Phys 3201 Fall 2018, Exam 1

October 14, 2018

We expect that you will complete this exam on your own without help from classmates, and without unfair use of the internet (e.g. it is not okay to ask physicsforums.com for help on these problems or to go hunting for solutions). The exam is, however, open notes and open book. Email dyanni3@gatech.edu with any questions or clarifications about what is being asked. The exam will be due Friday 09/21/2018 at 5:00 pm. To turn it in, follow the same guidelines as homework submissions- either drop it off in prof Goldman's office or email it to both of us. When you email it, if you would please make the subject line "Phys 3201 Exam 1." Thanks and good luck!

Problem 1

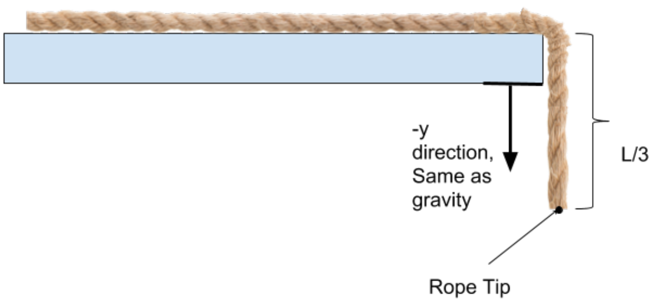

Figure 1: Rope draped over the edge of a table.

A rope of linear density ρ $[\frac{kg}{m}]$ rests dangling off the edge of a table as shown in figure 1. The total length of the rope is L, and initially the amount dangling off the edge is $\frac{1}{3}L$. The rope starts out at rest. Find an expression, $y(t)$, for the position of the tip of the rope over time in the following three scenarios:

a The coefficient of friction, μ, between the table and the rope is zero. $\mu = 0$

b The coefficient of friction, μ, between the table and the rope is one third. $\mu = \frac{1}{3}$

c The coefficient of friction, μ, between the table and the rope is two thirds. $\mu = \frac{2}{3}$

Notes

- Pretend that the table is infinitely thin

Problem 2

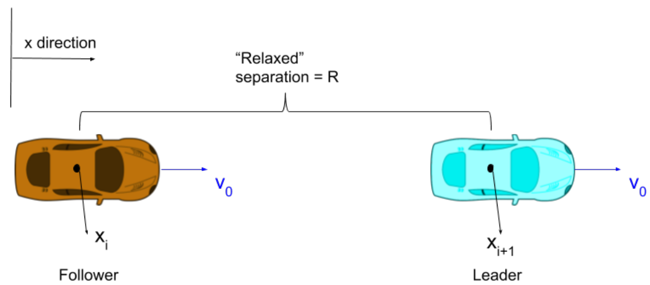

Figure 2: Leader and follower driving at a speed v_0, separated by a distance R just before the leader slams on the brakes. You can imagine a spring, of relaxed length R, between the two cars, except that *only the follower feels the spring force.*

You can capture a surprising amount of realistic traffic behavior by using force-based models. In other words, we model driver behavior by pretending that the equations of motion of each car are subject to two forces: A propulsion force that tends to accelerate the car toward the speed limit, and a repulsion force that prevents collisions. One simple (but not very realistic) choice for the repulsion force is a spring-like force felt only by the follower (see fig 2).

$$\ddot{x}_i = \epsilon^2 \left(x_{i+1} - x_i - R\right) \tag{1}$$

Two cars, a leader and a follower, are driving along at the speed limit v_0, separated by a safe distance. In this case that distance is R. Suddenly, a cat or something runs in the middle of the road so that the leader slams on the brakes and begins decelerating at a constant rate a. Assume that the follower has instantaneous reaction time, and begins decelerating according to equation 1.

a If the maximum braking deceleration is a, find a restriction on R so that the follower doesn't rear-end the leader *before the leader comes to a stop* (no matter what their initial speeds are). Your expression should be a funciton of ϵ and a.

For a more realistic force-based model see e.g. this paper or, for ant traffic, see Prof Goldman's work.

Notes

- ϵ is a fixed constant and $\epsilon > 0$
- a is a fixed constant and always acts to decrease a car's speed. In other words, the acceleration of the leader is $\ddot{x}(t)_{i+1} = -a$ with $a > 0$.

Problem 3

Dukat and Darheel are clamping down on Tosk with ever increasing force, F see Figure 3. Tosk hopes to escape into the 2$^{\text{nd}}$ dimension (i.e. into the $\hat{y}$ direction, as shown). Dukat, Tosk, and Darheel form an isosceles triangle of angle θ, as shown. Being a circle without any appendages, Tosk doesn't have many options available to her. But if she concentrates really hard she will break a sweat, which reduces the coefficient of friction, μ, between her and her oppressors.

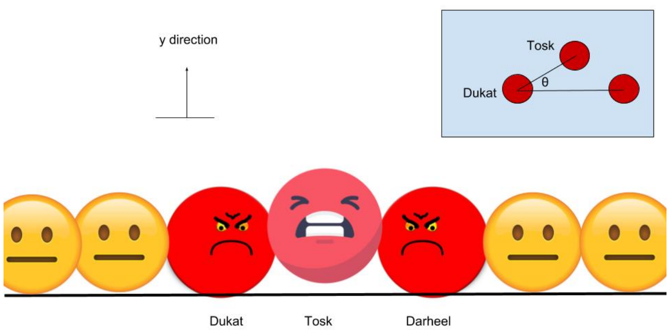

Figure 3: This is a picture of Tosk's escape

Find the maximum value of μ for Tosk to escape. Write your answer in terms of parameters given in the problem.

Let's say that gravity is going into the page, and that all of their weights are so small relative to the other forces in the problem that we can just ignore them anyway. Incredibly, this sort of picture is actually somewhat related to what happens at the growing front of some types of bacterial biofilms. See e.g. this review

Problem 4

Prof Goldman talked in class about life at low Reynold's number. Consider the equation of motion for a bacterium, Odo, swimming in a viscous media:

$$\cancel{f_{Odo} = m\ddot{x} - b\dot{x}}$$

$$f_{Odo} = m\ddot{x} + b\dot{x}$$

Where f_{Odo} is how much force Odo generates to keep swimming. If Odo stops supplying swimming force at time $t = 0$:

a write a general expression for the distance $x(t)$, that he will coast, given that he started at speed v_0

b If Odo's initial speed was 30 $[\frac{\mu m}{s}]$ (microns per second), and the ratio $\frac{m}{b} = \frac{1}{3}$ $[\mu s]$ (microseconds), find the total distance that Odo coasts.

c If Odo is 1 μm long, write the distance he coasts in units of his body size. Write his initial speed in units of his body size per second.

d repeat parts a, b, and c, but for David instead of Odo. David is coasting in the same fluid, weighs $\approx 10^{14}$ as much as Odo, and was swimming at an initial speed of 1 $[\frac{m}{s}]$. However, note that for David (since he is so much larger), the physics is a bit different and his swimming force follows a different form: $\cancel{f_{David} = m\ddot{x} - b\dot{x}^2}$ $f_{David} = m\ddot{x} + b\dot{x}^2$ where the ratio $\frac{m}{b}$ is about 0.1 $[m]$ (Note the difference in units). Find the distance that David coasts in the time it takes his speed to be reduced to 1% of its initial value. To write everything in units of David's body size, use that he is about 1 m tall (approximately correct).

Problem 5

Again consider the fidget spinner that I dropped off of Howey physics building a few weeks ago. If Howey has a height h, then how many revolutions did the fidget spinner make in total before hitting the ground?

Remember that I had the fidget spinner spinning at an angular frequency ω when I dropped it. Also remember that, if gravity is pointing in $-\hat{z}$ direction, then the spinner's axis of rotation was also in $-\hat{z}$ (i.e. it was rotating in the xy-plane).

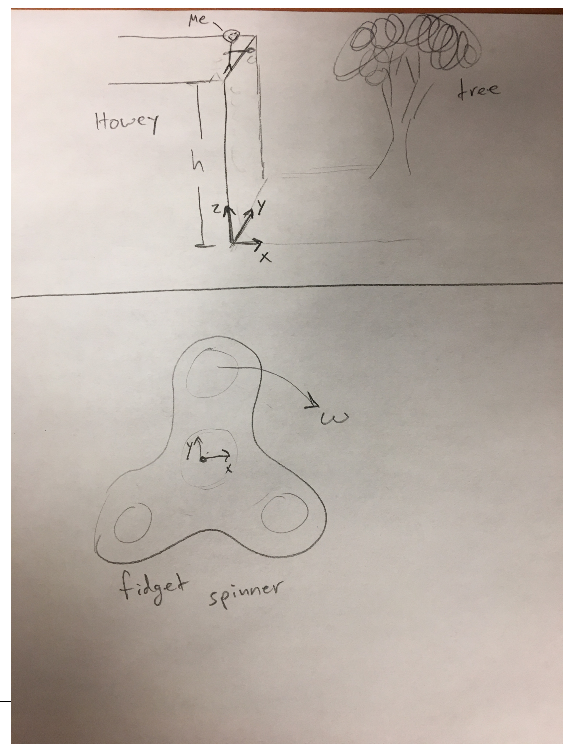

Figure 4: fidget spinner part 2