MATH103: Complex Analysis **Fall 2023**

Homework 6

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Chapter V (Power Series) Problems.

Section V.4 (Power Series Expansion of an Analytic Function), Problem 4

Suppose $f(z)$ is analytic at $z = 0$ and satisfies $f(z) = z + f(z)^2$. What is the radius of convergence of the power series expansion of $f(z)$ about $z = 0$?

Solution. ■

Section V.5 (Power Series Expansion at Infinity), Problem 4

Let E be a bounded subset of the complex plane $\mathbb{C}$ over which area integrals can be defined, and set

$$f(w) = \int\int_E \frac{dx\,dy}{w - z}, \quad w \in \mathbb{C} \setminus E$$

where $z = x+iy$. Show that $f(w)$ is analytic at ∞, and find a formula for the coefficients of the power series of $f(w)$ at ∞ in descending powers of w.

Solution. ■