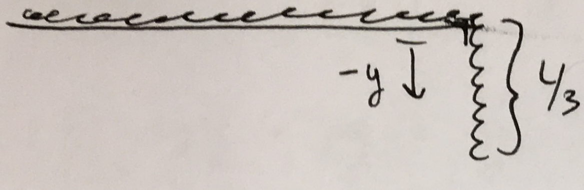
1 Difficult



rope total length $= L$

rope density $= \rho$

i) Find μ^*, the value of friction so that friction force balances gravitational force acting on dangling end of rope.

$f_f = \mu N = \mu \rho (L-y) g$ = friction force

@ balance: $\mu \rho (L-y) g = \rho g y$

$$\mu^* = \frac{y}{L-y} = \frac{L/3}{L - L/3} = \frac{1}{2}$$

$$\boxed{\text{so if } \mu > \tfrac{1}{2} \text{ rope tip will not move}}$$

ii) if $\mu < \frac{1}{2}$ use $\vec{F} = m\vec{a}$

$$\rho g y - \mu \rho (L-y) g = \rho L \ddot{y}$$

Rearranging: $\ddot{y} = \underbrace{\frac{g(1+\mu)}{L}}_{a^2} y - \underbrace{\mu g}_{b}$

$$\ddot{y} = a^2 y - b$$

$$\ddot{y} = a^2\left(y - \frac{b}{a^2}\right)$$

$$\ddot{u} = a^2(u)$$

$$u(t) = Ae^{at} + Be^{-at}$$

let $u = y - \frac{b}{a^2}$,

$\dot{u} = \dot{y}$

$\ddot{u} = \ddot{y}$

$$y(t) = Ae^{at} + Be^{-at} + \frac{\mu L}{1+\mu}$$

$$y(0) = \frac{L}{3} = A + B + \frac{\mu L}{1+\mu} \rightarrow A + B = \frac{L}{3} - \frac{\mu L}{1+\mu}$$

$$\dot{y}(0) = 0 = aA - aB \rightarrow A = B \rightarrow A = \frac{L}{6} - \frac{\mu L}{2(1+\mu)}$$

$$\boxed{y(t) = \left(\frac{L}{6} - \frac{\mu L}{2(1+\mu)}\right)\left(e^{\sqrt{\frac{g(1+\mu)}{L}}\,t} + e^{-\sqrt{\frac{g(1+\mu)}{L}}\,t}\right) + \frac{\mu L}{1+\mu}}$$

$$\mu = 0 \rightarrow y(t) = \frac{L}{6}\left(e^{\sqrt{\frac{g}{L}}\,t} + e^{-\sqrt{\frac{g}{L}}\,t}\right)$$

$$\mu = \frac{1}{3} \rightarrow y(t) = \left(\frac{L}{6} - \frac{L}{8}\right)\left(e^{\sqrt{\frac{g}{L}\cdot\frac{4}{3}}\,t} + e^{-\sqrt{\frac{4g}{3L}}\,t}\right) + \frac{L}{4}$$

$\mu = \frac{2}{3} \rightarrow \mu > \frac{1}{2}$ so $y(t) = \frac{L}{3}$ forever

$\boxed{2)}$ Difficult

$$\Delta x = x_{i+1} - x_i$$

$$\ddot{\Delta x} = \ddot{x}_{i+1} - \ddot{x}_i$$

$$\ddot{\Delta x} = -a - \varepsilon^2 (x_{i+1} - x_i - R)$$

$$\ddot{\Delta x} = -\varepsilon^2 \left(\Delta x - R + \frac{a}{\varepsilon^2} \right)$$

$$\ddot{u} = -\varepsilon^2 u \qquad \text{let} \quad u = \Delta x - R + \frac{a}{\varepsilon^2}$$

$$\ddot{u} = \ddot{\Delta x}$$

$$u(t) = A\cos(\varepsilon t) + B\sin(\varepsilon t) \rightarrow \quad u(0) = \Delta x(0) - R + \frac{a}{\varepsilon^2} = \frac{a}{\varepsilon^2}$$

$$\dot{u}(0) = \dot{\Delta x}(0) = 0$$

$$\left.\begin{aligned} \dot{u}(0) &= \varepsilon B = 0 \\ u(0) &= A = \frac{a}{\varepsilon^2} \end{aligned}\right\} \rightarrow u(t) = \frac{a}{\varepsilon^2}\cos(\varepsilon t) = \Delta x - R + \frac{a}{\varepsilon^2}$$

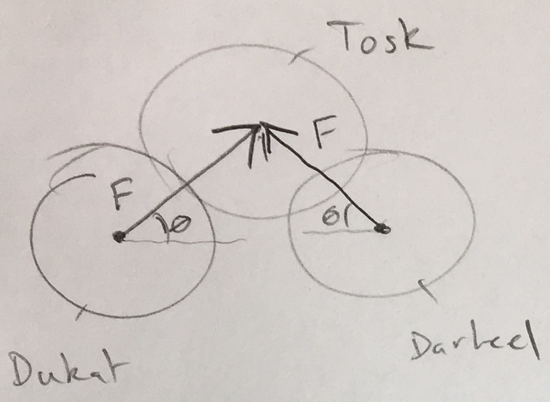
$$\boxed{\Delta x = \frac{a}{\varepsilon^2}\left(\cos(\varepsilon t) - 1 \right) + R} \quad \text{"equation 1"}$$

Eq. 1 is only valid while $\ddot{x}_{i+1} = -a$, until the leader comes to rest @ time $\frac{v_0}{a}$. Also, eq. 1 becomes invalid for $t > \frac{\pi}{2\varepsilon}$, when the follower reaches maximum braking deceleration. However, I've asked the class to ignore these constraints and find minimum velocity independent R such that no collision occurs before $t = \frac{v_0}{a}$ $\longrightarrow$

$$\Delta X_{min} = -\frac{2a}{\varepsilon^2} + R$$

So if $R > \frac{2a}{\varepsilon^2}$ no collision will occur.

$\boxed{3}$ Easy



Forces felt by Tosk:

$\hat{y}$: $2F\sin\theta \; - 2\mu F\cos\theta$

$\hat{x}$: $F\cos\theta - F\cos\theta = 0$

For Tosk to just barely slip:

$$2F\sin\theta - 2\mu F\cos\theta$$

$$\boxed{\mu = \tan\theta}$$

$\boxed{4}$ MEDIUM

$\boxed{\text{ODO}}$

a) when ODO stops swimming have $m\dot{v} + bv = 0$

$$\dot{v} = -\frac{b}{m} v$$

$$v = v_0 e^{-\frac{b}{m}t}$$

$$x = \int_0^t v\,dt' = -\frac{mv_0}{b} e^{-\frac{b}{m}t'} \Big|_0^t = -\frac{mv_0}{b} e^{-\frac{b}{m}t} + \frac{mv_0}{b}$$

$$\boxed{x(t) = \frac{mv_0}{b}\left(1 - e^{-\frac{b}{m}t}\right)}$$

b) $\frac{m}{b} = \frac{1}{3}\,\mu s$, $v_0 = 30\,\frac{\mu m}{s}$ $\rightarrow$ $\frac{mv_0}{b} = 10^{-11}$ meters $= 0.1$ Angstrom!

$$\lim_{t\to\infty} x(t) = \frac{mv_0}{b}\left(1 - e^{-\infty}\right) = \frac{mv_0}{b}$$

$\boxed{\text{ODO coasts 0.1 Angstrom}}$

c) $\boxed{v_0 = 30\,\frac{\text{body-length}}{\text{second}}}$, $x_{coast} = 10^{-11}\,\text{meters} \cdot \frac{1\ \text{body-length}}{10^{-6}\,\text{meters}}$

$\boxed{x_{coast} = 10^{-5}\ \text{body lengths}}$

$\boxed{4}$ Continued

$\boxed{\text{DAVID}}$

a) $m\dot{v} + bv^2 = 0 \quad \rightarrow \quad \frac{dv}{v^2} = -\frac{b}{m} dt$

$-\frac{1}{v} = -\frac{b}{m} t + \text{constant} \quad , \quad V(0) = V_0 \rightarrow \text{Constant} = -\frac{1}{V_0}$

$\frac{1}{v} = \frac{b}{m} t + \frac{1}{V_0} \quad \rightarrow \quad \frac{V_0}{V} = \frac{bV_0}{m} t + 1 \rightarrow \boxed{V = V_0 \left(\frac{bV_0}{m} t + 1 \right)^{-1}}$

let $\tau = \frac{m}{bV_0}$ then $X(t) = \int_0^t \frac{V_0}{\frac{t'}{\tau} + 1} dt'$, let $u = \frac{t'}{\tau} + 1$

$du = \frac{dt'}{\tau}$

$X(t) = V_0 \tau \int \frac{1}{u} du = V_0 \tau \log\left(\frac{t'}{\tau} + 1\right)\Big|_0^t = V_0 \tau \log\left(\frac{t}{\tau} + 1\right)$

$$\boxed{X(t) = \frac{m}{b} \log\left(\frac{bV_0 t}{m} + 1\right)}$$

b) $V_0 = 1 \frac{m}{s}$, $\frac{m}{b} = .1\,m$ For David to go to 1% of initial speed:

$0.01\, V_0 = V_0 \left(\frac{bV_0}{m} t + 1\right)^{-1} \Rightarrow \frac{1}{100} = \frac{1}{\frac{bV_0}{m} t^* + 1}$

$x(t^*) = 0.1 \log\left(\frac{bV_0}{m} t^* + 1\right) = 0.1 \log(100) = 0.46$ meters

$$\boxed{X_{coast} = 0.46 \text{ meters}}$$

$\boxed{4}$ continued

$$V_0 = 1 \ \frac{\text{body-length}}{\text{second}} \qquad X_{coast} = 0.46 \text{ body-lengths}$$

$\boxed{5}$

Number of revolutions is: $\dfrac{\theta(t)}{2\pi} = \dfrac{\omega t}{2\pi}$

time of flight: $h = \frac{1}{2} g t^2 \rightarrow t = \sqrt{\frac{2h}{g}}$

$$\boxed{N = \frac{\omega}{2\pi}\sqrt{\frac{2h}{g}}}$$