

MODULE-III

STATISTICAL INFERENCE 1

Topic Learning Objectives:

Upon Completion of this module, student will be able to:

- Demonstrate the validity of testing the hypothesis.
- Solve problems on testing the hypothesis and probability distribution functions two variables.
- Apply discrete and continuous distributions in analyzing the probability models arising in engineering field.

Hypothesis Testing

Introduction:

Statistical Inference is a branch of Statistics which uses probability concepts to deal with uncertainty in decision making. There are a number of situations where in we come across problems involving decision making. For example, consider the problem of buying 1 kilogram of rice, when we visit the shop, we do not check each and every rice grains stored in a gunny bag; rather we put our hand inside the bag and collect a sample of rice grains. Then analysis takes place. Based on this, we decide to buy or not. Thus, the problem involves studying whole rice stored in a bag using only a sample of rice grains.

First what is meant by hypothesis testing?

This means that testing of hypothetical statement about a parameter of population.

Conventional approach to testing:

The procedure involves the following:

1. First we set up a definite statement about the population parameter which we call it as null hypothesis, denoted by H_0 . **Null Hypothesis is the statement which is tested for possible rejection under the assumption that it is true.** Next we set up another hypothesis called alternate statement which is just opposite of null statement; denoted by H_1 which is just complimentary to the null hypothesis. Therefore, if we start with $H_0: \mu = \mu_0$ then alternate hypothesis may be considered as either one of the following statements; $H_1: \mu \neq \mu_0$, or $H_1: \mu > \mu_0$ or $H_1: \mu < \mu_0$.

As we are studying population parameter based on some sample study, one can not do the job with 100% accuracy since sample is drawn from the population and possible sample may not

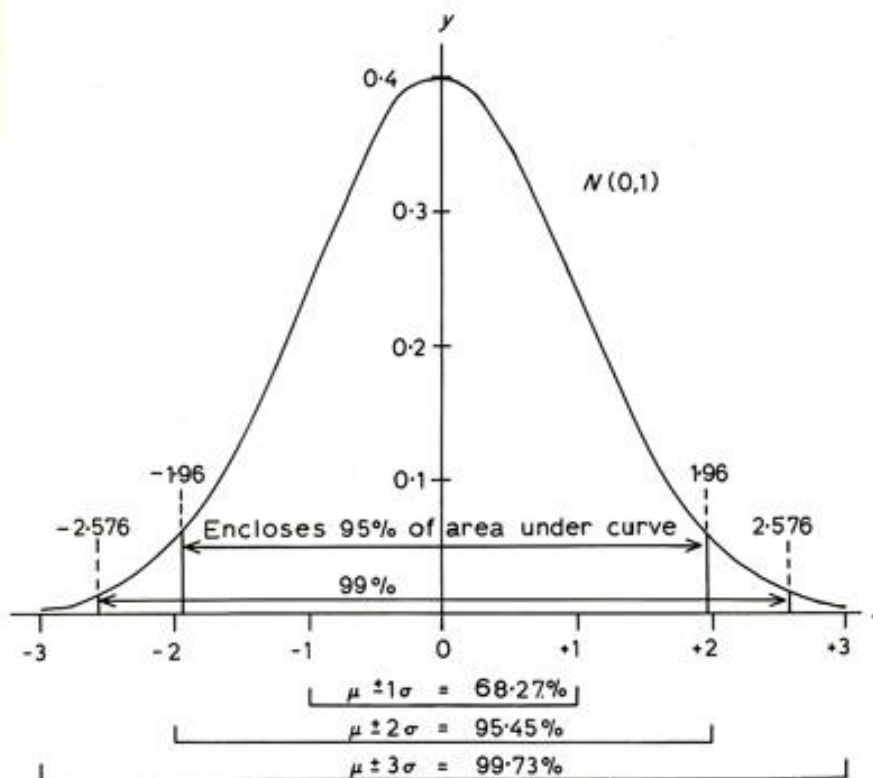
represent the whole population. Therefore, usually we conduct analysis at certain level of significance (lower than 100%). The possible choices include 99%, or 95% or 98% or 90%. **Usually we conduct analysis at 99% or 95% level of significance**, denoted by the symbol α . We test H_0 against H_1 at certain level of significance. The confidence with which a person rejects or accepts H_0 depends upon the significance level adopted. It is usually expressed in percentage forms such as 5% or 1% etc. Note that when α is set as 5%, then probability of rejecting null hypothesis when it is true is only 5%. It also means that when the hypothesis in question is accepted at 5% level of significance, then statistician runs the risk of taking wrong decisions, in the long run, is only 5%. The above is called II step of hypothesis testing.

Critical values or Fiducial limit values for a two tailed test:

Sl. No	Level of significance	Theoretical Value
1	$\alpha = 1\%$	2.58
2	$\alpha = 2\%$	2.33
3	$\alpha = 5\%$	1.96

Critical values or Fiducial limit values for a single tailed test (right and test)

Tabulated value	$\alpha = 1\%$	$\alpha = 5\%$	$\alpha = 10\%$
Right – tailed test	2.33	1.645	1.28
Left tailed test	-2.33	-1.645	-1.28



Setting a test criterion: The third step in hypothesis testing procedure is to construct a test criterion. This involves selecting an appropriate probability distribution for the particular test i.e. a proper probability distribution function to be chosen. Some of the distribution functions used are t, F, when the sample size is small (size lower than 30). However, for large samples, normal distribution function is preferred. Next step is the computation of statistic using the sample items drawn from the population. Usually, samples are drawn from the population by a procedure called random, where in each and every data of the population has the same chance of being included in the sample. Then the computed value of the test criterion is compared with the tabular value; as long the calculated value is lower then or equal to tabulated value, we accept the null hypothesis, otherwise, we reject null hypothesis and accept the alternate hypothesis. Decisions are valid only at the particular level significance of level adopted. During the course of analysis, there are two types of errors bound to occur. These are **(i) Type – I error and (ii) Type – II error.**

Type – I error: This error usually occurs in a situation, when the null hypothesis is true, but we reject it i.e. rejection of a correct/true hypothesis constitute type I error.

Type – II error: Here, null hypothesis is actually false, but we accept it. Equivalently, accepting a hypothesis which is wrong results in a type – II error. The probability of committing a type – I error is denoted by α where

$$\alpha = \text{Probability of making type I error} = \text{Probability [Rejecting } H_0 \mid H_0 \text{ is true]}$$

On the other hand, type – II error is committed by not rejecting a hypothesis when it is false. The probability of committing this error is denoted by β . Note that

$$\beta = \text{Probability of making type II error} = \text{Probability [Accepting } H_1 \mid H_1 \text{ is false]}$$

Critical region:

A region in a sample space S which amounts to Rejection of H_0 is termed as critical region.

One tailed test and two tailed test:

This depends upon the setting up of both null and alternative hypothesis.

A note on computed test criterion value:

1. When the sampling distribution is based on population of proportions/Mean, then test criterion may be given as

$$Z_{cal} = \frac{(\text{Expected results} - \text{Observed results})}{\text{Standard error of the distribution}}$$

Application of standard error:

1. S.E. enables us to determine the probable limit within which the population parameter may be expected to lie. For example, the probable limits for population of proportion are given by $p \pm 3\sqrt{pq/n}$. Here, p represents the chance of achieving a success in a single trial, q stands for the chance that there is a failure in the trial and n refers to the size of the sample.
2. The magnitude of standard error gives an index of the precision of the parameter.

Probable limits of population mean are:

$$95\% \text{ fiducial limits of population mean are } \bar{X} \pm 1.96 \frac{\sigma}{\sqrt{n}}$$

$$99\% \text{ fiducial limits of population mean are } \bar{X} \pm 2.58 \frac{\sigma}{\sqrt{n}}. \text{ Further, test criterion } z_{cal} = \left| \frac{\bar{x} - \mu}{S.E.} \right|$$

The binomial distribution is regarded as the sampling distribution of the number of successes in the sample. We know that the mean of this distribution is np and S.D is \sqrt{npq} . S.E proportion of successes $\sqrt{\frac{pq}{n}}$.

The associated standard normal variate Z be defined by $z = \frac{x-np}{\sqrt{npq}}$. The probable limits $p \pm 2.58 \sqrt{\frac{pq}{n}}$.

Problems:

1. A coin is tossed 400 times and the head turned up 216 times. Test the hypothesis that the coin is un-biased?

Solution: First we construct null and alternate hypotheses set up H_0 : The coin is not a biased one. Set up H_1 : Yes, the coin is biased. As the coin is assumed be fair and it is tossed 400 times, clearly we must expect 200 times heads occurring and 200 times tails. Thus, expected number of heads is 200. But the observed result is 216. There is a difference of 16. Further, standard error is $\sigma = \sqrt{npq}$. With $p = \frac{1}{2}$, $q = \frac{1}{2}$ and $n = 400$, clearly $\sigma = 10$. The test criterion is $z_{cal} = \frac{\text{difference}}{\text{standard error}} = \left| \frac{216-200}{10} \right| = 1.6$. If we choose $\alpha = 5\%$, then the tabulated value for a two tailed test is 1.96. Since, the calculated value is lower than the tabulated value; we accept the null hypothesis that coin is un-biased.

2. A person throws a 10 dice 500 times and obtains 2560 times 4, 5, or 6. Can this be attributed to fluctuations in sampling?

Solution: As in the previous problem first we shall set up H_0 : The die is fair and H_1 : The die is unfair. We consider that problem is based on a two-tailed test. Let us choose level of significance as $\alpha = 5\%$ then, the tabulated value is 1.96. Consider computing test criterion,

$z_{cal} = \left| \frac{\text{Expected value} - \text{observed result}}{\text{standard error}} \right|$; here, as the dice is tossed by a person 5000 times, and on the basis that die is fair, then chance of getting any of the 6 numbers is $\frac{1}{6}$. Thus, chance of getting either 4 or 5, or 6 is $p = \frac{1}{2}$. Also, $q = \frac{1}{2}$. With $n = 5000$, standard error, $\sigma = \sqrt{npq}$

= 35.36. Further, expected value of obtaining 4 or 5 or 6 is 2500. Hence,

$$z_{\text{cal}} = \left| \frac{2500 - 2560}{35.36} \right| = 1.7 \text{ which is lower than } 1.96. \text{ Hence, we conclude that die is a fair one.}$$

3. A sample of 1000 days is taken from meteorological records of a certain district and 120 of them are found to be foggy. What are the probable limits to the percentage of foggy days in the district?

Solution: Let p denote the probability that a day is foggy in nature in a district as reported by meteorological records. Clearly, $p = \frac{120}{1000} = 0.12$ and $q = 0.88$. With $n = 1000$, the probable limits to the percentage of foggy days is given by $p \pm 3\sqrt{pqn}$. Using the data available in this problem, one obtains the answer as $0.12 \pm 3\sqrt{0.12 \cdot 0.88 \cdot 1000}$. Equivalently, 8.91% to 15.07%.

4. A die was thrown 9000 times and a throw of 5 or 6 was obtained 3240 times. On the assumption of random throwing, do the data indicate that die is biased?

Solution: We set up the null hypothesis as H_0 : Die is un-biased. Also, H_1 : Die is biased. Let us take level of significance as $\alpha = 5\%$. Based on the assumption that distribution is normally distributed, the tabulated value is 1.96. The chance of getting each of the 6 numbers is same and it equals to $1/6$ therefore chance of getting either 5 or 6 is $1/3$. In a throw of 9000 times, getting the numbers either 5 or 6 is $\frac{1}{3} \times 9000 = 3000$. Now the difference in these two results is 240. With $p = 1/3$, $q = 2/3$, $n = 9000$, $S.E. = \sqrt{npq} = 44.72$. Now consider the test criterion $z_{\text{cal}} = \frac{\text{Difference}}{S.E.} = \frac{240}{44.72} = 5.367$ which is again more than the tabulated value. Therefore, we reject null hypothesis and accept the alternate that die is highly biased.

Tests of significance for large samples:

In the previous section, we discussed problems pertaining to sampling of attributes. It is time to think of sampling of other variables one may come across in a practical situation such as height weight etc. We say that a sample is small when the size is usually lower than 30, otherwise it is called a large one.

The study here is based on the following assumptions: (i) the random sampling distribution of a statistic is approximately normal and (ii) values given by the samples are sufficiently close

to the population value and can be used in its place for calculating standard error. When the standard deviation of population is known, then $S.E(\bar{X}) = \frac{\sigma_p}{\sqrt{n}}$

where σ_p denotes the standard deviation of population. When the standard deviation of the population is unknown, then $S.E(\bar{X}) = \frac{\sigma}{\sqrt{n}}$ where σ is the standard deviation of the sample.

Fiducial limits of population mean are:

95% fiducial limits of population mean are $\bar{X} \pm 1.96 \frac{\sigma}{\sqrt{n}}$

99% fiducial limits of population mean are $\bar{X} \pm 2.58 \frac{\sigma}{\sqrt{n}}$. Further, test criterion $z_{cal} = \left| \frac{\bar{X} - \mu}{S.E.} \right|$

Testing the hypothesis

	Significance of proportions	Significance of a sample mean	Significance of difference between proportions	Significance of difference between means
S.N.V	$z = \frac{\bar{x} - np}{\sqrt{npq}}$ <p>or</p> $z = \frac{\bar{x} - \mu}{(\sigma/\sqrt{n})}$		$z = \frac{p_1 - p_2}{\sqrt{pq \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}}$ $p = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$ $q = 1 - p$	$z = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$
Probable limits	$p \pm z_c \sqrt{pq/n}$ or $\bar{x} \pm z_c (\sigma/\sqrt{n})$		$(p_1 - p_2) \pm z_c \sqrt{pq \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}$	$(\bar{x} - \bar{y}) \pm z_c \left(\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} \right)$

Problems:

1. A sample of 100 tyres is taken from a lot. The mean life of tyres is found to be 39,350 kilometers with a standard deviation of 3,260. Could the sample come from a population with mean life of 40,000 kilometers? Establish 99% confidence limits within which the mean life of tyres is expected to lie.

Solution: First we shall set up null hypothesis, $H_0: \mu = 40,000$, alternate hypothesis as $H_1: \mu \neq 40,000$. We consider that the problem follows a two tailed test and

chose $\alpha = 5\%$. Then corresponding to this, tabulated value is 1.96. Consider the expression

for finding test criterion, $z_{cal} = \frac{\bar{x} - \mu}{S.E.}$. Here, $\mu = 40,000$, $\bar{x} = 39,350$ and $\sigma = 3,260$, $n =$

100. $S.E. = \frac{\sigma}{\sqrt{n}} = \frac{3,260}{\sqrt{100}} = 326$. Thus, $z_{cal} = 1.994$. As this value is slightly greater than 1.96,

we reject the null hypothesis and conclude that sample has not come from a population of 40,000 kilometers.

The 99% confidence limits within which population mean is expected to lie is given as $\bar{x} \pm 2.58 \times S.E.$ i.e. $39,350 \pm 2.58 \times 326 = (38,509, 40,191)$.

2. The mean life time of a sample of 400 fluorescent light bulbs produced by a company is found to be 1,570 hours with a standard deviation of 150 hours. Test the hypothesis that the mean life time of bulbs is 1600 hours against the alternative hypothesis that it is greater than 1,600 hours at 1% and 5% level of significance.

Solution: First we shall set up null hypothesis, $H_0: \mu = 1,600$ hours, alternate hypothesis as $H_1: \mu > 1,600$ hours. We consider that the problem follows a two tailed test and chose $\alpha = 5\%$. Then corresponding to this, tabulated value is 1.96. Consider the expression for finding test

criterion, $z_{cal} = \frac{\bar{x} - \mu}{S.E.}$. Here, $\mu = 1,600$, $\bar{x} = 1,570$, $n = 400$, $\sigma = 150$ hours so that using all

these values above, it can be seen that $z_{cal} = 4.0$ which is really greater than 1.96. Hence, we have to reject null hypothesis and to accept the alternate hypothesis.

3. A light bulb company claims that the 100-watt light bulb it sells has an average life of 1200 hours with a standard deviation of 100 hours. For testing the claim 50 new bulbs were selected randomly and allowed to burn out. The average lifetime of these bulbs was found to be 1180 hours. Is the company's claim is true at 5% level of significance?

Solution: Here, we are given that

Specified value of population mean $= \mu = 1200$ hours,

Population standard deviation $= \sigma = 100$ hours,

Sample size $= n = 50$

Sample mean $= \bar{X} = 1180$ hours.

$$H_0: \mu = 1200$$

$$H_1: \mu \neq 1200$$

Thus, for testing the null hypothesis the test statistic is given by

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{1180 - 1200}{\frac{100}{\sqrt{50}}} = -1.41$$

The critical (tabulated) values at 5% level of significance are $z_\alpha = -1.96$.

Hence, we have to accept the null hypothesis and to reject the alternate hypothesis.

4. A manufacturer of ball point pens claims that a certain pen manufactured by him has a mean writing-life at least 460 A-4 size pages. A purchasing agent selects a sample of 100 pens and put them on the test. The mean writing-life of the sample found 453 A-4 size pages with standard deviation 25 A-4 size pages. Should the purchasing agent reject the manufacturer's claim at 1% level of significance?

Solution: Here, we are given that

Specified value of population mean = $\mu = 460$,

Sample size = $n = 100$,

Sample mean = $\bar{X} = 453$,

Sample standard deviation = $\sigma = 25$

$$H_0: \mu \geq 460$$

$$H_1: \mu < 460$$

Thus, for testing the null hypothesis the test statistic is given by

$$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{453 - 460}{\frac{25}{\sqrt{100}}} = -2.8$$

The critical (tabulated) values at 1% level of significance is $z_\alpha = -2.33$.

Hence, we have to reject the null hypothesis and to accept the alternate hypothesis.

Test of significance of difference between the means of two samples

Consider two populations P1 and P2. Let S_1 and S_2 be two samples drawn at random from these two different populations. Suppose we have the following data about these two samples, say

Samples/Data	Sample size	Mean	Standard Deviation
S_1	n_1	\bar{x}_1	σ_1
S_2	n_2	\bar{x}_2	σ_2

then standard error of difference between the means of two samples S_1 and S_2 is

$S.E = \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$ and the test criterion is $Z_{cal} = \frac{\text{Difference of sample means}}{\text{Standard error}}$. The rest of the analysis is same as in the preceding sections.

When the two samples are drawn from the same population, then standard error is

$S.E = \sigma \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ and test criterion is $Z_{cal} = \frac{\text{Difference of sample means}}{\text{Standard error}}$.

When the standard deviations are un – known, then standard deviations of the two samples must be replaced. Thus, $S.E = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ where s_1 and s_2 are standard deviations of the two samples considered in the problem.

Problems:

1. Intelligence test on two groups of boys and girls gave the following data:

Data	Mean	Standard deviation	Sample size
Boys	75	15	150
Girls	70	20	250

Is there a significant difference in the mean scores obtained by boys and girls?

Solution: We set up null hypothesis as H_0 : there is no significant difference between the mean scores obtained by boys and girls. The alternate hypothesis is considered as H_1 : Yes, there is a significant difference in the mean scores obtained by boys and girls. We choose level of

significance as $\alpha = 5\%$ so that tabulated value is 1.96. Consider $z_{\text{cal}} = \frac{\text{Difference of means}}{\text{Standard Error}}$

The standard error may be calculated as $S.E = \sqrt{\frac{15^2}{150} + \frac{20^2}{250}} = 1.761$, The test criterion is

$z_{\text{cal}} = \frac{75 - 70}{1.761} = 2.84$. As 2.84 is more than 1.96, we have to reject null hypothesis and to accept

alternate hypothesis that there are some significant difference in the mean marks scored by boys and girls.

2. A man buys 50 electric bulbs of “Philips” and 50 bulbs of “Surya”. He finds that Philips bulbs give an average life of 1,500 hours with a standard deviation of 60 hours and Surya bulbs gave an average life of 1,512 hours with a standard deviation of 80 hours. Is there a significant difference in the mean life of the two makes of bulbs?

Solution: we set up null hypothesis, H_0 : there is no significant difference between the bulbs made by the two companies, the alternate hypothesis can be set as H_1 : Yes, and there could be some significant difference in the mean life of bulbs. Taking $\alpha = 1\%$ and $\alpha = 5\%$, the respective tabulated values are 2.58 and 1.96. Consider standard error is

$S.E = \sqrt{\frac{60^2}{50} + \frac{80^2}{50}} = 14.14$ so that $z_{\text{cal}} = \frac{1512 - 1500}{14.14} = 0.849$. Since the calculated value is

certainly lower than the two tabulated values, we accept the hypothesis there is no significant difference in the make of the two bulbs produced by the companies.

3. A random sample of size $N=100$ is taken from a population with standard deviation $\sigma = 5.1$. Given that the sample mean is $\bar{X} = 21.6$. Obtain the 95% confidence interval for the population mean μ .

Solution: $N=100$, $\sigma = 5.1$, $\bar{X} = 21.6$

Confidence limits for the population mean are

$$\bar{X} \pm Z_c \frac{s}{\sqrt{N}} = 21.6 \pm Z_c \frac{5.1}{\sqrt{100}}$$

For 95% confidence level, $Z_c = 1.96$

$$\therefore \bar{X} \pm Z_c \frac{s}{\sqrt{N}} = 21.6 \pm (1.96) \frac{5.1}{\sqrt{100}} = 21.6 \pm .9996$$

4. One type of aircraft is found to develop engine trouble in 5 flights out of 100 flights and another type in 7 flights out of 200 flights. Is there a significant difference in the two types of air crafts so far as engine defects are concerned?

Solution: Let P_1 be the proportion of type 1 aircrafts that develop engine trouble, then

$$P_1 = 5/100 = 0.05 \text{ in the sample of size } N_1 = 100.$$

Let P_2 be the proportion of type 2 aircrafts that develop engine trouble, then

$$P_2 = 7/200 = 0.035 \text{ in the sample of size } N_2 = 200.$$

Let us make an hypothesis that there is no difference in the two types of aircrafts.

Then the mean of the distribution of differences in proportions is zero; that is

$$\mu_{(p_1-p_2)} = (p_1 - p_2) = 0 \text{ and the standard deviation of the distribution is}$$

$$\sigma_{(p_1-p_2)} = \sqrt{\frac{p_1 q_1}{N_1} + \frac{p_2 q_2}{N_2}} = \sqrt{\frac{0.05(0.95)}{100} + \frac{0.035(0.965)}{200}} = 0.0254.$$

The corresponding Z-score is

$$Z = \frac{(p_1-p_2) - \mu_{(p_1-p_2)}}{\sigma_{(p_1-p_2)}} = \frac{0.05-0.035}{0.0254} = 0.591.$$

This Z - score is less than $Z_c = 1.96$ and $Z_c = 2.58$. Therefore, the hypothesis cannot be rejected. This means that the difference between the two types of aircrafts is not significant.

5. A sample height of 6400 soldiers has a mean of 172.34 cms and a standard deviation of 6.5 cms, while a sample of heights of 1600 sailors has a mean of 174.12 cms and a standard deviations of 6.4 cms. Does the data indicate that the sailors are on the average taller than soldiers. Use the left - tailed test.

Solution: Here,

$$N_1 = 6400, \bar{X}_1 = 172.34, s_1 = 6.5, N_2 = 1600, \bar{X}_2 = 174.12, s_2 = 6.4.$$

We test the Null hypothesis

H : There is no difference in the heights of soldiers and sailors (on the average), against the alternative hypothesis.

H_1 : Sailors are taller than soldiers, on the average; that is $\mu_2 > \mu_1$, of $\mu_1 - \mu_2 < 0$.

Under the hypothesis H , we have

$$\mu_{(\bar{X}_1 - \bar{X}_2)} = \mu_1 - \mu_2 = 0 \text{ and}$$

$$\sigma_{(\bar{X}_1 - \bar{X}_2)} = \sqrt{\frac{s_1^2}{N_1} + \frac{s_2^2}{N_2}} = \sqrt{\frac{(6.5)^2}{6400} + \frac{(6.4)^2}{1600}} = 0.179.$$

The corresponding Z – score is

$$z = \frac{(172.34 - 174.12) - 0}{0.179} = -9.94.$$

This score lies in the critical region $z < -z_c$ for the left – tailed test at both of 0.01 and 0.05 levels of significance. Therefore, we reject the null hypothesis H_0 . Consequently, we accept the alternative hypothesis H_1 . Thus, the data indicates that, on the average, the sailors are taller than soldiers.

6. In a sample of 500 men it was found that 60% of them had overweight. Find the 99% of confidence limits for the proportion of men in the population having overweight.

Solution: Probability of persons having over weight $p = 60\% = .60$

$$q = 40\% = .40$$

For 99% confidence level, $Z_c = 2.58$

$$\text{Probable limits are, } p \pm z_c \sigma_p = p \pm z_c \sqrt{\frac{pq}{N}} = 0.6 \pm (2.58) \sqrt{\frac{(0.6 \times 0.4)}{500}} = 0.6 \pm 0.057$$

7. In two samples of women from Punjab and Tamilnadu, the mean height of 1000 and 2000 women are 67.6 and 68.0 inches respectively. If population standard deviation of Punjab and Tamilnadu are same and equal to 5.5 inches then, can the mean heights of Punjab and Tamilnadu women be regarded as same at 1% level of significance?

Solution: We are given

$$n_1 = 1000, n_2 = 2000, \quad \bar{x} = 67.6, \bar{y} = 68.0, \sigma_1 = \sigma_2 = 5.5$$

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

Thus, for testing the null hypothesis the test statistic is given by

$$z = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$z = \frac{67.6 - 68.0}{\sqrt{\frac{(5.5)^2}{1000} + \frac{(5.5)^2}{2000}}} = -1.88$$

The critical (tabulated) values at 1% level of significance are $z_\alpha = -2.58$.

Hence, we have to accept the null hypothesis and to reject the alternate hypothesis.

8. A university conducts both face to face and distance mode classes for a particular course intended both to be identical. A sample of 50 students of face to face mode yields examination results mean and SD respectively as: $\bar{X} = 80.4, \sigma_1 = 12.8$ and other sample of 100 distance-mode students yields mean and SD of their examination results in the same course respectively as: $\bar{Y} = 74.3, \sigma_2 = 20.5$. Are both educational methods statistically equal at 5% level?

Solution: We are given that

$$n_1 = 50, \quad \bar{X} = 80.4, \sigma_1 = 12.8; \quad n_2 = 100, \quad \bar{Y} = 74.3, \quad \sigma_2 = 20.5$$

$$H_0: \mu_1 = \mu_2$$

$$H_1: \mu_1 \neq \mu_2$$

Thus, for testing the null hypothesis the test statistic is given by

$$z = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$z = \frac{80.4 - 74.3}{\sqrt{\frac{(12.8)^2}{50} + \frac{(20.5)^2}{100}}} = 2.23$$

The critical (tabulated) values at 5% level of significance are $z_\alpha = 1.96$.

Hence, we have to reject the null hypothesis and to accept the alternate hypothesis.

9. A machine produces a large number of items out of which 25% are found to be defective. To check this, company manager takes a random sample of 100 items and found 35 items defective. Is there an evidence of more deterioration of quality at 5% level of significance?

Solution: The company manager wants to check that his machine produces 25% defective items. Here, attribute under study is defectiveness. And we define our success and failure as getting a defective or non-defective item.

Let p_0 be population proportion of defectives items = 0.25; $q_0 = 1 - p_0 = 0.75$

p is observed proportion of defectives items in the sample = $35/100 = 0.35$

$$H_0: p \leq p_0$$

$$H_1: p > p_0$$

Thus, for testing the null hypothesis the test statistic is given by

$$z = \frac{p - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.35 - 0.25}{\sqrt{0.25 \times \frac{0.75}{100}}} = 2.31$$

The critical (tabulated) values at 5% level of significance are $z_{\alpha} = 1.645$.

Hence, we have to reject the null hypothesis and to accept the alternate hypothesis.

10. A die is thrown 9000 times and draw of 2 or 5 is observed 3100 times. Can we regard that die is unbiased at 5% level of significance.

Solution:

Let getting a 2 or 5 be our success, and getting a number other than 2 or 5 be a failure then in usual notions, we have

$$n = 9000, X = \text{number of successes} = 3100, p = \frac{3100}{9000} = 0.3444$$

Here, we want to test that the die is unbiased and we know that if die is unbiased then proportion or probability of getting 2 or 5 is

p_0 be probability of getting a 2 or 5

= Probability of getting 2 + Probability of getting 5

$$p_0 = \frac{2}{6} = \frac{1}{3}; q_0 = 1 - p_0 = \frac{2}{3}$$

$$H_0: p = p_0$$

$$H_1: p \neq p_0$$

Thus, for testing the null hypothesis the test statistic is given by

$$z = \frac{p - p_0}{\sqrt{\frac{p_0 q_0}{n}}} = \frac{0.3444 - 0.3333}{\sqrt{0.3333 \times \frac{0.6667}{900}}} = 2.22$$

The critical (tabulated) values at 5% level of significance are $z_{\alpha} = 1.96$.

Hence, we have to reject the null hypothesis and to accept the alternate hypothesis.

11. In a random sample of 100 persons from town A, 60 are found to be high consumers of wheat. In another sample of 80 persons from town B, 40 are found to be high consumers of wheat. Do these data reveal a significant difference between the proportions of high wheat consumers in town A and town B (at $\alpha = 0.05$)?

Solution: Here, attribute under study is high consuming of wheat. And we define our success and failure as getting a person of high consumer of wheat and not high consumer of wheat respectively. We are given that

n_1 = total number of persons in the sample of town A = 100

n_2 = total number of persons in the sample of town B = 80

X_1 = number of persons of high consumer of wheat in town A = 60

X_2 = number of persons of high consumer of wheat in town B = 40

The sample proportion of high wheat consumers in town A is

$$p_1 = \frac{X_1}{n_1} = \frac{60}{100} = 0.60$$

and the sample proportion of wheat consumers in town B is

$$p_2 = \frac{X_2}{n_2} = \frac{40}{80} = 0.50$$

$$H_0: p_1 = p_2$$

$$H_1: p_1 \neq p_2$$

The estimate of the combined proportion (P) of high wheat consumers in two towns is given by

$$P = \frac{X_1 + X_2}{n_1 + n_2} = \frac{60 + 40}{100 + 80} = \frac{5}{9}; \quad Q = 1 - P = \frac{4}{9}$$

Thus, for testing the null hypothesis the test statistic is given by

$$z = \frac{p_1 - p_2}{\sqrt{PQ \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} = \frac{0.60 - 0.50}{\sqrt{\frac{5}{9} \times \frac{4}{9} \left(\frac{1}{100} + \frac{1}{80} \right)}} = 1.34$$

The critical (tabulated) values at 5% level of significance are $z_\alpha = 1.96$.

Hence, we have to accept the null hypothesis and to reject the alternate hypothesis.

12. A machine produced 60 defective articles in a batch of 400. After overhauling it produced 30 defective in a batch of 300. Has the machine improved due to overhauling? (Take $\alpha = 0.01$).

Solution: Here, the machine produced articles and attribute under study is defectiveness. And we define our success and failure as getting a defective or non defective article.

Therefore, we are given that

X_1 = number of defective articles produced by the machine before overhauling = 60

X_2 = number of defective articles produced by the machine after overhauling = 30

$n_1 = 400$; $n_2 = 300$

Let p_1 be observed proportion of defective articles in the sample before the overhauling

$$p_1 = \frac{X_1}{n_1} = \frac{60}{400} = 0.15$$

and p_2 be observed proportion of defective articles in the sample after the overhauling

$$p_2 = \frac{X_2}{n_2} = \frac{30}{300} = 0.10$$

$$H_0: p_1 \leq p_2$$

$$H_1: p_1 > p_2$$

Since P is unknown, so the pooled estimate of proportion is given by

$$P = \frac{X_1 + X_2}{n_1 + n_2} = \frac{60 + 30}{400 + 300} = \frac{9}{70}; \quad Q = 1 - P = \frac{61}{70}$$

Thus, for testing the null hypothesis the test statistic is given by

$$z = \frac{p_1 - p_2}{\sqrt{PQ \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} = \frac{0.15 - 0.10}{\sqrt{\frac{9}{70} \times \frac{61}{70} \left(\frac{1}{400} + \frac{1}{300} \right)}} = 1.95$$

The critical (tabulated) values at 1% level of significance are $z_\alpha = 2.33$.

Hence, we have to accept the null hypothesis and to reject the alternate hypothesis.

Video links:

1. [Hypothesis Testing - Statistics - YouTube](#)
2. [Testing of Hypothesis for Difference of Two Population Means](#)
3. [Testing of Hypothesis for Difference of Two Population Proportions](#)