MATH103: Complex Analysis **Fall 2023**

Homework 5

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Chapter IV (Complex Integration and Analyticity) Problems.

Section IV.4 (The Cauchy Integral Formula), Problem 4

Let D be a bounded domain with smooth boundary ∂D, and let $z_0 \in D$. Using the Cauchy integral formula, show that there is a constant C such that

$$|f(z_0)| \leq C \sup\{|f(z)| : z \in \partial D\}$$

for any function $f(z)$ analytic on $D \cup \partial D$. By applying this estimate to $f(z)^n$, taking nth roots, and letting $n \to \infty$, show that the estimate holds with $C = 1$. *Remark.* This provides an alternative proof of the maximum principle for analytic functions.

Solution. ■

Section IV.5 (Liouville's Theorem), Problem 4

Suppose that $f(z)$ is an entire function such that $f(z)/z^n$ is bounded for $|z| \geq R$. Show that $f(z)$ is a polynomial of degree at most n. What can be said if $f(z)/z^n$ is bounded on the entire complex plane?

Solution. ■