

Homework 3

David Yang

-
1. Suppose a planet has m days in a year, and life forms have equal probability of being hatched on any of these days. For a random group of n lifeworms, find the expected proportion of the m possible hatch days that are represented. Find the value when $m = 365$ and $n = 365$ (hint: use indicator variables; the answer is close to $1 - e^{-1}$.)
 2. Suppose X has pmf $P(X = k) = \frac{c}{(1+|k|)^2}$ for $k = 0, \pm 1, \pm 2, \dots$. The constant $c = (2\psi'(1) - 1)^{-1}$, where $\psi'(\alpha) = \frac{d^2}{d\alpha^2} \log \Gamma(\alpha)$ is the trigamma function. Explain why $E(X)$ is not 0, despite the symmetry of this pmf.
 3. Suppose $X \sim \text{Gamma}(\alpha, \lambda)$, with pdf $f_x(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x} I(x > 0)$ for $\alpha > 0$ and $\lambda > 0$.
 - a) Find an expression for $E(X^k)$, for $k = 1, 2, \dots$, using integration by recognition.
 - b) $Y = \frac{1}{X}$ follows a reciprocal Gamma distribution. Find $E(Y)$, $E(Y^2)$, and $\text{Var}[Y]$, first using integration by recognition with the pdf found in HW2, and again using LOTUS with the pdf for X . Be sure to say if there are conditions when these are not defined.
 4. Suppose $V \sim \text{Gamma}(b, 1)$ and $X | V \sim \text{Gamma}(a, V)$.
 - a) Show that $X \sim F^*(a, b, 1)$.
 - b) Use the laws of total expectation and variance to find the mean and variance of the $F^*(a, b, 1)$ distribution. Be sure to say if there are conditions when these are not defined.