CS91T: Computational Geometry **Fall 2023**

Circularity of Convex Partitions

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1 Problem Setup

In this project, I explored [Open Problem 59], the problem of partitioning a polygon into pieces, each of which is as "circular" as possible. This problem was first introduced by [] and was expanded upon by Mirela Damian and Joseph O'Rourke.

One notion of the circularity of a given polygon is its *aspect ratio* – the more "circular" a partition, the closer its aspect ratio is to 1.

Definition 1 (Aspect Ratio)

The **aspect ratio** of a polygon is the ratio of the diameters of the smallest circumscribing circle to the largest inscribed disk.

The distinction between circle and disk is made to emphasize that the circumcircles can intersect, whereas the disks cannot.

In Open Problem 59 and this project, we focus on minimizing the aspect ratio across a set of convex partitions of a regular polygon.

Definition 2 (Partition)

A **partition** of a polygon $\mathcal{P}$ is a collection of polygonal pieces $P_1, P_2 \dots$ such that $\mathcal{P} = \cup P_i$ and no pair of pieces share an interior point.

In our problem, we add the restriction that our partition of a regular polygon P must be convex, meaning that each of its polygonal pieces are themselves convex. Finally, we study the circularity, an extension of aspect ratio, of our convex partition.

Definition 3 (Circularity)

The **circularity** of a convex partition of a polygon $\mathcal{P}$ is the maximum aspect ratio, across all polygonal pieces of the partition.

In summary, the problem is as follows:

Goal — Find the minimum circularity across all convex partitions of a regular polygon.

2 Related Problems

Calculating the circularity is difficult.

3 Approach and Methodology