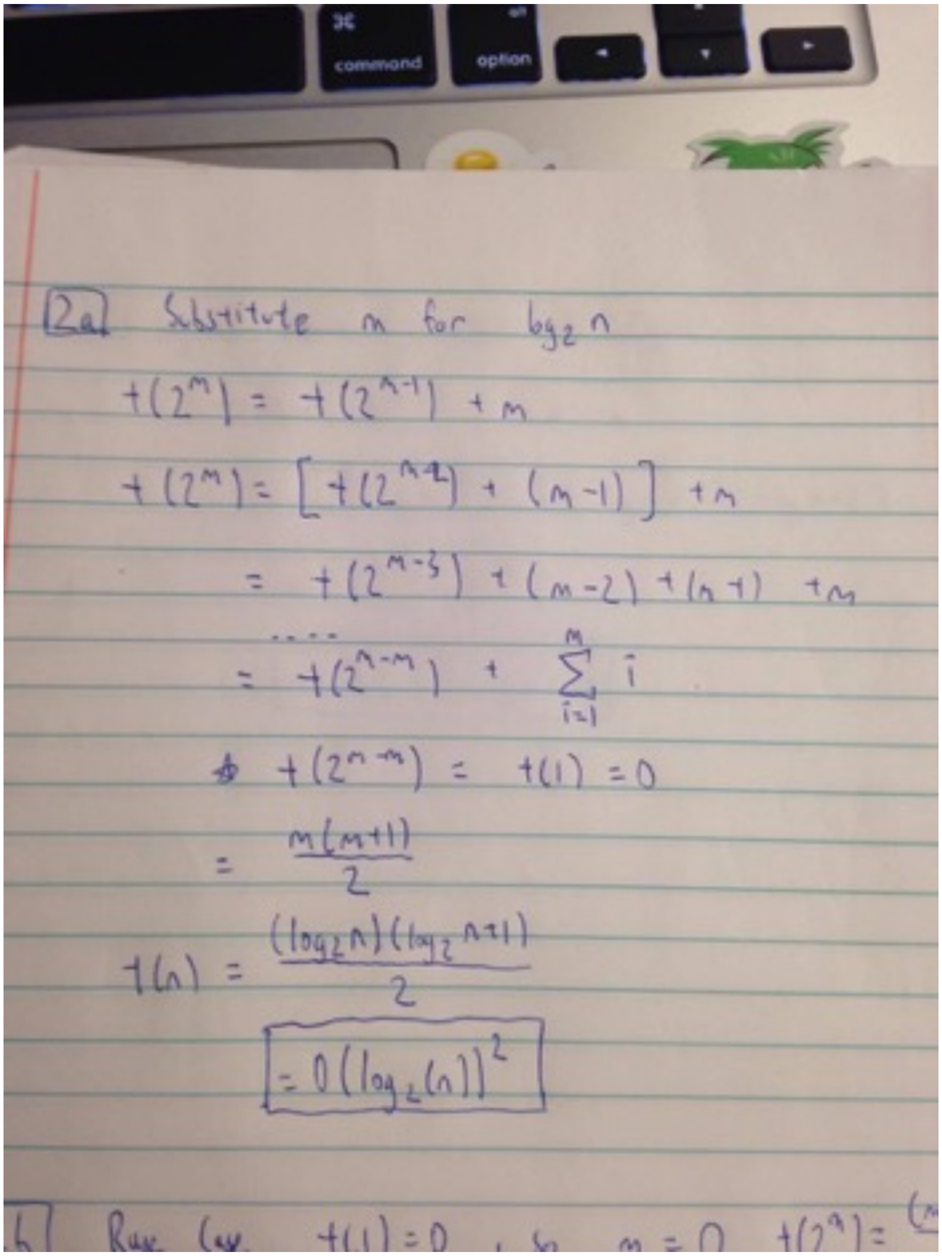
2a



2a Substitute m for $\log_2 n$

$$t(2^m) = t(2^{m-1}) + m$$

$$t(2^m) = [t(2^{m-2}) + (m-1)] + m$$

$$= t(2^{m-3}) + (m-2) + (m-1) + m$$

....

$$= t(2^{m-m}) + \sum_{i=1}^{m} i$$

$$t(2^{m-m}) = t(1) = 0$$

$$= \frac{m(m+1)}{2}$$

$$t(n) = \frac{(\log_2 n)(\log_2 n+1)}{2}$$

$$\boxed{= O(\log_2(n))^2}$$

b Base case $t(1)=0$, so $m=0$ $t(2^0) =$ (m

2b

$$t(2^{m-m}) = t(1) = 0$$

$$= \frac{m(m+1)}{2}$$

$$t(n) = \frac{(\log_2 n)(\log_2 n + 1)}{2}$$

$$\boxed{= O(\log_2(n))^2}$$

$\boxed{2b}$ Base Case $t(1) = 0$, so $m = 0$ $t(2^0) = \frac{(m)(m+1)}{2}$

$$t(2^0) = \frac{0(0+1)}{2}$$

$$0 = 0 \checkmark$$

Induction Step Assume $P(k)$ is true, prove $P(k+1)$

Prove $t(2^{k+1}) = \frac{(k+1)(k+2)}{2}$

$$t(2^{k+1}) = t(2^k) + k + 1$$

$$= \frac{(k)(k+1)}{2} + k + 1$$ by induction hypothesis

$$= \frac{k(k+1)}{2} + \frac{2k+2}{2}$$

$$= \frac{k^2 + 3k + 2}{2} \; \boxed{= \frac{(k+1)(k+2)}{2}} \checkmark$$

2c

2c log n denotes number of digits

$$t(n) = (n^2) \cdot n \cdot n \cdot n \ldots$$

$$= [(\log n)^2 \times 2] \cdot (\log n \times 2) \cdot (\log n \times 2) \ldots$$

$$= 2^n (\log n)^n$$

$$\boxed{= O(2^n (\log n)^n)}$$

2d

$$\therefore \ = O(10^n \cdot n^n)$$

$\boxed{2d}$

$$\sqrt{n^2+100n} - n \left(\frac{\sqrt{n^2+100n}+n}{\sqrt{n^2+100n}+n} \right)$$

$$t(n) = \frac{\cancel{n^2} + 100n - \cancel{n^2}}{\sqrt{n^2+100n}+n}$$

$$= \frac{\cancel{n}(100)}{\cancel{n}\left(\sqrt{1+\frac{100}{n}}+1\right)}$$

$$\leq \frac{100}{\sqrt{2}+1} \quad \text{for } n \geq 100$$

$$\leq \frac{100}{\sqrt{2}+1}$$