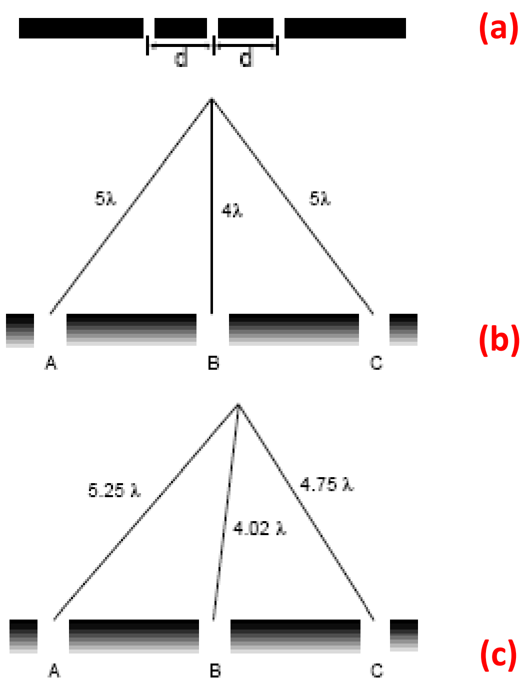


Figure (2.5): *(a) A triple slit.*
(b) There is a bright central maximum.
(c) At this point just off the central maximum, the path Lengths traveled by the three waves have changed.

Now let's compare with what would have happened if slit **C** had been covered, creating a plain old double slit. The waves coming from slits **A** and **B** would have been out of phase by 0.23 wavelengths, but this would not have caused very severe interference. The point in Figure (2.5-c) would have been quite brightly lit up.

To summarize, by adding a third slit narrows down the central fringe dramatically. The same is true for all the other fringes as well, and since the same amount of energy is concentrated in narrower diffraction fringes, each fringe is brighter and easier to see, Figure (2.6).

Figure (2.6): *A double-slit diffraction pattern (top), and a triple-slit pattern (bottom).*

This is an example of a more general fact about diffraction; if some feature of the diffracting object is repeated, the locations of the maxima and minima are unchanged, but they become narrower.

Taking this reasoning to its logical conclusion, a diffracting object with thousands of slits would produce extremely narrow fringes. Such an object is called a diffraction grating.

2.7 Fourier Series:

A Fourier series is an expansion of a periodic function $f(x)$ in terms of an infinite sum of *sines* and *cosines*. Fourier series make use of the orthogonality relationships of the sine and cosine functions. The computation and study of Fourier series is known as harmonic analysis and is extremely useful as a way to break up an arbitrary periodic function into a set of simple terms that can be plugged in, solved individually, and then recombined to obtain the solution to the original problem or an approximation to it to whatever accuracy is desired or practical. Examples of successive approximations to common functions using Fourier series are illustrated below.

In particular, since the superposition principle holds for solutions of a linear homogeneous ordinary differential equation, if such an equation can be solved in the case of a single sinusoid, the solution for an arbitrary function is immediately available by expressing the original function as a Fourier series and then plugging in the solution for each sinusoidal component. In some special cases where the Fourier series can be summed in closed form, this technique can even yield analytic solutions.

Any set of functions that form a complete orthogonal system have a corresponding generalized Fourier series analogous to the Fourier series. The computation of the (usual) Fourier series is based on the integral identities

$$\int_{-\pi}^{\pi} \sin(mx)\ \sin(nx)\, dx = \pi\delta_{mn} \quad (2.12)$$

$$\int_{-\pi}^{\pi} \cos(mx)\ \cos(nx)\, dx = \pi\delta_{mn} \quad (2.13)$$

$$\int_{-\pi}^{\pi} \sin(mx)\ \cos(nx)\, dx = 0 \quad (2.14)$$

$$\int_{-\pi}^{\pi} \sin(mx)\ dx = 0 \quad (2.15)$$

$$\int_{-\pi}^{\pi} \cos(mx)\, dx = 0 \quad (2.16)$$

for $m, n \neq 0$, where δ_{mn} is the Kronecker delta.

Using the method for a generalized Fourier series, the usual Fourier series involving sines and cosines is obtained by taking $f_1(x) = \cos x$ and $f_2(x) = \sin x$. Since these functions form a complete orthogonal system over $[\pi, -\pi]$, the Fourier series of a function $f(x)$ is given by:

$$f(x) = \tfrac{1}{2}a_o + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx) \quad (2.17)$$

where

$$a_o = \tfrac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx \quad (2.18)$$

$$a_n = \tfrac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\, dx \quad (2.19)$$

$$b_n = \tfrac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\, dx \quad (2.20)$$

and $n = 1,2,3, \ldots$. Note that the coefficient of the constant term a_o has been written in a special form compared to the general form for a generalized Fourier series in order to preserve symmetry with the definitions of a_n and b_n.

A Fourier series converges to the function $\bar{f}$ (equal to the original function at points of continuity or to the average of the two limits at points of discontinuity)

$$\bar{f} \equiv \begin{cases} \frac{1}{2}\left[\lim_{x \to x_0^-} f(x) + \lim_{x \to x_0^+} f(x)\right] & \text{for } -\pi < x_0 < \pi \\ \frac{1}{2}\left[\lim_{x \to -\pi^+} f(x) + \lim_{x \to \pi_-} f(x)\right] & \text{for } x_0 = -\pi, \pi \end{cases} \tag{2.21}$$

if the function satisfies so-called Dirichlet conditions. Dini's test gives a condition for the convergence of Fourier series, see Figure (2.7).

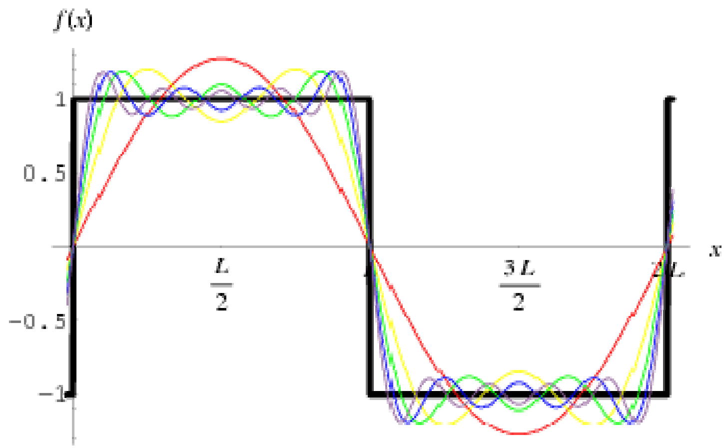


Fig.(2.7): *Illustration of Gibbs phenomenon near points of discontinuity.*

For a function $f(x)$ periodic on an interval $[-L, L]$ instead of $[-\pi, \pi]$, a simple change of variables can be used to transform the interval of integration from $[-\pi, \pi]$ to $[-L, L]$. Let

$$x = \frac{\pi \acute{x}}{L} \tag{2.22}$$

$$dx = \frac{\pi \, d\acute{x}}{L} \tag{2.23}$$

Solving for $\acute{x}$ gives $\acute{x} = Lx/\pi$, and plugging this in gives

$$f(\acute{x}) = \tfrac{1}{2}a_o + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi\acute{x}}{L}\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi\acute{x}}{L}\right) \tag{2.24}$$

Therefore,

$$a_o = \tfrac{1}{L}\int_{-L}^{L} f(\acute{x})d\acute{x} \tag{2.25}$$

$$a_n = \tfrac{1}{L}\int_{-L}^{L} f(\acute{x})\cos\left(\frac{n\pi\acute{x}}{L}\right)d\acute{x} \tag{2.26}$$

$$b_n = \tfrac{1}{L}\int_{-L}^{L} f(\acute{x})\sin\left(\frac{n\pi\acute{x}}{L}\right)d\acute{x} \tag{2.27}$$

Similarly, the function is instead defined on the interval $[0,2L]$, the above equations simply become

$$a_o = \tfrac{1}{L}\int_{0}^{2L} f(\acute{x})d\acute{x} \tag{2.28}$$

$$a_n = \tfrac{1}{L}\int_{0}^{2L} f(\acute{x})\cos\left(\frac{n\pi\acute{x}}{L}\right)d\acute{x} \tag{2.29}$$

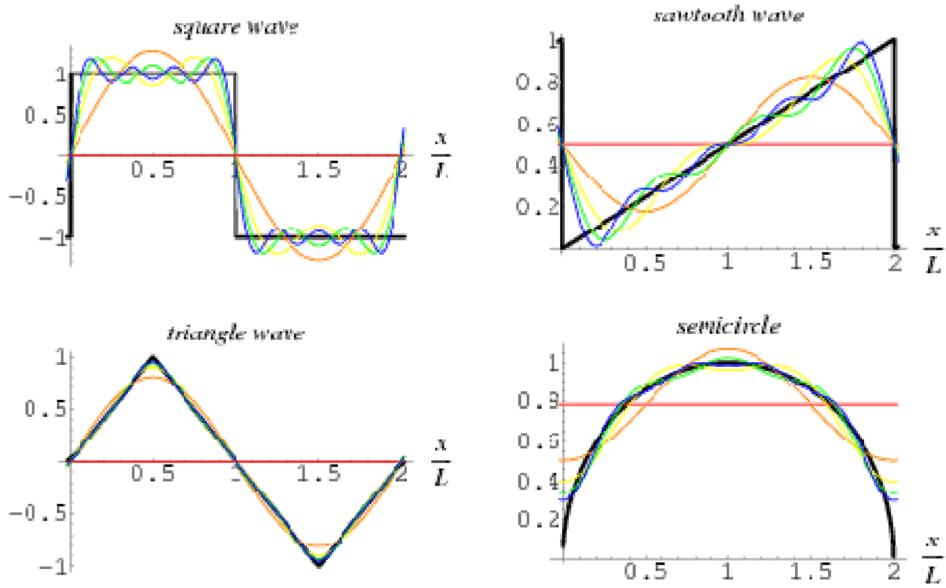
$$b_n = \tfrac{1}{L}\int_{0}^{2L} f(\acute{x})\sin\left(\frac{n\pi\acute{x}}{L}\right)d\acute{x} \tag{2.30}$$

In fact, for $f(x)$ periodic with period $2L$, any interval $(x_o, x_o + 2L)$ can be used, with the choice being one of convenience or personal preference.

One of the most common functions usually analyzed by this technique is the square wave. The function of Fourier series for a few common functions are summarized in the table (2.1), and the Figures of some common functions are sown in Figure (2.8).

Table(2.1): *Fourier series for some common functions.*

Function	$f(x)$	***Fourier series***
Fourier series--sawtooth wave	$\frac{x}{2L}$	$\frac{1}{2} - \frac{1}{\pi}\sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi x}{L}\right)$
Fourier series--square wave	$2\left[H\left(\frac{x}{L}\right) - H\left(\frac{x}{L} - 1\right)\right] - 1$	$\frac{4}{\pi}\sum_{n=1,3,5,\ldots}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi x}{L}\right)$
Fourier series--triangle wave	$T(x)$	$\frac{8}{\pi^2}\sum_{n=1,3,5,\ldots}^{\infty} \frac{(-1)^{(n-1)/2}}{n^2} \sin\left(\frac{n\pi x}{L}\right)$



Figure(2.8): *Fourier series for a few common functions*

If a function is even so that $f(x) = f(-x)$, then $f(x)\sin(nx)$ is odd. (This follows since $\sin(nx)$ is odd and an even function times an odd function is an odd function). Therefore, $b_n = 0$ for all n. Similarly, if a function is odd so that $f(x) = -f(-x)$, then $f(x)\cos(nx)$ is odd. (This follows since $\cos(nx)$ is even and an even function times an odd function is an odd function). Therefore, $a_n = 0$ for all n.

The notion of a Fourier series can also be extended to complex coefficients. Consider a real-valued function $f(x)$. Write

$$f(x) = \sum_{n=-\infty}^{\infty} A_n\, e^{inx} \tag{2.31}$$

2.8 Fast Fourier Transform:

The Fast Fourier Transform (FFT) is a discrete Fourier transform algorithm which reduces the number of computations needed for N points from $2N^2$ to $2N \lg N$, where lg is the base-2 logarithm. So for example a transform on 1024 points using the Discrete Fourier Transform DFT takes about 100 times longer than using the Fast Fourier Transform FFT, a significant speed increase.

If the function to be transformed is not harmonically related to the sampling frequency, the response of an FFT looks like a sinc function (although the integrated power is still correct). Aliasing (leakage) can be reduced by apodization using a tapering function. However, aliasing reduction is at the expense of broadening the spectral response.

FFTs were first discussed by Cooley and Tukey, although Gauss had actually described the critical factorization step as early as 1805. A discrete Fourier transform can be computed using an FFT by means of the Danielson-Lanczos lemma if the number of points N is a power of two. If the number of points N is not a power of two, a transform can be performed on sets of points corresponding to the prime factors of N which is slightly degraded in speed. Base-4 and base-8 fast Fourier transforms use optimized code, and can be 20-30% faster than base-2 fast Fourier transforms. prime factorization is slow when the factors are large, but discrete Fourier transforms can be made fast for n =2, 3, 4, 5, 7, 8, 11, 13, and 16 using the Winograd transform algorithm.

Fast Fourier transform algorithms generally fall into two classes: decimation in time, and decimation in frequency. The Cooley-Tukey FFT algorithm first rearranges the input elements in bit-reversed order, then builds the output transform (decimation in time). The basic idea is to break up a transform of length N into two transforms of length $N/2$ using the identity

$$\sum_{n=0}^{N-1} a_n e^{-2\pi i n k/N} = \sum_{n=0}^{N/2-1} a_{2n} e^{-2\pi i (2n) k/N} + \sum_{n=0}^{N/2-1} a_{2n+1} e^{-2\pi i (2n+1) k/N}$$

$$= \sum_{n=0}^{N/2-1} a_n^{\text{even}} e^{-2\pi i n k/(N/2)} + e^{-2\pi i k/N} \sum_{n=0}^{N/2-1} a_n^{\text{odd}} e^{-2\pi i n k/(N/2)},$$

sometimes called the Danielson-Lanczos lemma. The easiest way to visualize this procedure is perhaps via the Fourier matrix.

In the most general situation a 2-dimensional transform takes a complex array. The most common application is for image processing where each value in the array represents to a pixel, therefore the real value is the pixel value and the imaginary value is 0. The forward transform and the inverse transformation can be defined as:

$$F(u,v) = \frac{1}{MN} \sum_{x=0}^{M} \sum_{y=0}^{N} f(x,y)\, e^{-j\, 2\pi (u x / M + v y / N)}$$

$$f(x,y) = \sum_{u=0}^{M} \sum_{v=0}^{N} F(u,v)\, e^{j\, 2\pi (u x / M + v y / N)}$$

Chapter Three
Interference Simulation

3.1 Introduction:

The first serious challenge to the particle theory of light was made by the English scientist Thomas Young in 1803. He reasoned that if light were actually a wave phenomenon, as he suspected, then a phenomenon of interference effect should occur for light. This line of reasoning lead Young to perform an experiment which is nowadays referred to as Young's double-slit experiment.

The present works is an attempt to study the interference pattern produced by Young’s experiment of coherence light field using the Fast Fourier Transformation Matlab commend, hopefully to be clear. However, the Young’s double slit experiment is connect to so many basic concepts in optical physics (and still provides surprising new results to this day) that one post is hardly enough to describe all the interesting insights that can be gained by studying the experiment and its implications.

Simulation is an important feature in physics systems or any system that involves many processes. Most engineering simulations entail mathematical modeling and computer assisted investigation. There are many cases, however, where mathematical modeling is not reliable. Simulation of fluid dynamics problems often require both mathematical and physical simulations. In these cases the physical models require dynamic similitude. Physical and chemical simulations have also direct realistic uses, rather than research uses.

The power of simulation is that (even for easily solvable linear systems) a uniform model execution technique can be used to solve a large variety of systems without resorting to choose special-purpose and sometimes arcane solution methods to avoid simulation.

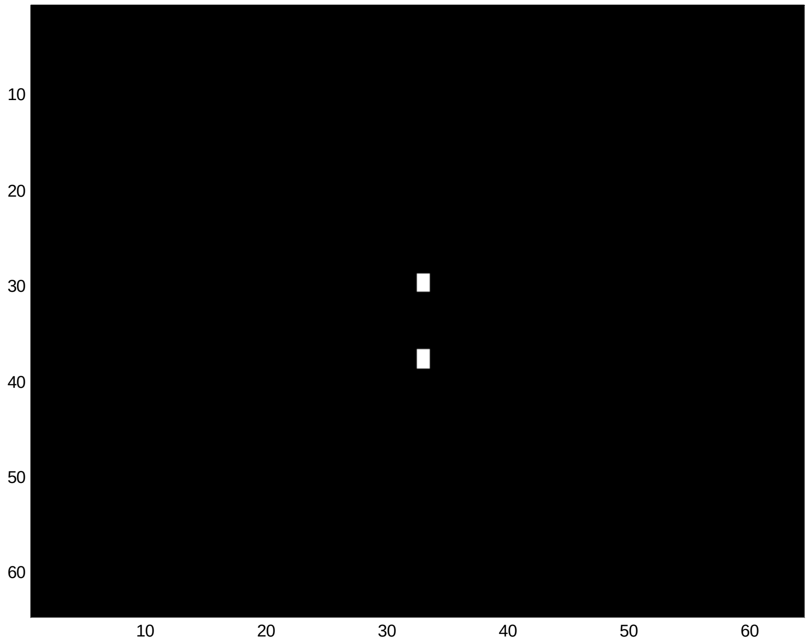
3.2 Double slit interference:

The interference of light from the two slits form a visible pattern on a screen, the pattern consist of a series of bright and dark parallel fringes, constructive interference occurs where a bright fringe appears, and destructive interference results in a dark fringe. However, at the center point, the interference is constructive, because the two waves travel the same distance, therefore, they arrive in phase.

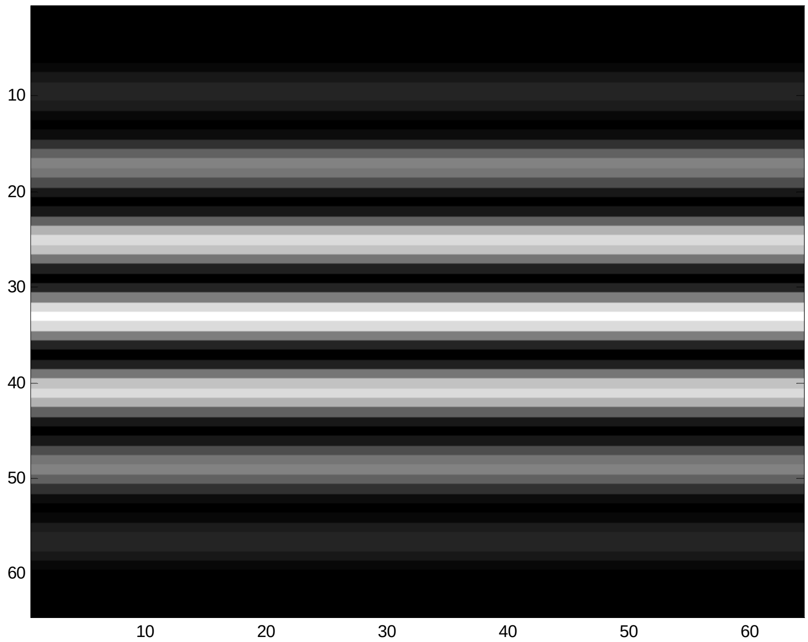
For the bright fringes, one of the waves travel farther than the other waves by integer multiple of wavelength, therefore, the two waves arrive in phase a bright fringe occurs, but in the dark fringes, the one wave travel one-half of a wavelength farther than the other wave then a dark fringe occurs.

The interpretation of an interference pattern was done by using FFT command from MATLAB. The process of splitting up the incident wave into two monochromatic waves was done be generation a 64x64 zeros matrix, and definite the values of two points in the middle of matrix to be one, as shown in Figure (3.1). The interference pattern was obtained by using the Fast Fourier Transformation to this matrix, and then taking the inverse Fast Fourier Transformation for the real part of the result (see Appendix).

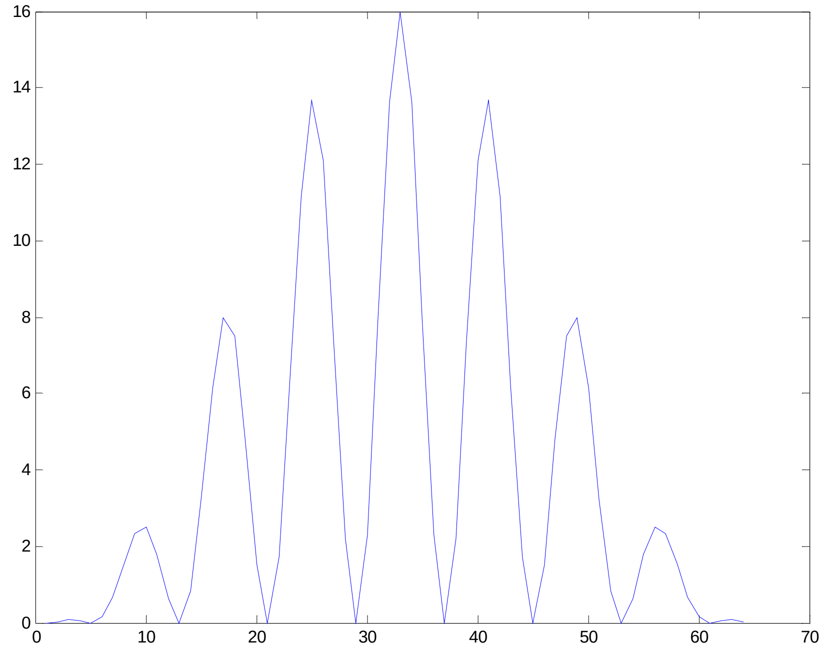
Figure (3.2) shows the interference pattern of light, the intensity minima as dark spots in space and maxima as bright spots, so the intensity pattern has only positive or zero values. The graph in Figures (3.3) and (3.4) shows the intensity for Young’s experiment as a function of distance. One observes that there is a maximum at the center, as it was expected theoretically.



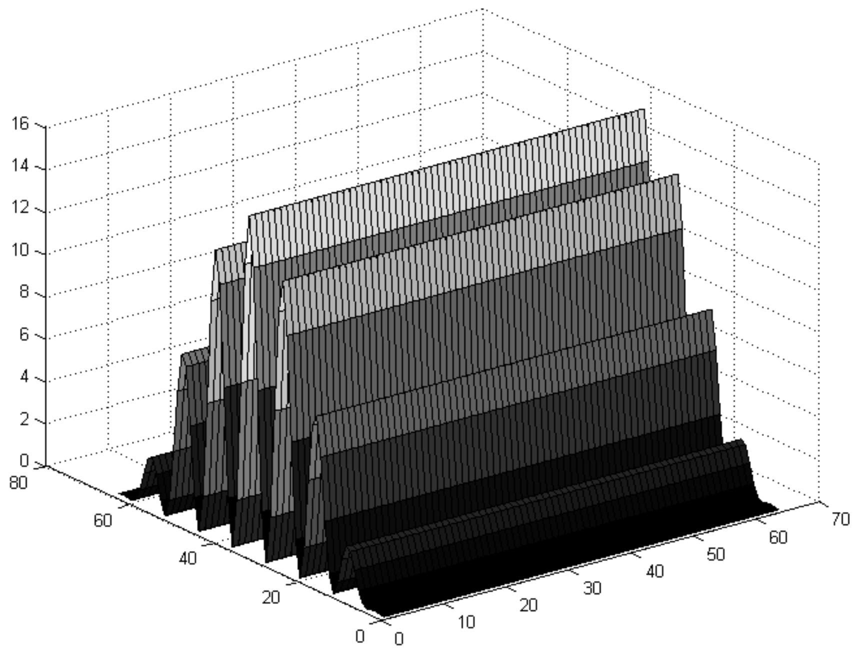
Fig(3.1): *Sketch of double slit .*



Fig(3.2): *Interference fringe of double slit.*

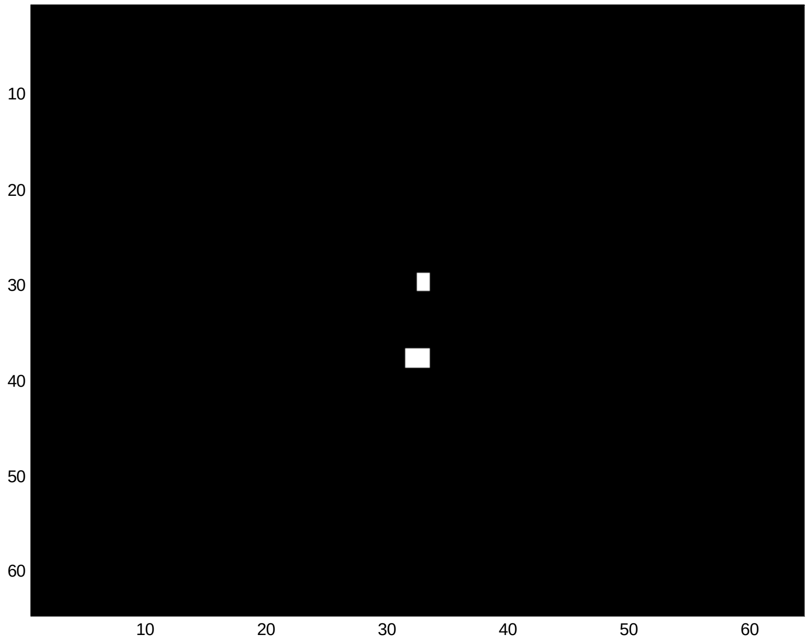


Fig(3.3): *The intensity of fringes in the center of the screen for double slit.*

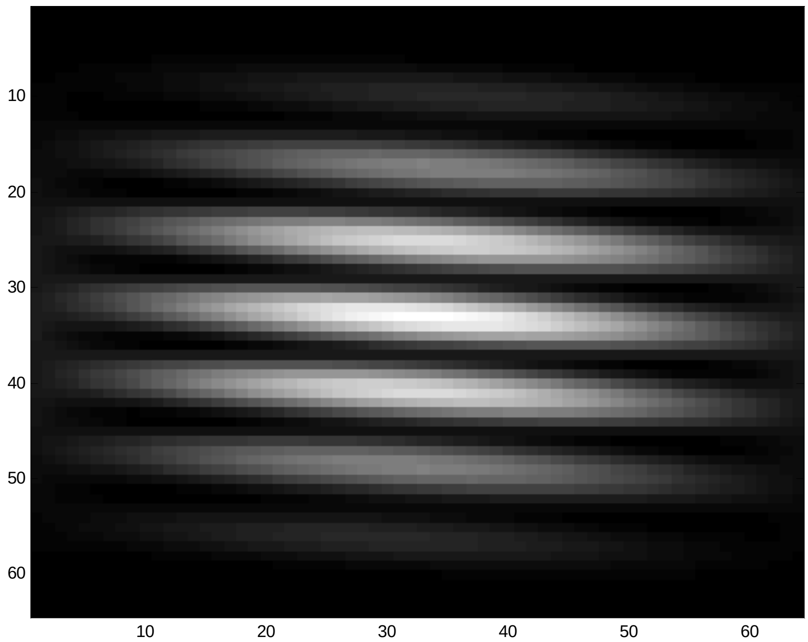


Fig(3.4): *The intensity of fringes for the double slit on the screen.*

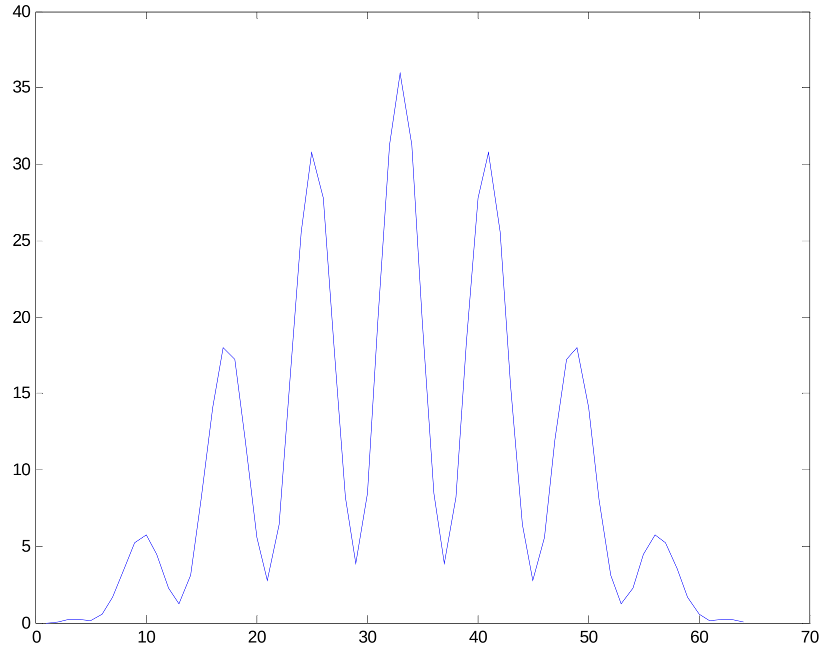
An attempted has been made to study the effect of width of the slits on the interference pattern. The sketch of the two different width slits was shown in Figure (3.5). The interference pattern that obtained for this configuration was shown in Figure (3.6). While Figures (3.7) and (3.8) shows the intensity as a function of distance. It was clear from these figures the intensity of all the bright fringes increases due to increase of the width of one slit.



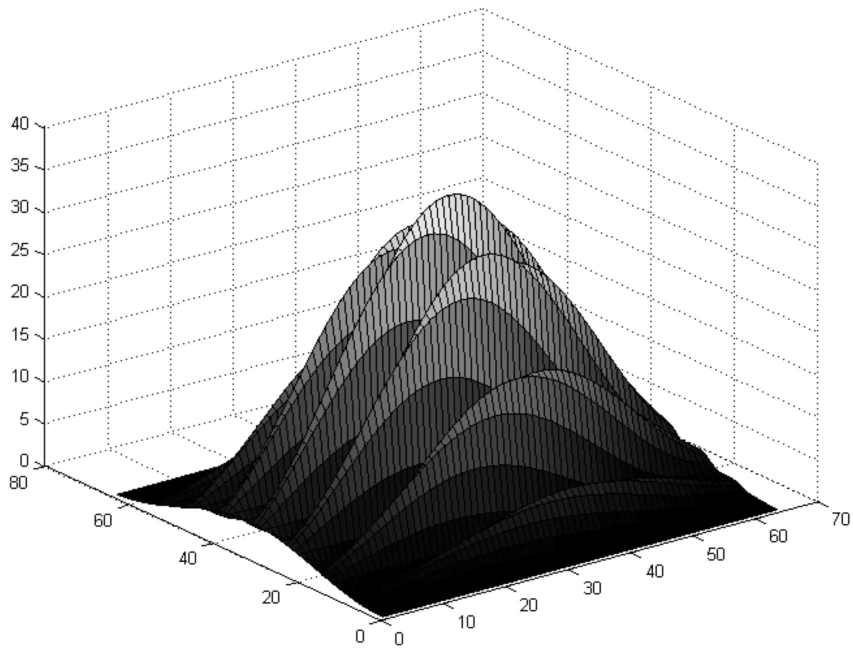
Fig(3.5):*The sketch of double slit of different width.*



Fig(3.6): *Interference fringe of different width double slit.*

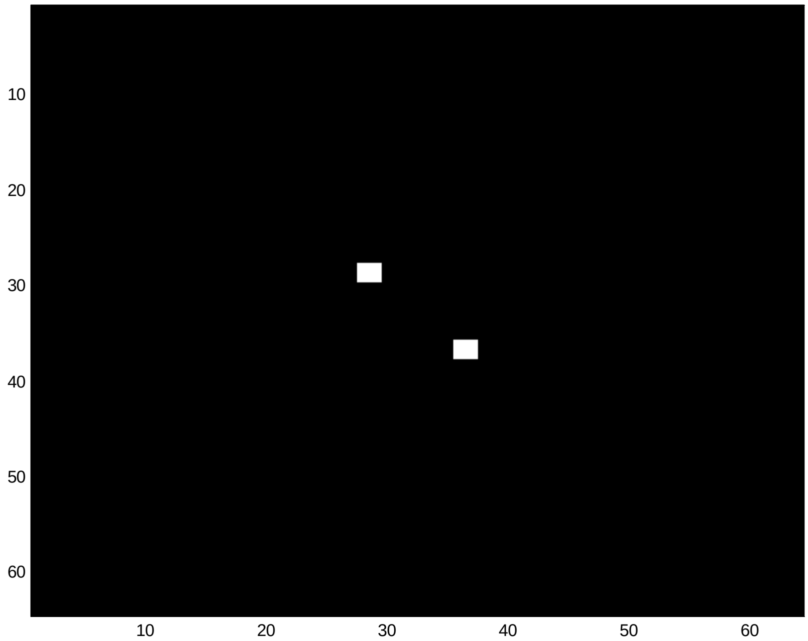


Fig(3.7): *The intensity of fringes in the center of the screen for different width double slit.*

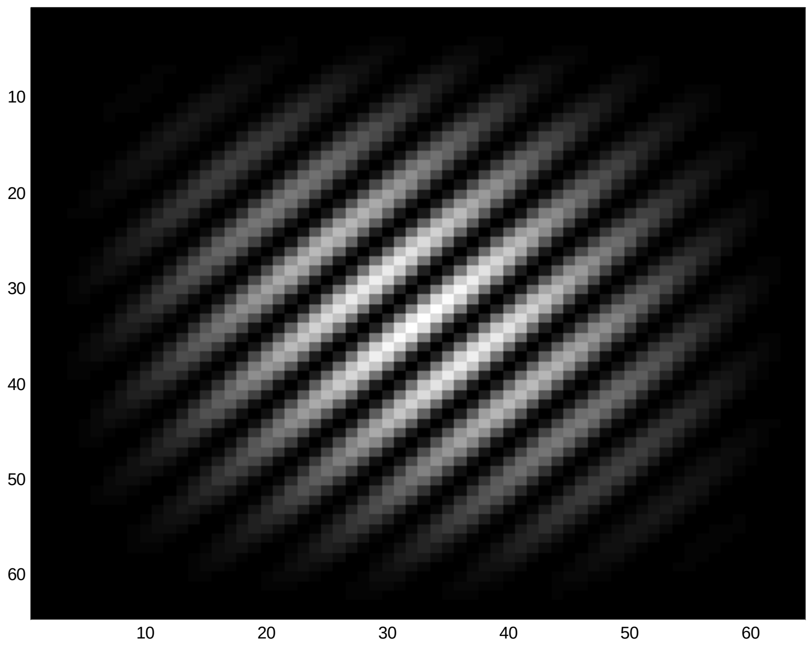


Fig(3.8): *The intensity of fringes for the double slit of different width on the screen.*

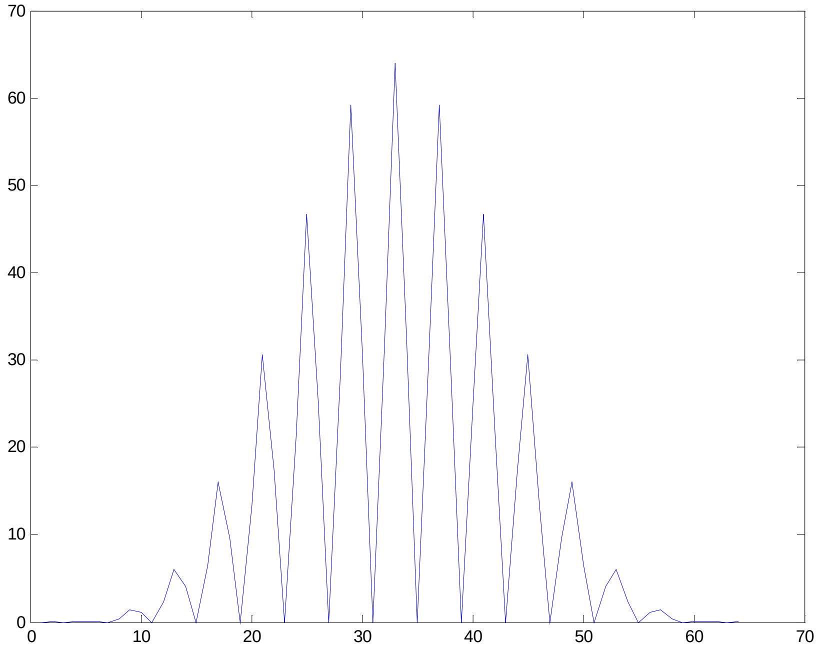
The interference simulation of diagonal double slits was also done as shown in Figure (3.9), the interference pattern on the screen for this configuration, was obtained by using the Fast Fourier Transformation as shown in Figure (3.10). The intensity pattern of this diagonal double slits, are shown in Figures (3.11) and (3.12). Also in this situation there is a maximum intensity at the center. While the fringes are declined by 45^o angle due to the slits direction.



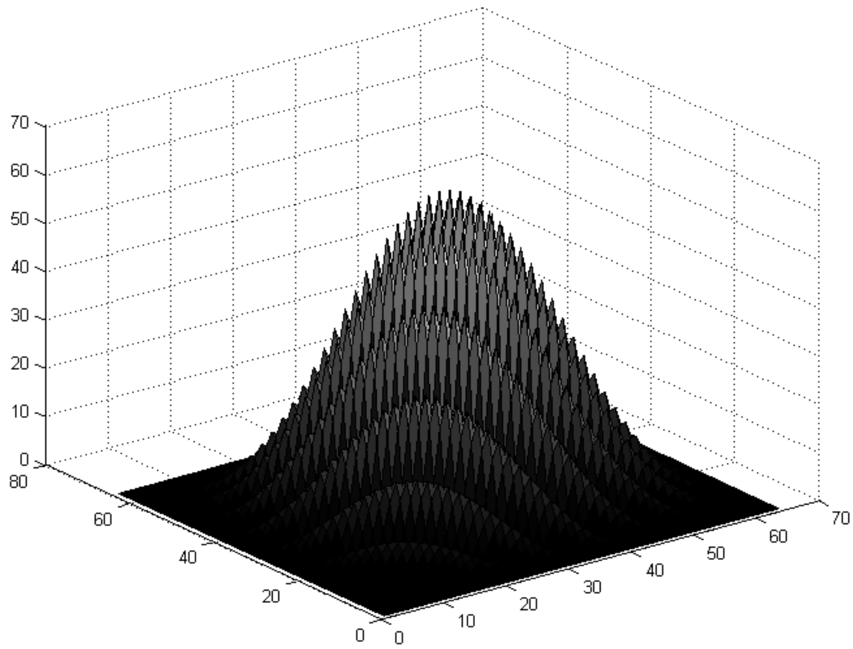
Fig(3.9):*The sketch of diagonal double slit.*



Fig(3.10): *Interference fringe of diagonal double slit.*



Fig(3.11): *The intensity of fringes in the center of the screen for diagonal double slit.*



Fig(3.12): *The intensity of fringes for the diagonal double slit.*

3.3 Interference in single slit:

If light travels to the centerline of the slit, their light arrives in phase and experiences constructive interference. Light from other element pairs symmetric to the centerline also arrive in phase. Although there is a progressive change in phase as in choosing element pairs closer to the centerline, this center position is nevertheless the most favorable location for constructive interference of light from the entire slit and has the highest light intensity.

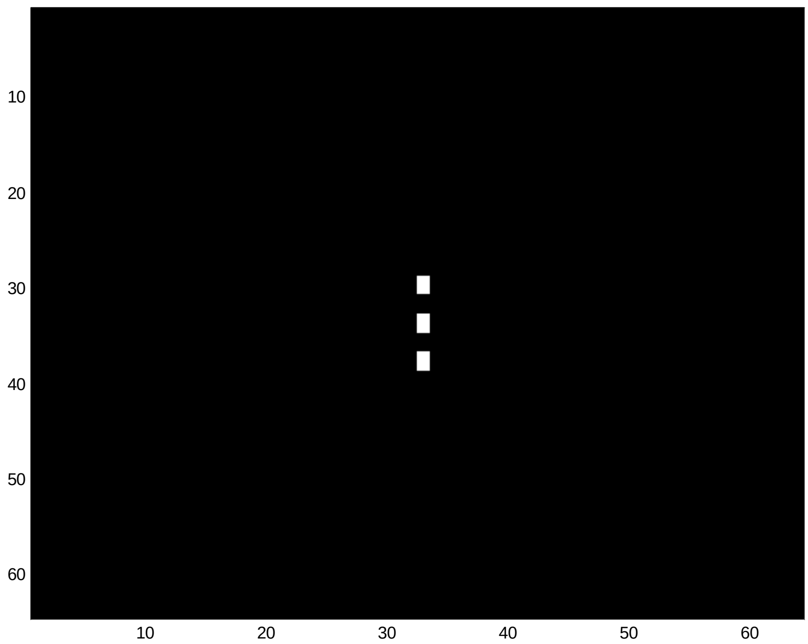
An element at one edge of the slit and one just past the centerline are chosen, and the condition for minimum light intensity is that light from these two elements arrive 180° out of phase, or a half wavelength different in pathlength. If those two elements suffer destructive interference, then choosing additional pairs of identical spacing, which progress downward across the slit, will give destructive interference for all those pairs and therefore an overall minimum in light intensity.

An attempted has been made to study the interference pattern for a single slit. Due to the assumption that the slit have constant values, the intensity of the interference pattern obtained for single slit shows also constant value everywhere, which is contrary to the theoretical aspects. This contravention can be corrected by taken the values of the slit as a Gaussian configuration instead of constant values. By this assumption, the effect of diffraction is taking in to account as well as the interference phenomena.

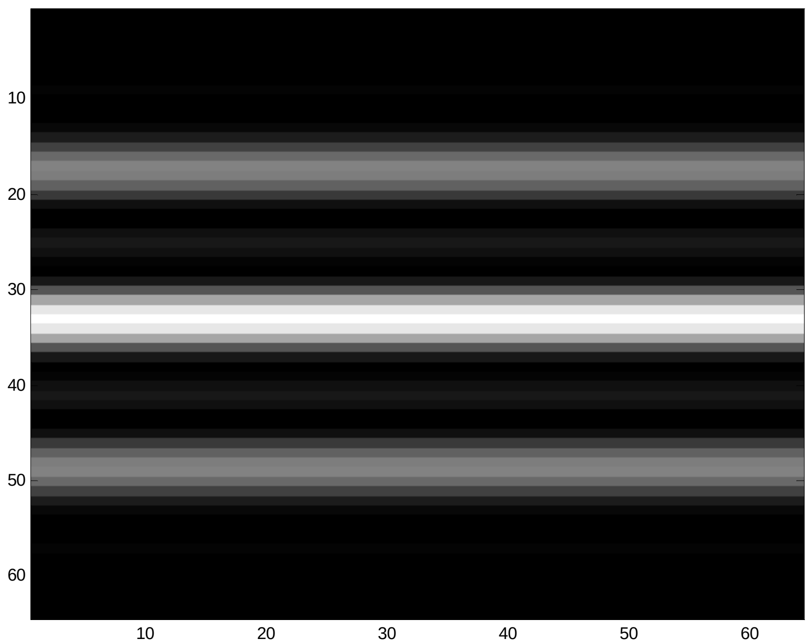
3.4 Interference in three slits:

Under the Fraunhofer conditions, the light curve of a multiple slit arrangement will be the interference pattern multiplied by the single slit diffraction envelope. This assumes that all the slits are identical. Increasing the number of slits not only makes the diffraction maximum sharper, but also much more intense, each fringe is easy to see, the width of fringe is narrower than in the two slit.

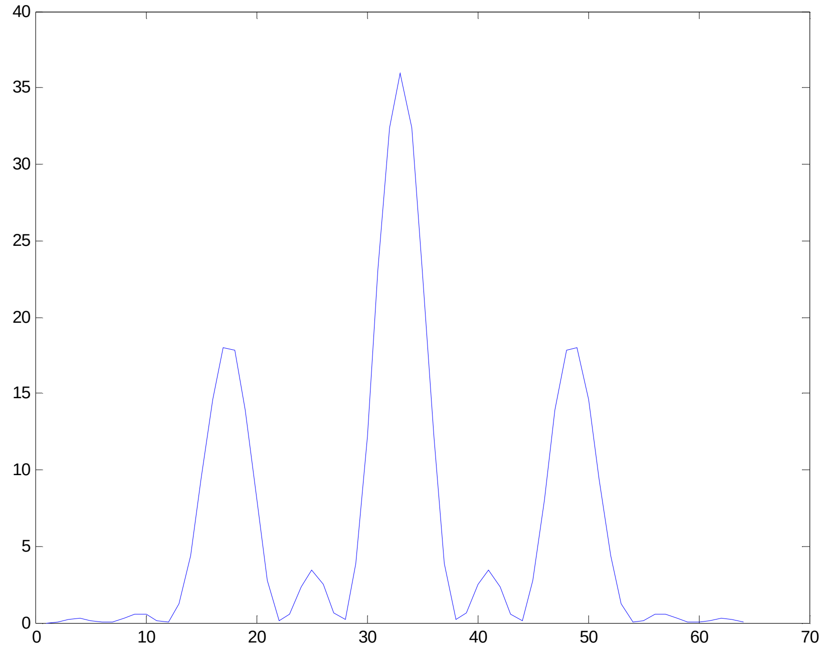
The interference presentation of three slits was also done using MATLAB program. The sketch of this configuration was shown in Figure (3.13). The interference pattern was obtained by using the Fast Fourier Transformation. The graph in Figure (3.14) represent the interference pattern of light. Figures (3.15) and (3.16) shows the intensity as a function distance. Comparing the intensity of the double-slit in Figure (3.3), and three-slits in Figure (3.15) notes that the intensity of three-slits interference higher than that of double slit.



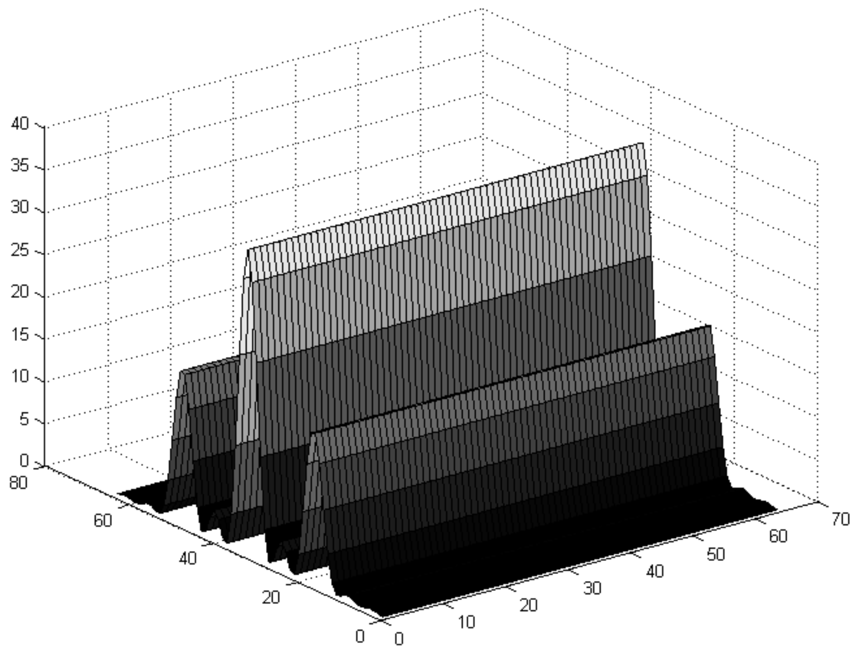
Fig(3.13):*The sketch of a three slits.*



Fig(3.14): *Interference fringe of a three slits.*



Fig(3.15): *The intensity of fringes in the center of the screen for a three slits.*

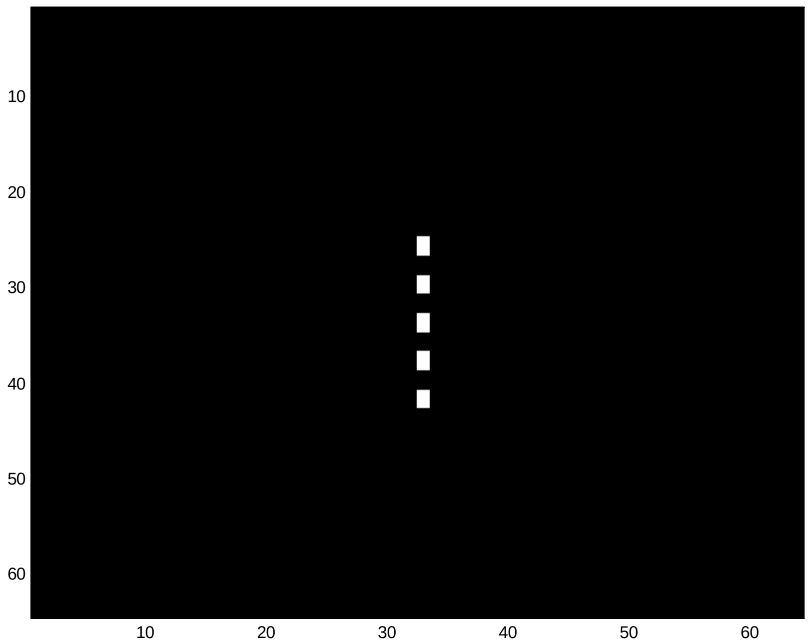


Fig(3.16): *The intensity of fringes for the diagonal double slit.*

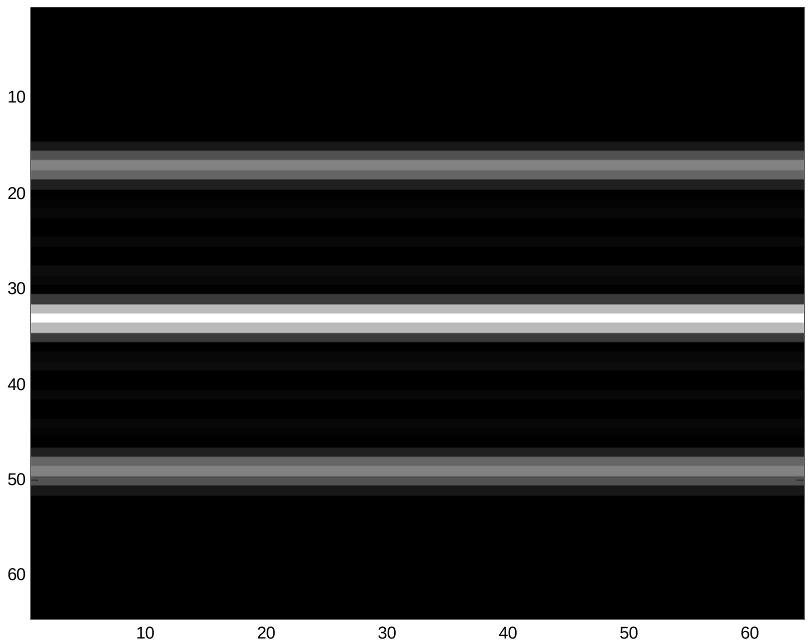
3.5 Interference in five slit:

As the number of slits increase, the peak width in the figure is decrease thus the intensity of the fringes is increase, the intensity of the central fringe is some larger than the other, if away from the central fringe the intensity of fringes decrease.

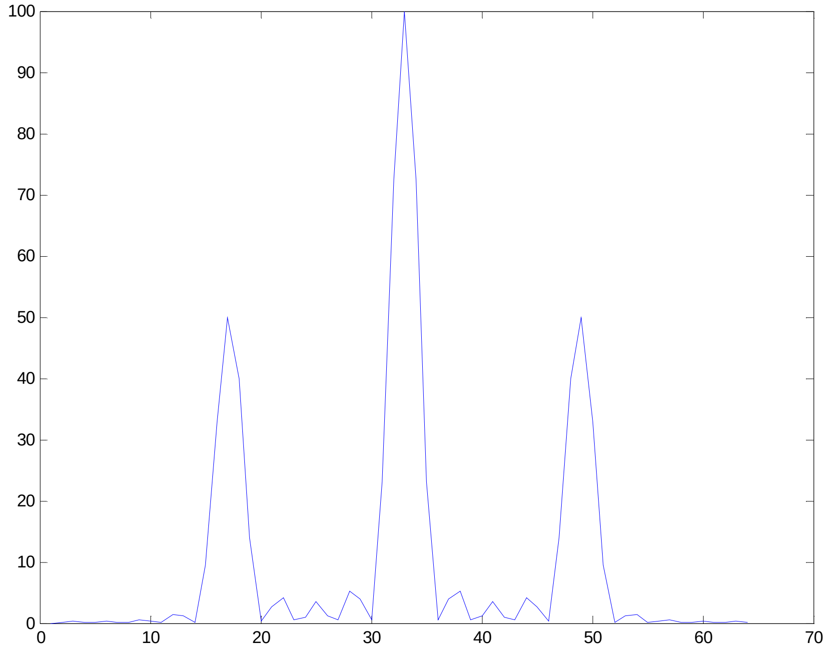
Figure (3.18) shows the interference pattern of light from configuration of the Figure (3.17). Comparing this result with those obtained for double slits realize that the width of the fringes decrease as the number of slits increase, while the intensity of three slits fringes greater than those of double slits, as shown in the Figures (3.19) and (3.20), this result are in a good agreement with theoretical.



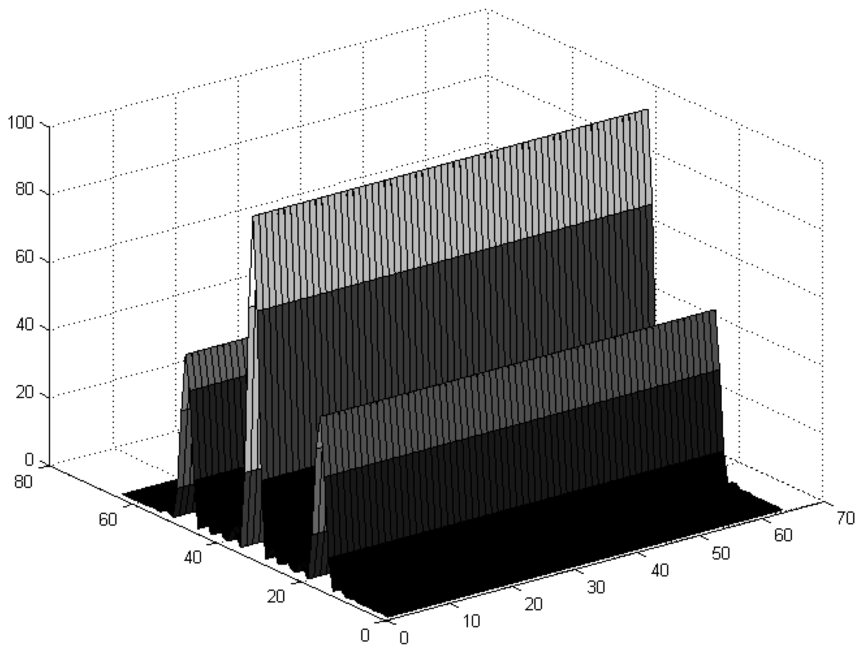
Fig(3.17):*The sketch of diagonal of five slits.*



Fig(3.18): *Interference fringe of five slits.*



Fig(3.19): *The intensity of fringes in the center of the screen for five slits.*

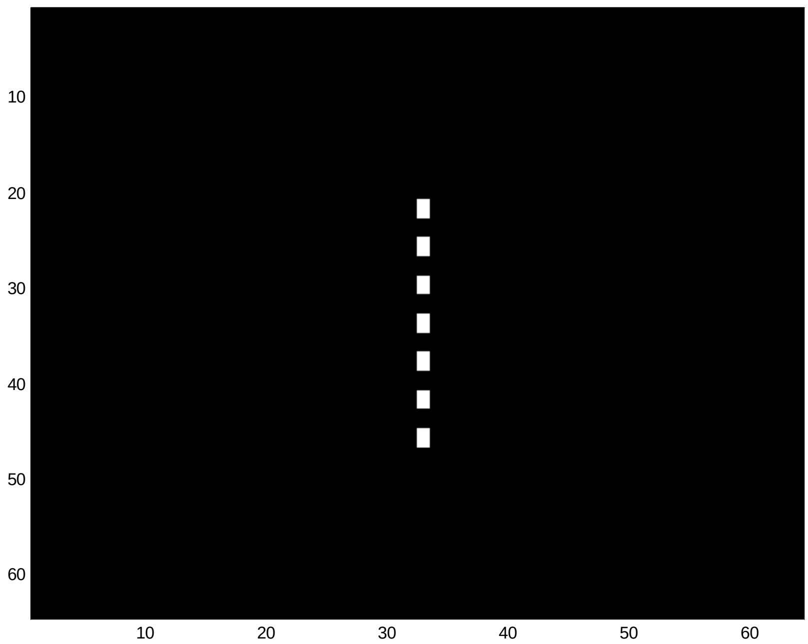


Fig(3.20): *The intensity of fringes for the five slits..*

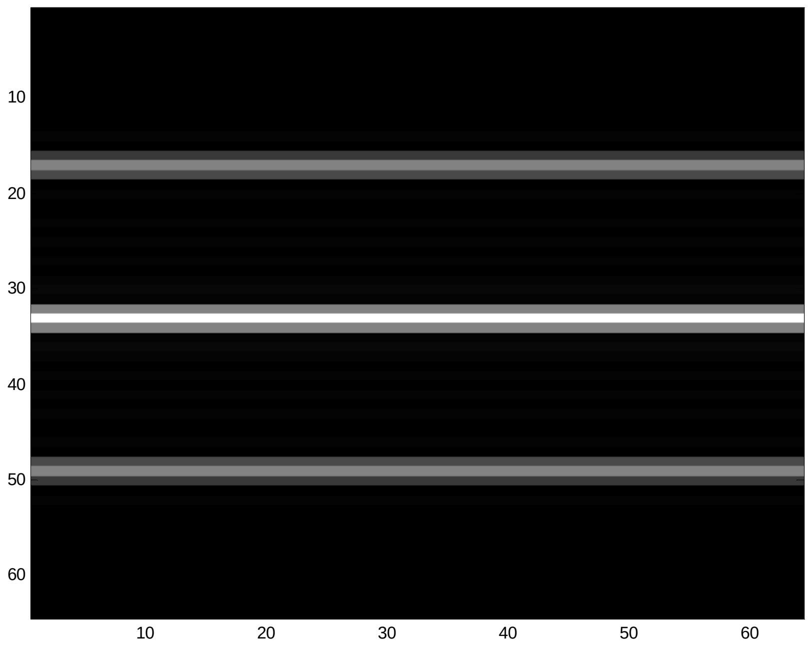
3.6 Interference in seven slit:

The progression to a larger number of slits shows a pattern of narrowing the high intensity peaks and a relative <u>increase in their peak intensity</u>. This progresses toward the <u>diffraction grating</u>, with a large number of extremely narrow slits. This gives very narrow and very high intensity peaks that are separated widely. Since the positions of the peaks depends upon the wavelength of the light, this gives high resolution in the separation of wavelengths.

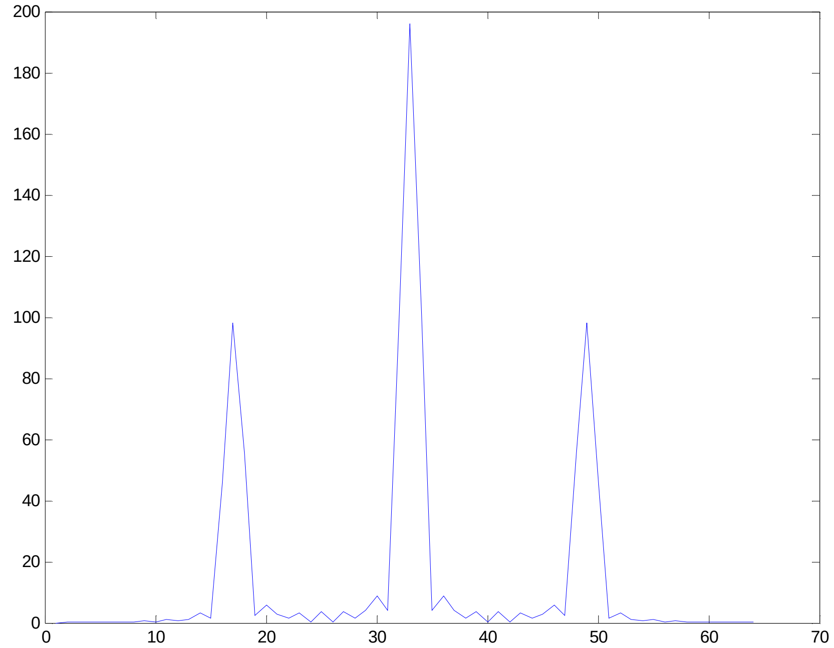
The interference presentation of seven slits was also done using MATLAB program. The sketch of this configuration was shown in Figure (3.21). The interference pattern was obtained by using the Fast Fourier Transformation. The graph in Figure (3.22) represent the interference pattern of light. Figures (3.23) and (3.24) shows the intensity as a function distance. All the results are in a good agreement with theoretical concepts.



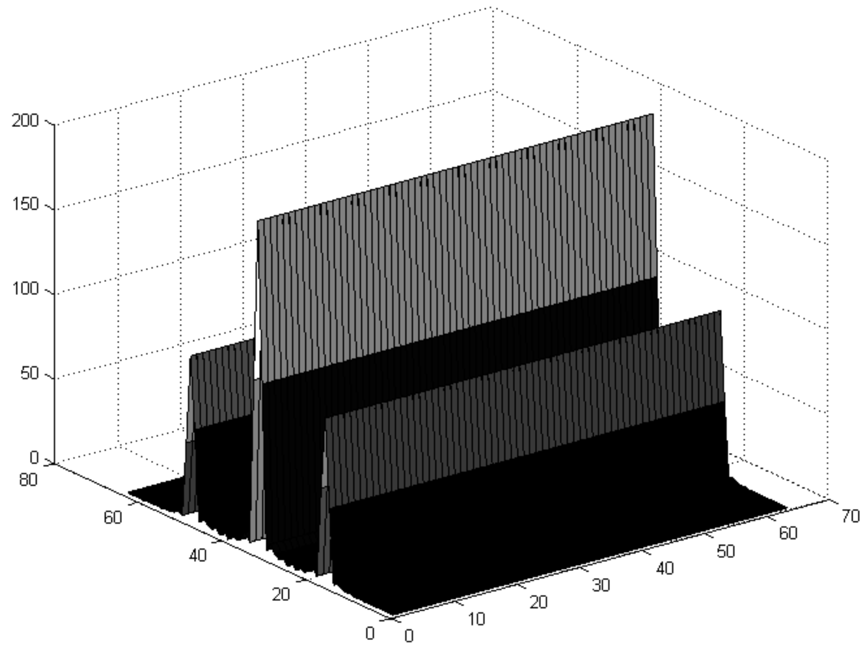
Fig(3.21):*The sketch of diagonal seven slits.*



Fig(3.22): *Interference fringe of seven slits.*



Fig(3.23): *The intensity of fringes in the center of the screen for seven slits.*



Fig(3.24): *The intensity of fringes for the seven slits.*

3.7 Conclusions:

An attempted has been made to describe the interference of two waves in a simple way, using command FFT from MATLAB. The classical experiment by Young was performed to demonstrate the wave interference theory of light. The result of the double-slit and motley-slits experiment shows the same tendency as that of theoretical. Whereas, the diffraction due to single slit can not be represented in this study, until the program to be modified by considering the value of the slit as a Gaussian configuration instead of constant values.

The simulation of double-slit experiment shows the intensity of the central fringe is some larger than the other, if away from the central fringe the intensity of fringes decrease. While the progression to a larger number of slits shows a pattern of narrowing the high intensity peaks and a relative increase in their peak intensity.

The result of this project shows that the FFT is a powerful technique to studies the interference and diffraction of the wave. For more reliability simulation the Gaussian function could be used to express the slits instead of the constant values which was established in present work.

References

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[6] http://skullsinthestars.com/2009/03/28/optics-basics-youngs-double-slit-experiment/

[7] http://en.wikipedia.org/wiki/Double-slit_experiment

[8] http://theory.uwinnipeg.ca/physics/light/node9.html

[9] http://www.matter.org.uk/schools/content/interference/laserinterference.html

[10] http://hyperphysics.phy-astr.gsu.edu/hbase/ems1.html

[11] http://class.phys.psu.edu/251Labs/10_Interference_&_Diffraction/Single_and_Double-Slit_Interference.pdf

Appendix

```
clear all
%sep=input(' slit separation value  ');
sep=4;
% Duble slits
g1=zeros(64,64);
g1(33-sep,33)=1;
g1(34-sep,33)=1;
g1(33+sep,33)=1;
g1(34+sep,33)=1;
colormap('gray');
imagesc(g1);
pause
gf1=fft2(g1,64,64);
for j=1:64
   for i=1:64
      rv=real(gf1(i,j));
      iv=imag(gf1(i,j));
      mod1(i,j)=(rv*rv+iv*iv);
   end
end
mod11=fftshift(mod1);
colormap('gray')
imagesc(mod11);
pause
plot(mod11(:,33));
pause
surf(mod11);
pause

% Effect of width of the slits
g2=zeros(64,64);
g2(33-sep,33)=1;
g2(34-sep,33)=1;
g2(33+sep,33)=1;
g2(34+sep,33)=1;
g2(33+sep,32)=1;
g2(34+sep,32)=1;
imagesc(g2);
pause

gf2=fft2(g2,64,64);
for j=1:64
```

```
    for i=1:64
        rv=real(gf2(i,j));
        iv=imag(gf2(i,j));
        mod2(i,j)=(rv*rv+iv*iv);
end
end
mod22=fftshift(mod2);
imagesc(mod22);
pause
plot(mod22(:,33));
pause
surf(mod22);
pause

% Diagonal double slits
g3=zeros(64,64);
g3(33-sep,33-sep)=1;
g3(33-sep,32-sep)=1;
g3(32-sep,32-sep)=1;
g3(32-sep,33-sep)=1;
g3(33+sep,33+sep)=1;
g3(33+sep,32+sep)=1;
g3(32+sep,32+sep)=1;
g3(32+sep,33+sep)=1;
imagesc(g3);
pause
gf3=fft2(g3,64,64);
k=0;
for j=1:64
    for i=1:64
        vr=real(gf3(i,j));
        vi=imag(gf3(i,j));
        mod3(i,j)=vr*vr+vi*vi;
    end
end
mod33=fftshift(mod3);
for j=1:64
    for i=1:64
        if i==j;
            k=k+1;
            v(k)=mod33(i,j);
        end
    end
end
```

```
imagesc(mod33);
pause
plot(v);
pause
surf(mod33);
pause

% Single slit
g4=zeros(64,64);
g4(33,33)=1;
imagesc(g4);
pause
fg4=fft2(g4,64,64);
for j=1:64
   for i=1:64
      rv=real(fg4(i,j));
      vi=imag(fg4(i,j));
      mod4(i,j)=rv*rv+vi*vi;
      if j==33;
         v(i)=mod4(i,j);
      end
   end
end
imagesc(mod4);
pause
plot(v);
pause
surf(mod4);
pause

% Multiple slits (three slits)
g5=zeros(64,64);
g5(33-sep,33)=1;
g5(34-sep,33)=1;
g5(33,33)=1;
g5(34,33)=1;
g5(33+sep,33)=1;
g5(34+sep,33)=1;
imagesc(g5);
pause
fg5=fft2(g5,64,64);
fg55=fftshift(fg5);
for j=1:64
   for i=1:64
      rv=real(fg55(i,j));
```

```
        vi=imag(fg55(i,j));
        mod5(i,j)=rv*rv+vi*vi;
        if j==33;
            v(i)=mod5(i,j);
        end
    end
end
imagesc(mod5);
pause
plot(v);
pause
surf(mod5);
pause

% Five slits
g6=zeros(64,64);
g6(33-2*sep,33)=1;
g6(33-sep,33)=1;
g6(33,33)=1;
g6(33+sep,33)=1;
g6(33+2*sep,33)=1;
g6(34-2*sep,33)=1;
g6(34-sep,33)=1;
g6(34,33)=1;
g6(34+sep,33)=1;
g6(34+2*sep,33)=1;
imagesc(g6);
pause
fg6=fft2(g6,64,64);
fg66=fftshift(fg6);
for j=1:64
    for i=1:64
        rv=real(fg66(i,j));
        vi=imag(fg66(i,j));
        mod6(i,j)=rv*rv+vi*vi;
        if j==33;
            v(i)=mod6(i,j);
        end
    end
end
imagesc(mod6);
pause
plot(v);
pause
surf(mod6);
```

```
pause

% Seven slits
g7=zeros(64,64);
g7(33-3*sep,33)=1;
g7(33-2*sep,33)=1;
g7(33-sep,33)=1;
g7(33,33)=1;
g7(33+sep,33)=1;
g7(33+2*sep,33)=1;
g7(33+3*sep,33)=1;
g7(34-3*sep,33)=1;
g7(34-2*sep,33)=1;
g7(34-sep,33)=1;
g7(34,33)=1;
g7(34+sep,33)=1;
g7(34+2*sep,33)=1;
g7(34+3*sep,33)=1;
imagesc(g7);
pause
fg7=fft2(g7,64,64);
fg77=fftshift(fg7);
for j=1:64
   for i=1:64
      rv=real(fg77(i,j));
      vi=imag(fg77(i,j));
      mod7(i,j)=rv*rv+vi*vi;
      if j==33;
         v(i)=mod7(i,j);
      end
   end
end
imagesc(mod7);
pause
plot(v);
pause
surf(mod7);
pause
```