YOUNG MATHEMATICIANS CONFERENCE NOTES

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1. Day 1: August 15th

1.1. The Failed Zero Forcing Number of a Graph.

Presented by Chirag Kaudan and Rachel Taylor.

Definition (Forcing Rule)
Let each vertex of a graph represent a person. Each person either knows or does not know a secret – if they do, their corresponding vertex is colored.

If all a person's friends except one friend knows the secret, then the secret is told to that friend as well.

Definition (Zero Forcing Number)
The zero forcing number of G, $Z(G)$, is the smallest cardinality of any set S of vertices on which repeated applications of the forcing rule results in all vertices joining S.

Definition (Failed Zero Forcing Number)
The failed zero forcing number of G, $F(G)$, is the maximum cardinality of any set of vertices on which repeated applications of the forcing rule will never result in all vertices joining the set.

Result — Using the theory of *modules* (a set of vertices such that every vertex in the module has the same neighorhood exclusing vertices in the module) in zero forcing graphs and a computer algorithm, they were able to show that there are 15 graphs with $F(G) = 2$ and 68 graphs with $F(G) = 3$.

1.2. Properties of Families of Graphs with Forbidden Induced Subgraphs.

Presented by Christian Pippin.

Definition (Induced Subgraphs)
H is an **induced subgraph** of G if the vertex set of H is a subset of the vertex set of G and for all $(u, v) \in E^H$, $(u, v) \in E^G$.

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There is a relation between indivisibility and the lex product.

Lemma 1 — If a family of graphs is closed under the lex product, the class is indivisible.

Theorem 2

For all n, $\mathrm{Forb}(P_n)$ is indivisible.

$\mathrm{Forb}(C_n)$ is indivisible for all $n \geq 5$.