Phys 3201 Fall 2018, Final Exam

December 16, 2018

We expect that you will complete this exam on your own without help from classmates, and without unfair use of the internet. The exam is, however, open notes and open book. Email dyanni3@gatech.edu with any questions or clarifications about what is being asked. The exam will be due at 5:00 pm, Dec 12, 5:00 pm. To turn it in, follow the same guidelines as homework submissions- either drop it off in prof Goldman's office or email it to both of us. Good luck!

Problem 1: Pendulum in non-uniformly accelerating frame

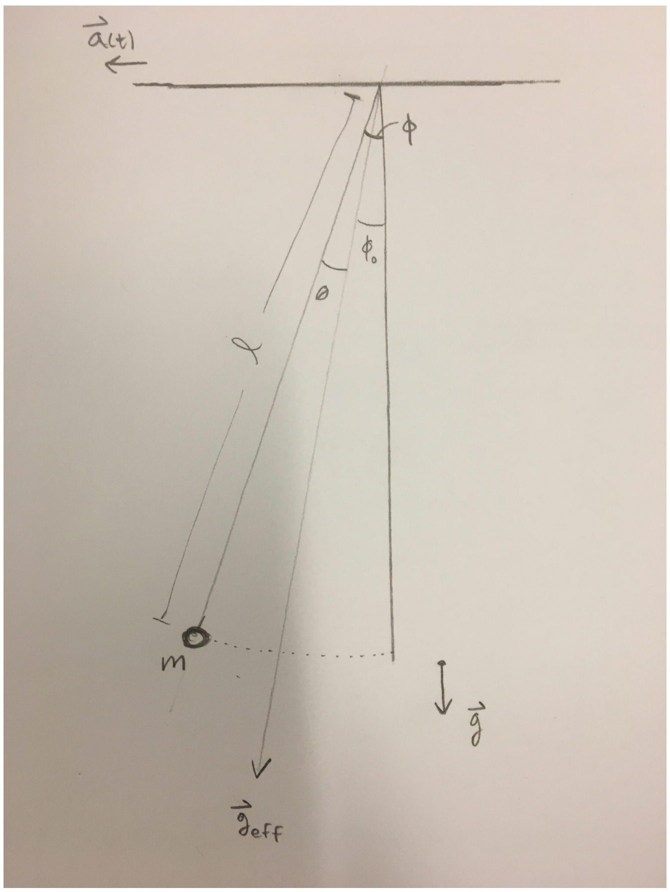

Figure 1: Pendulum in non-uniformly accelerating frame

A pendulum consisting of a bob of mass m suspended from a massless string of length ℓ is set up inside of a truck. The pendulum starts at $\phi = 0$ and at rest, when the truck begins to accelerate with $a(t)$.

a Draw a free-body-diagram of the forces acting on the bob, including fictitious forces. List the forces in the direction of string tension and the direction of the bob's movement.

b Show that

$$\ddot{\phi} = \frac{a(t)}{\ell}\cos(\phi) - \frac{g}{\ell}\sin(\phi) \tag{1}$$

c Show that

$$\ddot{\phi} = -\frac{g_e}{\ell}\sin(\phi - \phi_0) \tag{2}$$

with $\phi_0 = \tan^{-1}\left(\frac{a(t)}{g}\right)$, and $g_e = \sqrt{a(t)^2 + g^2}$. Hint: use an angle difference trig identity

d If the acceleration is uniform, $a(t) = a_0$ after time zero, we can make the transformation $\theta = \phi - \phi_0$, and write

$$\ddot{\theta} = -\frac{g_e}{\ell}\sin(\theta) \tag{3}$$

If $a_0 = 3\ \frac{\mathrm{m}}{\mathrm{s}}$, is it reasonable to make a small angle approximation $\sin(\theta) \approx \theta$? What is the percent error associated with this approximation?

e Using equation 3 and a small angle approximation, find the maximum velocity of the bob.

f How much work did the truck's engine do to generate the oscillation of the bob?

g EXTRA CREDIT: Numerically evaluate equation 1 using $a(t) = 3\tanh(.3\,t)\ \frac{\mathrm{m}}{\mathrm{s}}$, plot the results, compare to the results using $a(t) = 3\ \frac{\mathrm{m}}{\mathrm{s}}$ and comment on them. Is the amplitude as expected, is the frequency as expected, is the phase as expected? Why or why not?

Problem 2: Triforce

The triforce is cast in pure gold and has a total mass m. All of the triangles are equilateral and the largest triangle has a side length h. The whole thing is very thin so you can pretend it's two dimensional. The solid parts have a uniform density

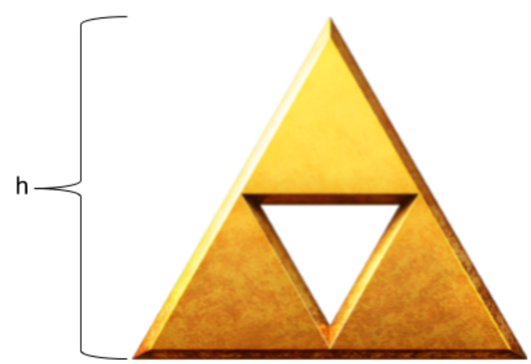

Figure 2: Triforce

a Find the center of mass of the triforce.

b Find the moment of inertia about the axis going into the page and passing through the center of mass.

c Find all three principal moments of inertia. Hint: one of them is the one you calculated in part [b]. For partial credit find the sum of the other two. For full credit find all of them.

d I get the triforce spinning like a fidget-spinner at an angular velocity ω and then drop it from Howey physics building. As I drop it, I nudge it so that the initial angular momentum is displaced by an angle α from the spin axis. Find the precession rate of the triforce. Note α is small, and hint: see KK Torque-Free-Precession example in chapter 8.

Problem 3: Rocket and observer (non-relativistic)

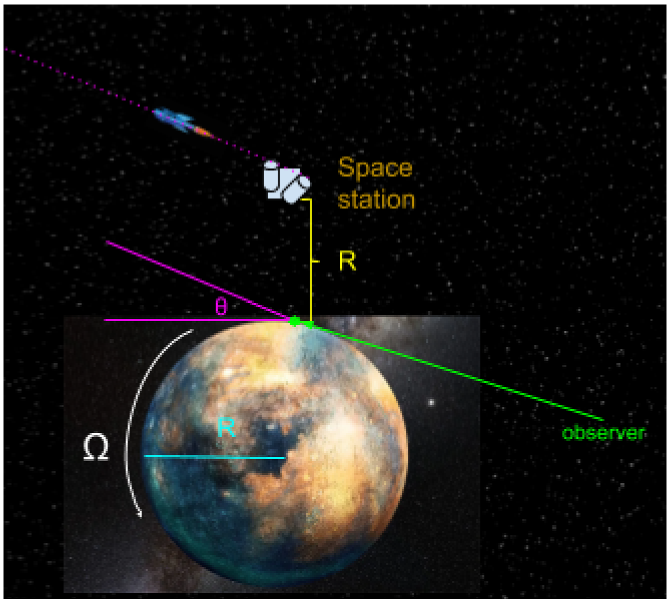

Figure 3: Rocket

A rocket of initial mass M burns fuel at a constant rate $K = -\frac{dm}{dt}$, and ejects exhaust at a speed u which is constant *relative to the rocket*. It takes off from a space station that is in synchronous orbit with a planet whose surface is a distance R below it. It starts from rest relative to the space station and begins accelerating in a direction θ from the plane parallel to the planet's surface. It keeps accelerating until half its initial mass has been burned off. An observer on the planet's surface directly below the space station watches the rocket's trajectory. The planet rotates at a rate Ω and has a radius R. Find the angular momentum of the rocket about the observer, as a function of time. Ignore the effect of the planet's gravity on the rocket.