YOUNG MATHEMATICIANS CONFERENCE NOTES

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1. Day 1: August 15th

1.1. The Failed Zero Forcing Number of a Graph.

Presented by Chirag Kaudan and Rachel Taylor.

Definition (Forcing Rule)
Let each vertex of a graph represent a person. Each person either knows or does not know a secret – if they do, their corresponding vertex is colored.

If all a person's friends except one friend knows the secret, then the secret is told to that friend as well.

Definition (Zero Forcing Number)
The zero forcing number of G, $Z(G)$, is the smallest cardinality of any set S of vertices on which repeated applications of the forcing rule results in all vertices joining S.

Definition (Failed Zero Forcing Number)
The failed zero forcing number of G, $F(G)$, is the maximum cardinality of any set of vertices on which repeated applications of the forcing rule will never result in all vertices joining the set.

Result — Using the theory of *modules* (a set of vertices such that every vertex in the module has the same neighorhood exclusing vertices in the module) in zero forcing graphs and a computer algorithm, they were able to show that there are 15 graphs with $F(G) = 2$ and 68 graphs with $F(G) = 3$.

Date: August 15th - 17th, 2023.

1.2. Properties of Families of Graphs with Forbidden Induced Subgraphs.

Presented by Christian Pippin.

Definition (Induced Subgraphs)
H is an **induced subgraph** of G if the vertex set of H is a subset of the vertex set of G and for all $(u, v) \in E^H$, $(u, v) \in E^G$.

There is a relation between indivisibility and the lex product.

Lemma — If a family of graphs is closed under the lex product, the class is indivisible.

Theorem
For all n, $\text{Forb}(P_n)$ is indivisible.

$\text{Forb}(C_n)$ is indivisible for all $n \geq 5$.

1.3. Generalized Stick Fragmentation and Benford's Law.

Presented by Xinyu Fang and Maxwell Sun.

Theorem (Benford's Law)
In base B, the probability of observing a value with first digit d is

$$\log_B\left(\frac{d+1}{d}\right).$$

If this property holds, a dataset is said to exhibit **weak Benford behavior**; if, moreover, the logarithms of the values modulo 1 are equidistributed, then the set exhibits **strong Benford behavior**.

Definition (Significand and Mentissa)
The **significand** of x base B is $S_B(x) \in [1, B)$ such that $x = S_B(x) \cdot B^k$ for some k.

The **mantissa** is the analogue but for fractional pieces.

To setup the problem, consider a stick breaking model where you begin with a stick of length ℓ. At each step, the stick is broken into smaller segments and this process continues until some termination condition is reached. The problem the authors studied is known as a *Discrete Breaking Problem with a Stopping Set*: if the stick length falls into the given stopping set, it is considered "dead" and should not be broken further.

Result — The authors give some results on when a given set of end lengths is Strong Benford (for which types of stopping sets).

In particular, they conjecture that if you break each stick into k parts and stop at $(k-1) \cdot \frac{n}{k}$ residue classes modulo n where $k \mid n$, then the results are Strong Benford.

1.4. **Geometry of the Numerical and Berezin Range.**

Presented by Edwin Xie and Caroline Norman.

Definition (Numerical Range)
The **numerical range** f of a bounded linear operator T on a complex Hilbert space H is defined as

$$W(T) = \{\langle Tf, f\rangle : ||f|| = 1.\}$$

Properties of the numerical range include unitary invariance, shift and scale, and that it satisfies the Topelitz-Hausdorff property.

Definition (Berezin Range)
On the Hardy-Hilbert space $H^2(\mathbb{D})$ of the open unit disk $\mathbb{D}$, the **Berezin range** is defined as

$$B(T) = \{\langle T\hat{k}_p, \hat{k}_p\rangle : p \in \mathbb{D}\}$$

where $\hat{k}_p$ is the normalized reproducing kernel.

Result — The authors work with the Berezin Range on a Hardy Space and prove some results about the Berezin Range.