MATH104: Topology **Fall 2023**

Homework 10

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Chapter 12 (Classification of Surfaces) Problems.

Section 74 (Classification of Surfaces), 74.7

If $m > 1$, show the fundamental group of the m-fold projective plane is not abelian. [*Hint*: There is a homomorphism mapping this group onto the group $\mathbb{Z}/2 * \mathbb{Z}/2$.]

Section 78 (Constructing Compact Surfaces), 78.2(a)(b)

Let H^2 be the subspace of $\mathbb{R}^2$ consisting of all points (x_1, x_2) with $x_2 \geq 0$. A 2-*manifold with boundary (or surface with boundary)* is a Hausdorff space X with a countable basis such that each point x of X has a neighborhood homeomorphic with an open set of $\mathbb{R}^2$ or H^2. The *boundary* of X (denoted ∂X) consists of those points x such that x has no neighborhood homeomorphic with an open set of $\mathbb{R}^2$.

a) **Show that no point of H^2 of the form $(x_1, 0)$ has a neighborhood (in H^2) that is homeomorphic to an open set of $\mathbb{R}^2$.**

b) **Show that $x \in \partial X$ if and only if there is a homeomorphism h mapping a neighborhood of x onto an open set of H^2 such that $h(x) \in \mathbb{R} \times 0$.**