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Second Order Taylor Polyphase Reconstruction of Periodic-Nonuniform Samples in Time-Interleaved ADCs

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Abstract This paper focuses on a filter bank approach for reconstructing periodic-nonuniform samples using second order Taylor polyphase decomposition for a four-channel time-interleaved ADC (TIADC). The proposed method is based on a parametric linear least squares approach to find the coefficients of synthesis filters used for signal reconstruction of periodic-nonuniform samples. The coefficients of synthesis filters found by linear least squares approach satisfy the general condition for an alias-free filter bank.

Keywords Time mismatch error · Filter bank · TIADCs · Periodic non-uniform sampling reconstruction · Synthesis filters · Second order Taylor expansion · Polyphase decomposition · Alias free filter bank · Linear least squares method

1 Introduction

In high frequency applications using several analog-to-digital converters (ADCs) in parallel operating at the Nyquist frequency on the average is a common approach [1–6]. This type of ADC is called time-interleaved analog-to-digital converter (TIADC). In such architectures a fast ADC is replaced by slower ADCs operating in parallel. Ideally, the slow ADCs should sample the signal at uniformly spaced instants. Unfortunately, due to component mismatch this can not be achieved in practice. The resulting time mismatch error generates periodic-nonuniform sampling pattern [7–9]. Other kind of mismatch errors such as gain mismatch and amplitude offset errors are discussed in [10]. The correction of errors due to component mismatch can be accomplished using analog approaches, but this procedure is not straightforward in practice and requires tuning the operation of ADCs in parallel [11]. Instead, a digital domain correction approach using synthesis filter banks is preferred.

Using filter bank theory, the outputs of different ADCs are modeled as outputs from an analysis filter bank, and reconstruction is performed by a corresponding synthesis filter bank. The main idea is to find a set of synthesis filters that can reconstruct the signal from its periodic-nonuniform samples. It has been proved that perfect reconstruction (PR) using FIR synthesis filters is impossible [9]. However, using FIR filters as synthesis filters one can approximately reconstruct the signal from its periodic-nonuniform samples. The higher the order of FIR filters used as synthesis filter bank the better reconstruction [9]. Finding a closed form for perfect reconstruction of periodic-nonuniform samples for a non-periodic signal is still a challenging problem.

A method based on non-maximally decimated filter banks for reconstruction of periodic-nonuniform samples is proposed in [8]. However, this method requires extra number of samples $N > M$ in every MT_s second resulting over-sampling factor $\frac{N}{M} > 1$ which is not desirable.

Another method applies maximally decimated filter banks based on polyphase decomposition [9]. In this method, the first type polyphase decomposition matrix of

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analysis filters and the second type polyphase decomposition matrix of synthesis filters are multiplied to apply general condition of an alias-free filter bank, then the coefficients of the synthesis filters are found using a linear least squares algorithm.

In our approach rather than using a direct fractional delay which is windowed and shifted to model analysis filters [9], we use a second order Taylor approximation of the analysis filters. The proposed method has the advantage that the coefficients of all-pass filter (APF) and first and second order differentiators used for second order Taylor approximation are fixed. Fractional delays are applied as delay matrices separately. So, the coefficients of reconstruction filters can be found in a simpler way.

2 Signal Reconstruction

In this section we present a new reconstruction method based on polyphase decomposition of the second order Taylor approximation for the analysis filters.

2.1 General Filter Bank Model

A continuous time signal $x_c(t)$ which is bandlimited with bandwidth $\frac{\alpha\pi}{T_s}$ $[\frac{radians}{sec}]$ is considered as the input to the filter bank. The discrete time representation for $x_c(t)$ is given by $x[n] = x_c(nT)$ where T_s is the sampling period. The DTFT of $x[n]$ is

$$X(e^{j\omega}) = \frac{1}{T_s}\sum_{k=-\infty}^{+\infty} X_c\left(j\left(\Omega - \frac{2\pi k}{T_s}\right)\right) \quad \text{where } \Omega = \tfrac{\omega}{T_s} \tag{1}$$

In this paper we only consider frequency components in the range $|\omega| \leq \pi$. Assuming $f_s = \frac{1}{T_s}$ satisfies the Nyquist criterion, the spectrum of $X(e^{j\omega})$ in the frequency range $|\omega| \leq \pi$ is equivalent to the spectrum of $X_c(j\Omega)$ in the frequency range $|\Omega| \leq \frac{\alpha\pi}{T_s}$ $[\frac{rad}{sec}]$. Therefore, we use a discrete-time filter bank model throughout the paper.

A filter bank used for approximate perfect reconstruction (APR) of a periodic non-uniformly sampled signal is shown in Fig. 1. This model allows for sampling patterns with period MT_s containing M samples. Analysis filters $H_i(e^{j\omega})$ model time error mismatch δ_i for $i = 0, 1, ..., M-1$, using sample time shifters. The second half of the filter bank includes up-samplers and reconstruction filters $F_i(e^{j\omega})$. After up-sampling and reconstruction of the signals from each channel, they are combined to form the reconstructed signal $\hat{x}[n]$ as the output of filter bank.

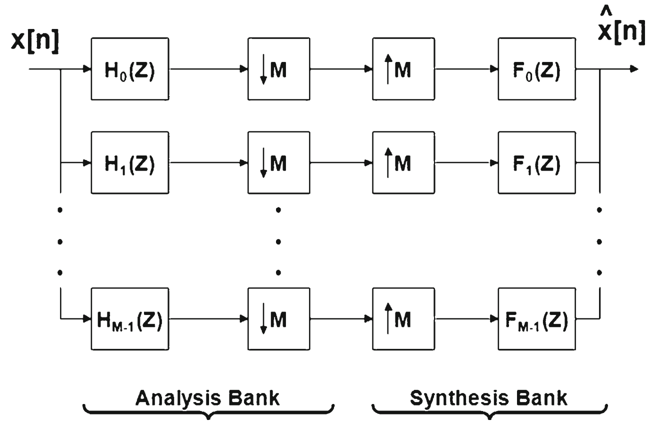

Figure 1 General filter bank model for reconstruction of periodic non-uniformly sampled signal.

2.2 Analysis Bank

Analysis filters are used to model the periodic non-uniform sampling pattern of the input signal $x[n]$. The analysis filters have the frequency response

$$H_i\left(e^{j\omega}\right) = e^{j\omega d_i} \tag{2}$$

where d_i is equal to

$$d_i = i + \delta_i \tag{3}$$

where δ_i represents fractional delay. By using the inverse DTFT of $H_i(e^{j\omega})$, the impulse response $h_i[n]$ can be written like

$$h_i[n] = \frac{\sin(\pi(n+d_i))}{\pi(n+d_i)} = \operatorname{sinc}(n+d_i) \tag{4}$$

So, the output signal after down-samplers $\widetilde{x}_i[n]$ is

$$\widetilde{x}_i[n] = (h_i[n] \star x[n]) \downarrow M \tag{5}$$

One way of approximating $h_i[n]$ by a finite impulse response (FIR) filter is to truncate $h_i[n]$ by windowing and appropriate time shift such that fractional delays will be located at the center of gravity of the window. The reason we do not use this approach is that using Taylor approximation of a fractional delay filter one can decompose the effect of fractional delays δ_i and the coefficients. This will lead to a more efficient approach to find the coefficients of synthesis filters $F_i(e^{j\omega})$ using linear least squares method which shall be explained later. So, we use second-order Taylor approximation for analysis filters as follows.

$$H_i(e^{j\omega}) = e^{j\omega(i+\delta_i)} \simeq e^{j\omega i}\sum_{k=0}^{2}\frac{(j\omega\delta_i)^k}{k!} \tag{6}$$

equivalently, $H_i(e^{j\omega})$ can be written as

$$H_i\left(e^{j\omega}\right) \simeq e^{j\omega i} + \delta_i \times \left(j\omega e^{j\omega i}\right) + \delta_i^2 \times \left(-\frac{1}{2}\omega^2 e^{j\omega i}\right) \tag{7}$$

The first, second, and third terms in the above equation are the frequency response of an all-pass filter (APF), first order differentiator, and second order differentiator respectively defined, for $|\omega| \leq \pi$, as

$$\begin{aligned} A_i(e^{j\omega}) &= e^{j\omega i} \\ D_i(e^{j\omega}) &= j\omega e^{j\omega i} \\ D_i^{(2)}(e^{j\omega}) &= -\tfrac{1}{2}\omega^2 e^{j\omega i} \end{aligned} \tag{8}$$

So, the impulse response of the analysis filters can be computed by the inverse DTFT of Eq. 7, as

$$h_i[n] \simeq a_i[n] + \delta_i \times d_i[n] + \delta_i^2 \times d_i^{(2)}[n] \tag{9}$$

where $a_i[n]$, $d_i[n]$, and $d_i^{(2)}[n]$ are the impulse response of $L_i(e^{j\omega})$, $D_i(e^{j\omega})$, and $D_i^{(2)}(e^{j\omega})$ respectively. In the proposed implementation, the impulse responses may be approximated directly using inverse DTFT and applying a Kaiser window with an appropriate β to reduce the transition ripples. See the Appendix for the inverse DTFT of Eq. 8.

2.3 Formation of FIR Synthesis Filters

Design of synthesis filters are made based on polyphase representation of analysis and synthesis filters $H_i(e^{j\omega})$ and $F_i(e^{j\omega})$ [9, 12]. Using the condition for perfect reconstruction

$$\boldsymbol{R}(z)\boldsymbol{E}(z) = \boldsymbol{P}(z) \tag{10}$$

where $\boldsymbol{R}(z)$ is a $M \times M$ matrix, representing type II polyphase decomposition of synthesis filters, and $\boldsymbol{E}(z)$ is a $M \times M$ matrix, representing type I polyphase decomposition of analysis filters. Based on our model $\boldsymbol{E}(z)$ is decomposed to three terms as

$$\boldsymbol{E}(z) = \boldsymbol{A}(z) + \boldsymbol{\Delta}\boldsymbol{D}(z) + \boldsymbol{\Delta}^2\boldsymbol{D}^{(2)}(z) \tag{11}$$

where

$$\boldsymbol{\Delta} = \text{diag}\,\{\delta_0, \delta_1, ..., \delta_{M-1}\} \tag{12}$$

$$\boldsymbol{\Delta}^2 = \text{diag}\left\{\delta_0^2, \delta_1^2, ..., \delta_{M-1}^2\right\}, \tag{13}$$

and $\boldsymbol{A}(z)$, $\boldsymbol{D}(z)$, and $\boldsymbol{D}^{(2)}(z)$ are $M \times M$ matrices, representing type I polyphase decomposition of APF, first order differentiator, and second order differentiator. $\boldsymbol{P}(z)$ is a matrix that satisfies the perfect reconstruction condition, and it is defined as

$$\boldsymbol{P}(z) = cz^{-m_0}\begin{bmatrix} \boldsymbol{0} & \boldsymbol{I}_{M-r} \\ z^{-1}\boldsymbol{I}_r & \boldsymbol{0} \end{bmatrix} \tag{14}$$

for some integer r with $0 \leq r \leq M-1$, some integer m_0, and some constant $c \neq 0$. Under this condition the reconstructed signal is $\hat{x}[n] = cx[n-n_0]$, where $n_0 = Mm_0 + r + M - 1$. Note that because an exact solution for Eq. 10 requires matrix inversion of $\boldsymbol{E}(z)$, this leads to an IIR form for the synthesis filters due to the fact that zeros of $\boldsymbol{E}(z)$ will become poles of each entries of matrix $\boldsymbol{R}(z)$. So, PR using FIR filters as the synthesis filters is not possible [9]. However, a very good approximation of Eq. 10 can be achieved using the linear least squares method, this leads to a very good approximation for a perfect reconstruction solution using FIR filters as the filters used in the synthesis part [9].

The effect of fractional delays appears as the coefficients δ_i and δ_i^2 of the i^{th} row of $\boldsymbol{D}(z)$ and $\boldsymbol{D}^{(2)}(z)$ respectively. So, the analysis filters can be written like

$$\begin{bmatrix} H_0(z) \\ \vdots \\ H_{M-1}(z) \end{bmatrix} = \boldsymbol{E}(z^M)\begin{bmatrix} 1 \\ \vdots \\ z^{-(M-1)} \end{bmatrix} \tag{15}$$

where $\boldsymbol{E}(z^M)$ consists of three terms

$$\boldsymbol{A}(z) = \begin{bmatrix} A_{0,0}(z) & \dots & A_{0,M-1}(z) \\ \vdots & \ddots & \vdots \\ A_{M-1,0}(z) & \dots & A_{M-1,M-1}(z) \end{bmatrix} \tag{16}$$

$$\boldsymbol{D}(z) = \begin{bmatrix} D_{0,0}(z) & \dots & D_{0,M-1}(z) \\ \vdots & \ddots & \vdots \\ D_{M-1,0}(z) & \dots & D_{M-1,M-1}(z) \end{bmatrix} \tag{17}$$

$$\boldsymbol{D}^{(2)}(z) = \begin{bmatrix} D_{0,0}^{(2)}(z) & \dots & D_{0,M-1}^{(2)}(z) \\ \vdots & \ddots & \vdots \\ D_{M-1,0}^{(2)}(z) & \dots & D_{M-1,M-1}^{(2)}(z) \end{bmatrix}. \tag{18}$$

Similarly, the synthesis bank is given by

$$\left[F_0(z) \dots F_{M-1}(z)\right] = \left[z^{-(M-1)} \dots 1\right]\boldsymbol{R}\left(z^M\right) \tag{19}$$

where $\boldsymbol{R}(z)$ is

$$\boldsymbol{R}(z) = \begin{bmatrix} R_{0,0}(z) & \dots & R_{0,M-1}(z) \\ \vdots & \ddots & \vdots \\ R_{M-1,0}(z) & \dots & R_{M-1,M-1}(z) \end{bmatrix}. \tag{20}$$

Using these definitions (10) can be rewritten as follows.

$$\begin{aligned} \boldsymbol{R}(z)\boldsymbol{A}(z) + \boldsymbol{R}(z)\boldsymbol{\Delta}\boldsymbol{D}(z) + \boldsymbol{R}(z)\boldsymbol{\Delta}^2\boldsymbol{D}^{(2)}(z) \\ = \boldsymbol{P}(z) \end{aligned} \tag{21}$$

Convolution is equivalent to multiplication of two polynomials. In Eq. 21, the entries of the matrix after multiplication can be achieved by convolution between the coefficients of polynomials in the Z domain. So, it is possible to transfer the above equation from the Z domain to a simple matrix equation based on the coefficients. This can be done by defining the convolution as a matrix operation as follows.

$$\boldsymbol{A}_{ij}^T = \left[\boldsymbol{a}_{ij}^{(0)}, \cdots, \boldsymbol{a}_{ij}^{(s-1)}\right] \tag{22}$$

where

$$\boldsymbol{a}_{ij}^{(k)} = \left[\underbrace{0, \cdots, 0}_{k}, a_{ij}(0), \cdots, a_{ij}(n-1), \underbrace{0, \cdots, 0}_{t-(n+k)} \right]^T \tag{23}$$

Matrix $\boldsymbol{A}_{ij}$ is formed such that it converts the multiplication between two polynomials to convolution between their coefficients. $\boldsymbol{A}_{ij}$ is defined as a $s \times t$ matrix where the entries $a_{ij}(0), a_{ij}(1), ..., a_{ij}(n-1)$ are the coefficients of polynomial $\boldsymbol{A}_{ij}(z)$, i.e. the i^{th} row and j^{th} column of matrix $\boldsymbol{A}(z)$ defined as Eq. 16. In a same way, we define

$$\boldsymbol{D}_{ij}^T = \left[\boldsymbol{d}_{ij}^{(0)}, \cdots, \boldsymbol{d}_{ij}^{(s-1)} \right] \times \delta_i \tag{24}$$

where

$$\boldsymbol{d}_{ij}^{(k)} = \left[\underbrace{0, \cdots, 0}_{k}, d_{ij}(0), \cdots, d_{ij}(n-1), \underbrace{0, \cdots, 0}_{t-(n+k)} \right]^T \tag{25}$$

$$\left(\boldsymbol{D}_{ij}^{(2)}\right)^T = \left[\left(\boldsymbol{d}_{ij}^{(2)}\right)^{(0)}, \cdots, \left(\boldsymbol{d}_{ij}^{(2)}\right)^{(s-1)} \right] \times \delta_i^2 \tag{26}$$

where

$$\left(\boldsymbol{d}_{ij}^{(2)}\right)^{(k)} = \left[\underbrace{0, \cdots, 0}_{k}, d_{ij}^{(2)}(0), \cdots, d_{ij}^{(2)}(n-1), \underbrace{0, \cdots, 0}_{t-(n+k)} \right]^T \tag{27}$$

Similarly, we define the unknown polynomial elements of $\boldsymbol{R}(z)$ with length s row vectors

$$\boldsymbol{R}_{ij} = \left[r_{ij}(0) \;\; r_{ij}(1) \;\; \ldots \;\; r_{ij}(s-1) \right] \tag{28}$$

and polynomial elements of $\boldsymbol{P}(z)$ with length t row vectors according to

$$\boldsymbol{P}_{ij} = \begin{cases} [0 \ldots 0 \; 1 \; 0 \ldots 0] & \boldsymbol{P}_{ij}(z) = z^{-k} \\ [0 \ldots 0 \; 0 \; 0 \ldots 0] & \boldsymbol{P}_{ij}(z) = 0 \end{cases} \tag{29}$$

In Eq. 29, the $(k+1)^{th}$ entry is equal to one for the case of $\boldsymbol{P}_{ij}(z) = z^{-k}$, note in the definition for $\boldsymbol{P}(z)$ we assumed $c = 1$. For the case of $\boldsymbol{P}_{ij}(z) = 0$, $\boldsymbol{P}_{ij}$ is a simple row matrix with zero entries. Using these definitions, Eq. 9 can be written as a matrix equation of the coefficients instead of matrix equation in the Z domain as

$$\boldsymbol{RE} = \boldsymbol{P} \tag{30}$$

where

$$\boldsymbol{E} = \boldsymbol{A} + \boldsymbol{D} + \boldsymbol{D}^{(2)}, \tag{31}$$

and $\boldsymbol{A}$,$\boldsymbol{D}$,$\boldsymbol{D}^{(2)}$ are $sM \times tM$ matrices with entries $\boldsymbol{A}_{ij}$, $\boldsymbol{D}_{ij}$,$\boldsymbol{D}_{ij}^{(2)}$ defined in Eqs. 22, 24, and 26 respectively. $\boldsymbol{R}$ is an unknown matrix of size $M \times sM$ with entries $\boldsymbol{R}_{ij}$. Finally, $\boldsymbol{P}$ is a $M \times tM$ matrix with entries $\boldsymbol{P}_{ij}$. Equation 30 can be solved using M linear least squares equation, for $i = 0, \cdots, M-1$, defined as

$$J(\boldsymbol{R}_i) = \left\| \boldsymbol{P}_i^T - \boldsymbol{E}^T \boldsymbol{R}_i^T \right\|^2 \tag{32}$$

$\boldsymbol{R}_i^T$ and $\boldsymbol{P}_i^T$ are column vectors of the length $Ms \times 1$ and $Mt \times 1$ respectively, defined as

$$\boldsymbol{R}_i^T = \left[\boldsymbol{R}_{i,0} \;\; \boldsymbol{R}_{i,1} \;\; \ldots \;\; \boldsymbol{R}_{i,M-1} \right]^T \tag{33}$$

$$\boldsymbol{P}_i^T = \left[\boldsymbol{P}_{i,0} \;\; \boldsymbol{P}_{i,1} \;\; \ldots \;\; \boldsymbol{P}_{i,M-1} \right]^T \tag{34}$$

Finally, using linear least squares method, we obtain

$$\boldsymbol{R}_i^T = \left(\boldsymbol{E}\boldsymbol{E}^T \right)^{-1} \boldsymbol{E}\boldsymbol{P}_i^T \tag{35}$$

Finding the matrix $\boldsymbol{R}$, one can compute synthesis filters $\boldsymbol{F}_i(z)$ using Eq. 19. Note that the effect of fractional delays can be decomposed from first and second order differentiator coefficients as follows.

$$\boldsymbol{D} = \boldsymbol{F}(\boldsymbol{\delta})\boldsymbol{D}_f \tag{36}$$

$$\boldsymbol{D}^{(2)} = \boldsymbol{F}(\boldsymbol{\delta}^2)\boldsymbol{D}_f^{(2)} \tag{37}$$

where $\boldsymbol{D}_f$ and $\boldsymbol{D}_f^{(2)}$ are $sM \times tM$ matrices of first and second order differentiators formed in the same way as $\boldsymbol{D}$ and $\boldsymbol{D}^{(2)}$ but with the fixed coefficients which are independent from fractional delays. Fractional delay matrices $\boldsymbol{F}(\boldsymbol{\delta})$ and $\boldsymbol{F}(\boldsymbol{\delta}^2)$ are $sM \times sM$ matrices formed as

$$\boldsymbol{F}(\boldsymbol{\delta}) = \begin{bmatrix} \boldsymbol{f}_d(\boldsymbol{\delta}) \\ \vdots \\ \boldsymbol{f}_d(\boldsymbol{\delta}) \end{bmatrix} \tag{38}$$

$$\boldsymbol{F}(\boldsymbol{\delta}^2) = \begin{bmatrix} \boldsymbol{f}_d(\boldsymbol{\delta}^2) \\ \vdots \\ \boldsymbol{f}_d(\boldsymbol{\delta}^2) \end{bmatrix} \tag{39}$$

where $\boldsymbol{f}_d(\boldsymbol{\delta})$ and $\boldsymbol{f}_d(\boldsymbol{\delta}^2)$ are $1 \times sM$ row vectors defined as

$$\boldsymbol{f}_d(\boldsymbol{\delta}) = \left[\underbrace{0, ..., 0}_{s} \; \delta_1, ..., \delta_1 \; \ldots \; \delta_{M-1}, ..., \delta_{M-1} \right] \tag{40}$$

$$\boldsymbol{f}_d(\boldsymbol{\delta}^2) = \left[\underbrace{0, ..., 0}_{s} \; \delta_1^2, ..., \delta_1^2 \; \ldots \; \delta_{M-1}^2, ..., \delta_{M-1}^2 \right] \tag{41}$$

3 Simulation Results

In the simulations presented in this section fractional delay filters with the frequency response $H_i(\omega)$ are obtained by inverse DTFT of second order Taylor series defined in Eq. 8.

A 39-taps FIR filter is used to limit the impulse response of the APF, first and second order differentiators. The result is windowed and time shifted using a Kaiser window with $\beta = 5.65$. The synthesis filters are designed for the case of a four-channel ($M = 4$) TIADC. The quantization error and thermal noise in each channel are modeled as bandlimited zero-mean WGN, the interested reader is referred to [7] on page 3 for more explanations. The noise power in each channel is assumed to be 70dB which is a typical value for a 12bits ADC, the reason is

$$\text{ENOB} = \frac{70 - 1.76}{6.02} \approx 12 \tag{42}$$

where ENOB is the effective number of bits. The performance of synthesis filters for reconstruction of PNUS is investigated through the sum of two sinusoids with unit amplitude and respective frequencies of $0.1 f_s$ and $0.3 f_s$. The fractional delays used for the simulations for a four-channel TIADC are $\delta_0 = 0$, $\delta_1 = 0.01$, $\delta_2 = -0.02$, $\delta_3 = -0.01$.

In a four-channel TIADC, the effect of increasing the order of synthesis filters is shown by using two set of the synthesis filters. The first set of synthesis filters are of the order 20 and the second set of synthesis filters are of the order 48. Figure 2, shows the distorted and reconstructed power spectrum using synthesis filters of the order 20, and two sinusoids with the frequencies at $0.1 f_s$ and $0.3 f_s$.

The effect of increasing the order of the synthesis filters on the distortion peaks is shown in Fig. 3. In Fig. 3, the amplitude of the distortion peaks are reduced considerably comparing with synthesis filters of the order 20 presented in Fig. 2.

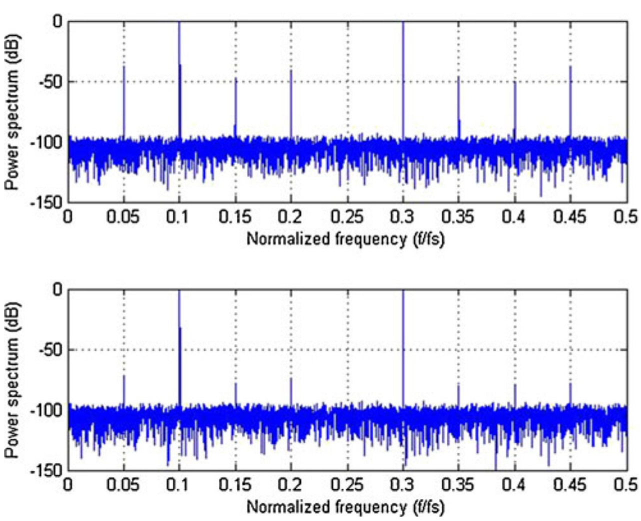

Figure 2 (*top*) Distorted and (*bottom*) reconstructed power spectrum using the synthesis filters with the order 20 for two sinusoids at $0.1 f_s$ and $0.3 f_s$ and down-sampling factor equal to 4.

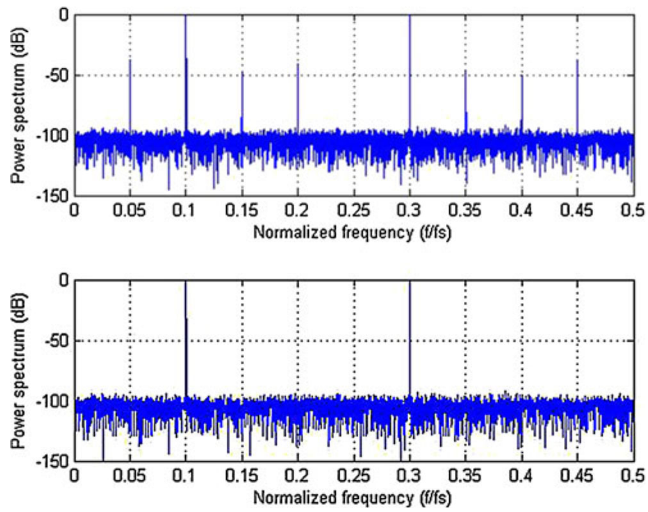

Figure 3 (*top*) Distorted and (*bottom*) reconstructed power spectrum using the synthesis filters with the order 48 for two sinusoids at $0.1 f_s$ and $0.3 f_s$ and down-sampling factor equal to 4.

Reconstruction performance curves are defined as: signal-to-noise and distortion ratio (SNDR) and signal-to-error ratio (SER). SER and SNDR values for 10 different reconstruction filter orders is shown in Fig. 4 for 40000 samples. It is clear that the saturation region for SER should be greater than SNDR, and this is verified in Fig. 4 by the simulations. After a certain reconstruction filter order increasing the reconstruction filter order does not affect SNDR and SER significantly, this is called sufficient order of SNDR and SER in this paper. Based on Fig. 4, sufficient order of SNDR and SER are 40 and 50 respectively. Note that SNDR and SER plots are for a single sinusoid at $0.3 f_s$ as the input signal.

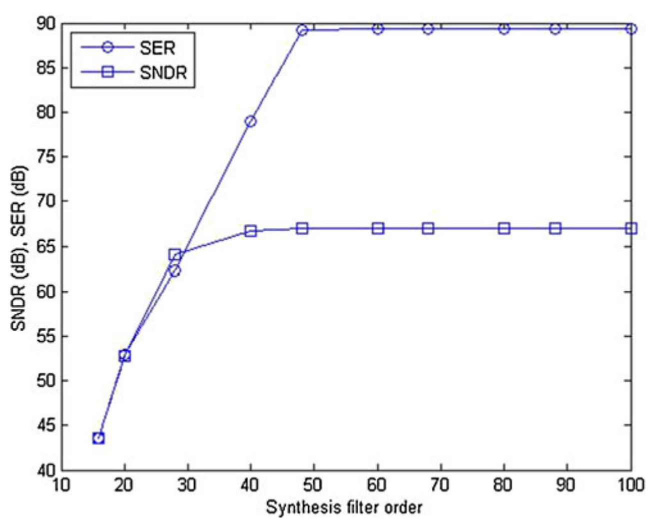

Figure 4 SNDR and SER plots versus the order of synthesis filters for four-channel TIADC.

4 Conclusion

A new reconstruction method has been proposed for TIADC with an arbitrary number of channels. This method decompose the effect of fractional delays in the analysis bank using second order Taylor expansion. A linear least squares algorithm based on the proposed method is used to find the coefficients of the reconstruction filter bank. Using the proposed method it is easier to find the solution for the linear least squares problem considering the fact that the effect of fractional delays is separated from the fixed filter coefficients. The performance curves (SNDR and SER) show a significant improvement using the presented method.

Appendix

The inverse DTFT of an all-pass filter with the frequency response $A_i(e^{j\omega})$ is

$$a_i[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega(i+n)} d\omega \tag{43}$$

Computing the above integral is very simple and leads to

$$a_i[n] = \frac{\sin \pi(i+n)}{\pi(i+n)} \tag{44}$$

where $n = 0, ..., N-1$, and N is the length of APF. Note, by defining

$$n' = n - \frac{N}{2}, \tag{45}$$

we need to replace n by n' in Eq. 44 to put the center of the gravity of APF , $a_i[n]$, in the center of a Kaiser window of the length N with the maximum value located at $\frac{N}{2}$. The inverse DTFT of first order differentiator $D_i(e^{j\omega})$ is

$$d_i[n] = \frac{\cos \pi(i+n)}{(i+n)} - \frac{\sin \pi(i+n)}{\pi(i+n)^2} \tag{46}$$

Again, we need to replace n by n' in Eq. 46 to put the center of the gravity of first order differentiator, $d_i[n]$, in the center of a Kaiser window of the length N with the maximum value located at $\frac{N}{2}$. Finally, the inverse DTFT of second order differentiator $D_i^{(2)}(e^{j\omega})$ is

$$d_i^{(2)}[n] = \frac{\sin \pi(i+n)}{\pi(i+n)^3} - \frac{\cos \pi(i+n)}{(i+n)^2} - \frac{\pi \sin \pi(i+n)}{2(i+n)} \tag{47}$$

Like the previous cases, we need to replace n by n' in Eq. 47 to put the center of the gravity of second order differentiator, $d_i^{(2)}[n]$, in the center of a Kaiser window of the length N with the maximum value located at $\frac{N}{2}$.

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