MATH104: Topology **Fall 2023**

Homework 1

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Chapter 1 (Set Theory and Logic) Problems.

Section 2 (Functions), 2.5

In general, let us denote the *identity function* for a set C by i_C. That is, define $i_C \colon C \to C$ to be the function given by the rule $i_C(x) = x$ for all $x \in C$. Given $f \colon A \to B$, we say that a function g is a *left inverse* for f if $g \circ f = i_A$; and we asy that $h \colon B \to A$ is a *right inverse* for f if $f \circ h = i_B$.

a) **Show that if f has a left inverse, f is injective; and if f has a right inverse, f is surjective.**

b) **Give an example of a function that has a left inverse but no right inverse.**

c) **Give an example of a function that has a right inverse but no left inverse.**

d) **Can a function have more than one left inverse? More than one right inverse?**

e) **Show that if a function f has both a left inverse g and right inverse h, then f is bijective and $g = h = f^{-1}$.**