MATH104: Topology **Fall 2023**

Homework 3

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Chapter 3 (Connectedness and Compactness) Problems.

Section 22 (Connected Spaces), 22.4(a)

Define an equivalence relation on the plane $X = \mathbb{R}^2$ as follows:

$$x_0 \times y_0 \sim x_0 \times y_1 \text{ if } x_0 + y_0^2 = x_1 + y_1^2.$$

Let X^* be the corresponding quotient space. It is homeomorphic to a familiar space; what is it? [*Hint:* Set $g(x \times y) = x + y^2$.]

Solution. ■

Section 26 (Compact Spaces), 26.5

Let A and B be disjoint compact subspaces of the Hausdorff space X. Show that there exist disjoint open sets U and V containing A and B, respectively.