Vectors and Kinematics

August 27, 2018

Problem 1- Triple Product 1

Show that

$$\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$$

Problem 2- Fidget Spinner

Last night I put an LED in one of the three holes of a fidget spinner, and then set the fidget spinner spinning at an angular velocity ω. I then dropped the spinner from the top of Howey – height h. Note that I dropped the spinner so that its rotation was in the horizontal, xy plane, not the xz plane. In other words the LED will corkscrew as it falls, not tumble.

a

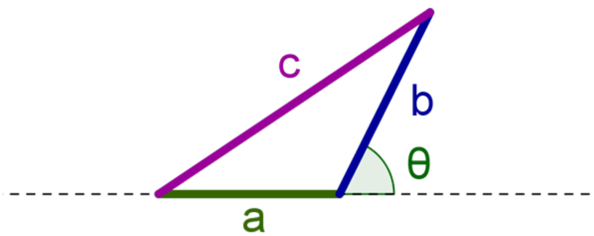
Describe the position and velocity of the LED at an arbitrary time after the drop, t_0, in the reference frame of someone standing on the ground watching the LED.

b

Describe the position and velocity of the LED at an arbitrary time after the drop, t_0, in the reference frame of the fidget spinner's center of mass. Is this an inertial reference frame? How could you prove that it is or isn't?

Note that I gave some incorrect advice to one of the groups during class. Think about whether Newton's laws hold

Problem 3- Law of Cosines



Find the length of C using the dot product.

Problem 4- Triple Product 2

a

Show that the volume of a parallelepiped formed by the vectors $\vec{A}$, $\vec{B}$, and $\vec{C}$ is given by

$$V = \vec{A} \cdot (\vec{B} \times \vec{C})$$

b

Find $\vec{C} \cdot (\vec{A} \times \vec{B})$ in terms of V.

Problem 5- Polar Coordinates

a

Write the 2D polar coordinate system unit vectors, $\hat{r}$ and $\hat{\theta}$, in terms of the 2D Cartesian unit vectors $\hat{x}$ and $\hat{y}$, and in terms of θ.

b

Write the time derivatives of $\hat{r}$ and $\hat{\theta}$ entirely in terms of θ, r, and $\dot{\theta}$.

c

Do the same exercise in spherical polar coordinates