MATH104: Topology **Fall 2023**

Homework 10

David Yang

Chapter 13 (Classification of Covering Spaces) Problems.

Section 79 (Equivalence of Covering Spaces), 79.5(b)

Let $T = S^1 \times S^1$ be the torus; let $x_0 = b_0 \times b_0$. Prove the following:

Theorem. If E is a covering space of T, then E is homeomorphic either to $\mathbb{R}^2$, or to $S^1 \times \mathbb{R}$, or to T.

(Hint: You may use the following result from algebra: if F is a free abelian group of rank 2 and N is a nontrivial subgroup, then there is a basis a_1, a_2 for F such that either (1) ma_1 is a basis for N, for some positive integer m, or (2) ma_1, na_2 is a basis for N, where m and n are positive integers.)

Solution. ■

Section 81 (Covering Transformations), 81.4

Let G be a group of homeomorphisms of X. The action of G on X is said to be fixed-point free if no element of G other than the identity e has a fixed point. Show that if X is Haussdorf, and if G is a finite group of homeomorphisms of X whose action is fixed-point free, then the action of G is properly discontinuous.

Solution. ■