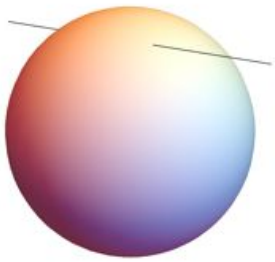
EXAM 3

We expect that you will complete this exam on your own without help from classmates, and without unfair use of the internet. The exam is, however, open notes and open book, and obviously see the links embedded in the pdf. Email dyanni3@gatech.edu with any questions or clarifications about what is being asked. The exam will be due at 5:00 pm, Friday November 30th. Drop it off in Dr Goldman's office or in class email it to both of us with the subject "exam 3". Good luck!

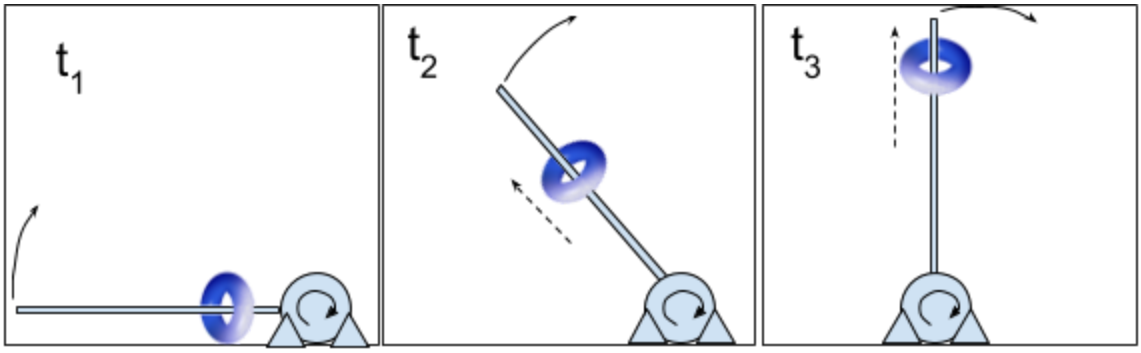
1) Two cylinders, A and B, roll without slipping down an incline. They have the same shape and both have uniform density, but different masses and moments of inertia. The mass of cylinder A is greater than mass of cylinder B; $m_A = 2\, m_B$. Which cylinder reaches the bottom first?

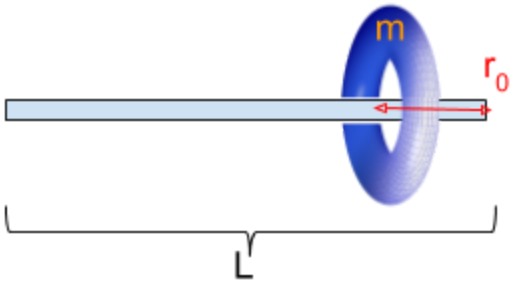
2) A sphere of radius R is centered at the origin. Find the moment of inertia about the rotation axis parallel to the x-axis given by $y = 0,\ z = \sqrt{\frac{3}{5}} \cdot R$, as shown.



3) Show that for any object the largest principal moment of its inertia tensor is less than or equal to the sum of the other two.

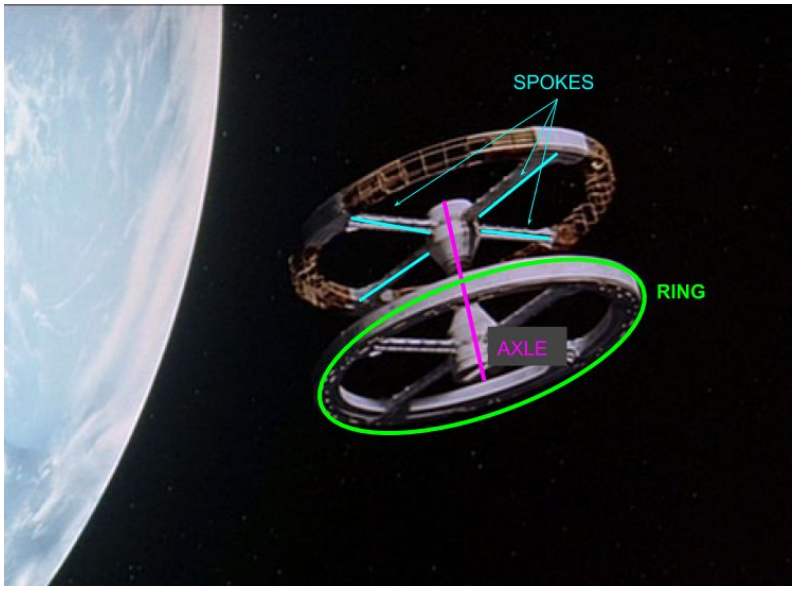
4) A doughnut shaped bead of mass *m* starts at rest a distance r_0 along a massless rigid rod of length L . A motor forces the rod to rotate at a constant angular speed ω. As the rod rotates the bead slides along it until eventually it will fly off the free end. Ignore gravity. The coefficient of friction between the bead and the rod is μ.





a) Find the force of the rod on the bead. *Hint: read K&K, this is related to Coriolis force*
b) Find the work done by the friction force as the bead moves from its initial position to the free end of the rod
c) How much work was done by the motor from the initial time until the bead flies off the end of the rod?

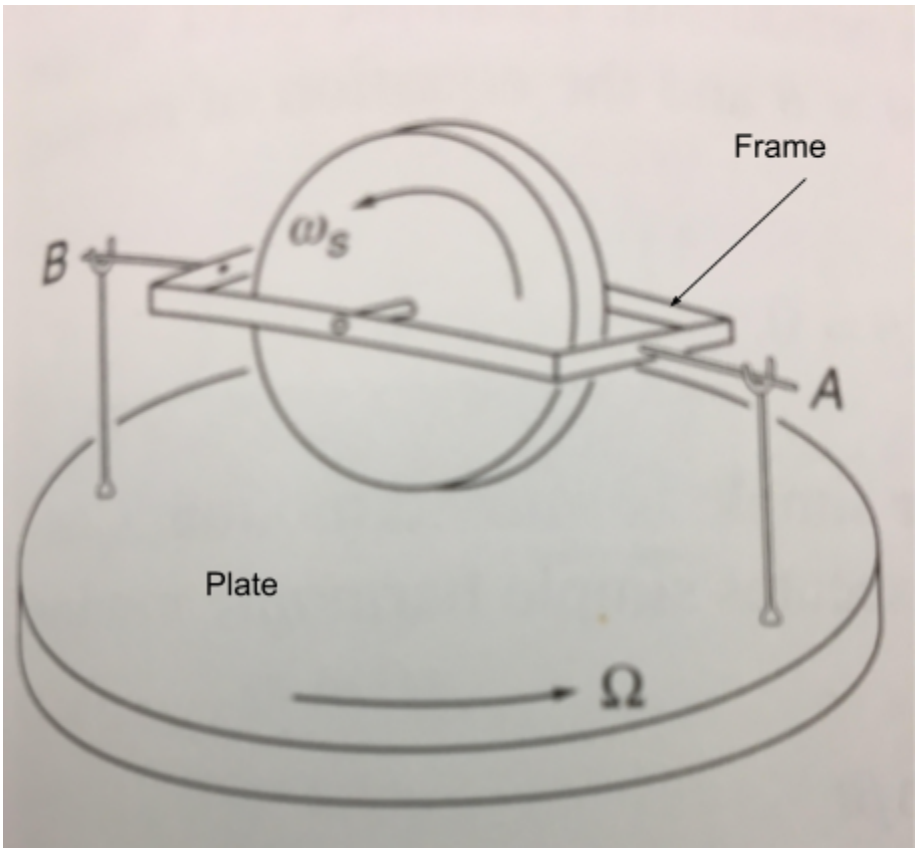
5) A space station is arranged as two rings each of radius R separated by an axle length l as shown (see also this clip from *2001: A Space Odyssey*)



Each ring has a mass M, ignore the mass of the axle and the "spokes". The rings rotate at an angular velocity $\vec{\omega}$ set so

that the centripetal acceleration is $g = 9.81\ m\,s^{-2}$, for the comfort of the residents. At a time t_0, the space station begins to accelerate toward a distant starchild at a rate $\vec{a}$. The angle between $\vec{\omega}$ and $\vec{a}$ is ϕ. Find the precession rate of the axle in terms of the parameters given.

6) Consider a gyrocompass as shown below. It consists of a flywheel that spins at $\vec{\omega}_s$ housed in a frame that is free to rotate about the A-B axis. The entire apparatus rests on a plate which is itself rotating at a constant rate Ω. If we set up a gyrocompass frame in Atlanta (latitude 33.75° N) with the A-B axis pointing vertically (toward the sky), then the rotation of the earth supplies Ω, and $\vec{\omega}_s$ will reorient to point north. In fact the spin axis, $\frac{\vec{\omega}_s}{|\vec{\omega}_s|}$, will oscillate in the horizontal plane about the direction of the north pole at some frequency. Find that frequency and solve for the angular speed of the earth in terms of $\vec{\omega}_s$. *Hint: read K&K and use a small angle approximation*



7) If I weigh *mg* at the North Pole, how much do I weigh on the equator assuming the earth is spherical with radius *R*? What about if I'm in Atlanta and run really fast (at a speed *v*) to the east?

8) OPTIONAL Extra credit (5 points) A Foucault pendulum is an incredibly clever device that makes use of the fictitious forces that arise in rotating frames to demonstrate the earth's rotation. See example 9.11 of K&K for an introduction. Flat Earthers, obviously, have a difficult time rationalizing the results of Foucault pendulum type experiments and generally outright reject them as faked. However, they sometimes raise interesting objections. One argument by a flat earther goes as follows (taken from a Quora post here): "In fact, if Foucault's pendulum premises were valid, we

could use the earth's rotation to generate electricity, and solve the world's energy problems. " Briefly (250 to 500 words) describe how a Foucault pendulum works, and comment on this flat earther's argument. You may include a sketch if you like.