STAT111: Mathematical Statistics II **Spring 2024**

Homework 3

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1. **Suppose V_1 and V_2 are independent $\text{Gamma}(1, \lambda)$ random variables that represent waiting times in a Poisson process with rate λ events per unit time. Let $X = V_1$ be the time of the first event and let $Y = V_1 + V_2$ be the time of the second event.**

2. **Suppose X and Y have joint pdf $f_{xy}(x, y) = I(0 < x < 1, -x < y < x)$.**

   a) **Explain how you can tell, without finding the marginal densities, that the conditional densities are Uniform. Write out the conditional densities $f_{x|y}(x \mid y)$ and $f_{y|x}(y \mid x)$.**

   b) **Explain how you can tell, without finding the marginal densities, that X and Y are not independent. Find the marginal pdf's $f_x(x)$ and $f_y(y)$ and verify that $f_{xy}(x, y) \neq f_x(x)f_y(y)$.**

   c) **Show that X and Y are uncorrelated.**

   a) **Suppose X_1 and X_2 are Bernoulli random variables with expectations p_1 and p_2. Show that X_1 and X_2 are independent if and only if they are uncorrelated. This shows the Bernoulli distribution is special like the multivariate Normal distribution in that uncorrelated implies independence.**

   b) **Suppose $Y = X_1 + X_2$ with X_1 and X_2 independent. If you learn that Y and X_1 are Normal variables, prove that X_2 is also a Normal random variable.**