Машинное обучение
на примере глубокого обучения в компьютерного зрения

Занятие 3
Backpropagation и Нейронные сети

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На прошлом занятии: **Классификация изображений**

Ключевая задача компьютерного зрения

К какому классу принадлежит изображение?
классы: человек, животное, автомобиль …

кот

На прошлом занятии: **Линейный классификатор**

Image

Array of **32x32x3** numbers
(3072 numbers total)

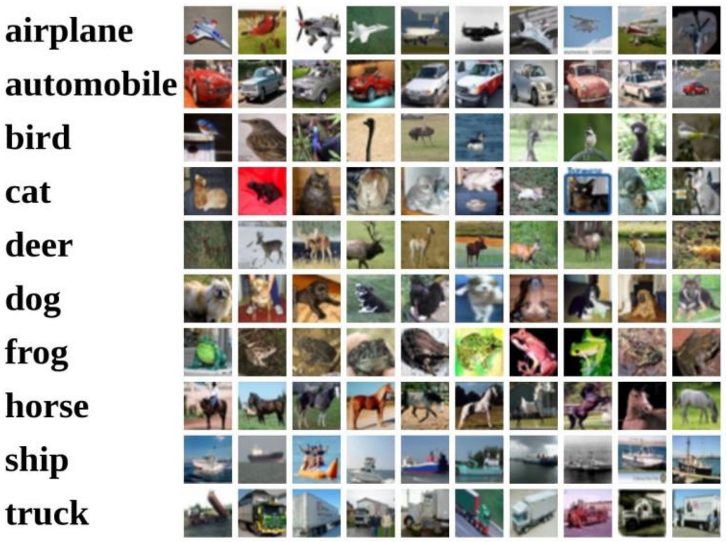
CIFAR-10
50,000 training images
10,000 testing images
10 classes

$$\underbrace{s = f(x,W)}_{10\times1} = \underbrace{\boxed{W}}_{10\times3072}\overbrace{\boxed{x}}^{3072\times1} + \underbrace{\boxed{b}}_{10\times1}$$

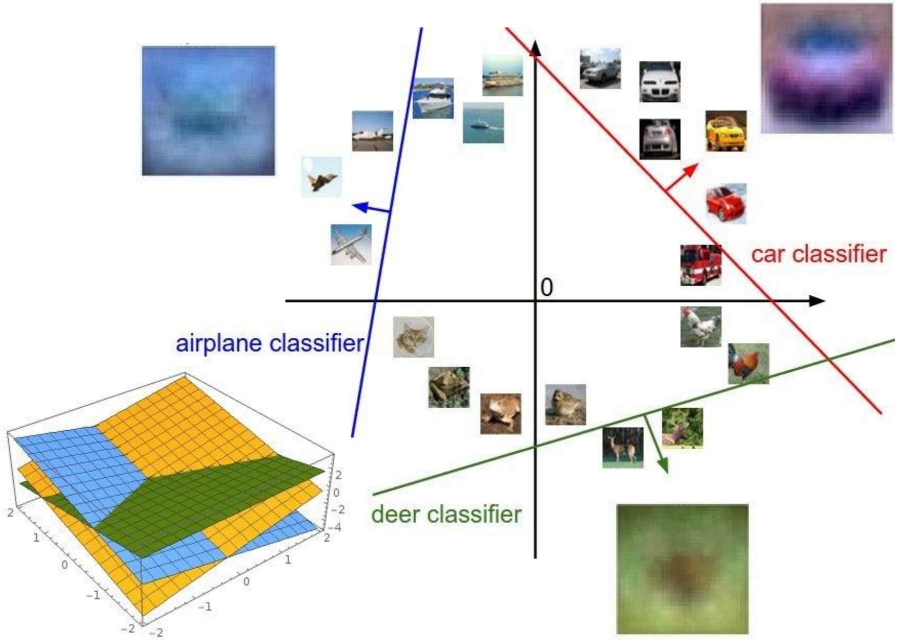
s – scores
W – weights or parameters
x – image pixels
b – bias

На прошлом занятии: **Интерпретация линейного классификатора**

CIFAR-10



$$f(x,W) = Wx + b$$



Example trained weights of a linear classifier trained on CIFAR-10:

На прошлом занятии: **Функции ошибки**

Image

x_i - image
y_i - label, element of a set {0, 1, …}

scores $s = f(x_i, W) = [s_0, \dots s_{y_i}, \dots]$

Loss over dataset:

$$L = \frac{1}{N}\sum_{i=1}^{N} L_i$$

Multiclass SVM (hinge) loss:

$$\boxed{L_i = \sum_{i \neq y_i} \max(0, s_i - s_{y_i} + 1)}$$

Cross-entropy (softmax) loss:

$$\boxed{L_i = -\log \frac{e^{s_{y_i}}}{\sum_j e^{s_j}}}$$

На прошлом занятии: **Регуляризация**

Softmax or SVM

λ - regularization strength (hyperparameter)

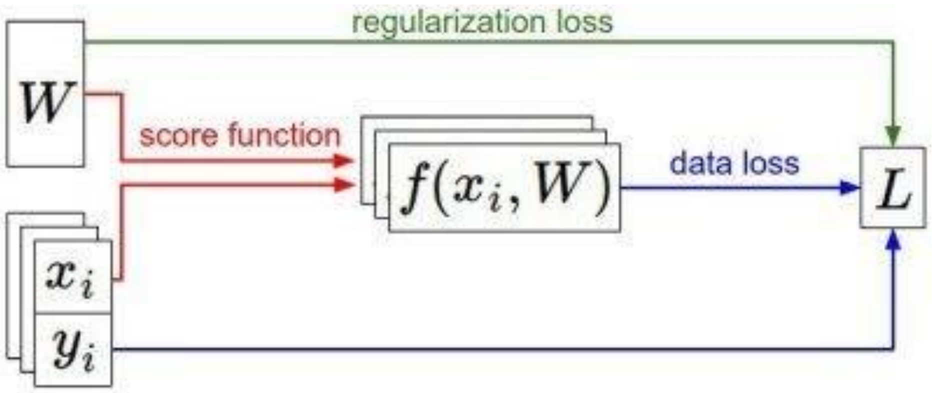
Full loss $$L = \underbrace{\frac{1}{N}\sum_{i=1}^{N} L_i}_{\text{Data loss}} + \underbrace{\lambda R(W)}_{\text{Regularization}}$$

How do we find the best W?

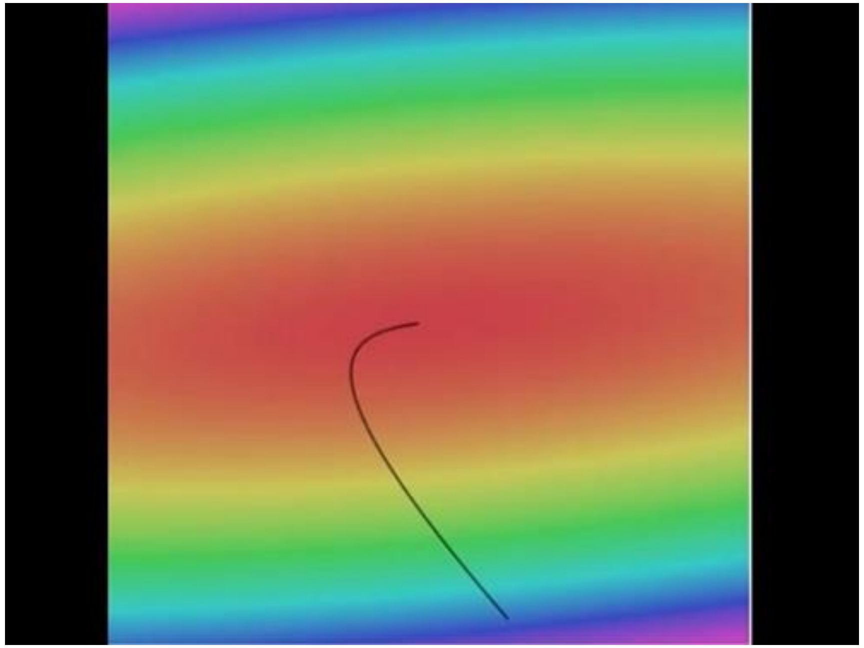
L2 regularization $R(W) = \sum_k \sum_l W_{k,l}^2$

L1 regularization $R(W) = \sum_k \sum_l |W_{k,l}|$

Elastic net (L1 + L2) $R(W) = \sum_k \sum_l \beta W_{k,l}^2 + |W_{k,l}|$



Оптимизация



```
# Vanilla Minibatch Gradient Descent

while True:
  data_batch = sample_training_data(data, 256) # sample 256 examples
  weights_grad = evaluate_gradient(loss_fun, data_batch, weights)
  weights += - step_size * weights_grad # perform parameter update
```

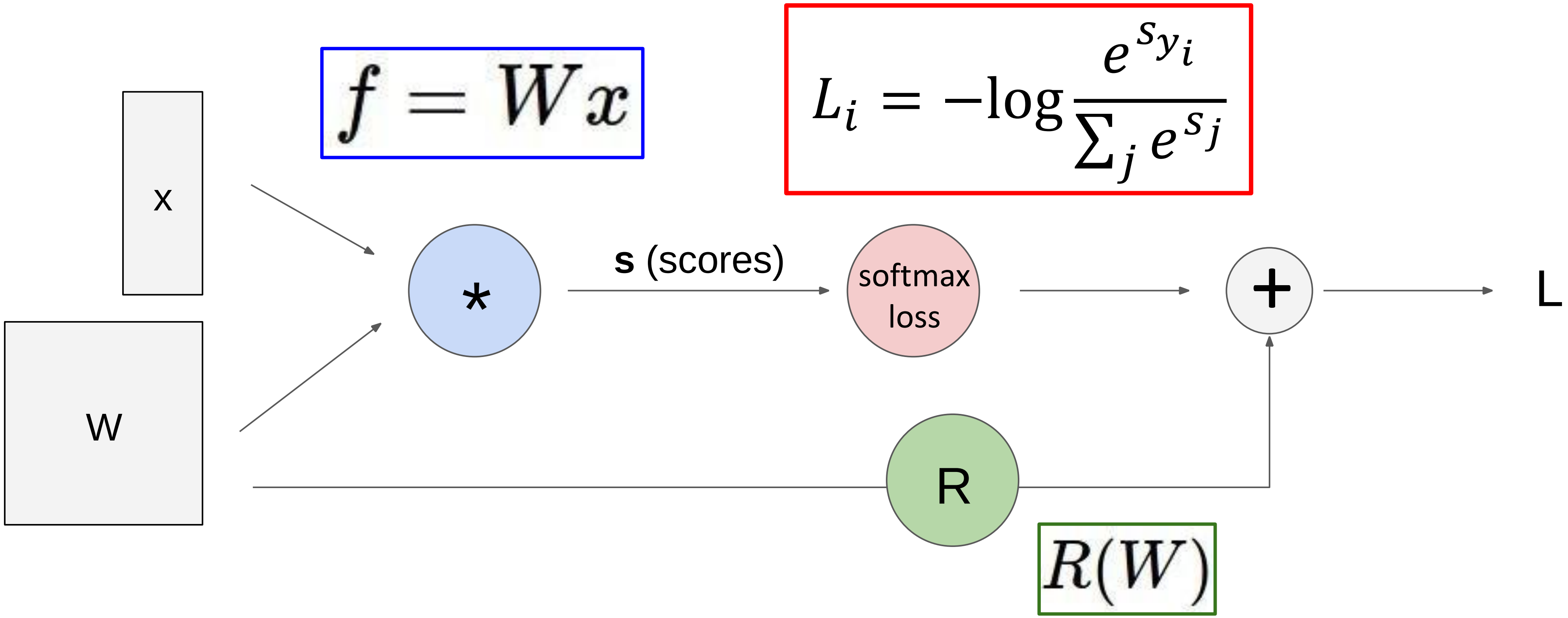
Метод градиентного спуска

$$\frac{dL(w)}{dw} = \lim_{h \to 0} \frac{L(w+h) - L(w)}{h}$$

Численные градиенты: медленно, не точно, быстро реализовать

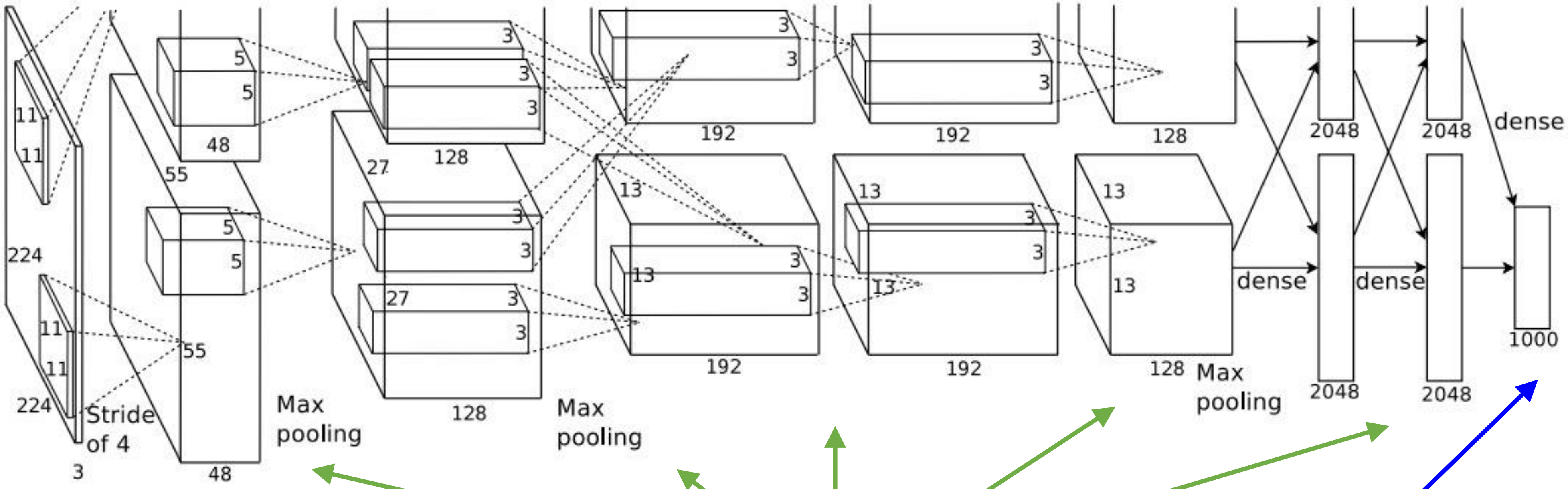
Аналитические градиенты: быстро, точно, можно ошибиться

Вычислительный граф



Вычислительный граф: **Convolutional Network**

AlexNet



input image

weights

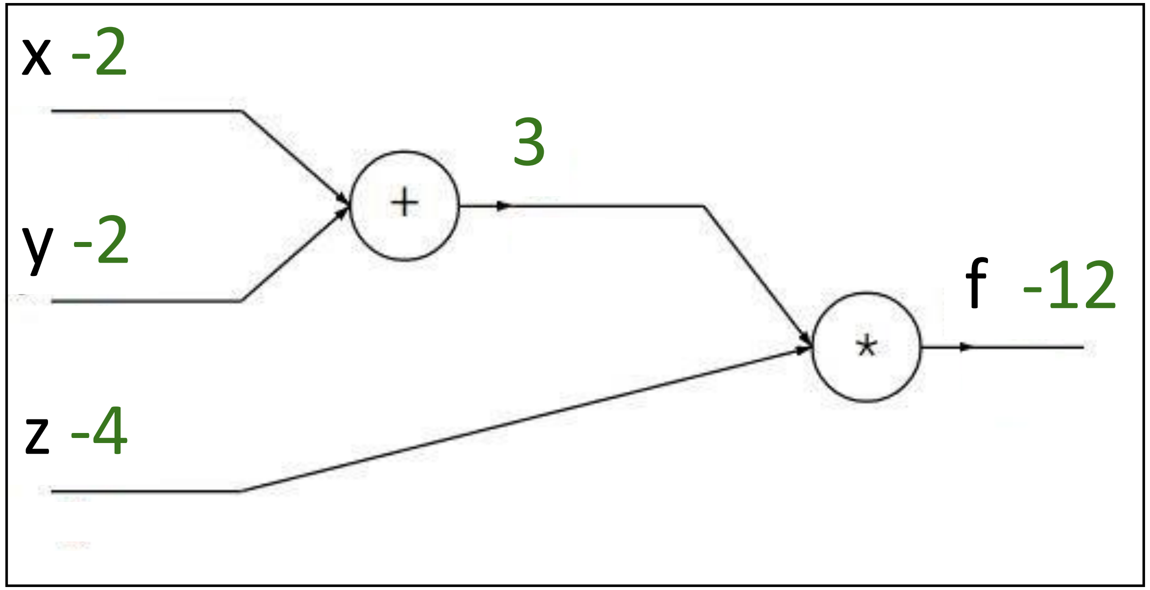
loss

Backpropagation

Backpropagation: пример

$$f(x, y, z) = (x + y)z$$

e.g. x = -2, y = 5, z = -4



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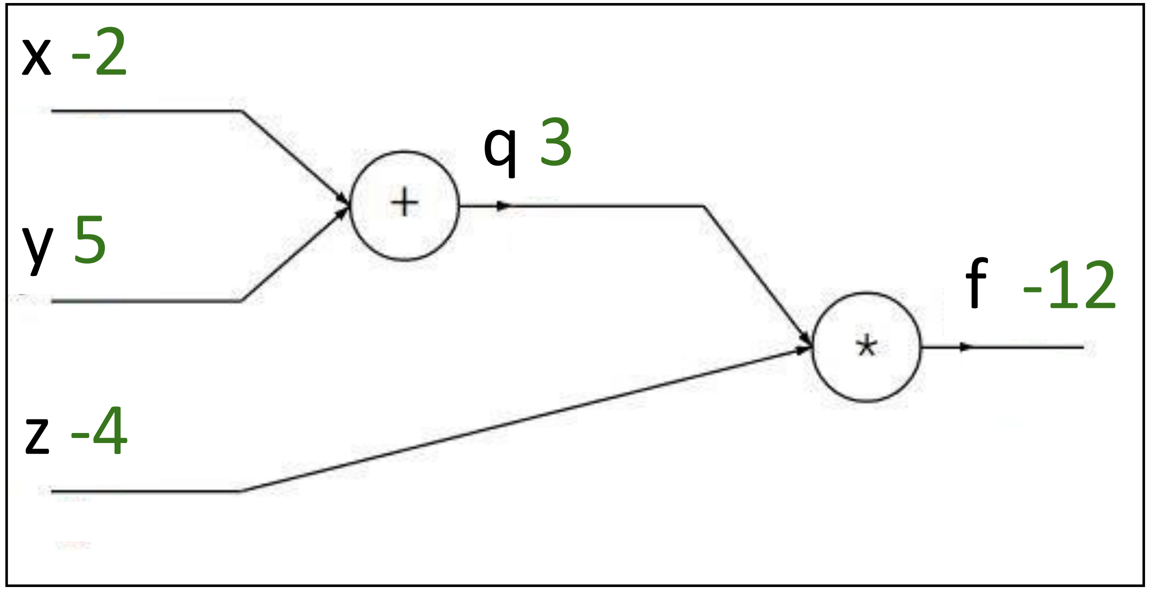
Want: $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}$



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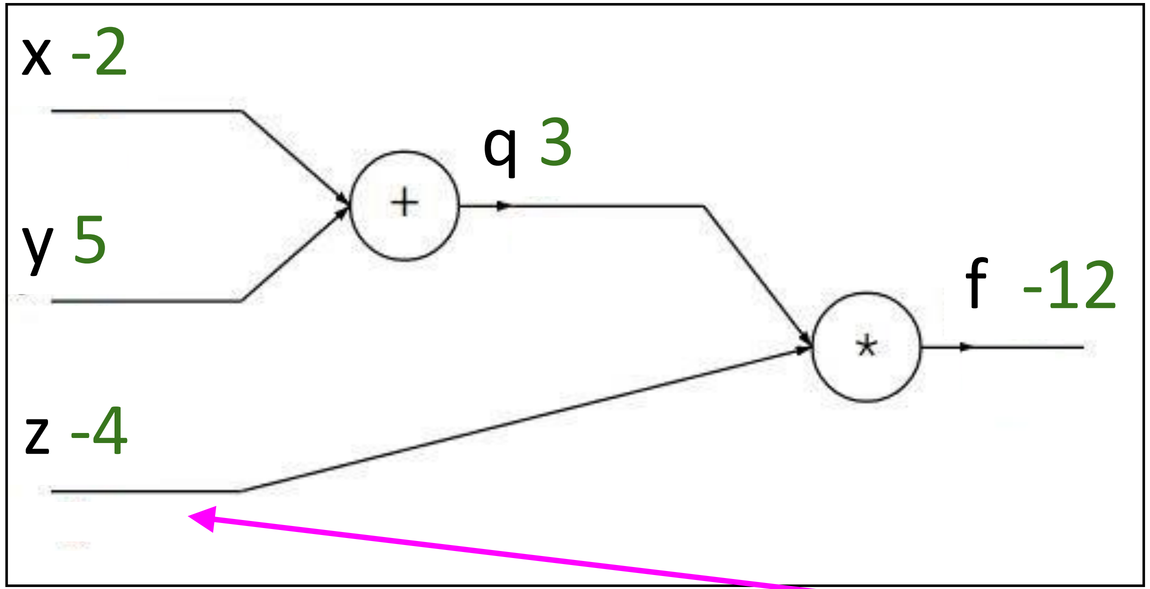

$$\boxed{\frac{\partial f}{\partial z}}$$

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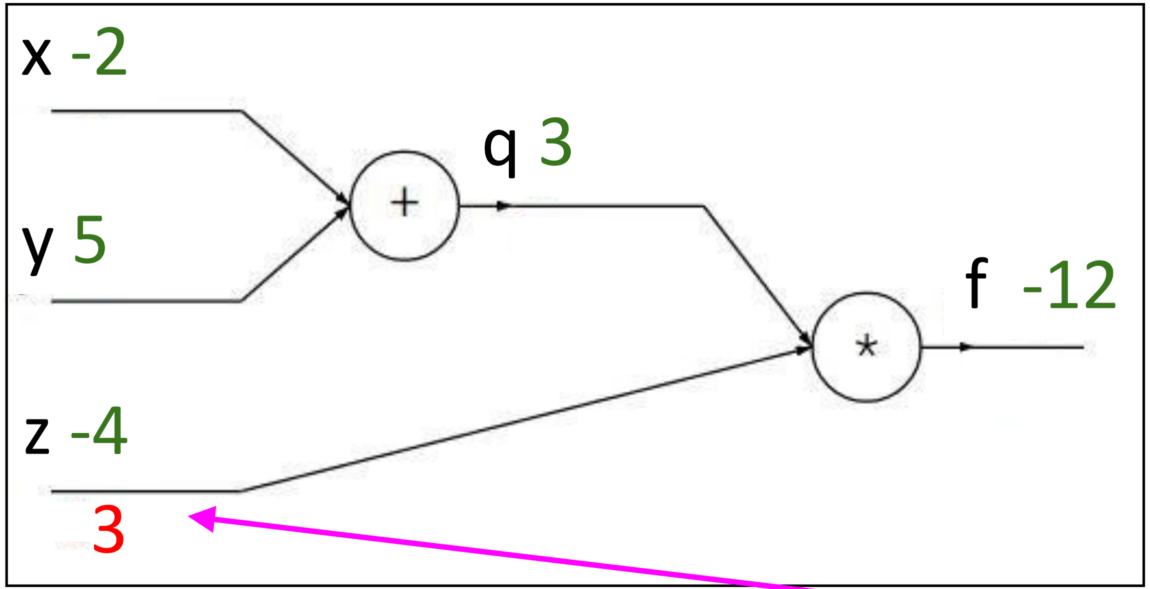

$\frac{\partial f}{\partial z}$

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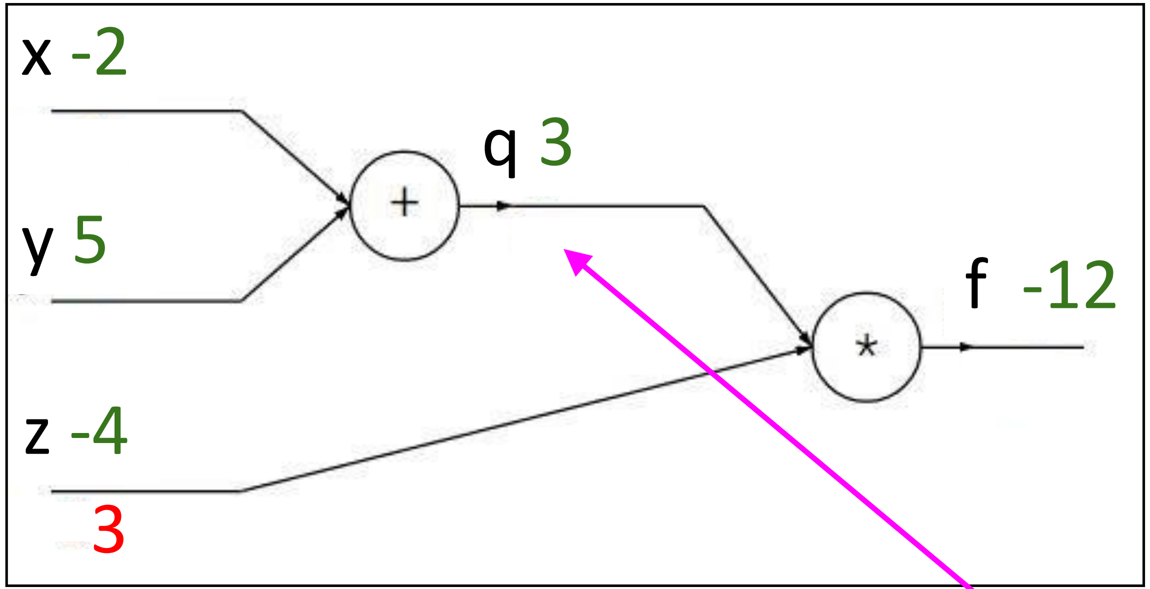

$$\frac{\partial f}{\partial q}$$

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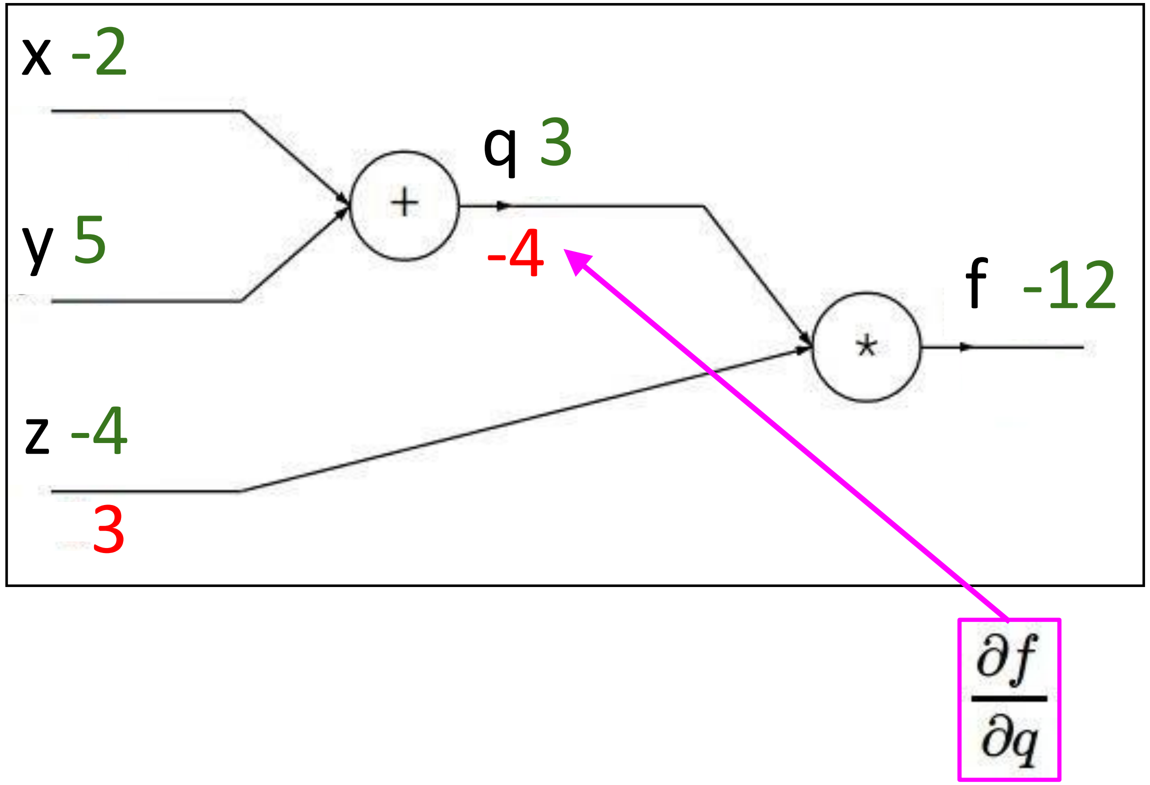

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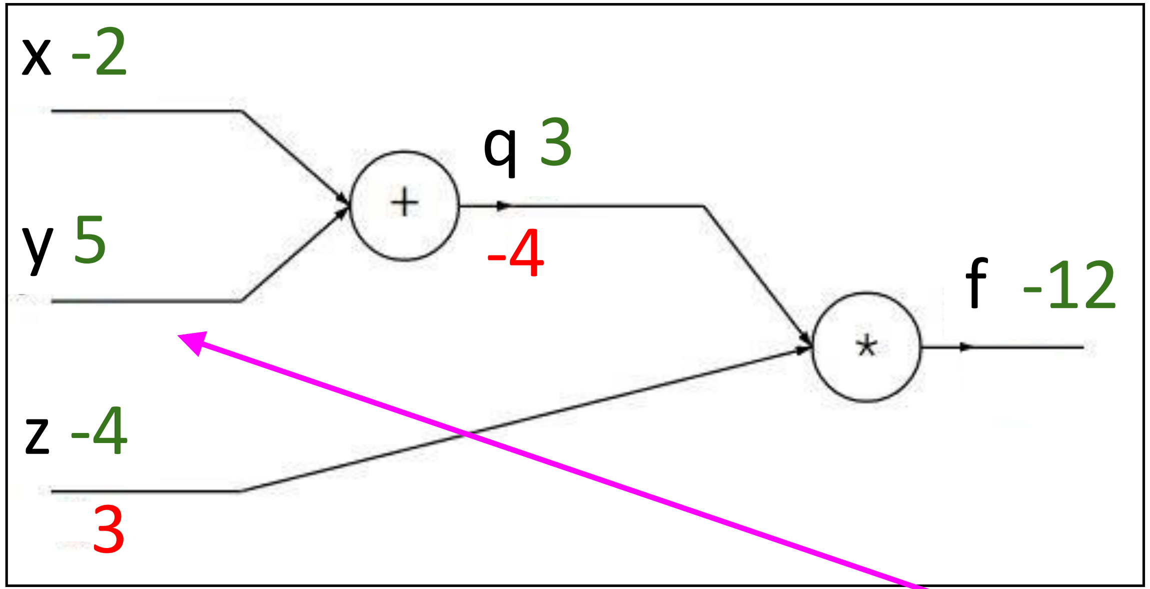

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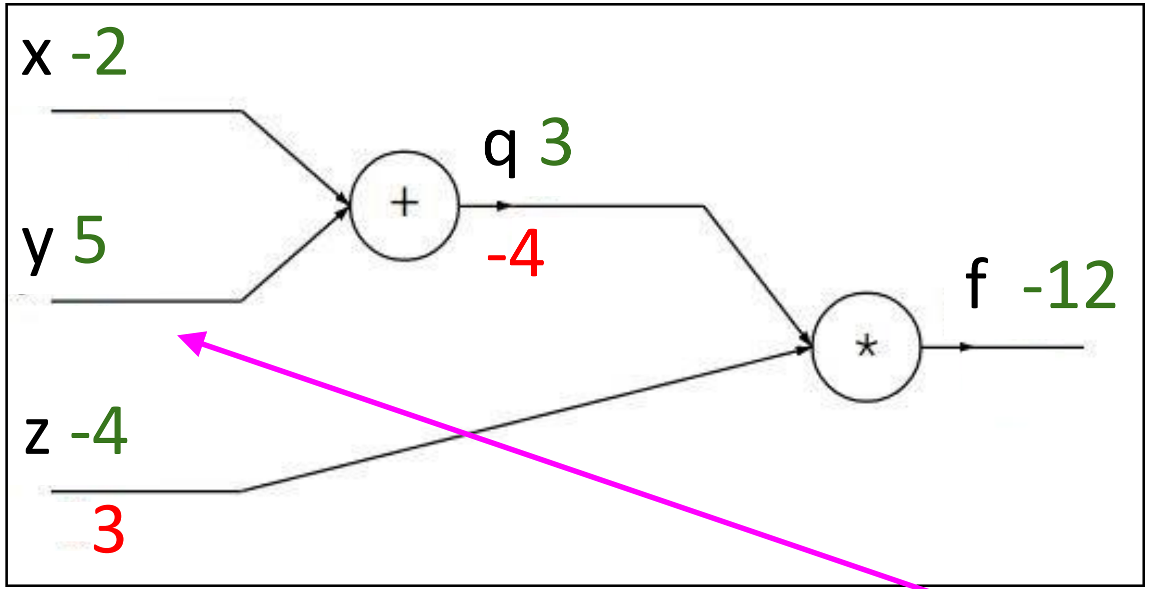

$\frac{\partial f}{\partial y}$

Правило дифференцирования:

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial q}\frac{\partial q}{\partial y}$$

Входящий градиент

Локальный градиент

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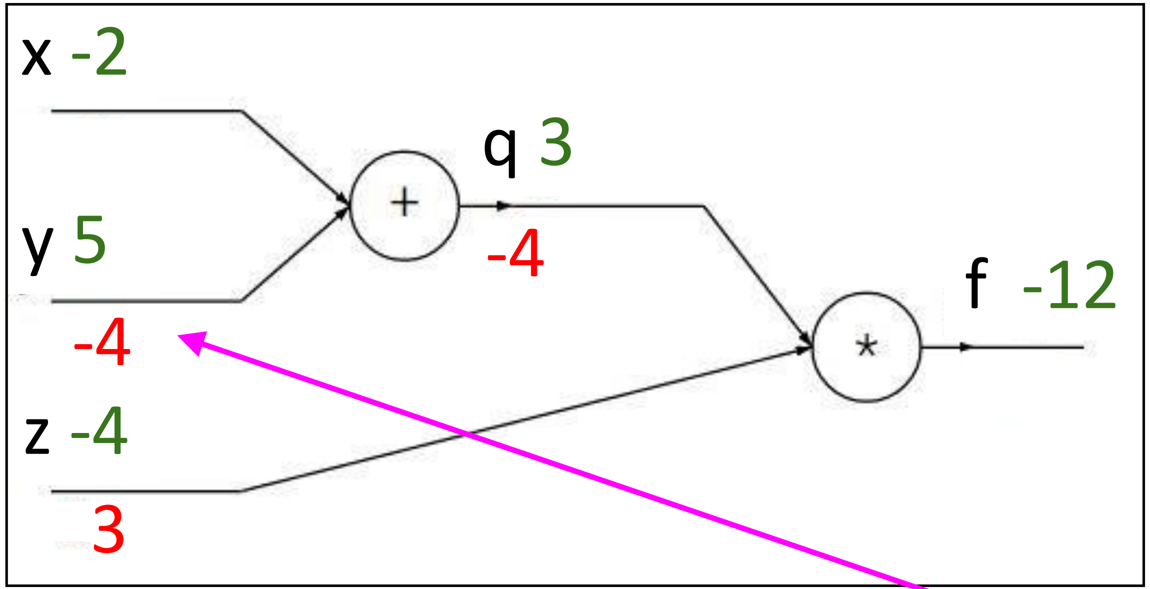

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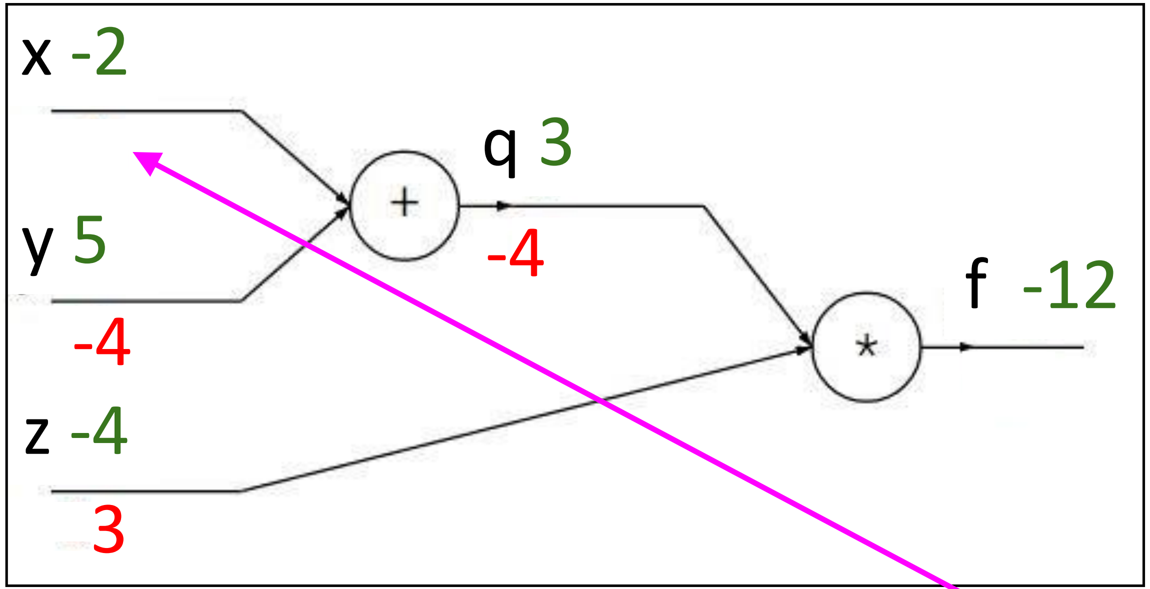

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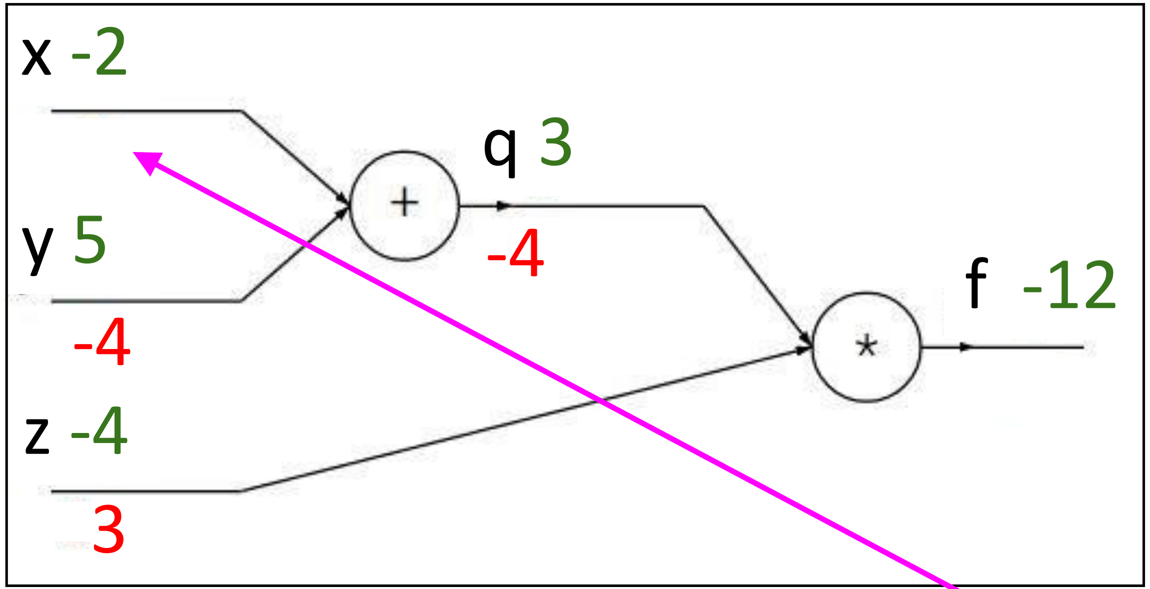

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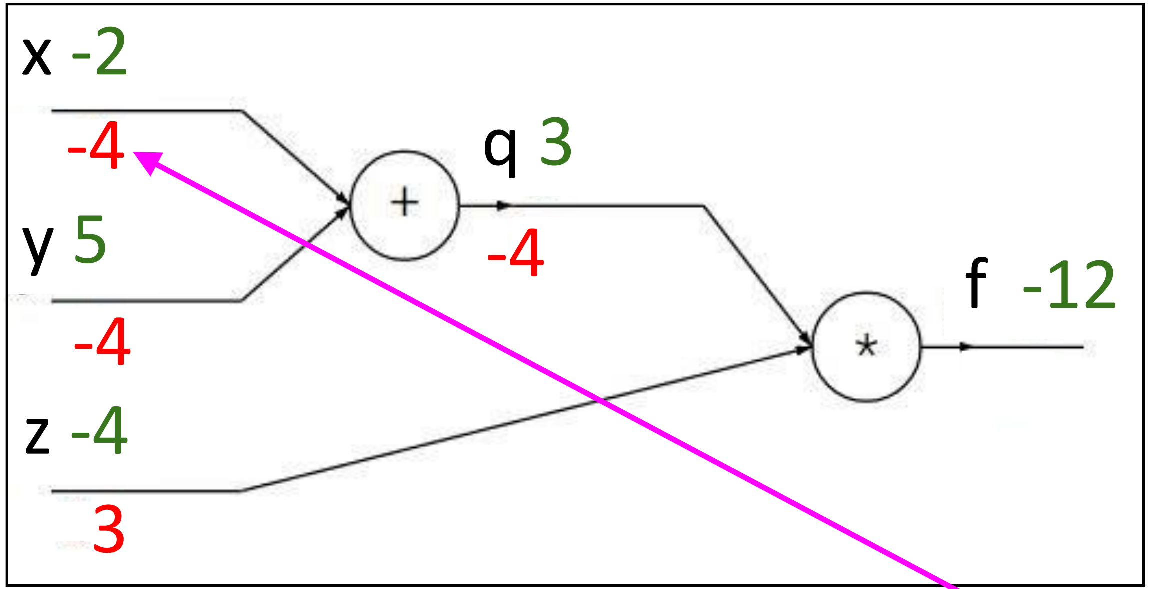

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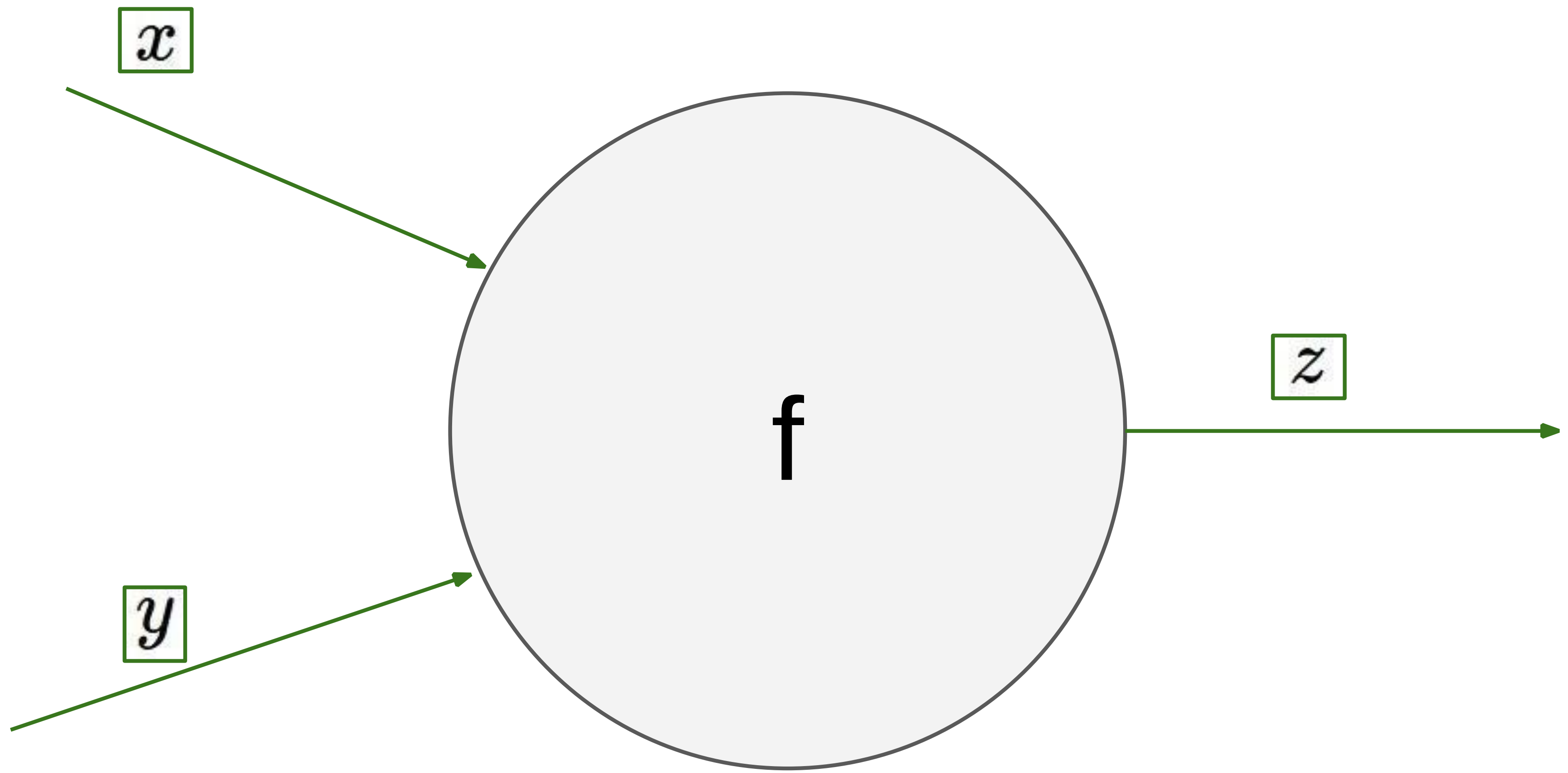
Входящий градиент

Локальный градиент

Backpropagation: общий случай



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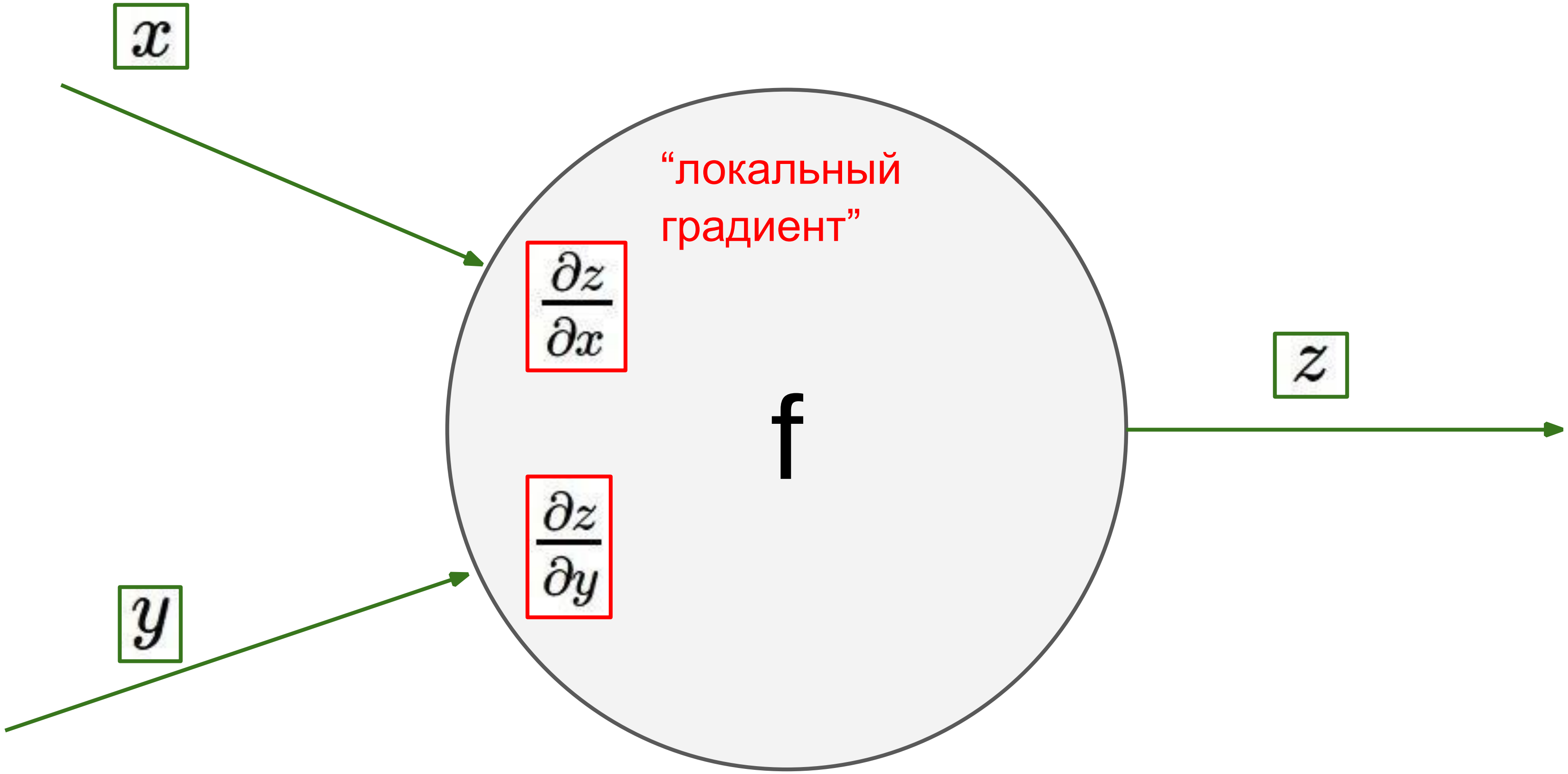

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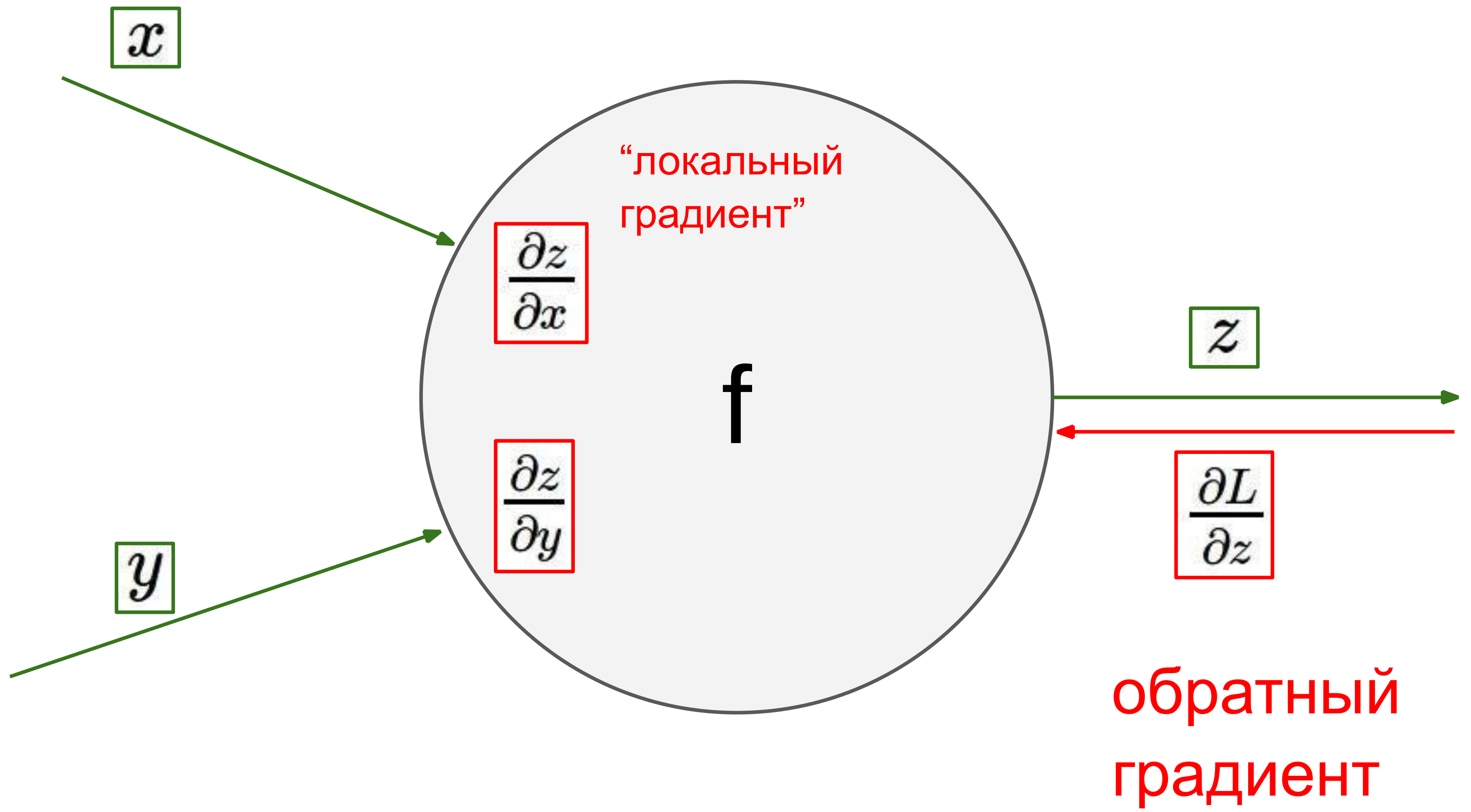

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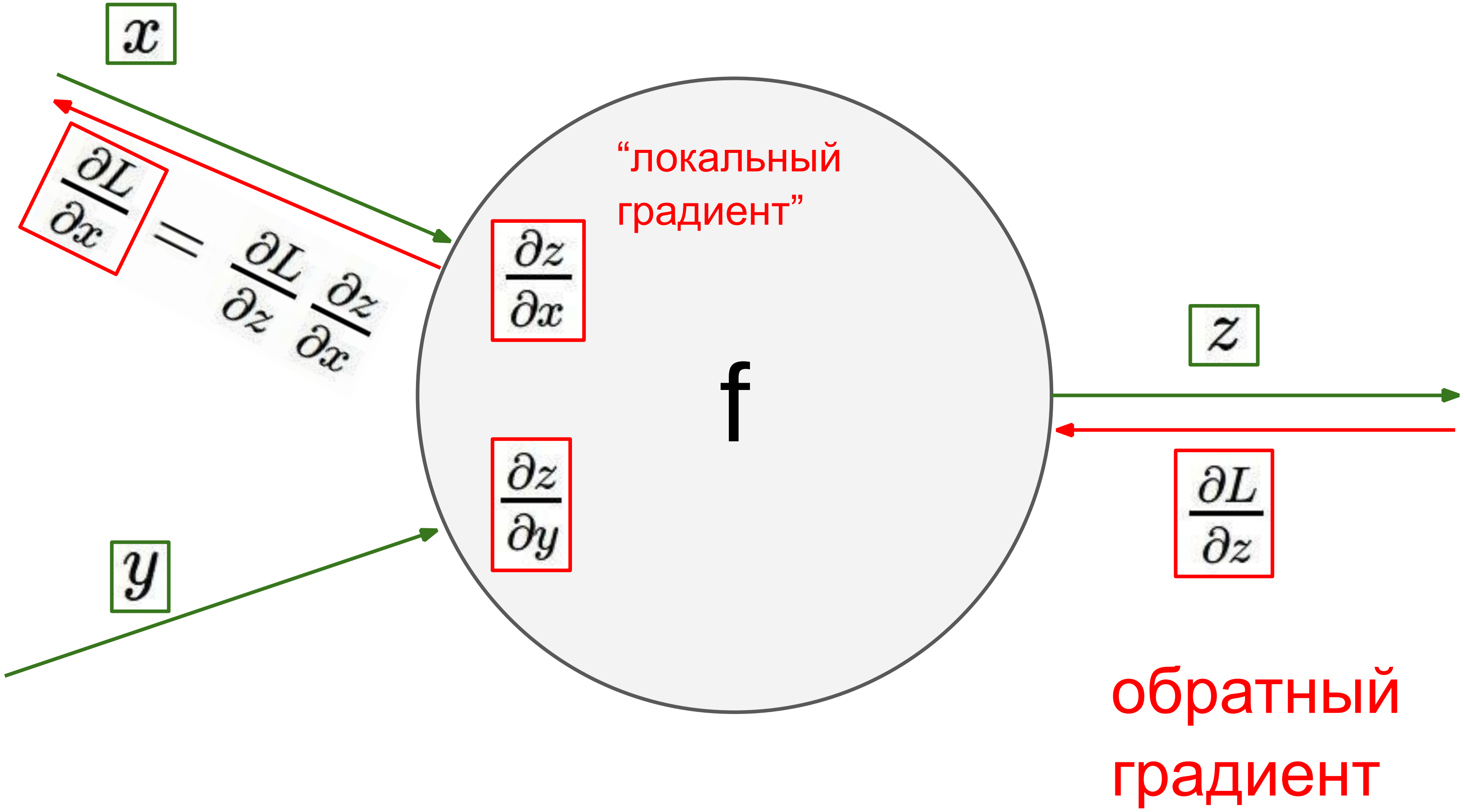

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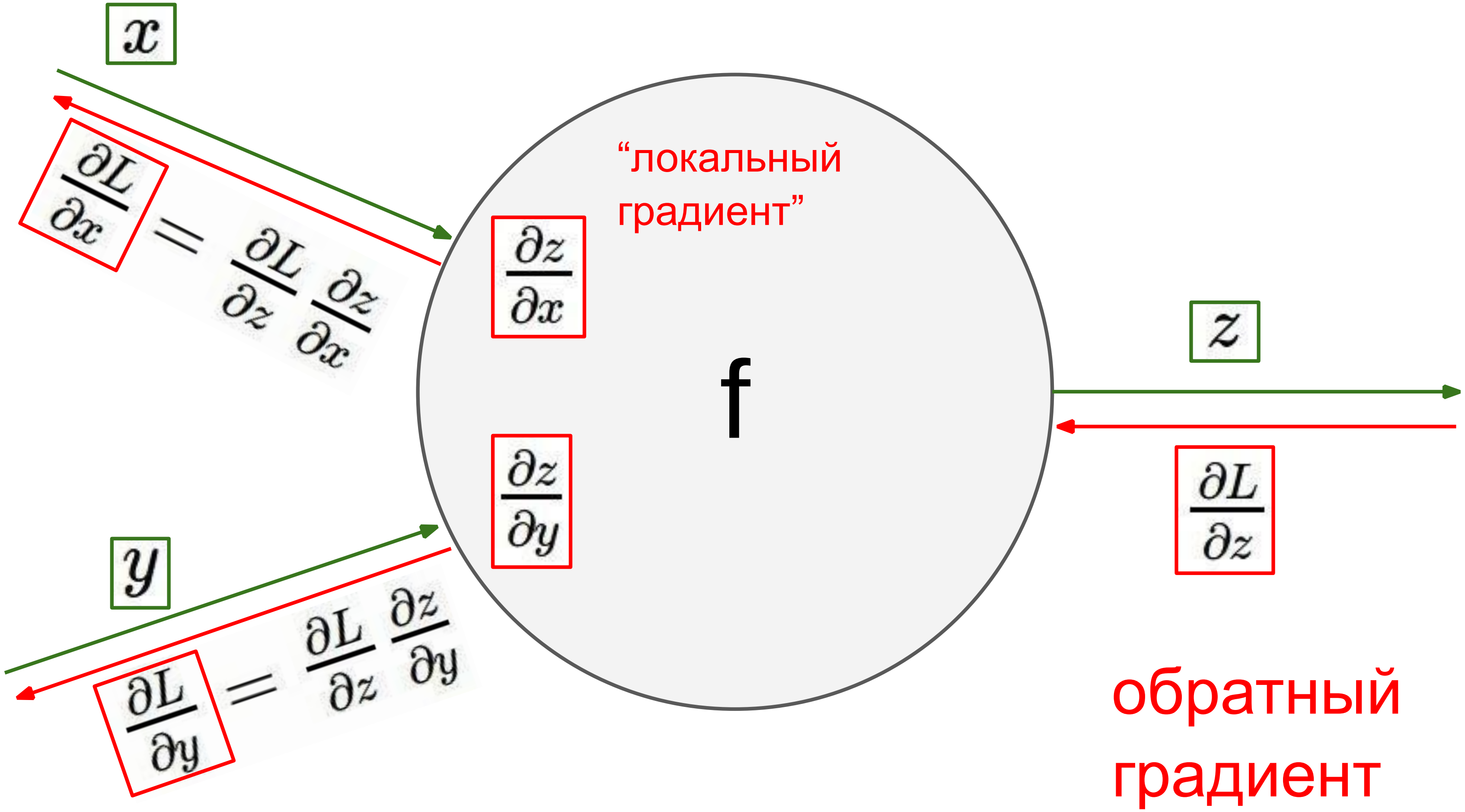

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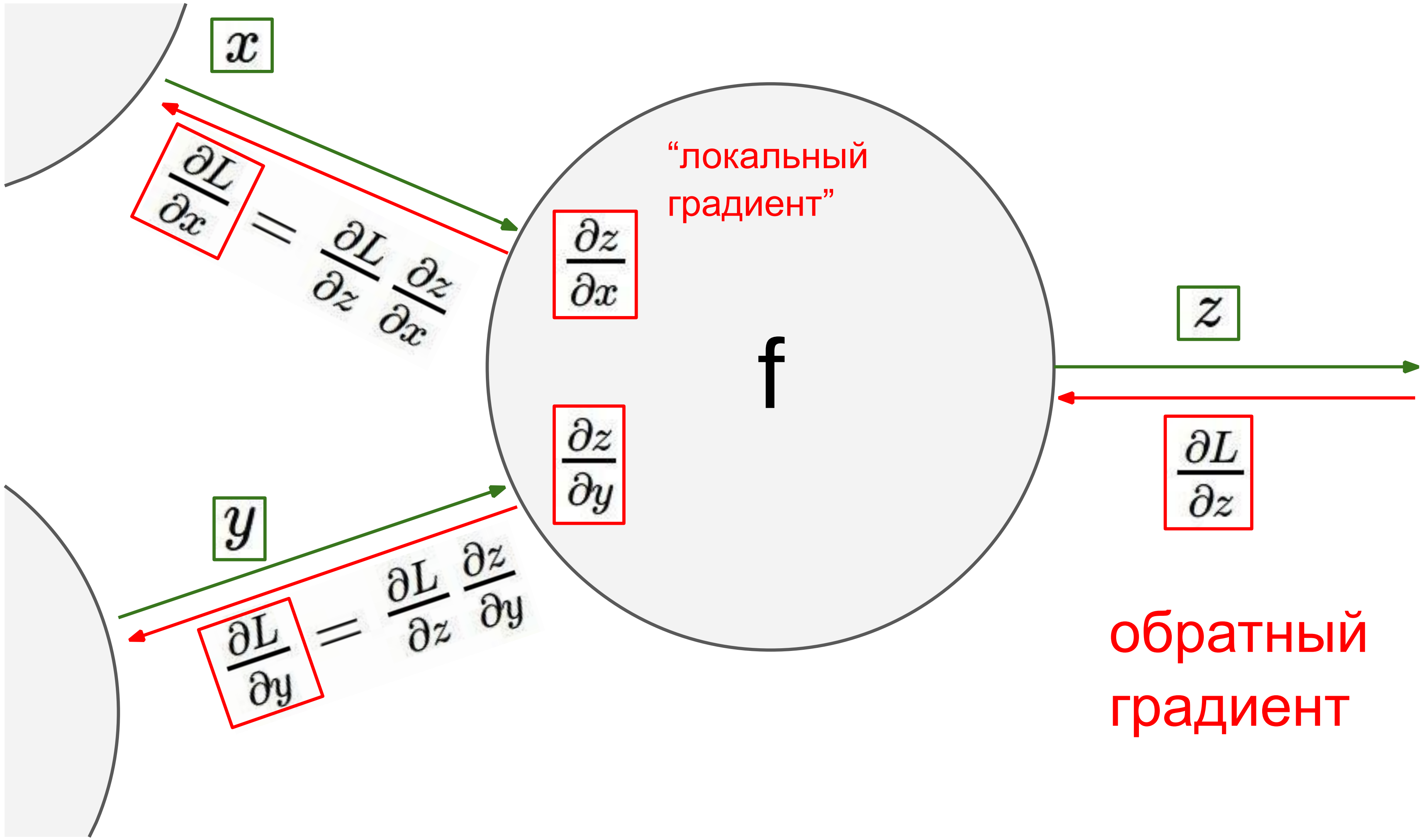
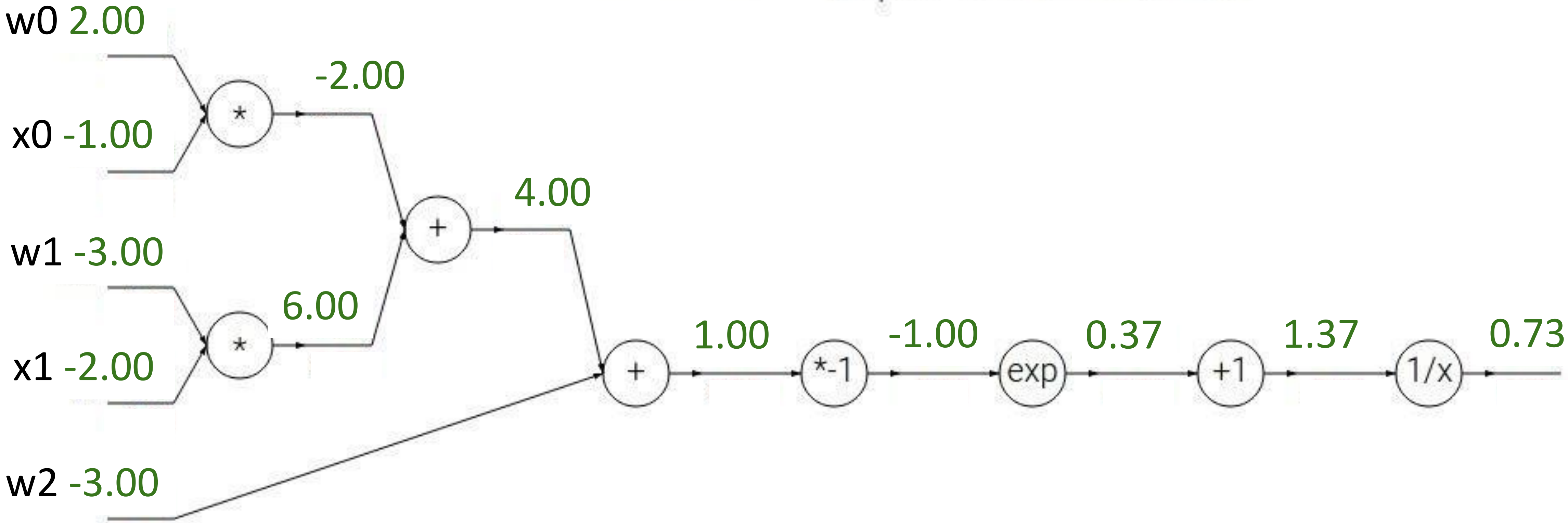


Пример:

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



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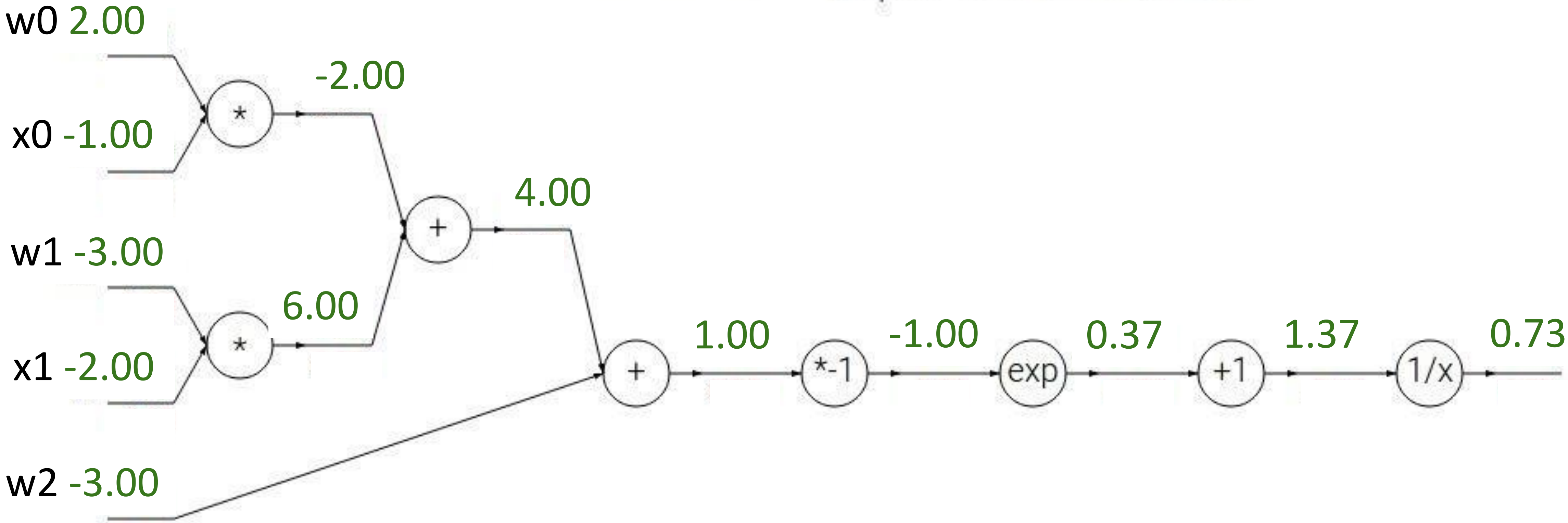

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$

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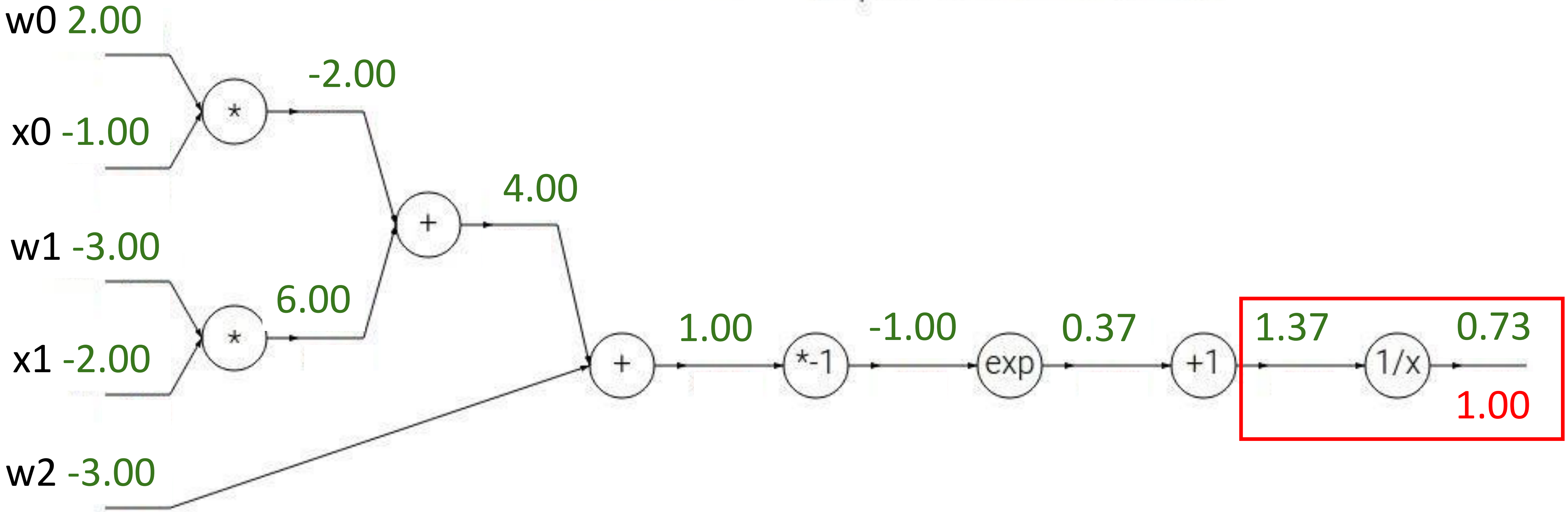

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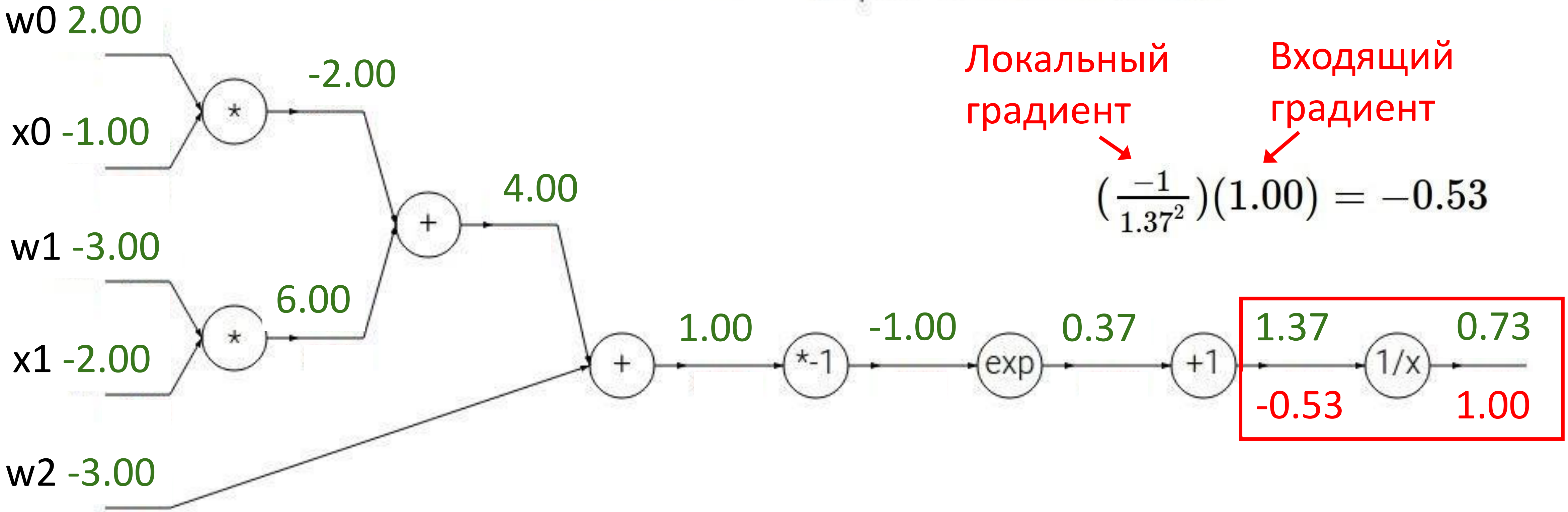

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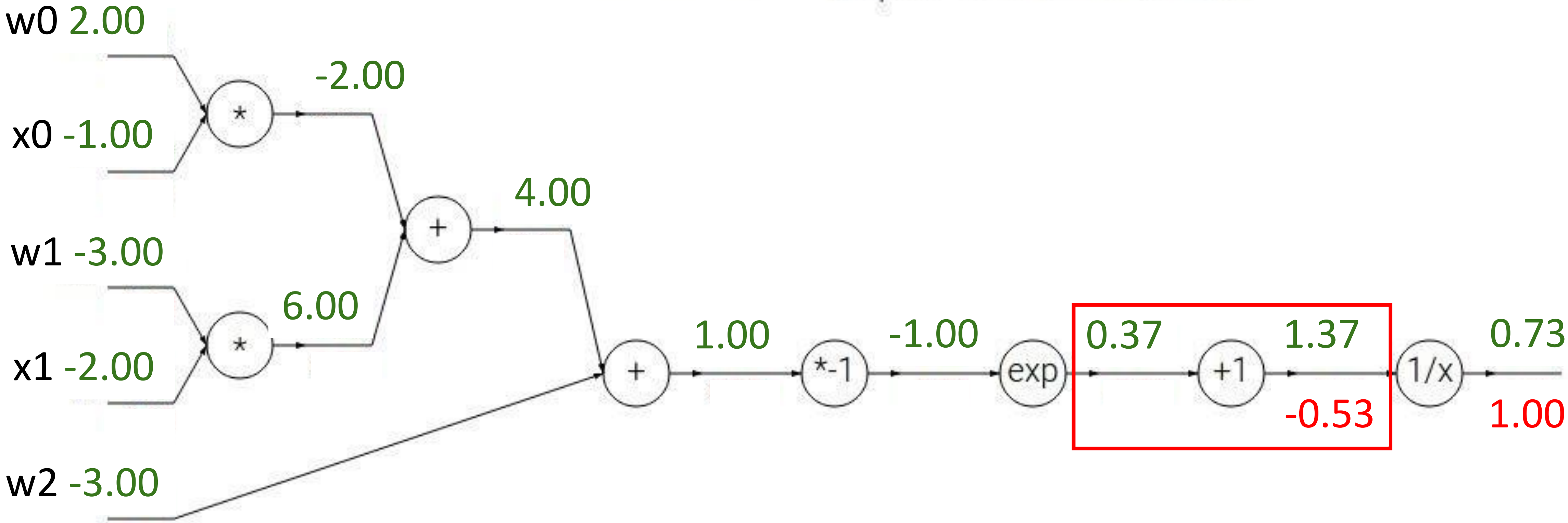

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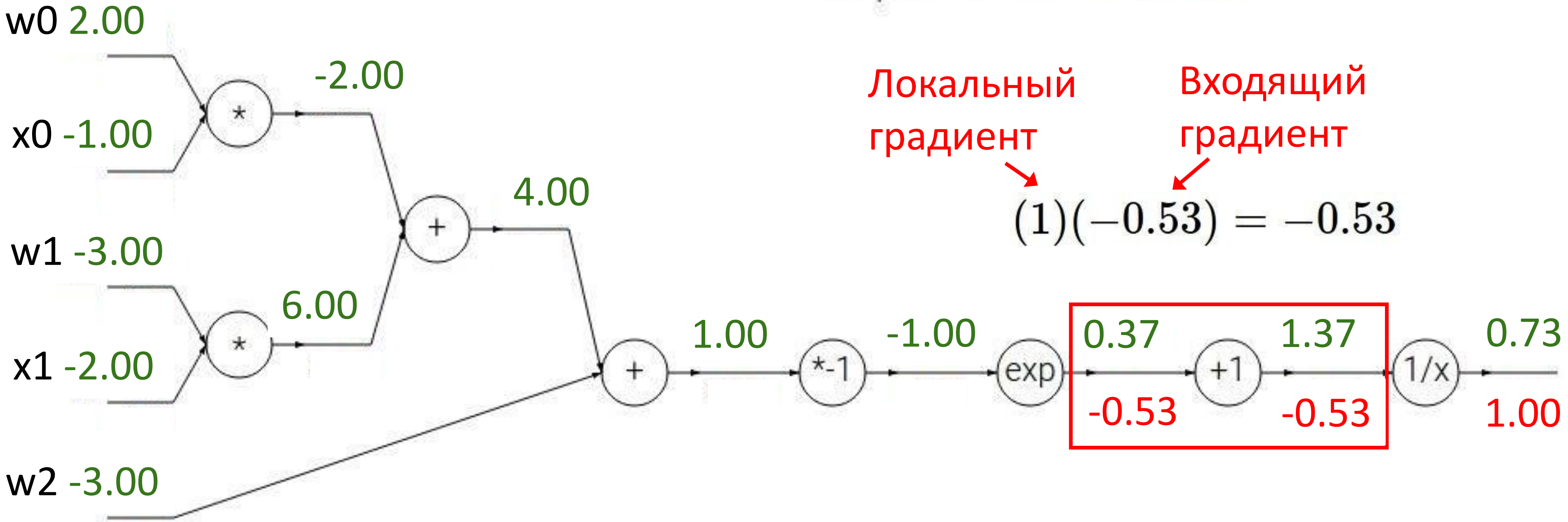

$$(1)(-0.53) = -0.53$$

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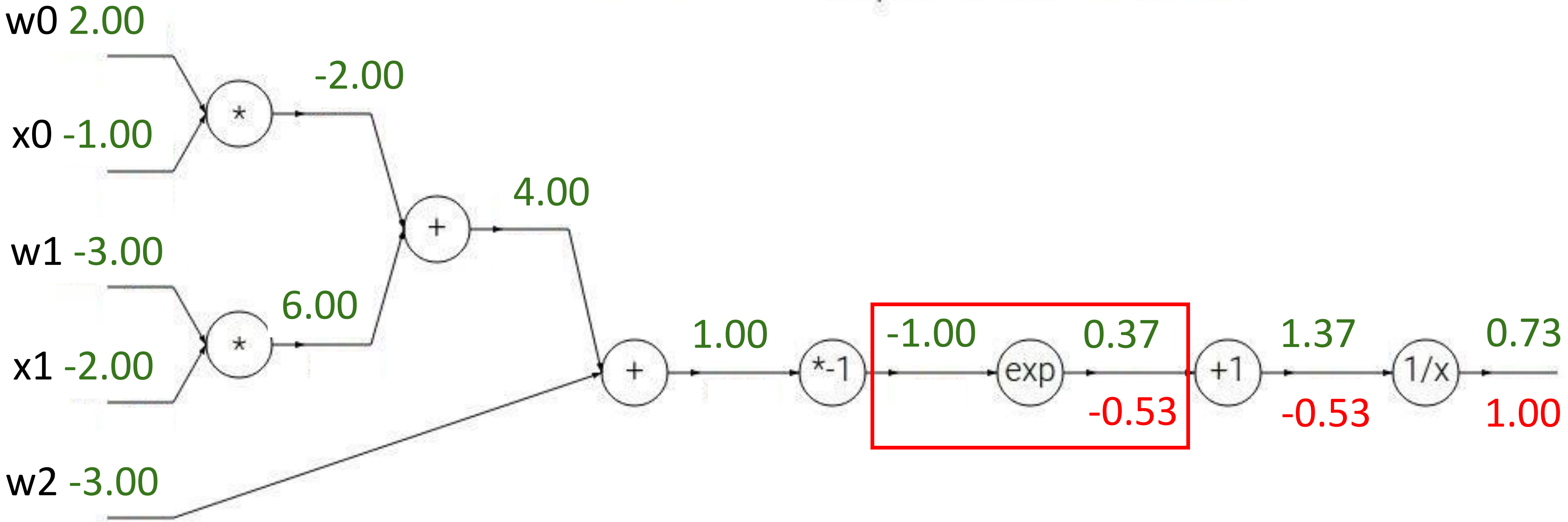

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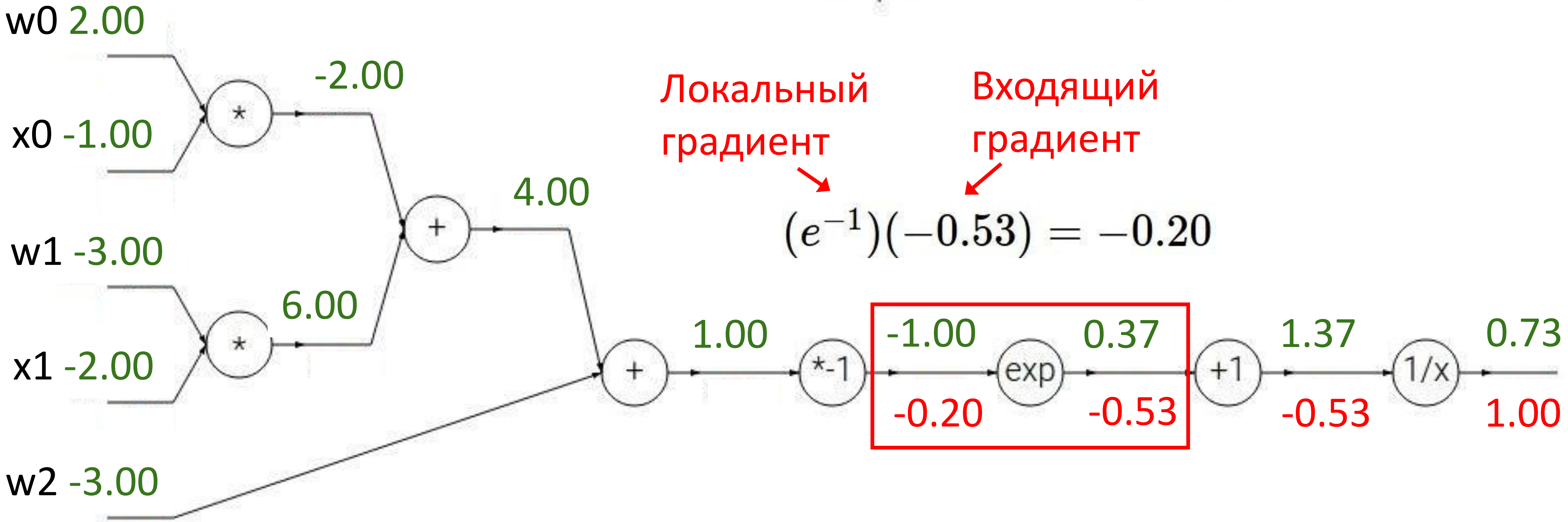

$$(e^{-1})(-0.53) = -0.20$$

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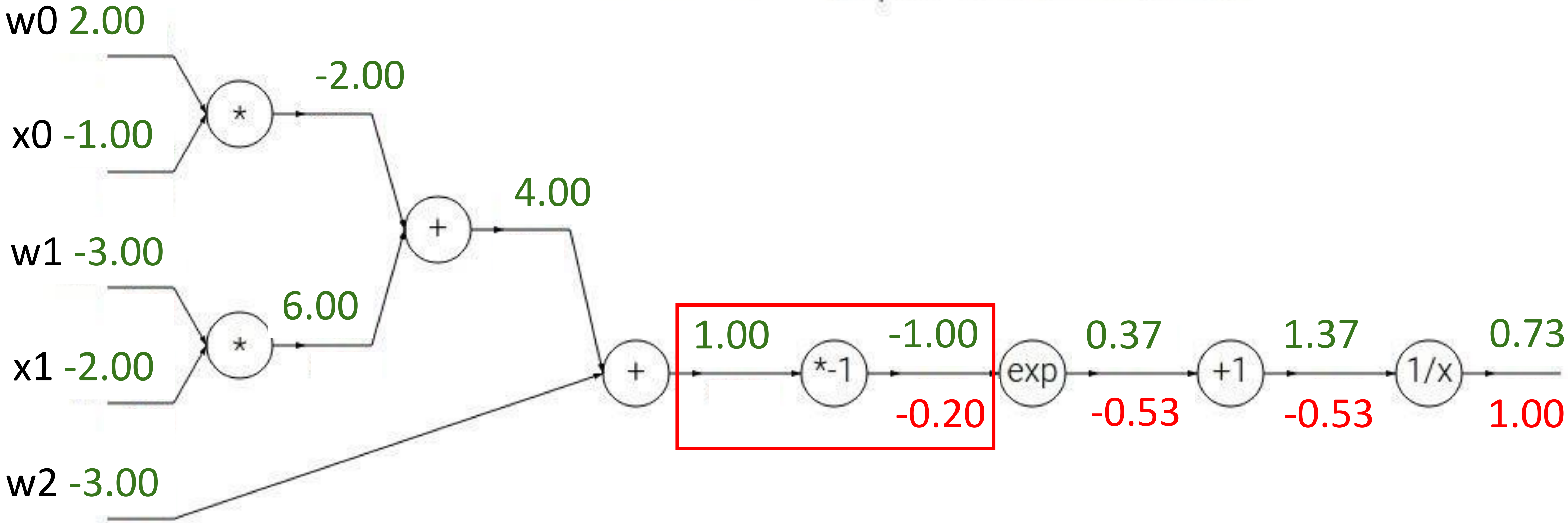

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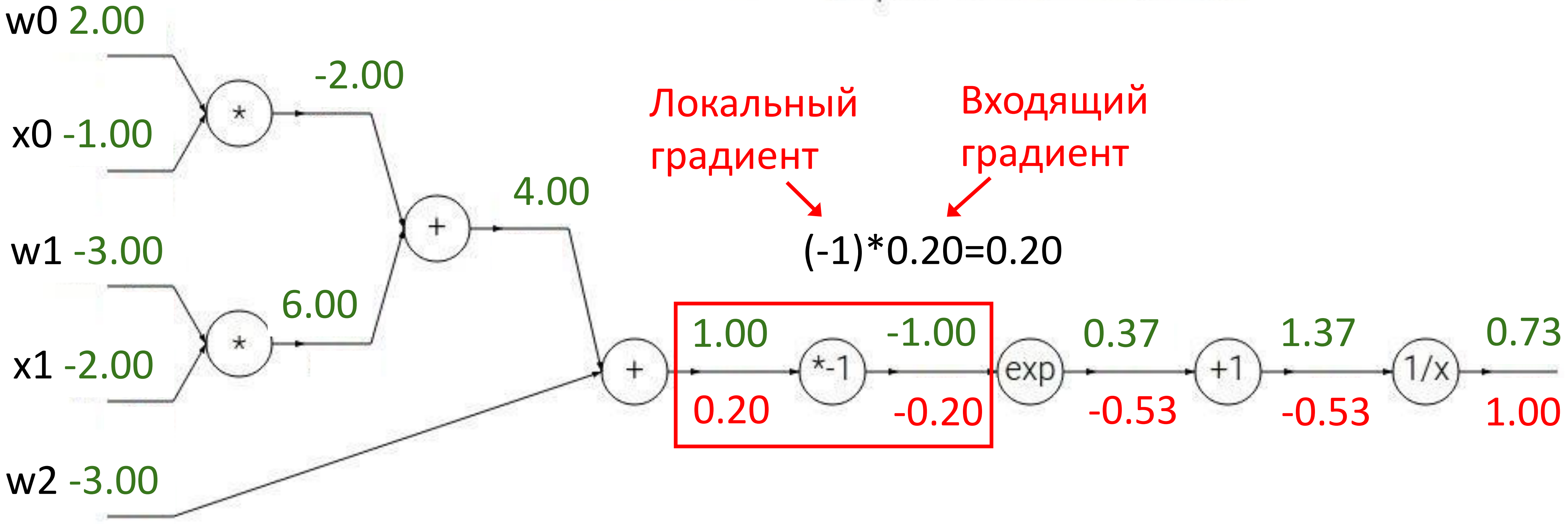


$$f(x)=e^x \quad \rightarrow \quad \frac{df}{dx}=e^x$$

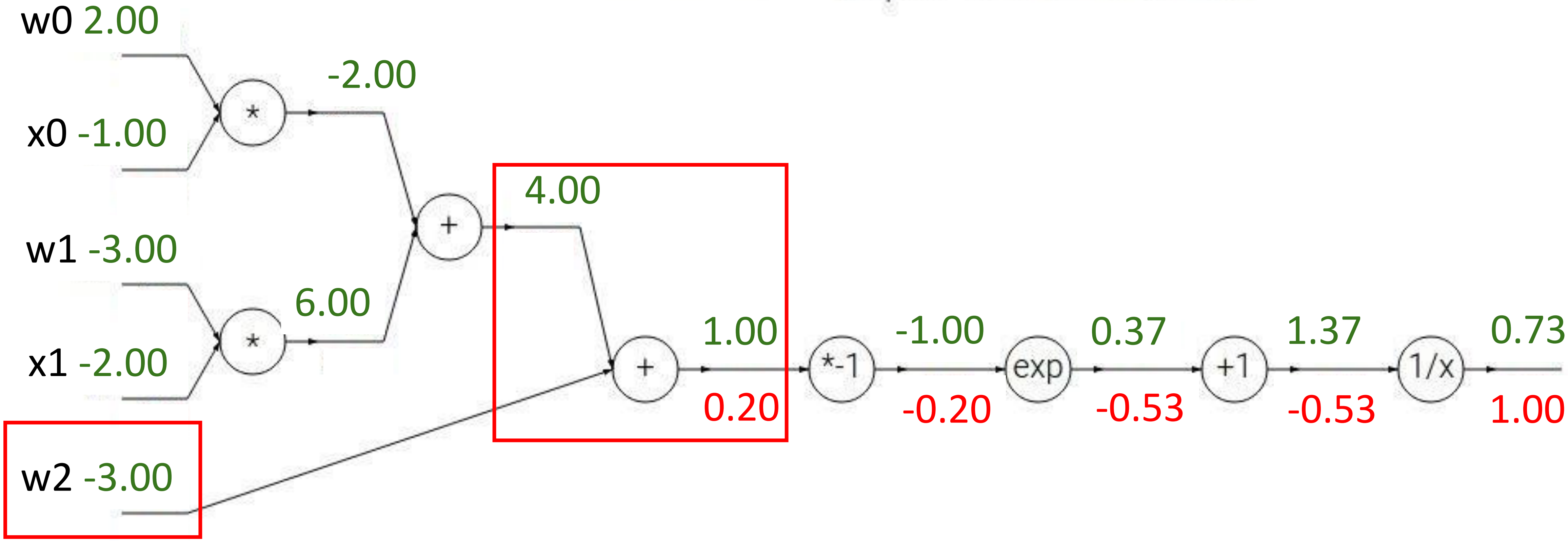
$$\boxed{f_a(x)=ax \quad \rightarrow \quad \frac{df}{dx}=a}$$

$$f(x)=\frac{1}{x} \quad \rightarrow \quad \frac{df}{dx}=-1/x^2$$

$$f_c(x)=c+x \quad \rightarrow \quad \frac{df}{dx}=1$$

Пример:

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

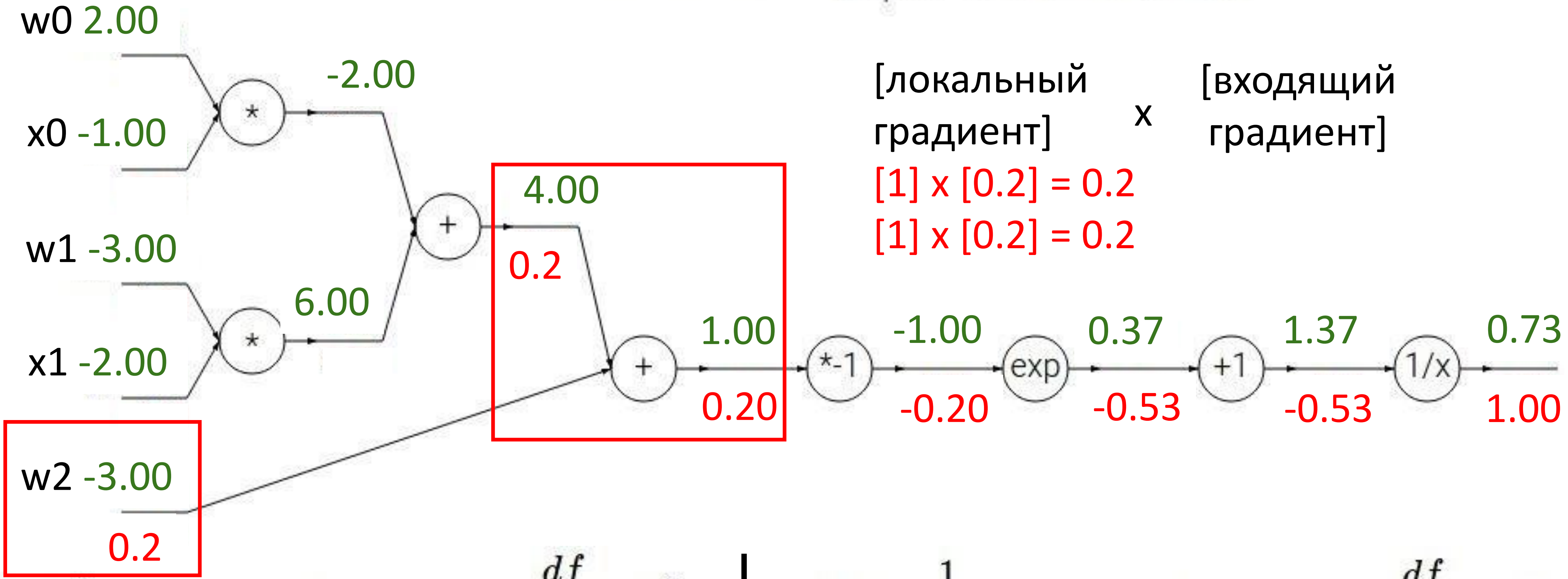


$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x \qquad\qquad f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a \qquad\qquad f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$$

Пример:

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



[локальный градиент] x [входящий градиент]

[1] x [0.2] = 0.2

[1] x [0.2] = 0.2

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a$$

$$f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$$

Пример:

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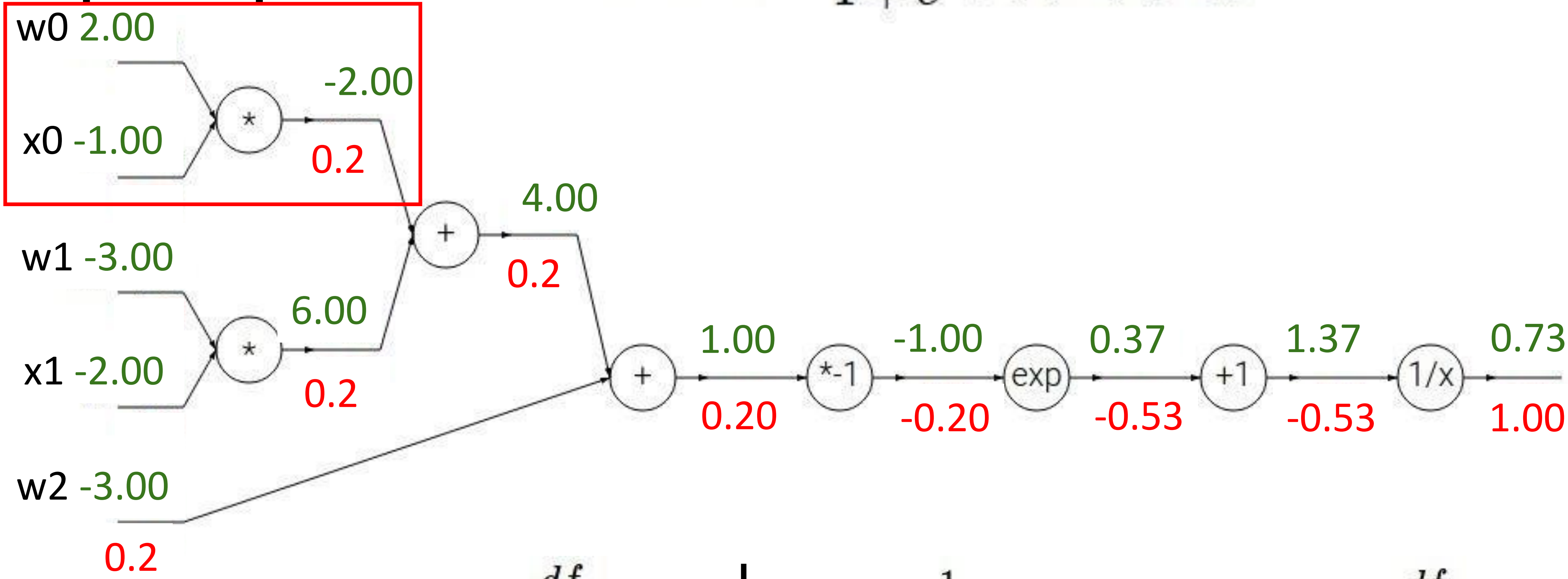

$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x$$

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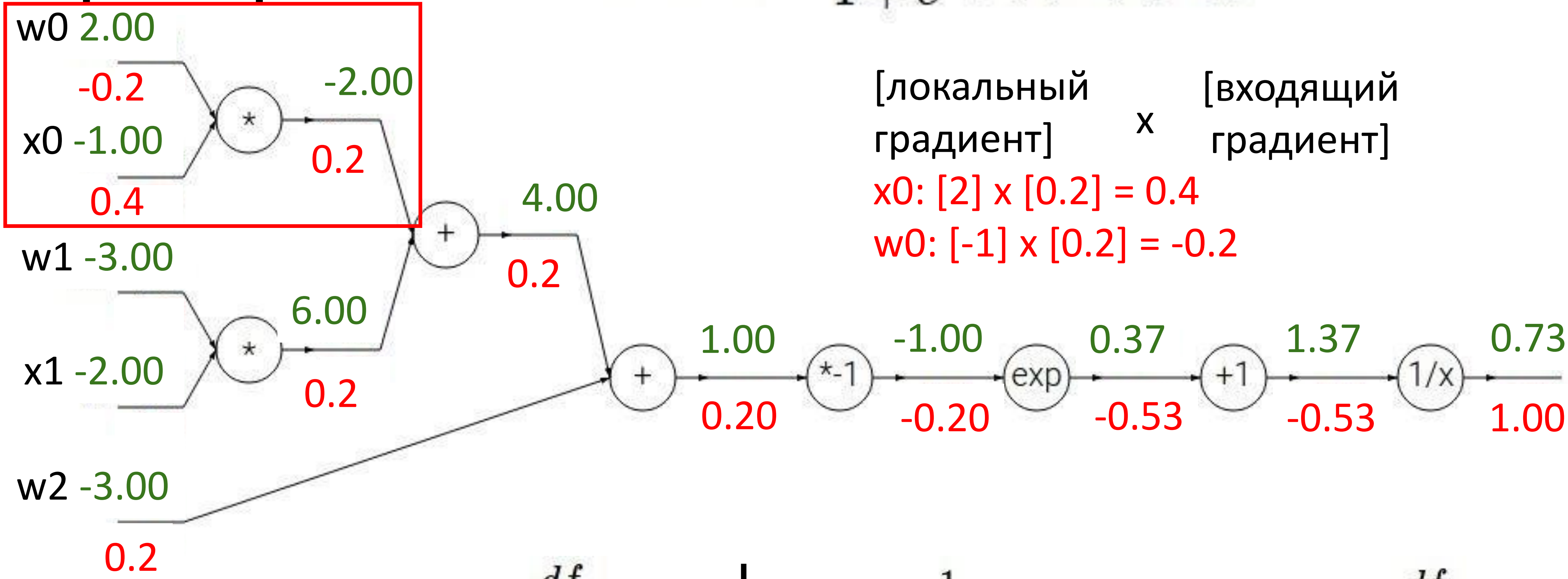
Пример:

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$



[локальный градиент] x [входящий градиент]

x0: [2] x [0.2] = 0.4

w0: [-1] x [0.2] = -0.2

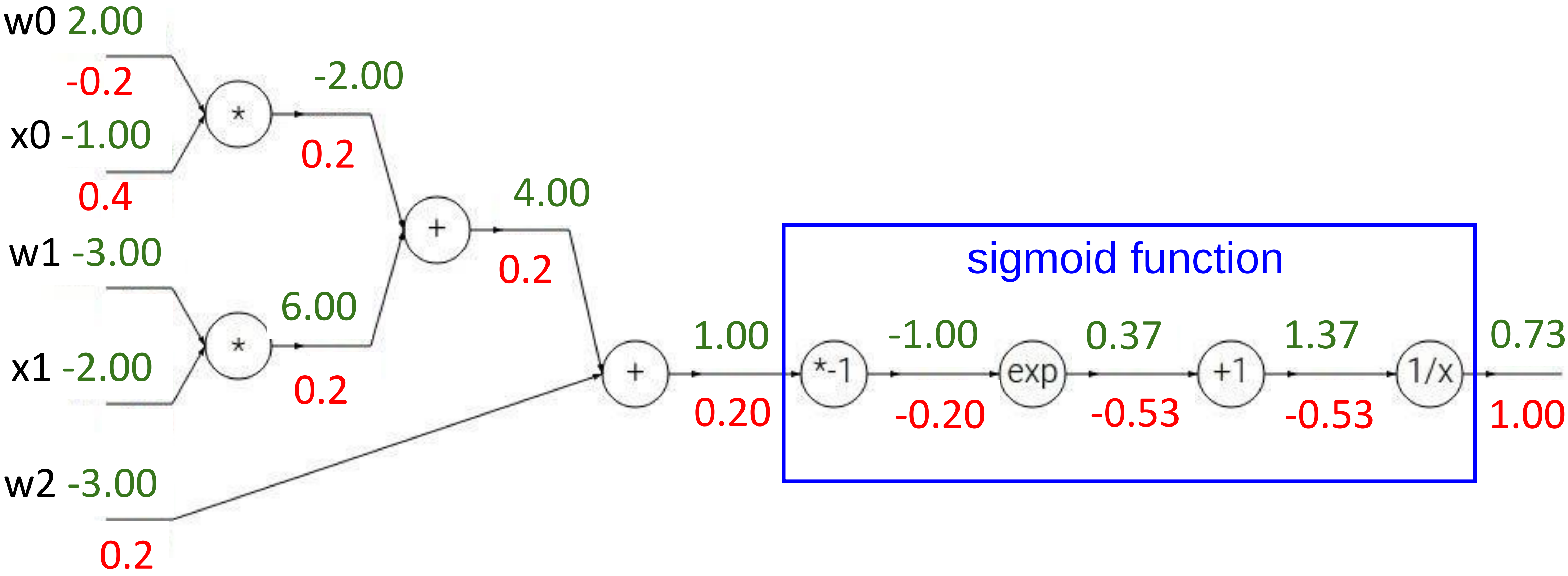
$$f(x) = e^x \quad \rightarrow \quad \frac{df}{dx} = e^x \qquad\qquad f(x) = \frac{1}{x} \quad \rightarrow \quad \frac{df}{dx} = -1/x^2$$

$$f_a(x) = ax \quad \rightarrow \quad \frac{df}{dx} = a \qquad\qquad f_c(x) = c + x \quad \rightarrow \quad \frac{df}{dx} = 1$$

$$f(w,x) = \frac{1}{1+e^{-(w_0x_0+w_1x_1+w_2)}}$$

$$\boxed{\sigma(x) = \frac{1}{1+e^{-x}}}$$ sigmoid function

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1+e^{-x})^2} = \left(\frac{1+e^{-x}-1}{1+e^{-x}}\right)\left(\frac{1}{1+e^{-x}}\right) = (1-\sigma(x))\sigma(x)$$



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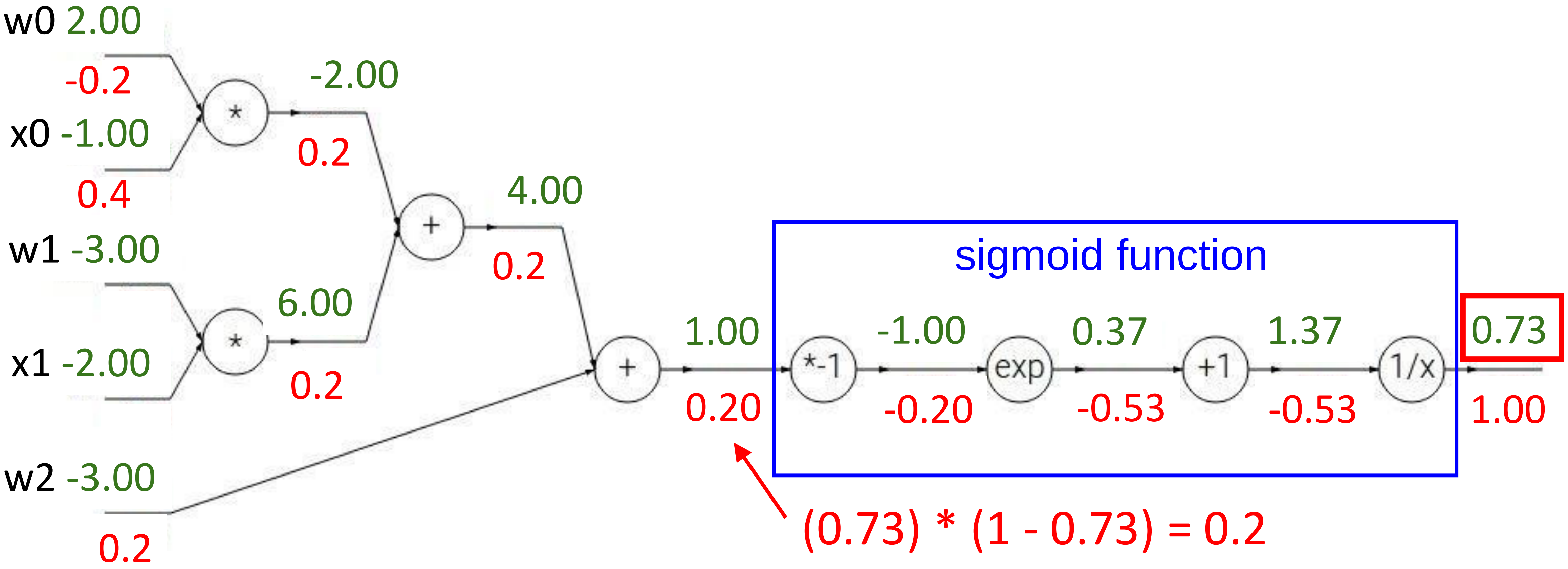
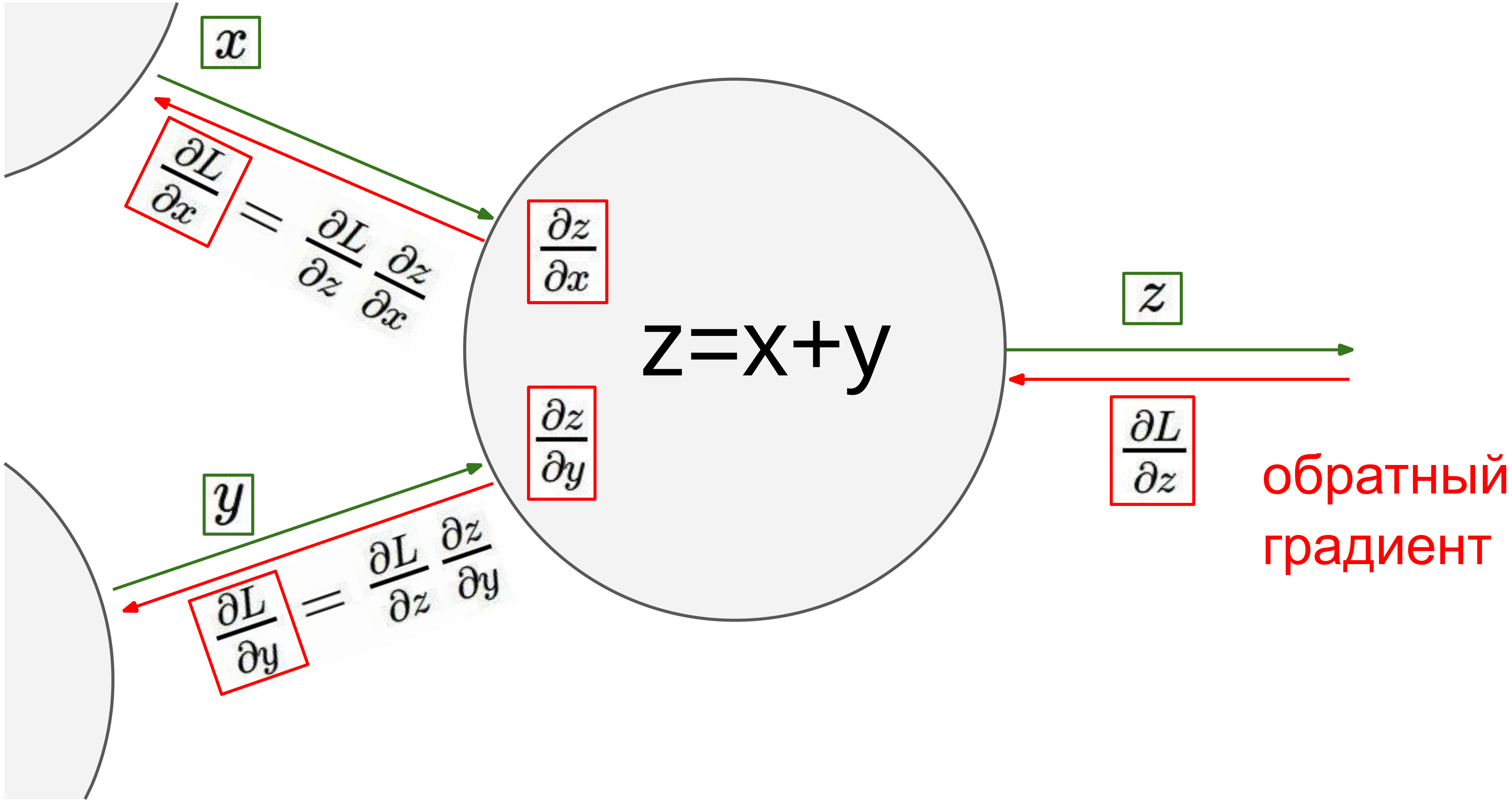
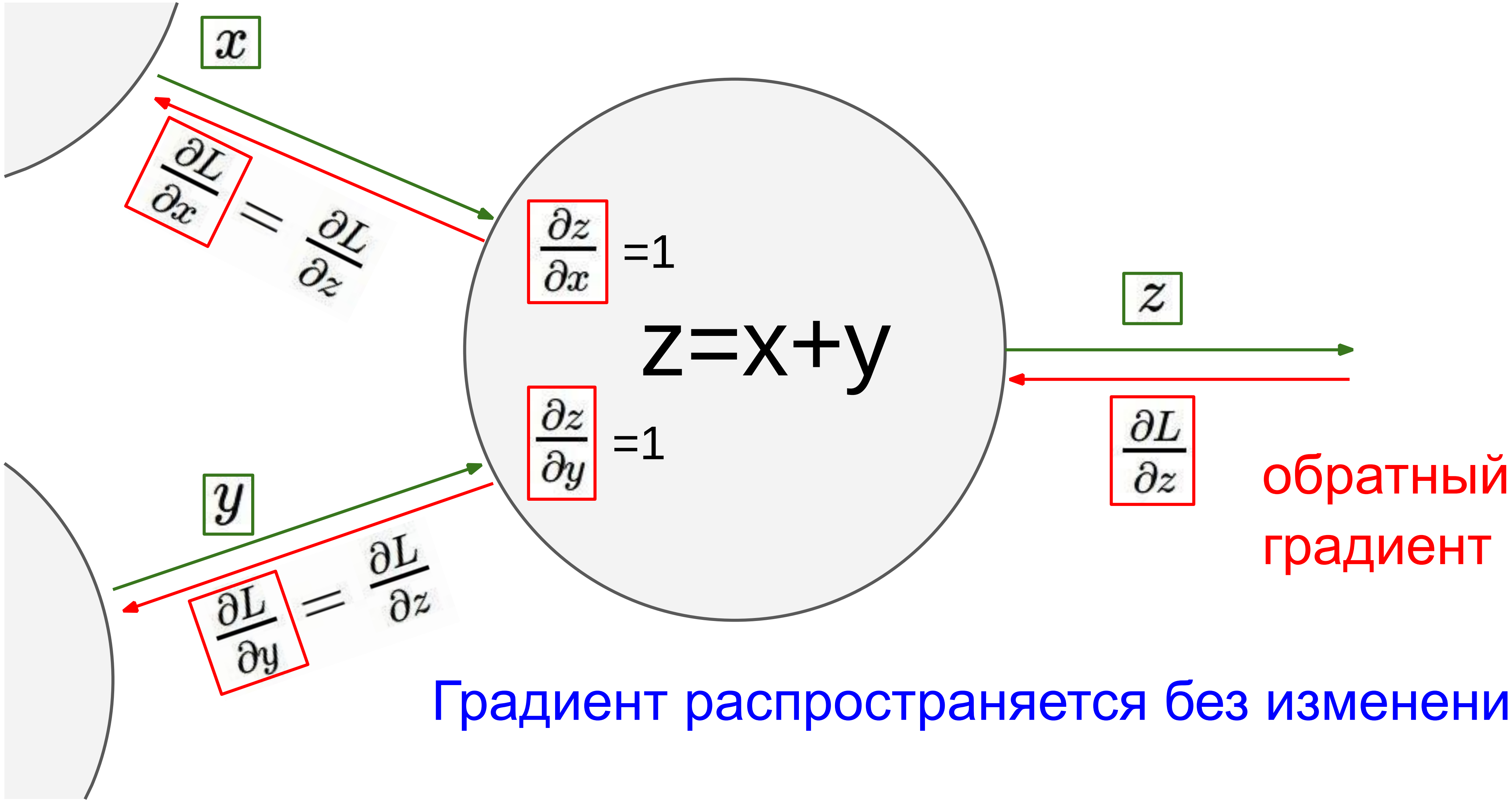


Обратный градиент при суммировании

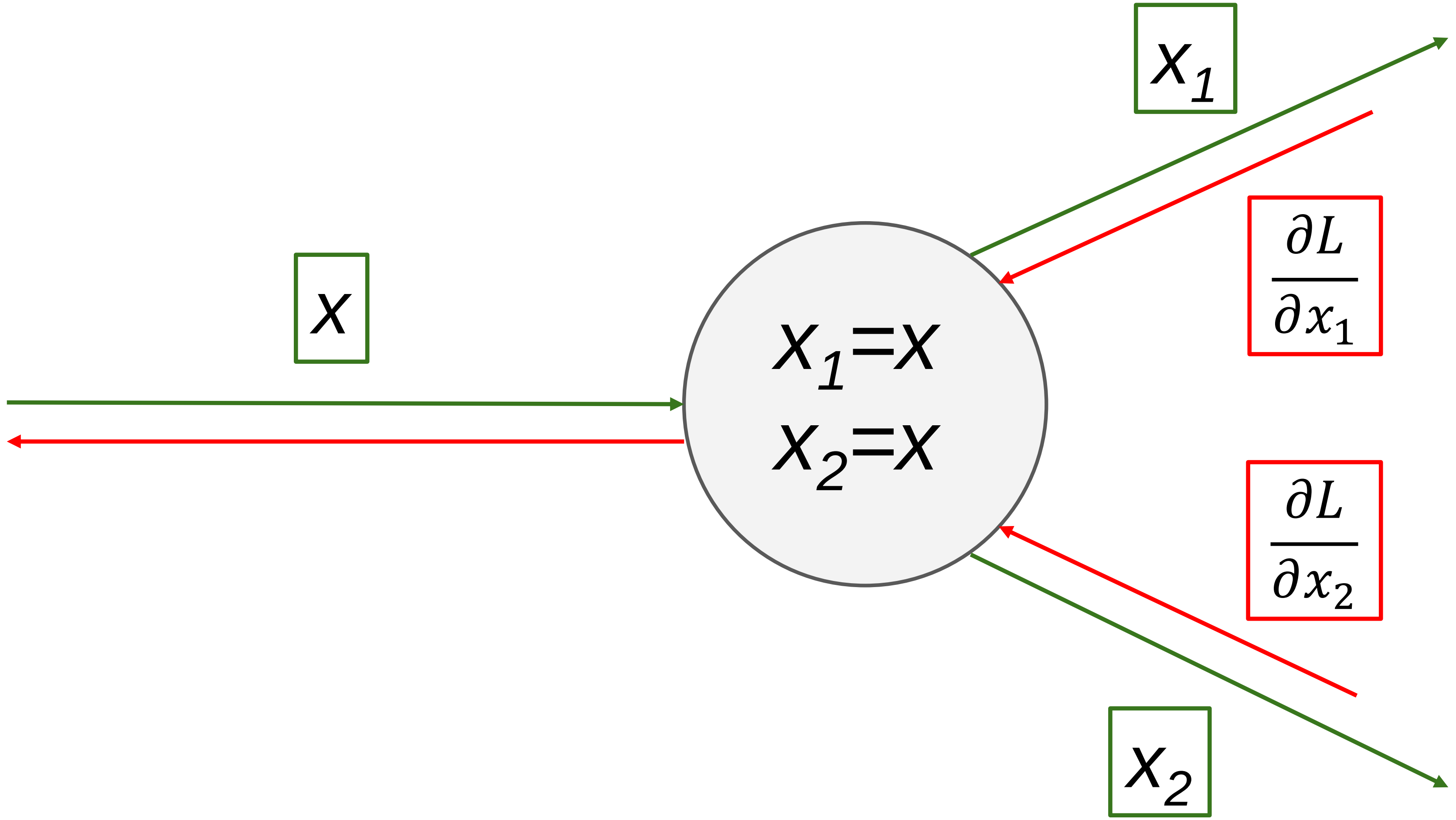


Обратный градиент при суммировании

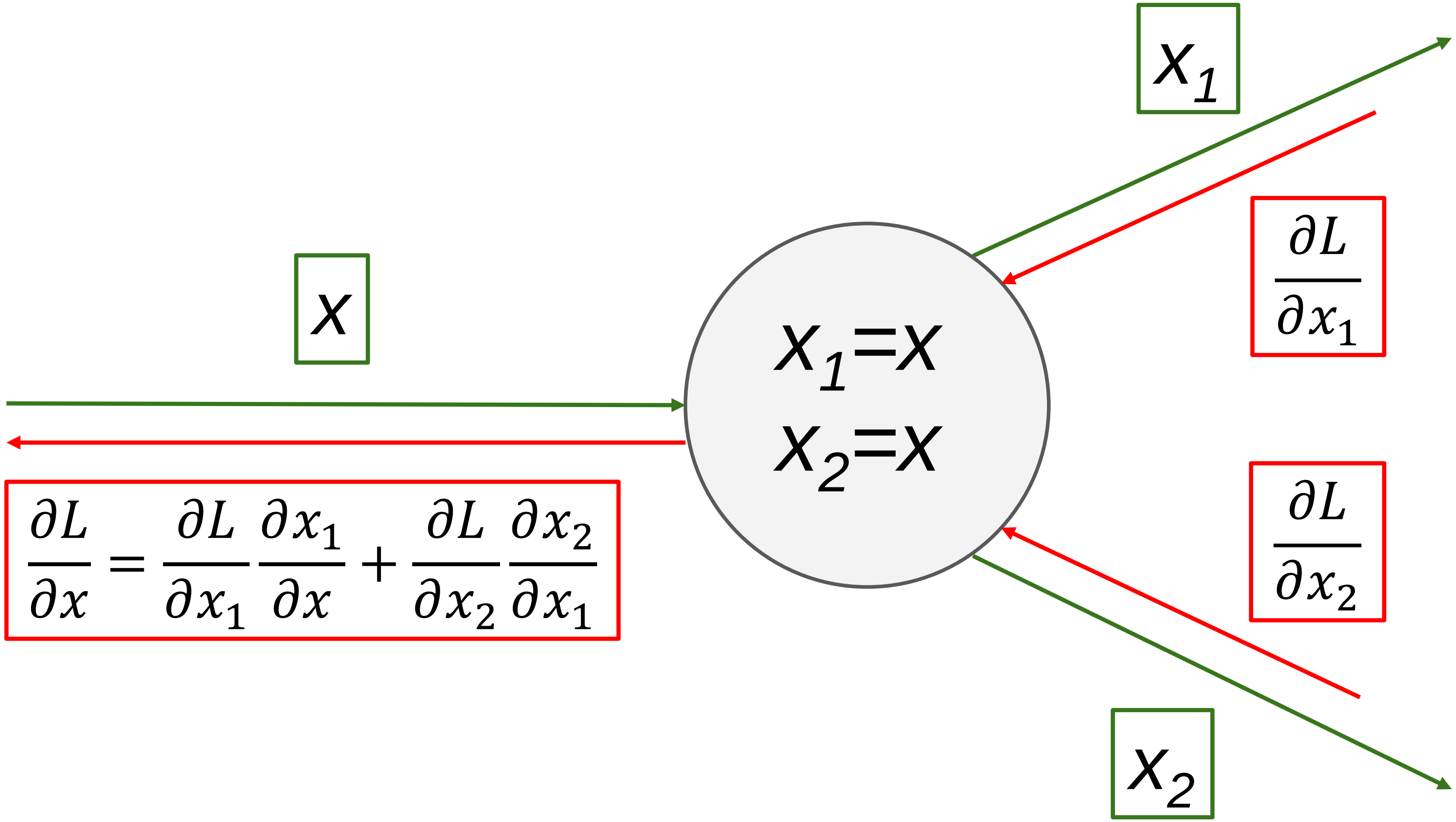


Градиент распространяется без изменений

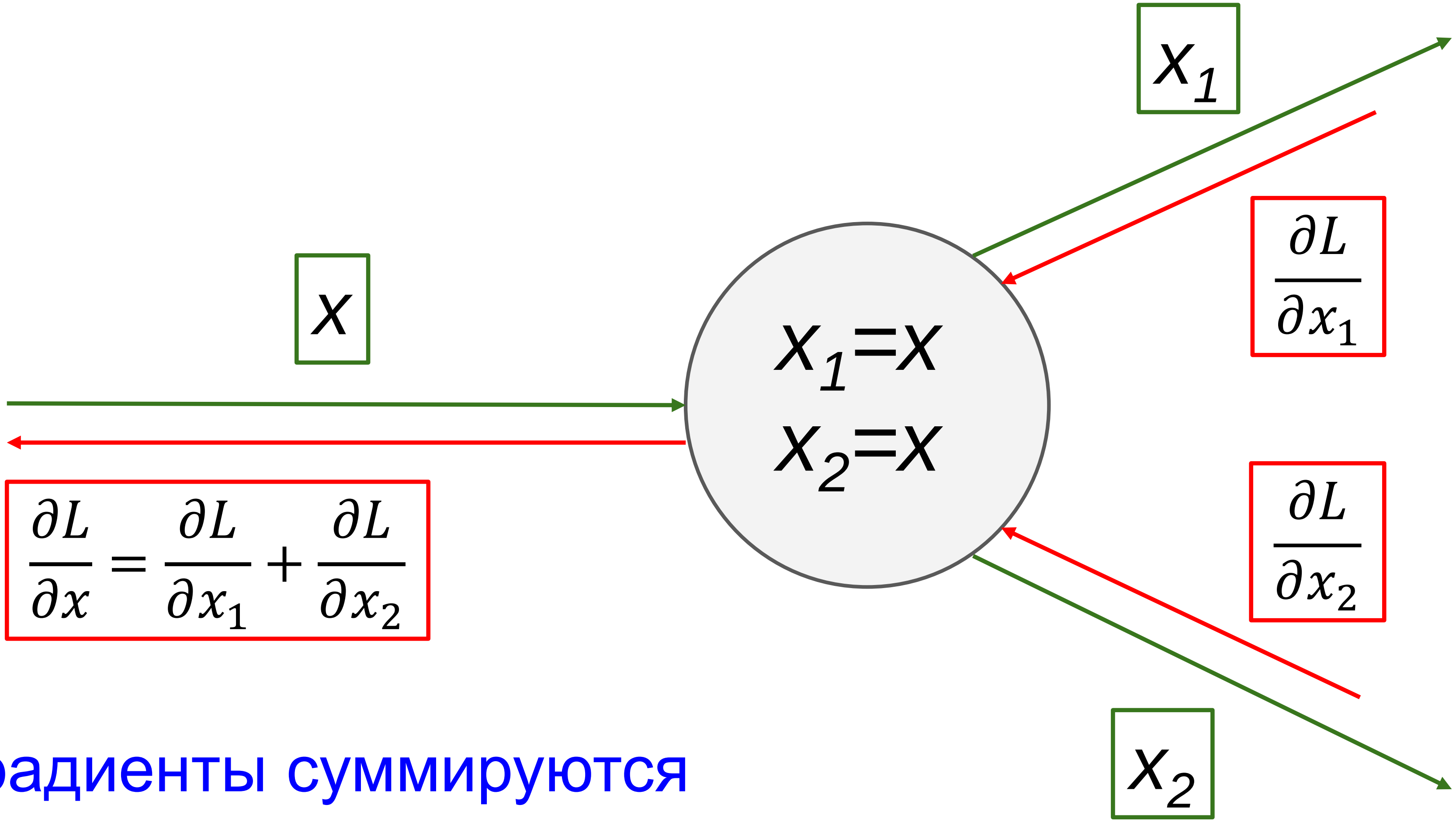
Обратный градиент при переиспользовании переменной



Обратный градиент при переиспользовании переменной



Обратный градиент при переиспользовании переменной

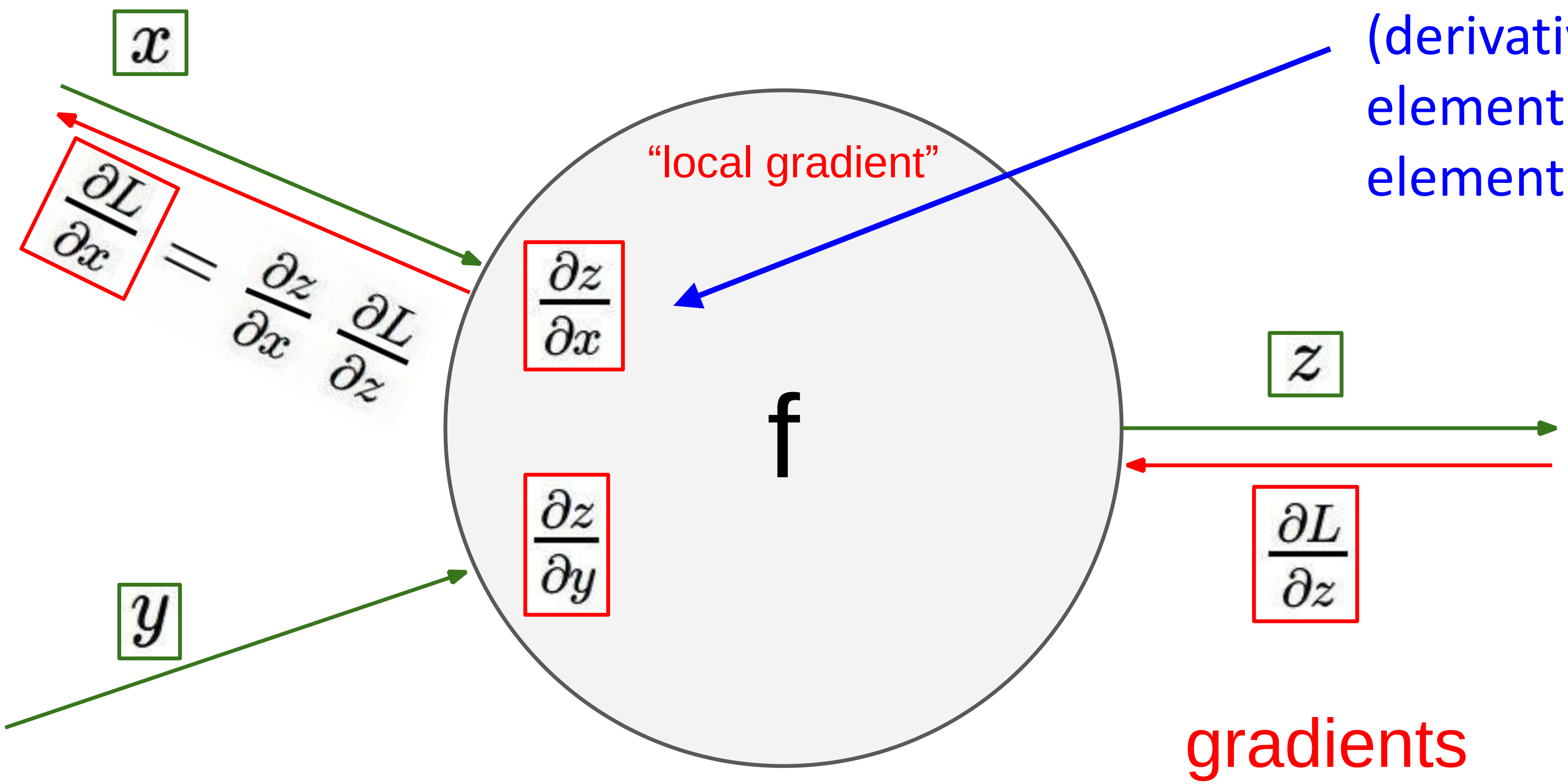


$$\frac{\partial L}{\partial x} = \frac{\partial L}{\partial x_1} + \frac{\partial L}{\partial x_2}$$

Градиенты суммируются

Backpropagation: векторный случай

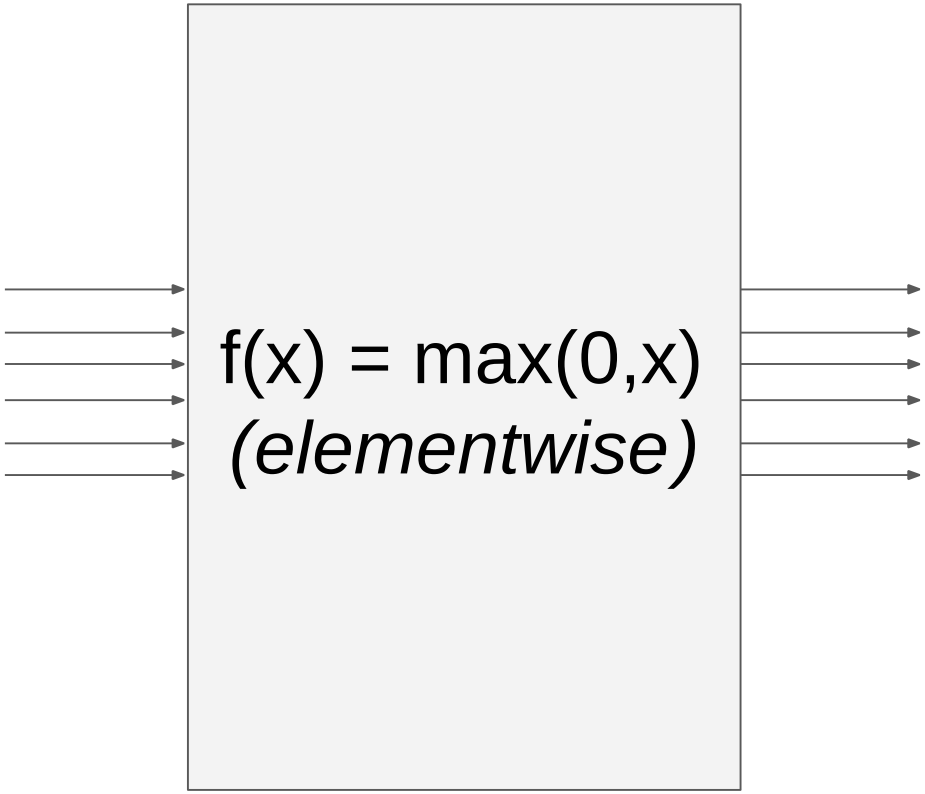
(x,y,z are now vectors)



This is now the **Jacobian matrix** (derivative of each element of z w.r.t. each element of x)

Векторные операции

4096-d
input vector



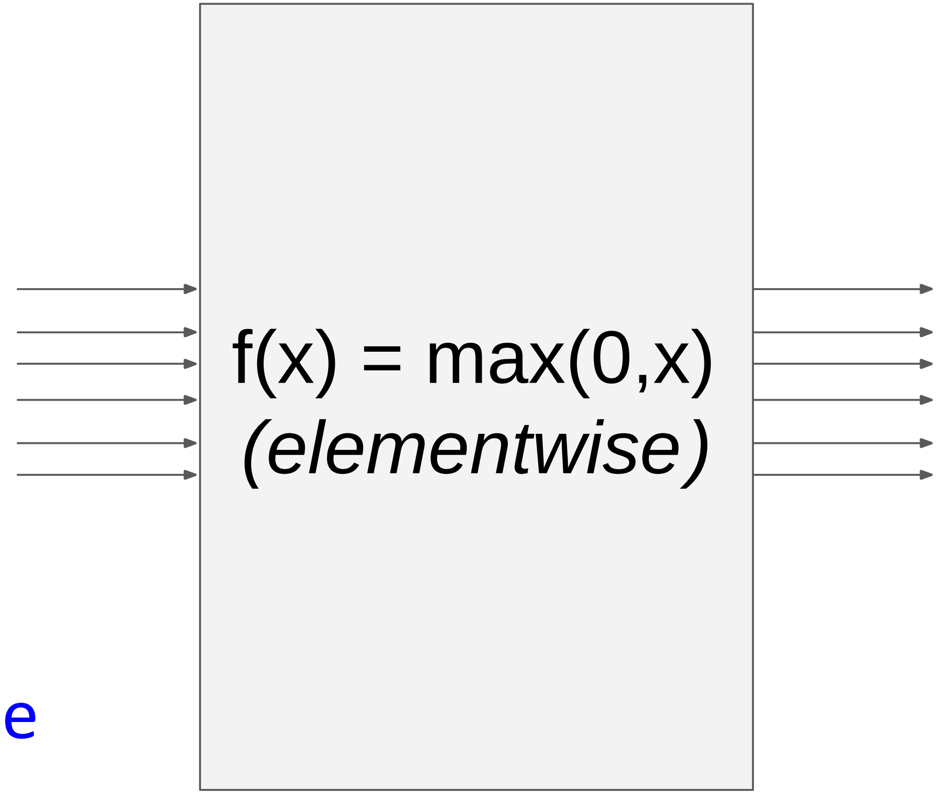
4096-d
output vector

Векторные операции

$$\frac{\partial L}{\partial x} = \boxed{\frac{\partial f}{\partial x}} \frac{\partial L}{\partial f}$$

Jacobian matrix

4096-d
input vector



4096-d
output vector

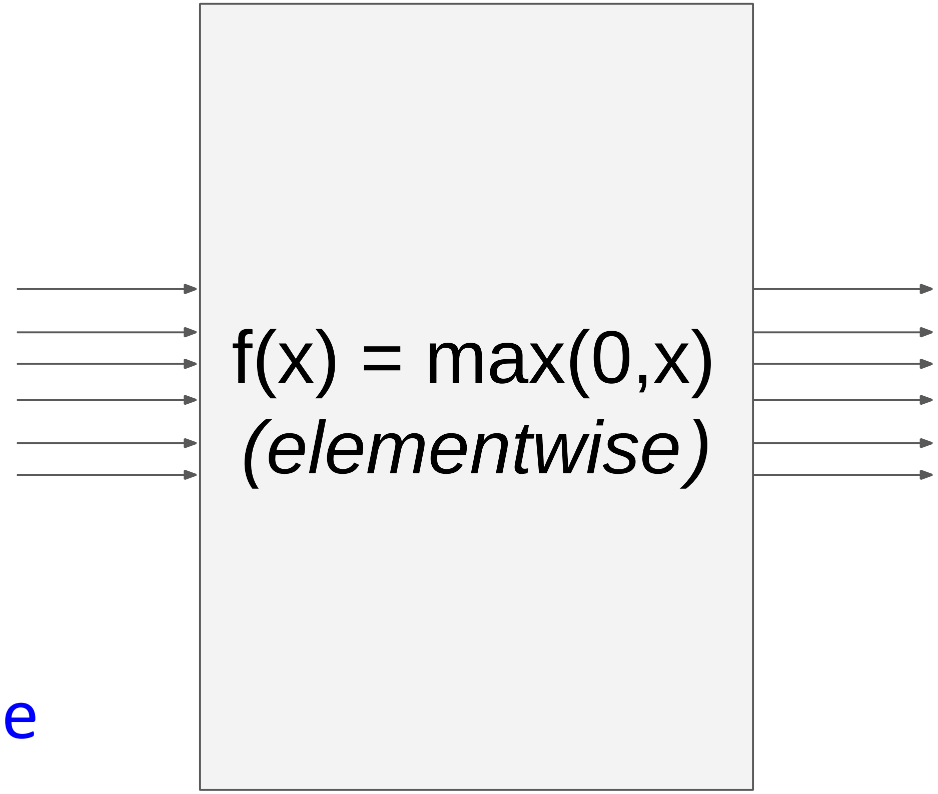
Q: what is the size of the Jacobian matrix?

Векторные операции

$$\frac{\partial L}{\partial x} = \boxed{\frac{\partial f}{\partial x}} \frac{\partial L}{\partial f}$$

Jacobian matrix

4096-d
input vector

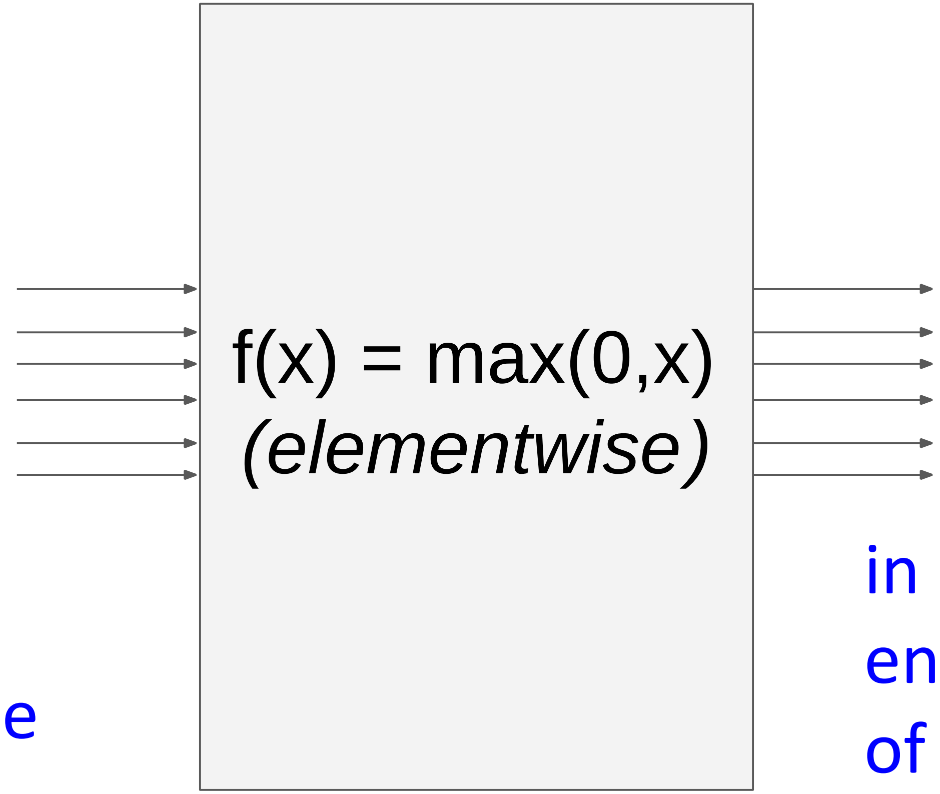


4096-d
output vector

Q: what is the
size of the
Jacobian matrix?
[4096 x 4096!]

Векторные операции

4096-d
input vector



4096-d
output vector

Q: what is the size of the Jacobian matrix?
[4096 x 4096!]

in practice we process an entire minibatch (e.g. 100) of examples at one time:

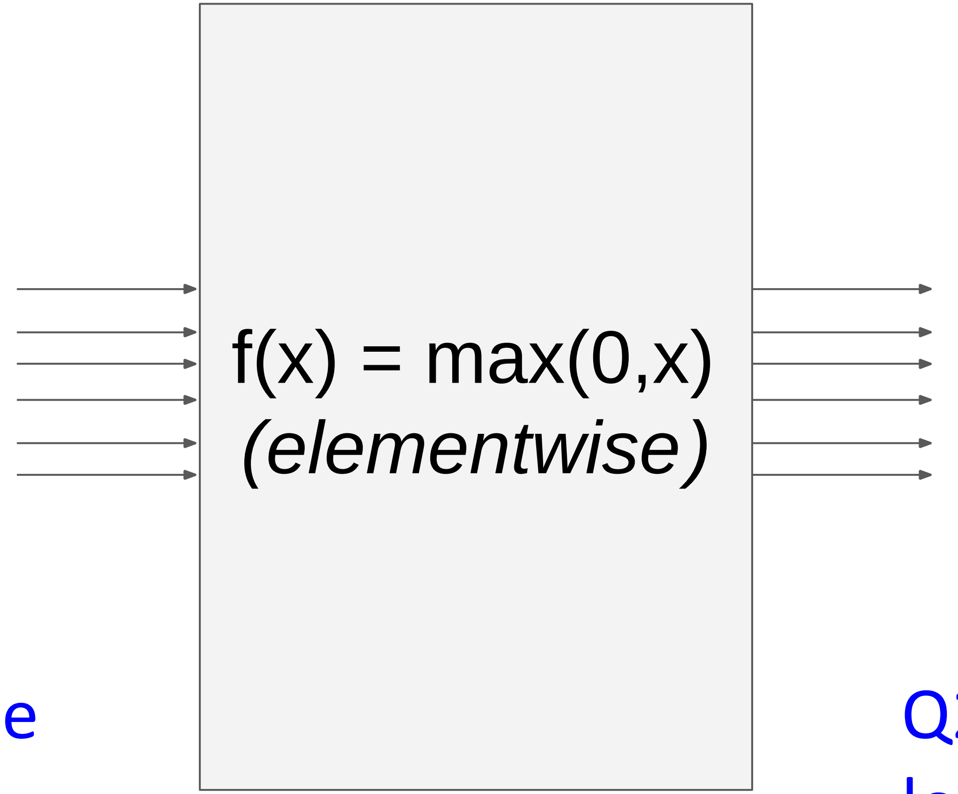
i.e. Jacobian would technically be a [409,600 x 409,600] matrix

Векторные операции

$$\frac{\partial L}{\partial x} = \boxed{\frac{\partial f}{\partial x}} \frac{\partial L}{\partial f}$$

Jacobian matrix

4096-d
input vector



4096-d
output vector

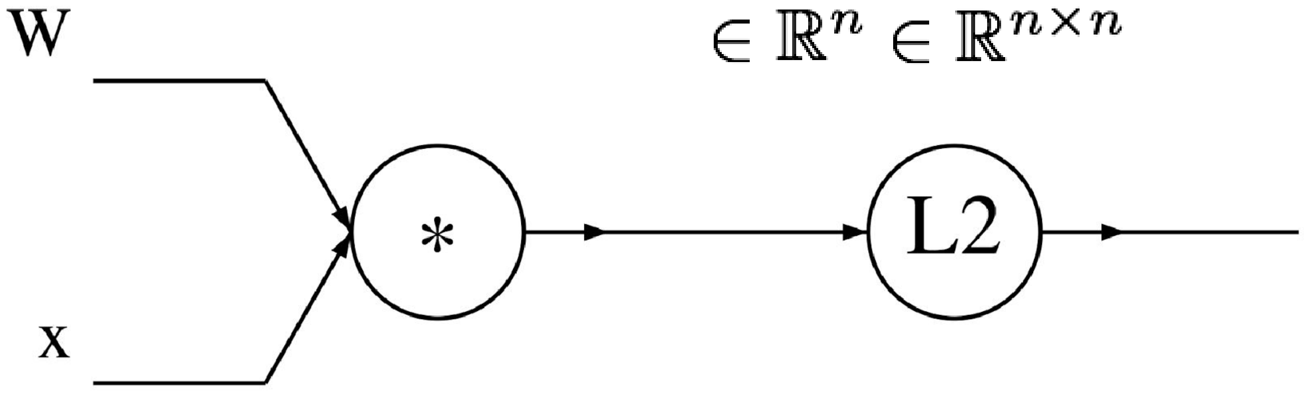
Q: what is the
size of the
Jacobian matrix?
[4096 x 4096!]

Q2: what does it
look like?

A vectorized example: $f(x, W) = ||W \cdot x||^2 = \sum_{i=1}^{n} (W \cdot x)_i^2$

A vectorized example: $f(x, W) = ||W \cdot x||^2 = \sum_{i=1}^{n} (W \cdot x)_i^2$

$\in \mathbb{R}^n \in \mathbb{R}^{n \times n}$



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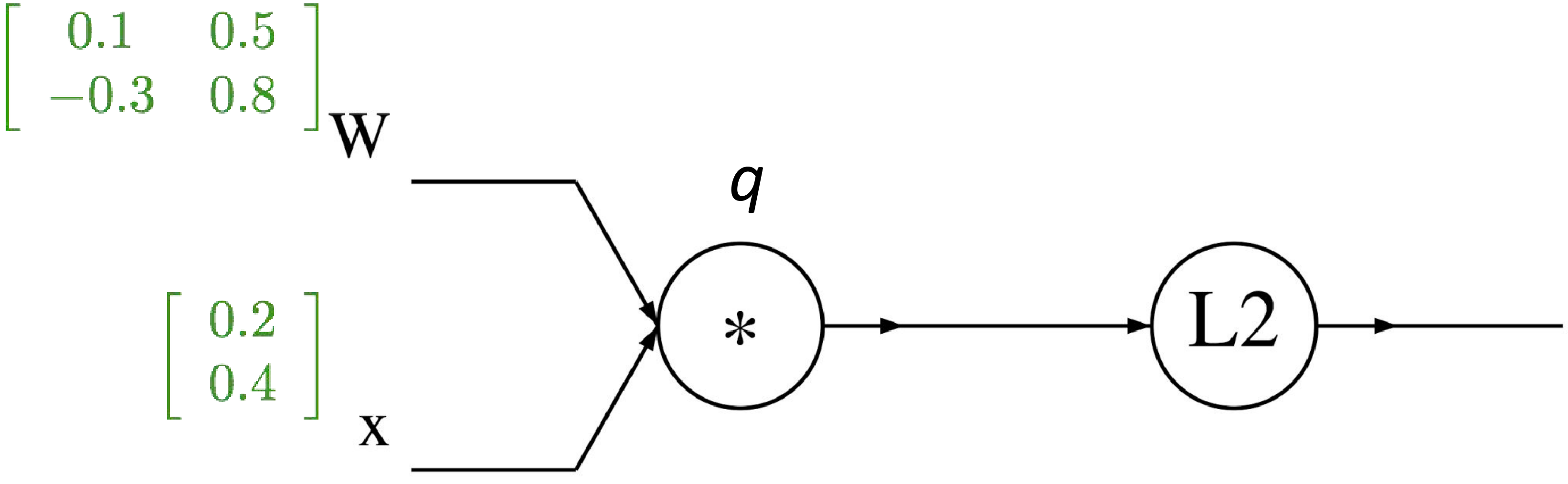

$$q = W \cdot x = \begin{pmatrix} W_{1,1}x_1 + \cdots + W_{1,n}x_n \\ \vdots \\ W_{n,1}x_1 + \cdots + W_{n,n}x_n \end{pmatrix}$$

$$f(q) = ||q||^2 = q_1^2 + \cdots + q_n^2$$

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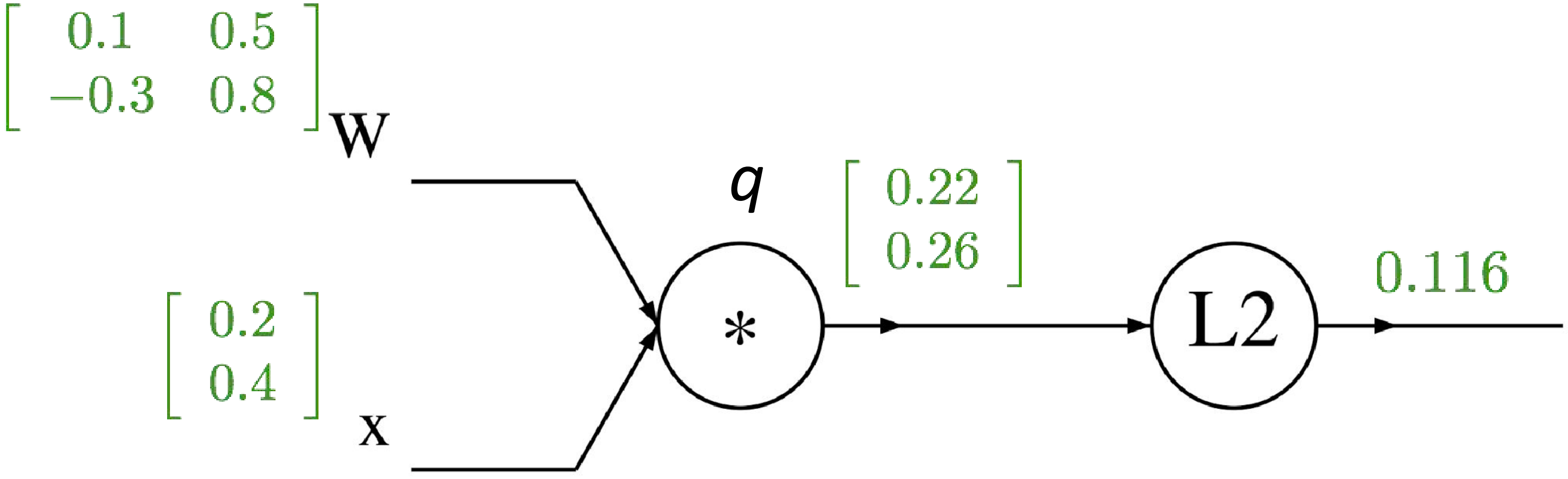

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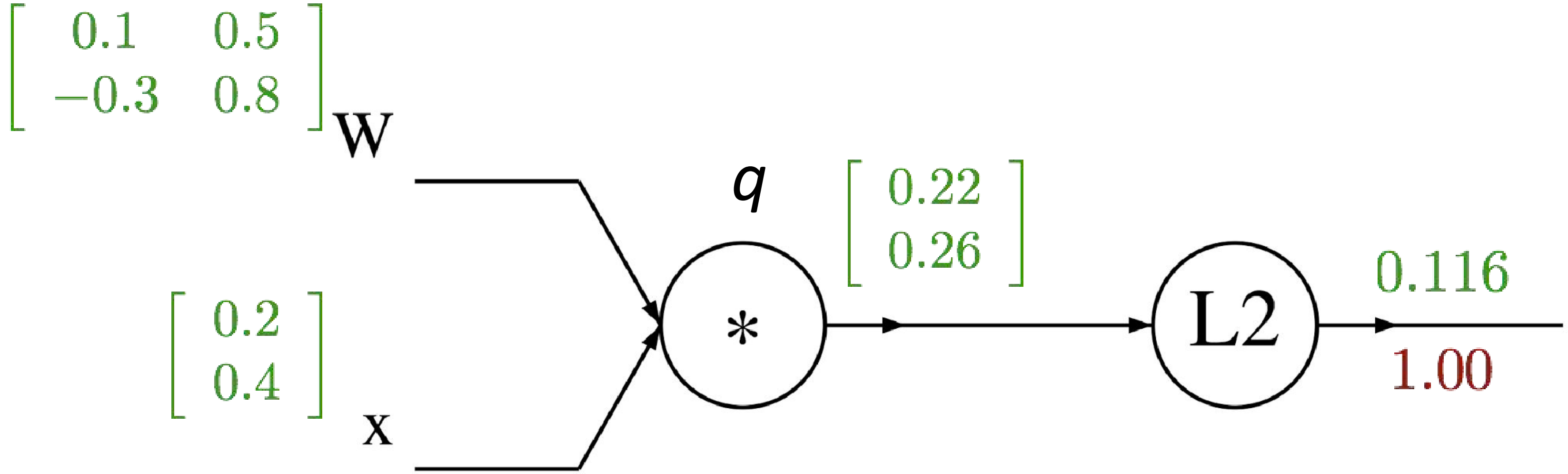

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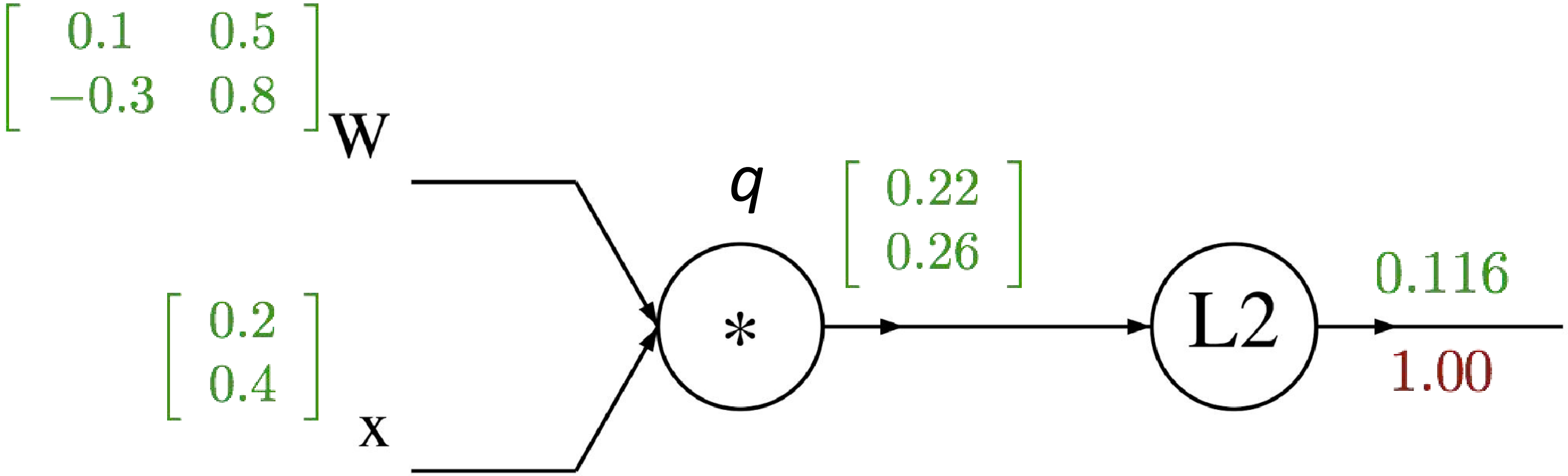

$$\frac{\partial f}{\partial q_i} = 2q_i$$

$$\nabla_q f = 2q$$

$$q = W \cdot x = \begin{pmatrix} W_{1,1}x_1 + \cdots + W_{1,n}x_n \\ \vdots \\ W_{n,1}x_1 + \cdots + W_{n,n}x_n \end{pmatrix}$$

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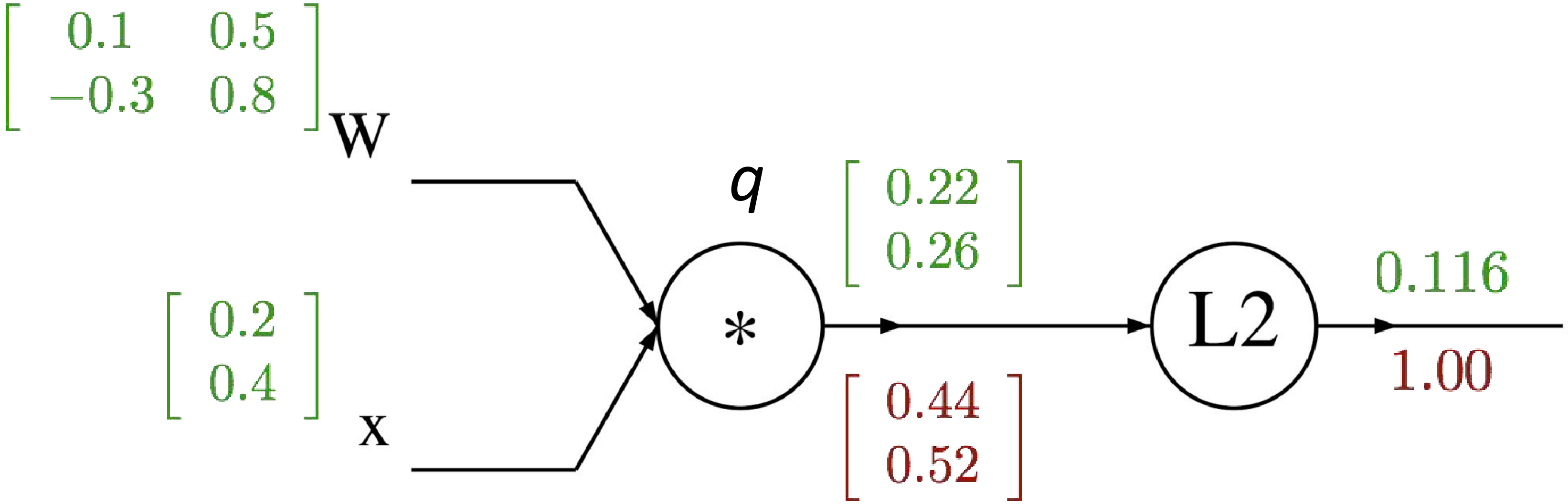

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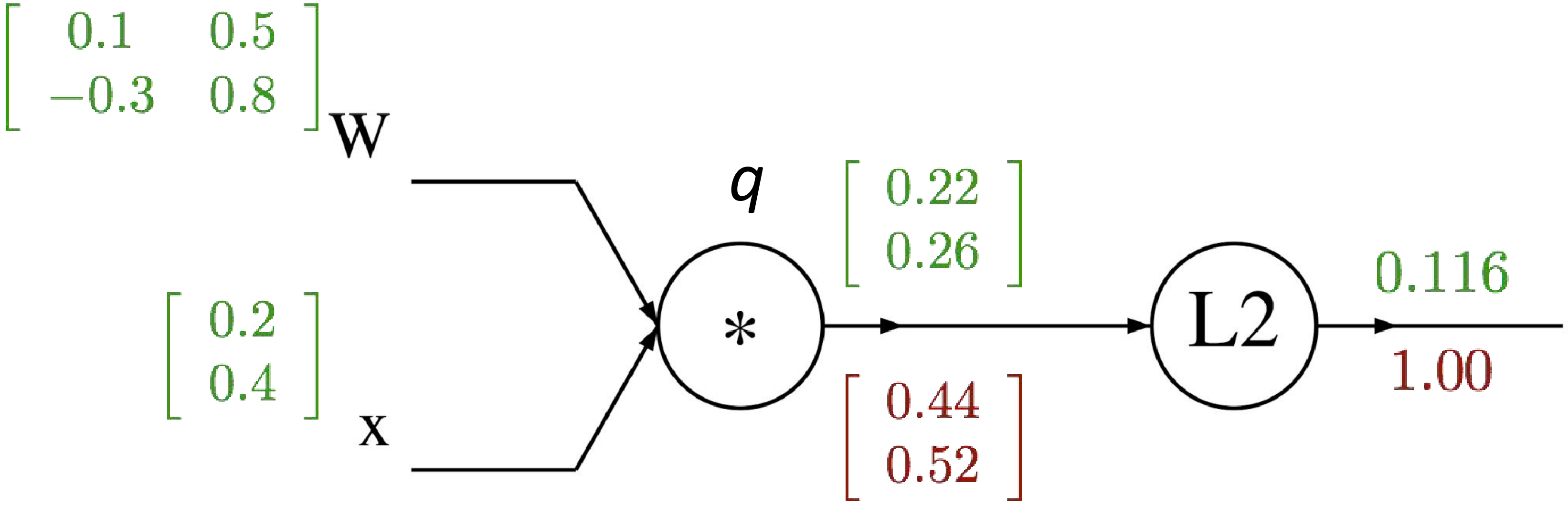

$$\frac{\partial q_k}{\partial W_{i,j}} = \mathbf{1}_{k=i} x_j$$

$$q = W \cdot x = \begin{pmatrix} W_{1,1}x_1 + \cdots + W_{1,n}x_n \\ \vdots \\ W_{n,1}x_1 + \cdots + W_{n,n}x_n \end{pmatrix}$$

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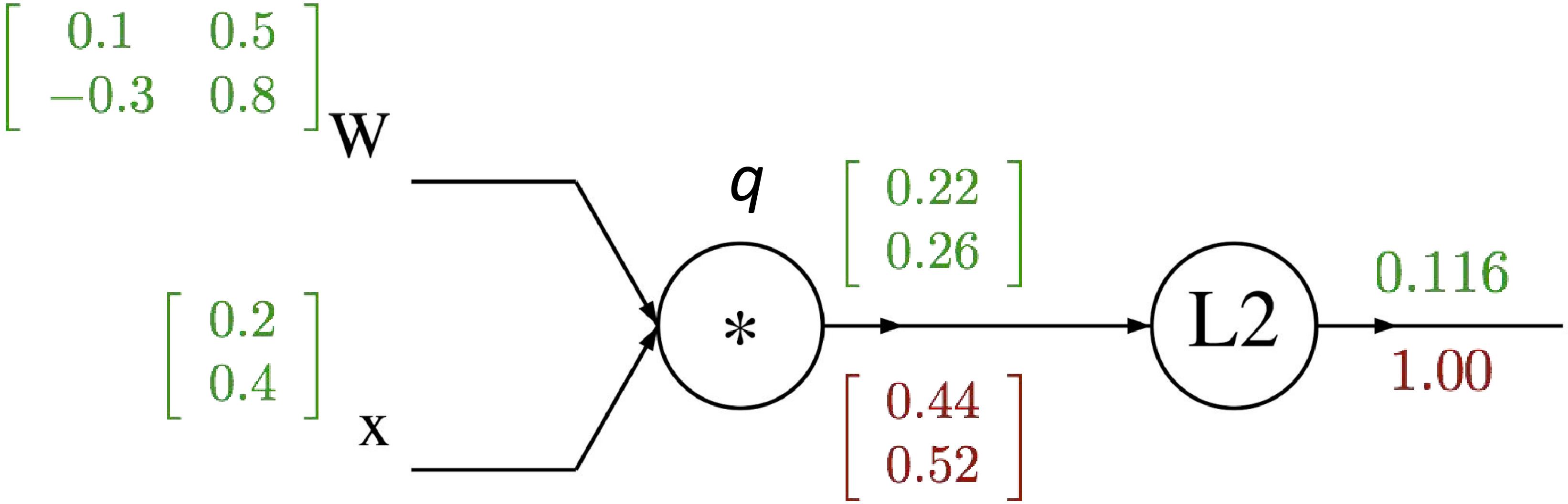

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$$\begin{aligned} \frac{\partial f}{\partial W_{i,j}} &= \sum_k \frac{\partial f}{\partial q_k} \frac{\partial q_k}{\partial W_{i,j}} \\ &= \sum_k (2q_k)(\mathbf{1}_{k=i} x_j) \\ &= 2q_i x_j \end{aligned}$$

$$f(q) = ||q||^2 = q_1^2 + \cdots + q_n^2$$

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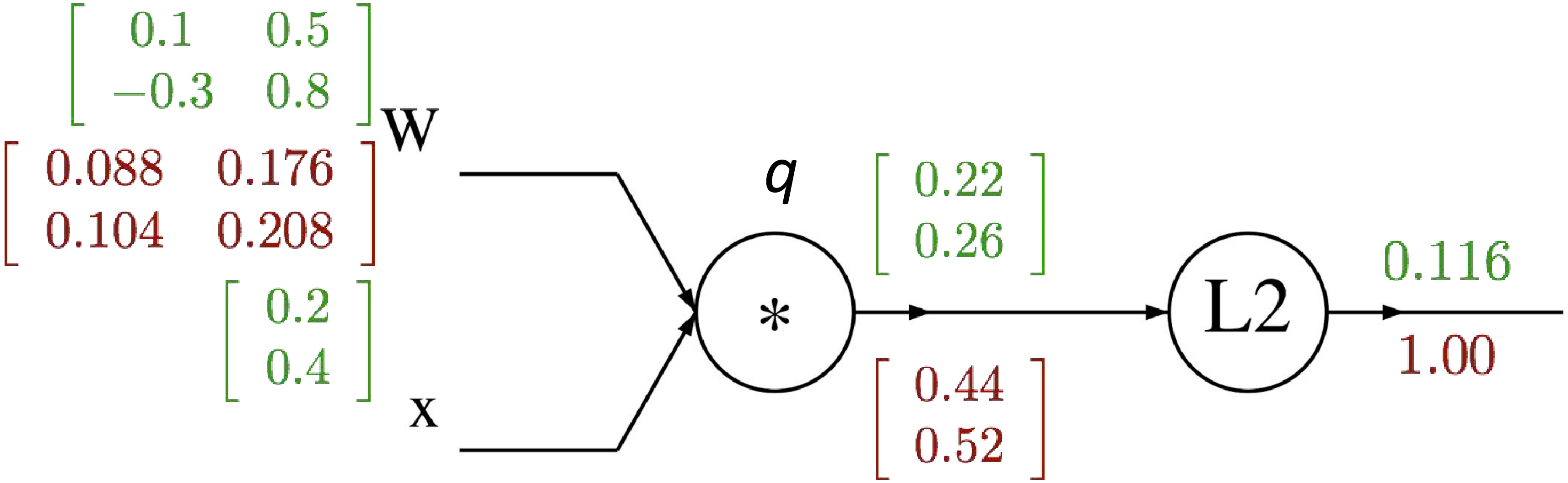

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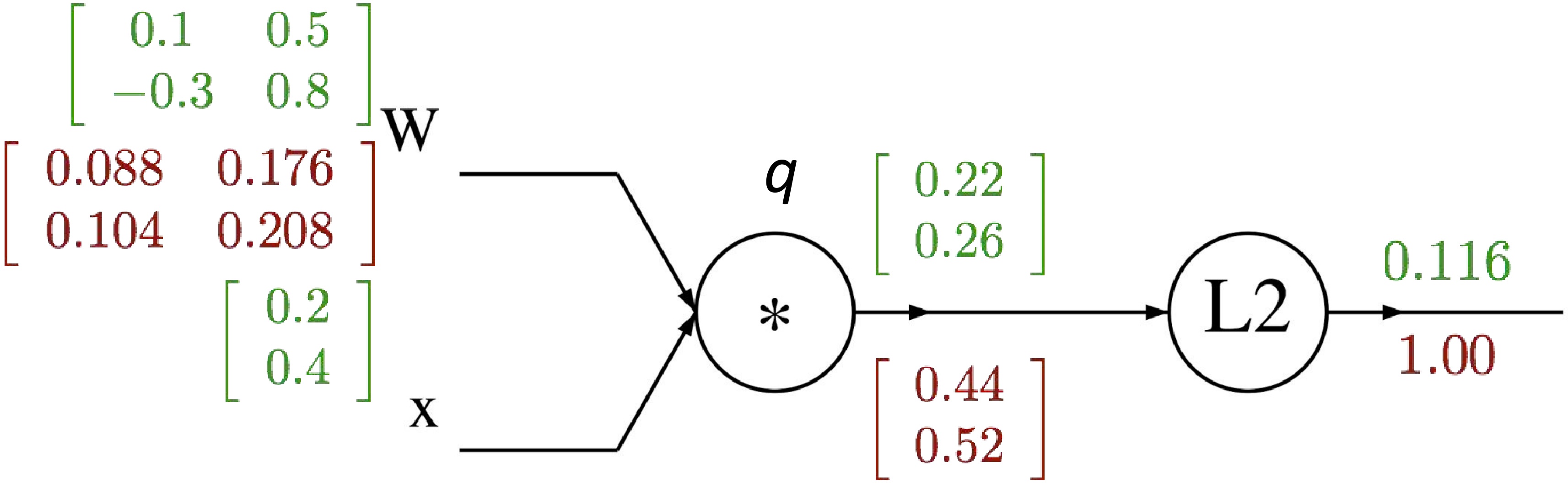
A vectorized example: $f(x, W) = ||W \cdot x||^2 = \sum_{i=1}^{n} (W \cdot x)_i^2$

$$\nabla_W f = 2q \cdot x^T$$



$$\frac{\partial q_k}{\partial W_{i,j}} = \mathbf{1}_{k=i} x_j$$

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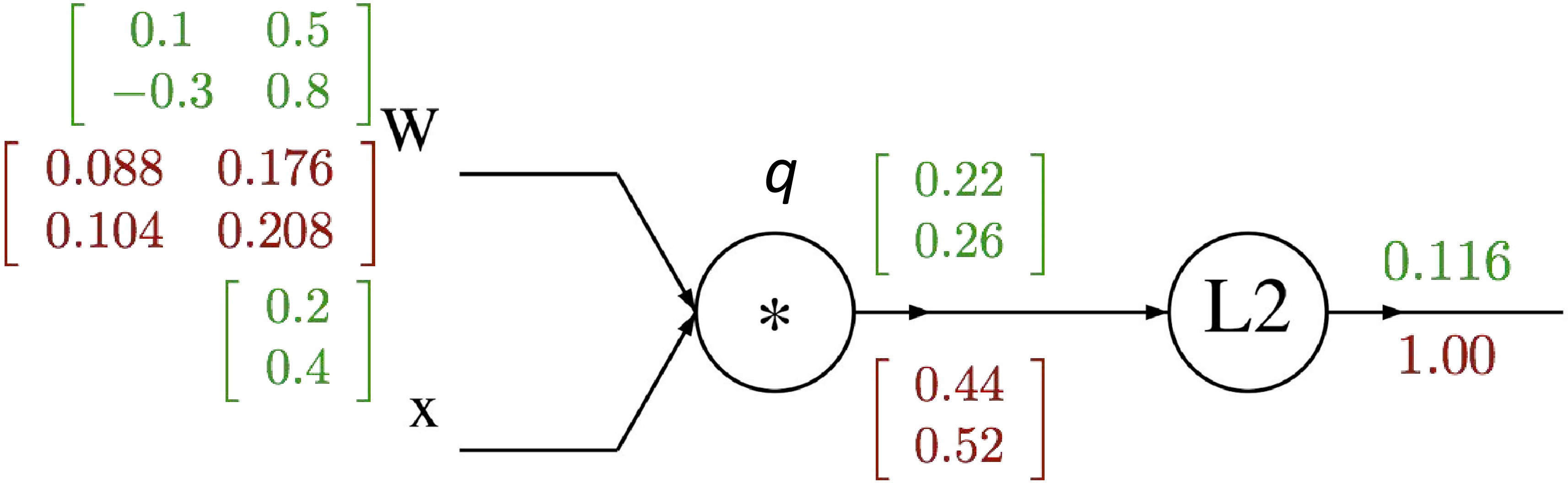

$$\nabla_W f = 2q \cdot x^T$$

Всегда проверяйте размерность градиента

Размерность градиента должна быть такой-же как размерность переменной, для которой он вычисляется

$$q = W \cdot x = \begin{pmatrix} W_{1,1}x_1 + \cdots + W_{1,n}x_n \\ \vdots \\ W_{n,1}x_1 + \cdots + W_{n,n}x_n \end{pmatrix}$$

$$f(q) = ||q||^2 = q_1^2 + \cdots + q_n^2$$

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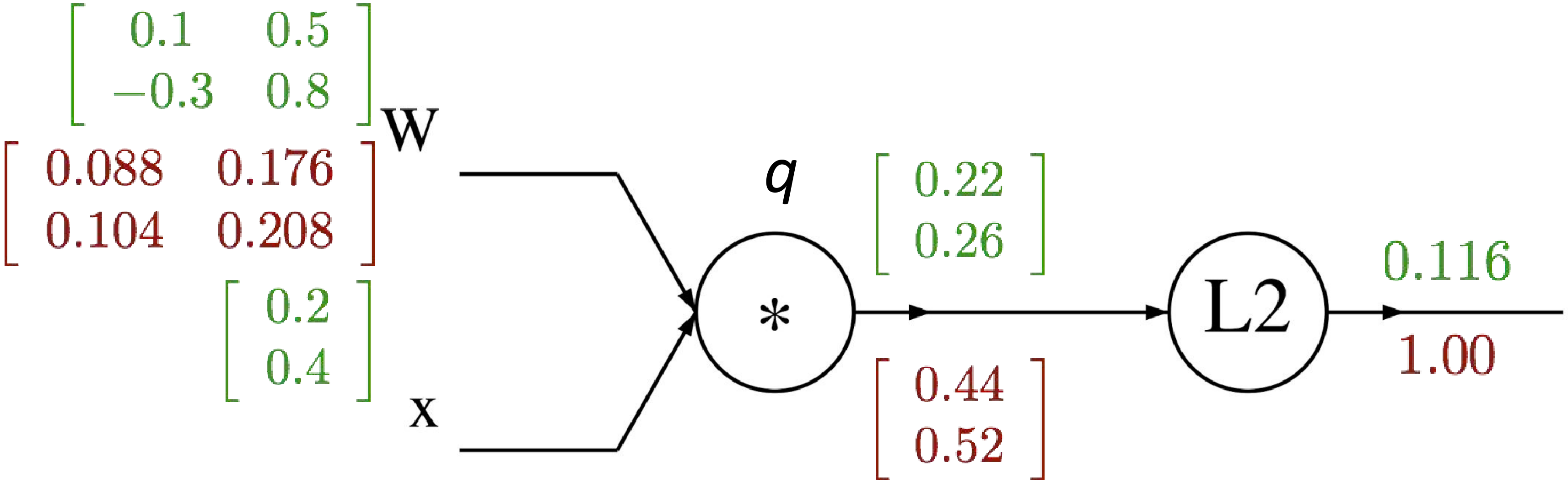
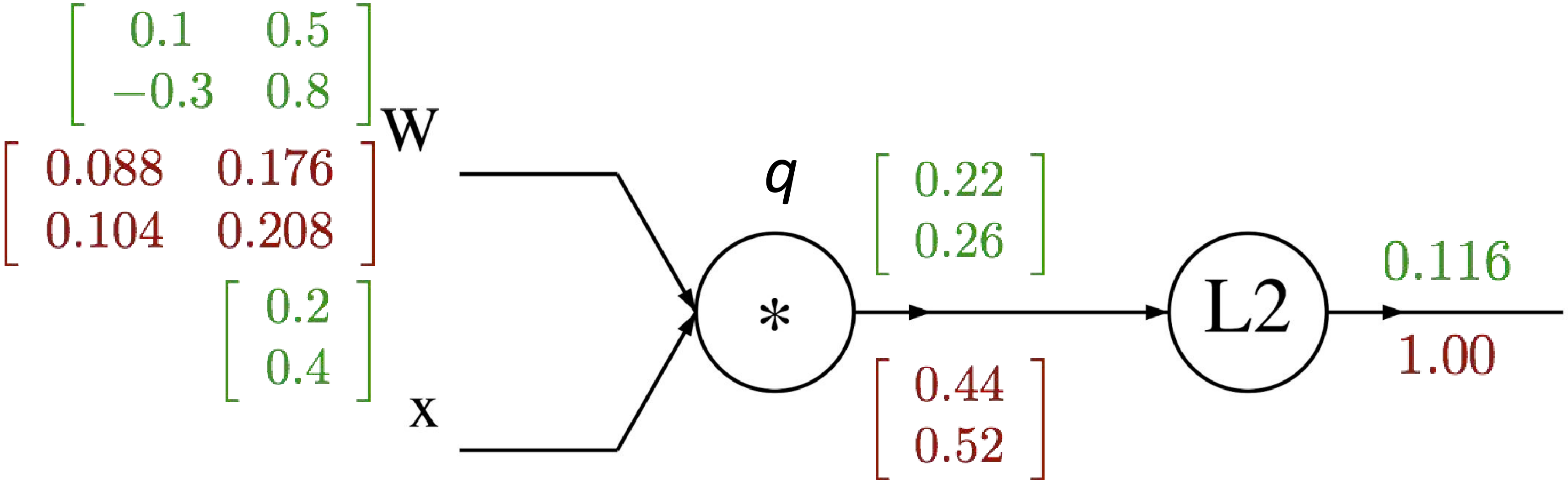


$$\frac{\partial q_k}{\partial x_i} = W_{k,i}$$

$$q = W \cdot x = \begin{pmatrix} W_{1,1}x_1 + \cdots + W_{1,n}x_n \\ \vdots \\ W_{n,1}x_1 + \cdots + W_{n,n}x_n \end{pmatrix}$$

$$f(q) = ||q||^2 = q_1^2 + \cdots + q_n^2$$

A vectorized example: $f(x, W) = ||W \cdot x||^2 = \sum_{i=1}^{n} (W \cdot x)_i^2$



$$q = W \cdot x = \begin{pmatrix} W_{1,1}x_1 + \cdots + W_{1,n}x_n \\ \vdots \\ W_{n,1}x_1 + \cdots + W_{n,n}x_n \end{pmatrix}$$

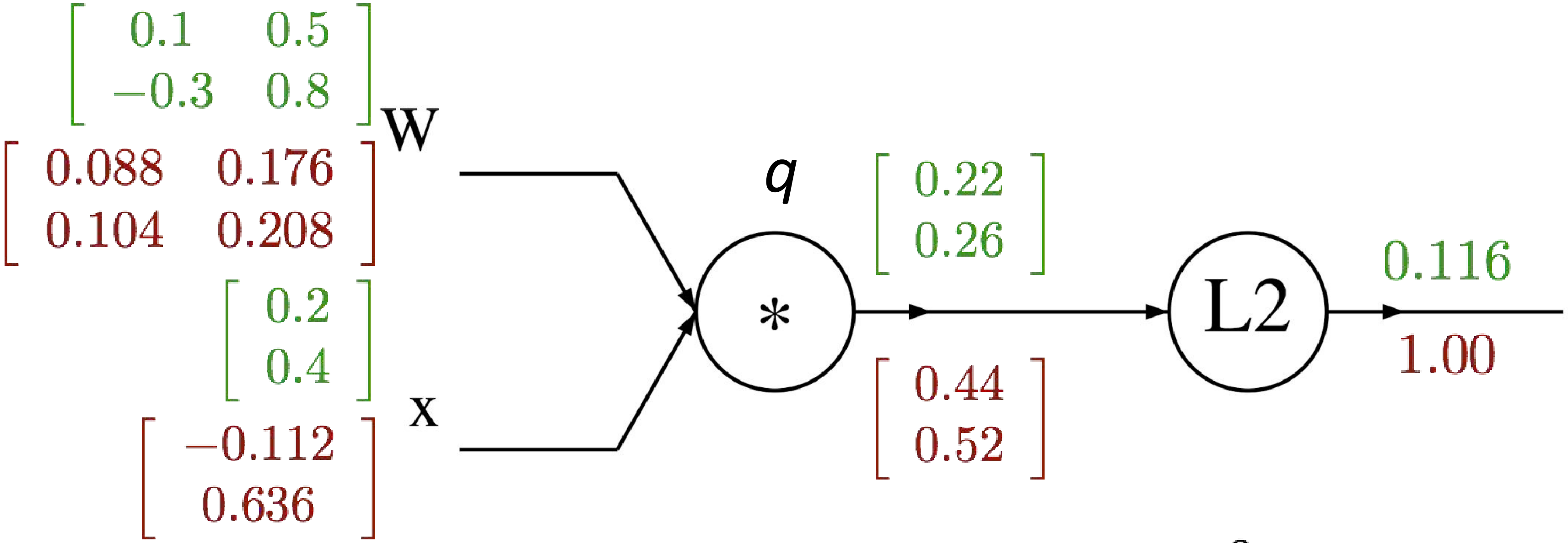
$$f(q) = ||q||^2 = q_1^2 + \cdots + q_n^2$$

$$\frac{\partial q_k}{\partial x_i} = W_{k,i}$$

$$\frac{\partial f}{\partial x_i} = \sum_k \frac{\partial f}{\partial q_k} \frac{\partial q_k}{\partial x_i}$$

$$\frac{\partial f}{\partial x_i} = \sum_k 2 q_k W_{k,i}$$

A vectorized example: $f(x, W) = ||W \cdot x||^2 = \sum_{i=1}^n (W \cdot x)_i^2$



$$\nabla_x f = 2W^T \cdot q$$

$$q = W \cdot x = \begin{pmatrix} W_{1,1}x_1 + \cdots + W_{1,n}x_n \\ \vdots \\ W_{n,1}x_1 + \cdots + W_{n,n}x_n \end{pmatrix}$$

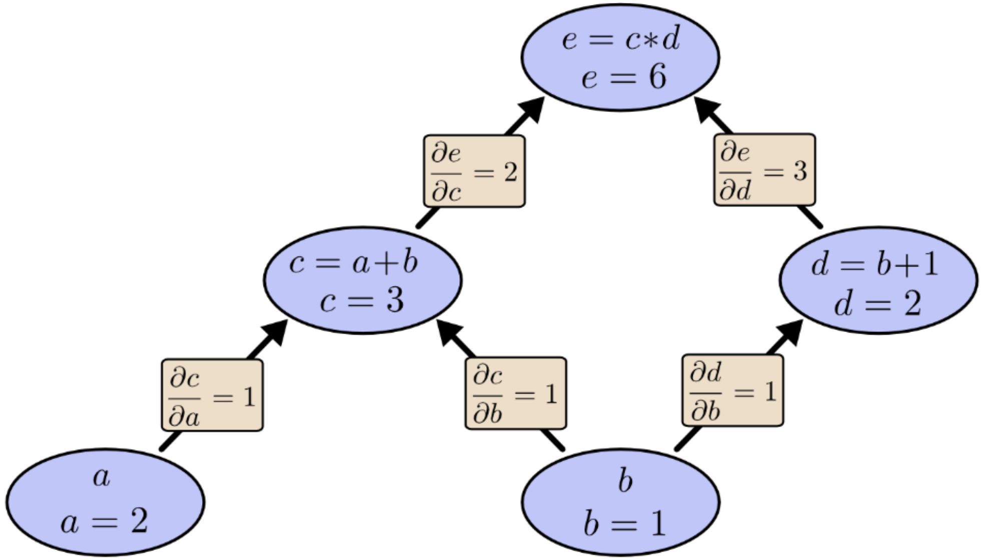
$$f(q) = ||q||^2 = q_1^2 + \cdots + q_n^2$$

$$\frac{\partial q_k}{\partial x_i} = W_{k,i}$$

$$\frac{\partial f}{\partial x_i} = \sum_k \frac{\partial f}{\partial q_k} \frac{\partial q_k}{\partial x_i}$$

$$\frac{\partial f}{\partial x_i} = \sum_k 2 q_k W_{k,i}$$

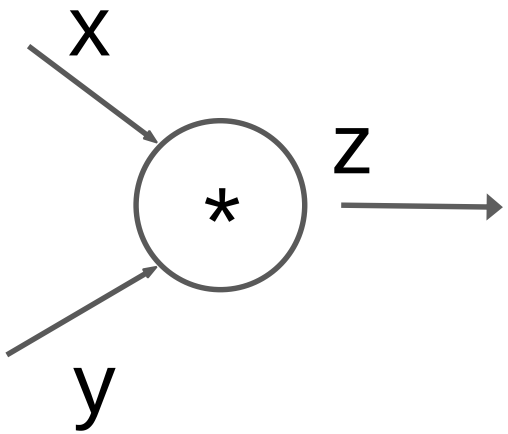
Modularized implementation: forward / backward API



Graph (or Net) object *(rough psuedo code)*

```
class ComputationalGraph(object):
    #...
    def forward(inputs):
        # 1. [pass inputs to input gates...]
        # 2. forward the computational graph:
        for gate in self.graph.nodes_topologically_sorted():
            gate.forward()
        return loss # the final gate in the graph outputs the loss
    def backward():
        for gate in reversed(self.graph.nodes_topologically_sorted()):
            gate.backward() # little piece of backprop (chain rule applied)
        return inputs_gradients
```

Modularized implementation: forward / backward API



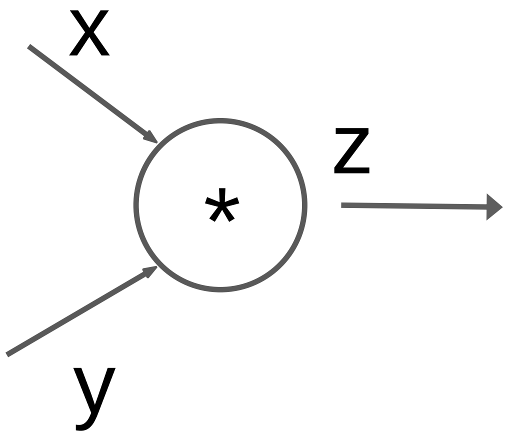
(x,y,z are scalars)

```python
class MultiplyGate(object):
    def forward(x,y):
        z = x*y
        return z
    def backward(dz):
        # dx = ... #todo
        # dy = ... #todo
        return [dx, dy]
```

$$\frac{\partial L}{\partial z}$$

$$\frac{\partial L}{\partial x}$$

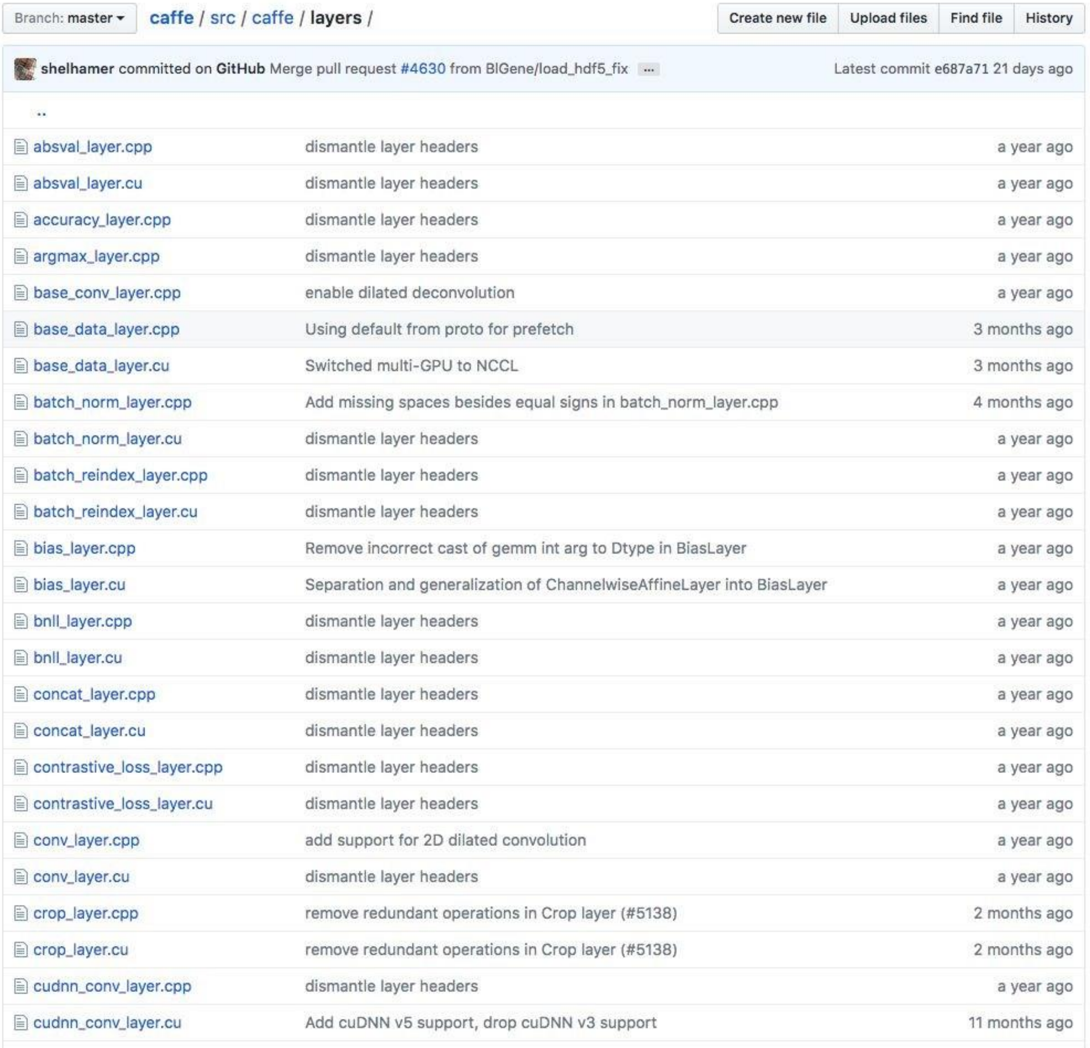
Modularized implementation: forward / backward API



(x,y,z are scalars)

```
class MultiplyGate(object):
    def forward(x,y):
        z = x*y
        self.x = x # must keep these around!
        self.y = y
        return z
    def backward(dz):
        dx = self.y * dz # [dz/dx * dL/dz]
        dy = self.x * dz # [dz/dy * dL/dz]
        return [dx, dy]
```

Example: Caffe layers


Branch: master ▾ caffe / src / caffe / layers /

Create new file | Upload files | Find file | History

shelhamer committed on GitHub Merge pull request #4630 from BlGene/load_hdf5_fix ... Latest commit e687a71 21 days ago

..

absval_layer.cpp	dismantle layer headers	a year ago
absval_layer.cu	dismantle layer headers	a year ago
accuracy_layer.cpp	dismantle layer headers	a year ago
argmax_layer.cpp	dismantle layer headers	a year ago
base_conv_layer.cpp	enable dilated deconvolution	a year ago
base_data_layer.cpp	Using default from proto for prefetch	3 months ago
base_data_layer.cu	Switched multi-GPU to NCCL	3 months ago
batch_norm_layer.cpp	Add missing spaces besides equal signs in batch_norm_layer.cpp	4 months ago
batch_norm_layer.cu	dismantle layer headers	a year ago
batch_reindex_layer.cpp	dismantle layer headers	a year ago
batch_reindex_layer.cu	dismantle layer headers	a year ago
bias_layer.cpp	Remove incorrect cast of gemm int arg to Dtype in BiasLayer	a year ago
bias_layer.cu	Separation and generalization of ChannelwiseAffineLayer into BiasLayer	a year ago
bnll_layer.cpp	dismantle layer headers	a year ago
bnll_layer.cu	dismantle layer headers	a year ago
concat_layer.cpp	dismantle layer headers	a year ago
concat_layer.cu	dismantle layer headers	a year ago
contrastive_loss_layer.cpp	dismantle layer headers	a year ago
contrastive_loss_layer.cu	dismantle layer headers	a year ago
conv_layer.cpp	add support for 2D dilated convolution	a year ago
conv_layer.cu	dismantle layer headers	a year ago
crop_layer.cpp	remove redundant operations in Crop layer (#5138)	2 months ago
crop_layer.cu	remove redundant operations in Crop layer (#5138)	2 months ago
cudnn_conv_layer.cpp	dismantle layer headers	a year ago
cudnn_conv_layer.cu	Add cuDNN v5 support, drop cuDNN v3 support	11 months ago

=

Caffe Sigmoid Layer

```cpp
#include <cmath>
#include <vector>

#include "caffe/layers/sigmoid_layer.hpp"

namespace caffe {

template <typename Dtype>
inline Dtype sigmoid(Dtype x) {
  return 1. / (1. + exp(-x));
}

template <typename Dtype>
void SigmoidLayer<Dtype>::Forward_cpu(const vector<Blob<Dtype>*>& bottom,
    const vector<Blob<Dtype>*>& top) {
  const Dtype* bottom_data = bottom[0]->cpu_data();
  Dtype* top_data = top[0]->mutable_cpu_data();
  const int count = bottom[0]->count();
  for (int i = 0; i < count; ++i) {
    top_data[i] = sigmoid(bottom_data[i]);
  }
}

template <typename Dtype>
void SigmoidLayer<Dtype>::Backward_cpu(const vector<Blob<Dtype>*>& top,
    const vector<bool>& propagate_down,
    const vector<Blob<Dtype>*>& bottom) {
  if (propagate_down[0]) {
    const Dtype* top_data = top[0]->cpu_data();
    const Dtype* top_diff = top[0]->cpu_diff();
    Dtype* bottom_diff = bottom[0]->mutable_cpu_diff();
    const int count = bottom[0]->count();
    for (int i = 0; i < count; ++i) {
      const Dtype sigmoid_x = top_data[i];
      bottom_diff[i] = top_diff[i] * sigmoid_x * (1. - sigmoid_x);
    }
  }
}

#ifdef CPU_ONLY
STUB_GPU(SigmoidLayer);
#endif

INSTANTIATE_CLASS(SigmoidLayer);


}  // namespace caffe
```

forward()

$$\boxed{\sigma(x) = \frac{1}{1+e^{-x}}}$$

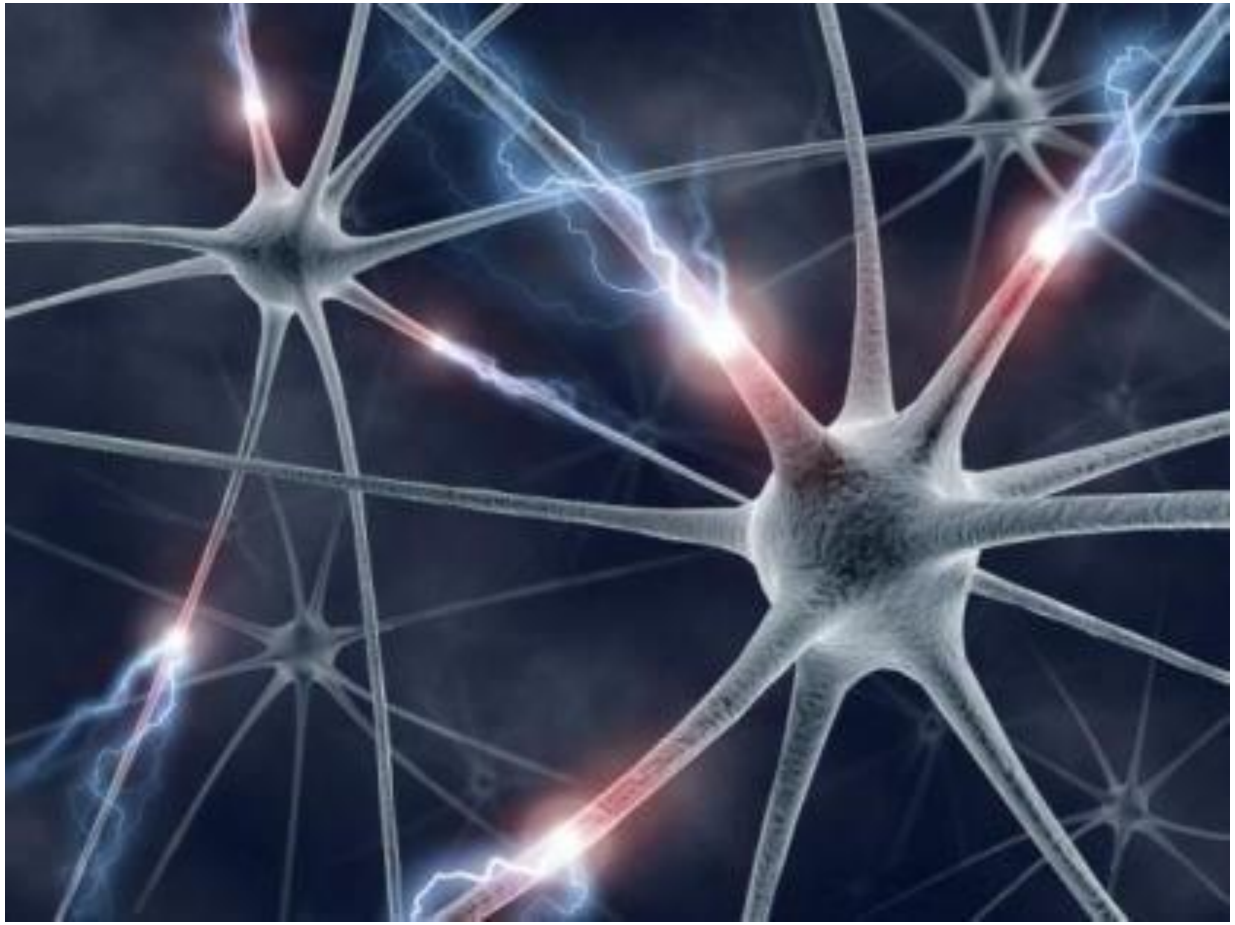
backward()

$$\boxed{(1-\sigma(x))\,\sigma(x)}$$ * top_diff (chain rule)

Backpropagation summary

- neural nets will be very large: impractical to write down gradient formula by hand for all parameters
- **backpropagation** = recursive application of the chain rule along a computational graph to compute the gradients of all inputs/parameters/intermediates
- implementations maintain a graph structure, where the nodes implement the **forward()** / **backward()** API
- **forward**: compute result of an operation and save any intermediates needed for gradient computation in memory
- **backward**: apply the chain rule to compute the gradient of the loss function with respect to the inputs

Neural Networks



Neural Networks

(**Before**) Linear score function: $f = Wx$

Neural Networks

(**Before**) Linear score function: $f = Wx$

(**Now**) 2-layer Neural Network: $f = W_2 \max(0, W_1 x)$

Neural Networks

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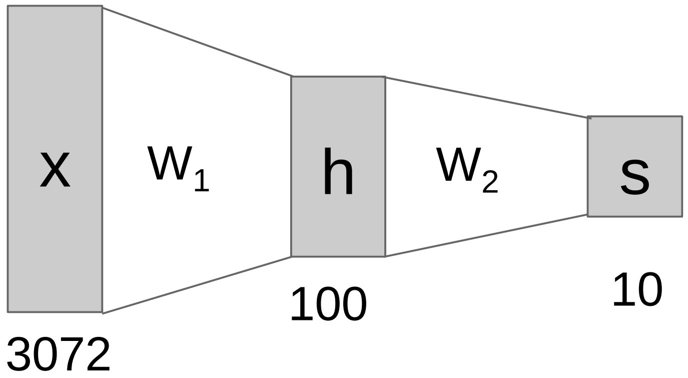
(**Now**) 2-layer Neural Network: $f = W_2 \max(0, W_1 x)$

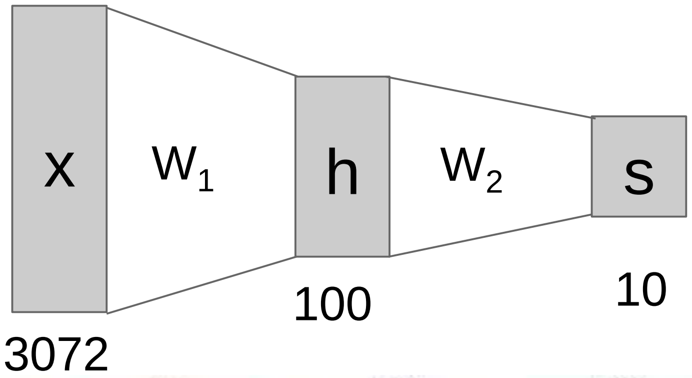


hidden layer

Neural Networks

(**Before**) Linear score function: $f = Wx$

(**Now**) 2-layer Neural Network: $f = W_2 \max(0, W_1 x)$



Neural Networks

(**Before**) Linear score function: $f = Wx$

(**Now**) 2-layer Neural Network: $f = W_2 \max(0, W_1 x)$

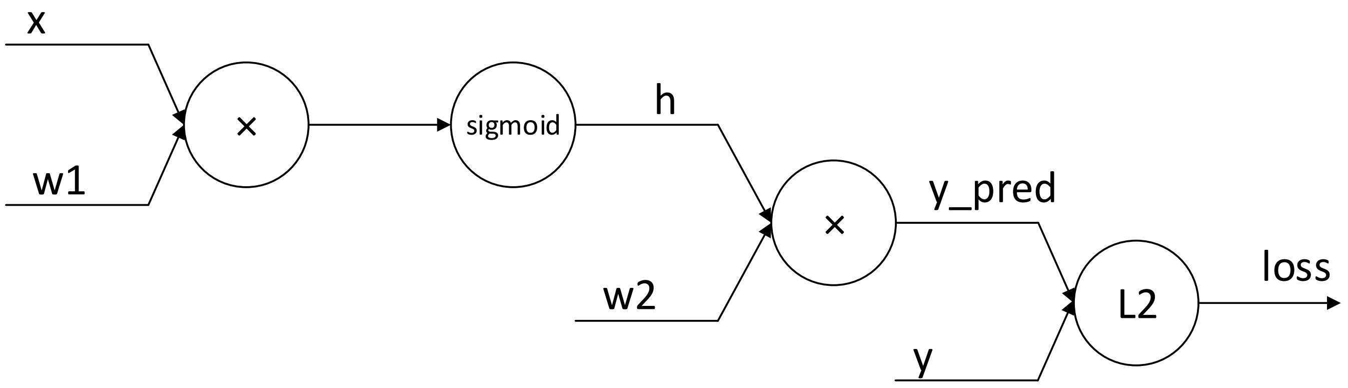
we can go deeper

3-layer Neural Network $f = W_3 \max(0, W_2 \max(0, W_1 x))$

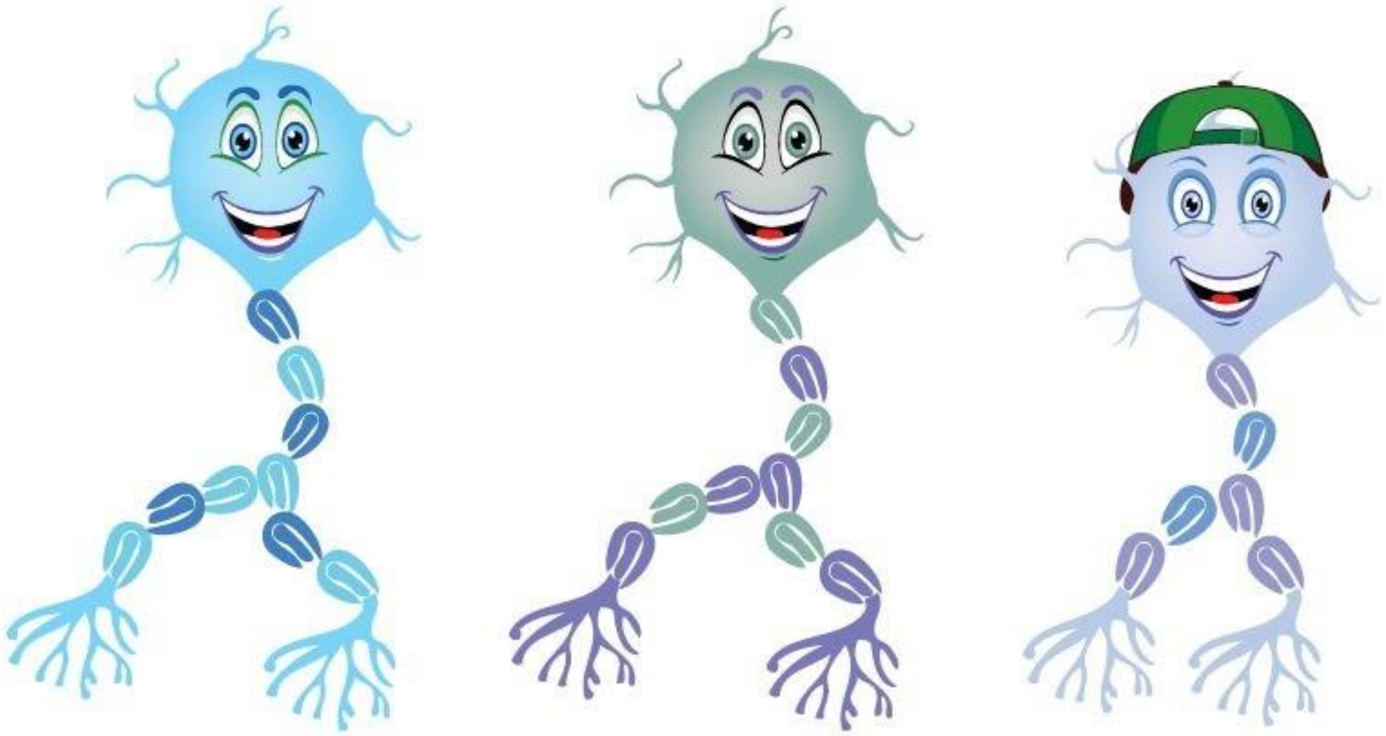
Full implementation of training a 2-layer Neural Network needs ~20 lines:

```python
import numpy as np
from numpy.random import randn

N, D_in, H, D_out = 64, 1000, 100, 10
x, y = randn(N, D_in), randn(N, D_out)
w1, w2 = randn(D_in, H), randn(H, D_out)

for t in range(2000):
  h = 1 / (1 + np.exp(-x.dot(w1)))
  y_pred = h.dot(w2)
  loss = np.square(y_pred - y).sum()
  print(t, loss)

  grad_y_pred = 2.0 * (y_pred - y)
  grad_w2 = h.T.dot(grad_y_pred)
  grad_h = grad_y_pred.dot(w2.T)
  grad_w1 = x.T.dot(grad_h * h * (1 - h))

  w1 -= 1e-4 * grad_w1
  w2 -= 1e-4 * grad_w2
```

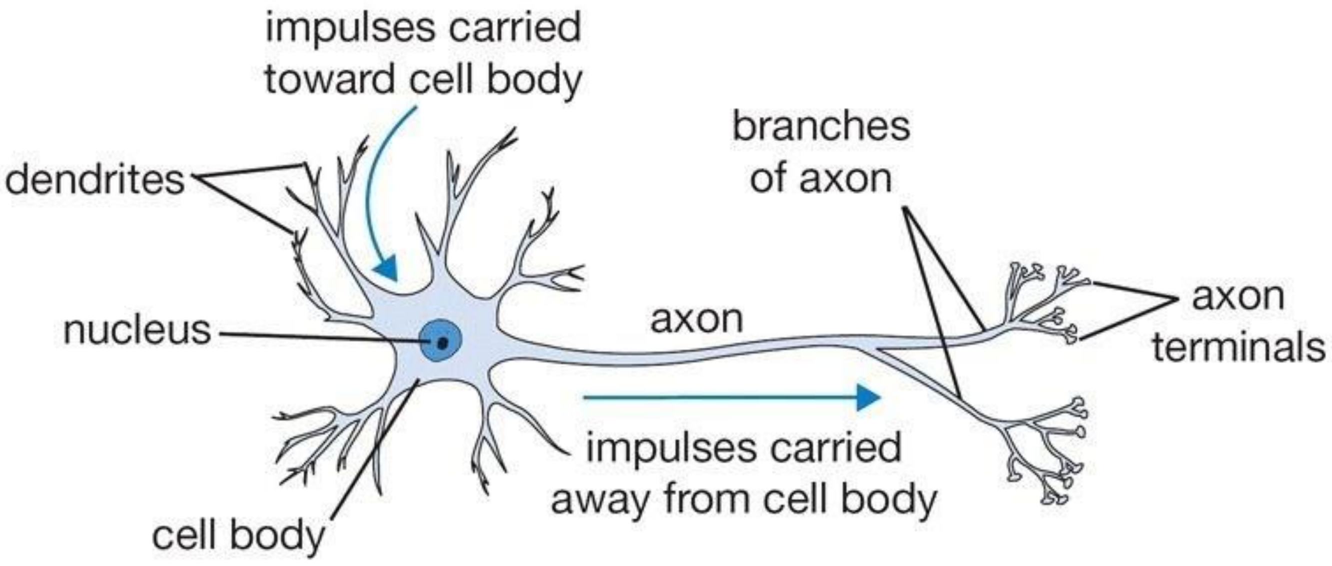
$$\sigma(x) = \frac{1}{1+e^{-x}}$$ sigmoid function



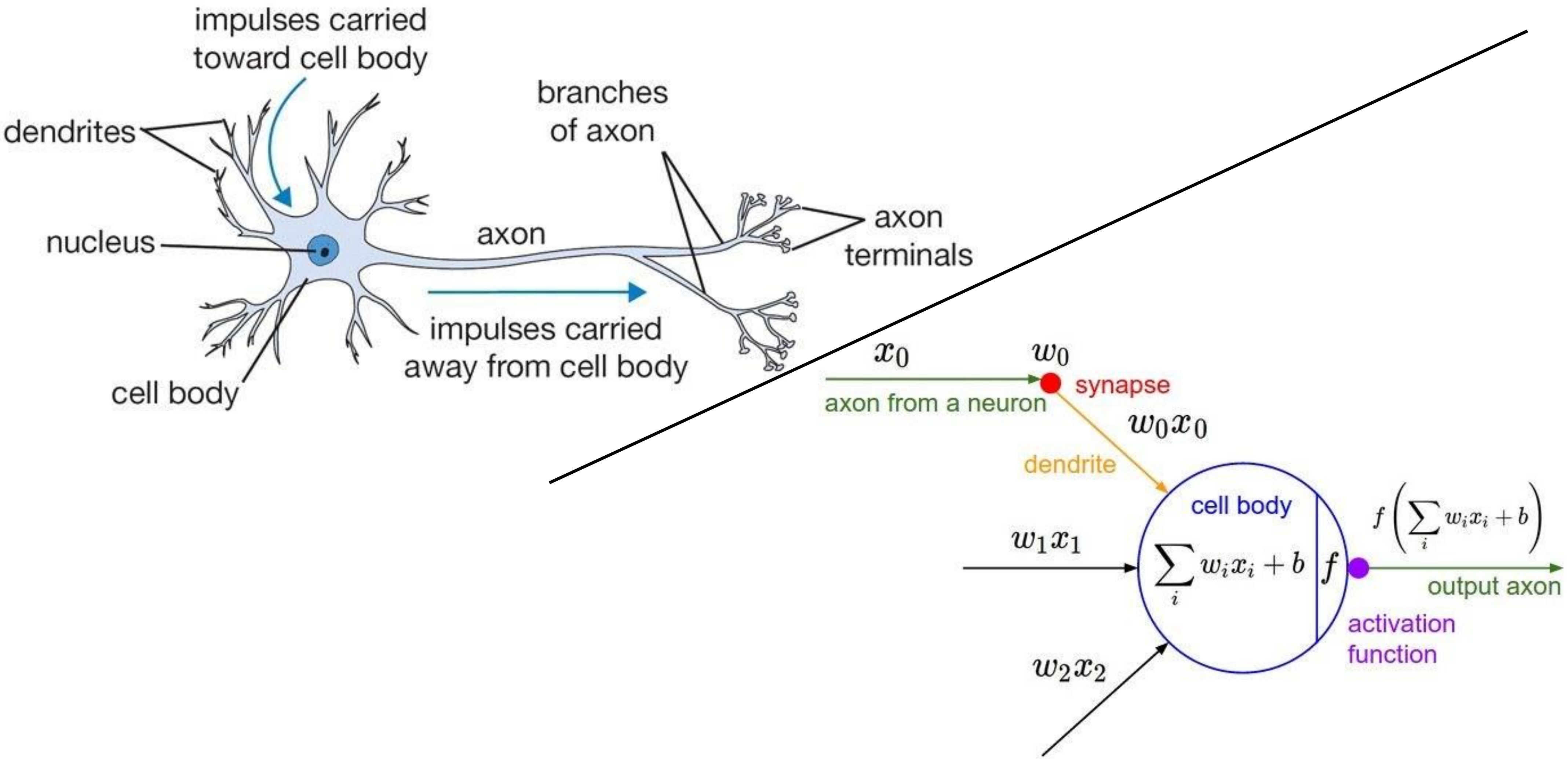
Artificial neuron



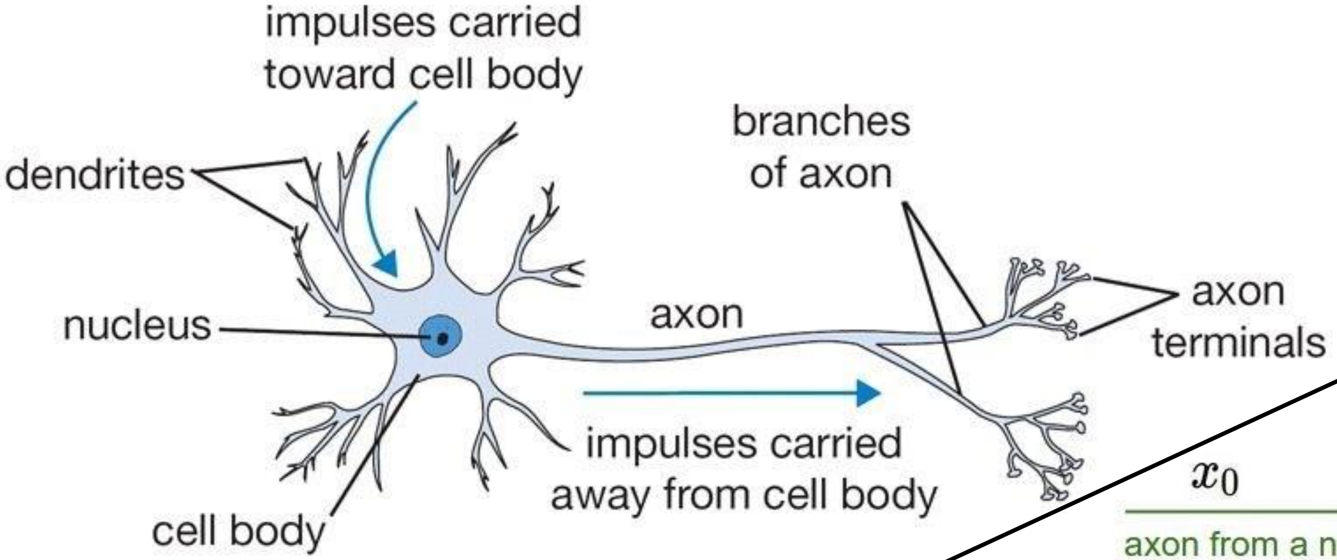
Biological neuron

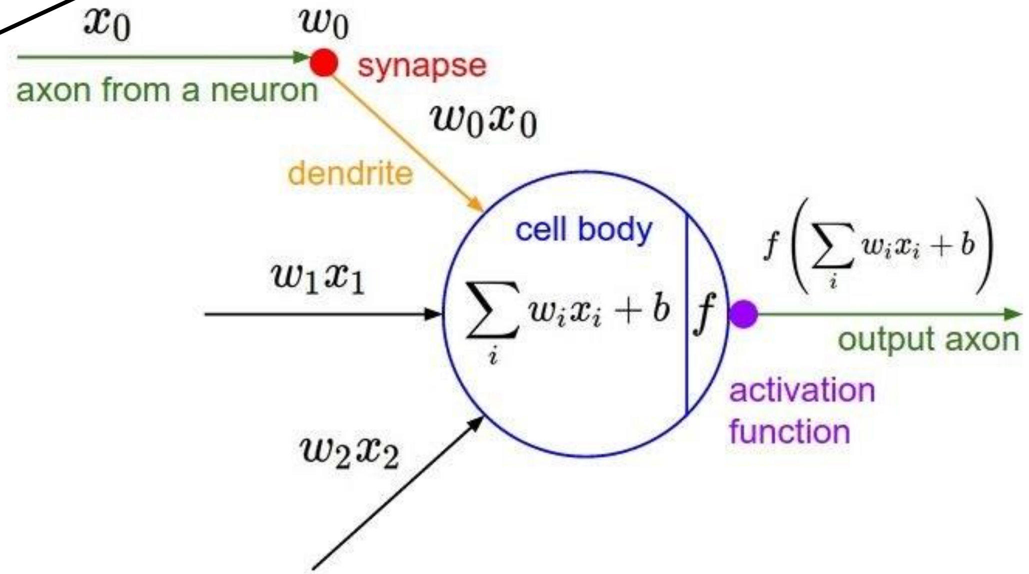


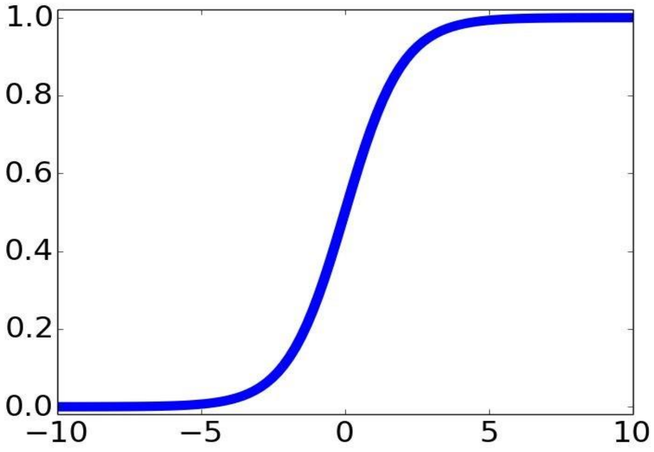
Artificial neuron



Artificial neuron





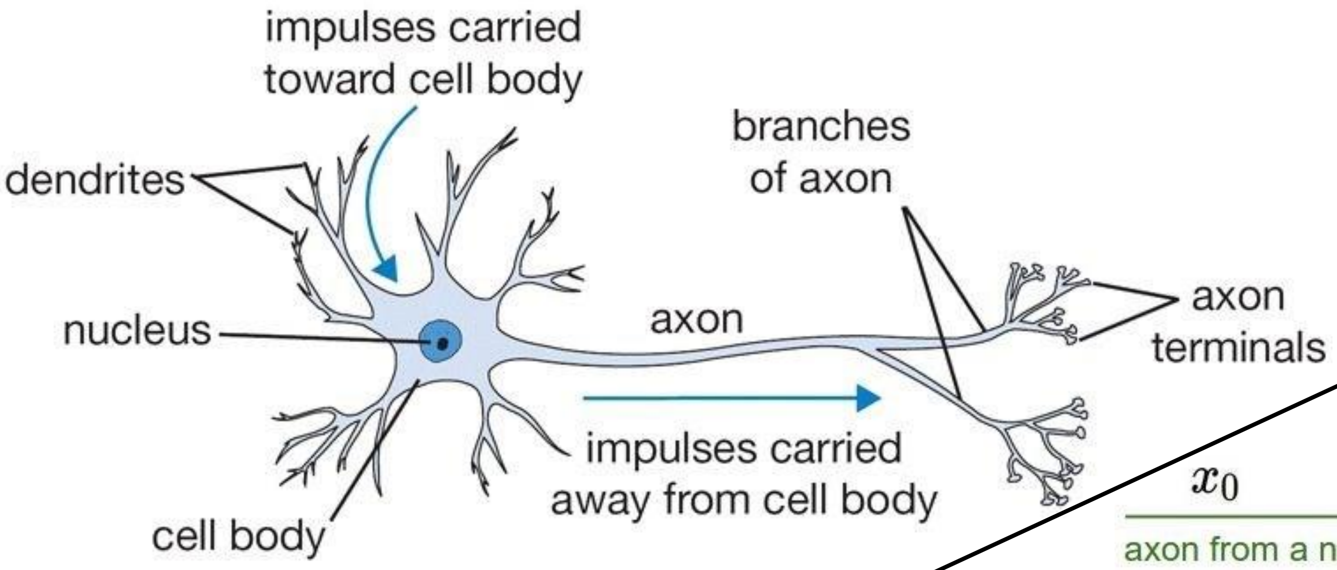


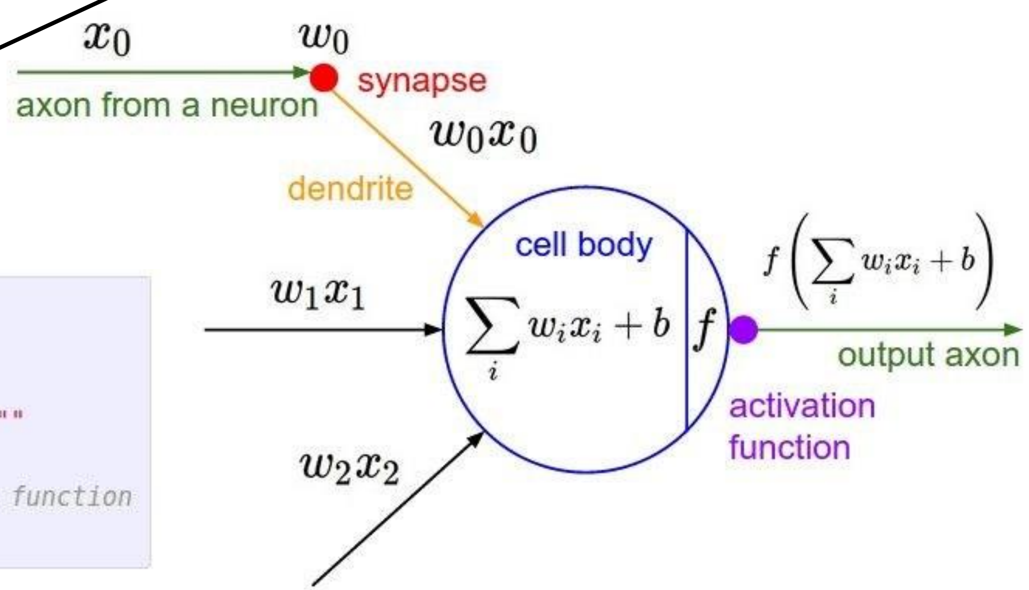
sigmoid activation

function

$$\frac{1}{1+e^{-x}}$$

Artificial neuron



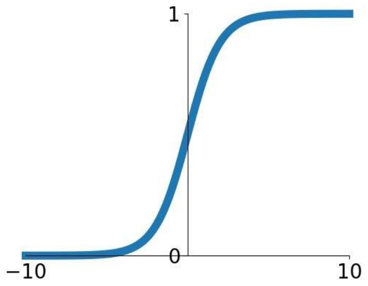



```python
class Neuron:
  # ...
  def neuron_tick(inputs):
    """ assume inputs and weights are 1-D numpy arrays and bias is a number """
    cell_body_sum = np.sum(inputs * self.weights) + self.bias
    firing_rate = 1.0 / (1.0 + math.exp(-cell_body_sum)) # sigmoid activation function
    return firing_rate
```

Активационные функции

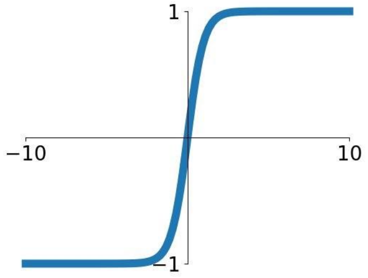
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



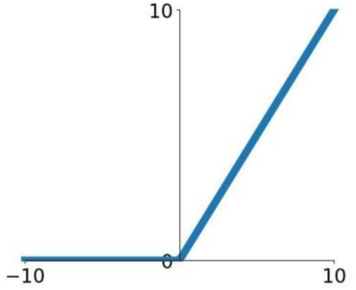
tanh

$$\tanh(x)$$



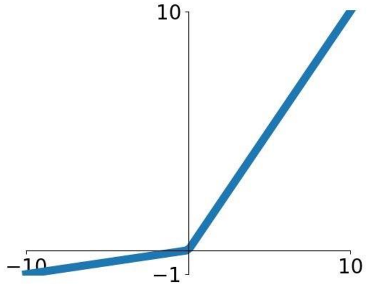
ReLU

$$\max(0, x)$$



Leaky ReLU

$$\max(0.1x, x)$$

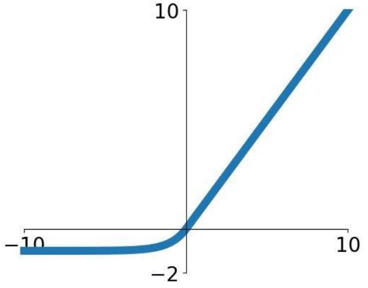


Maxout neuron

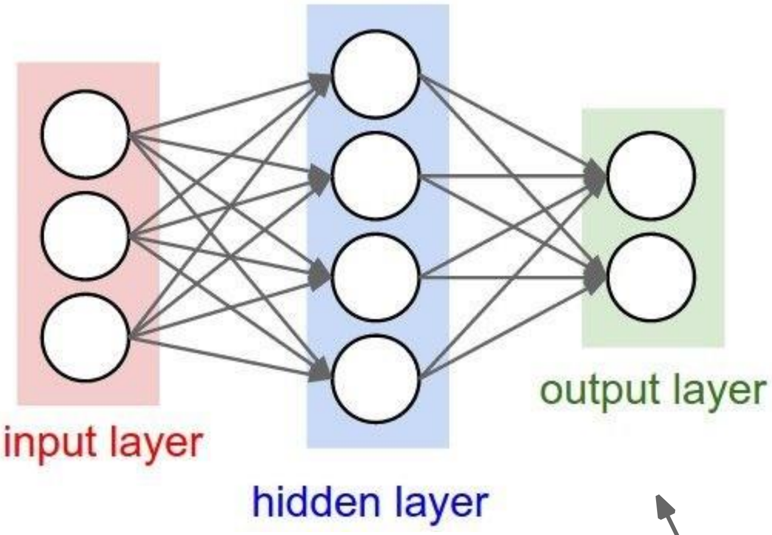
$$\max(w_1^T x + b_1, w_2^T x + b_2)$$

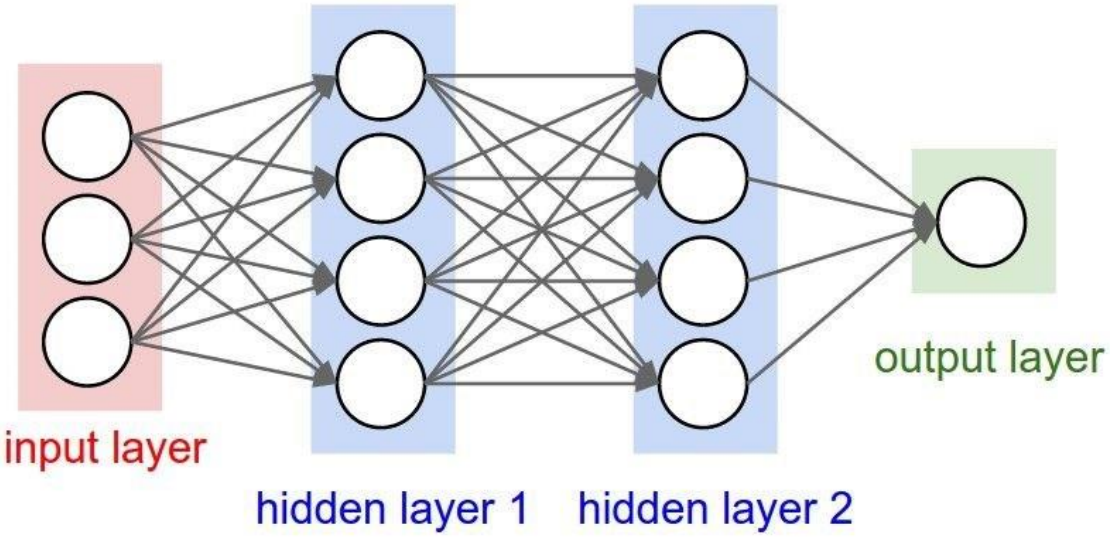
ELU

$$\begin{cases} x & x \geq 0 \\ \alpha(e^x - 1) & x < 0 \end{cases}$$



Neural networks: fully-connected architectures



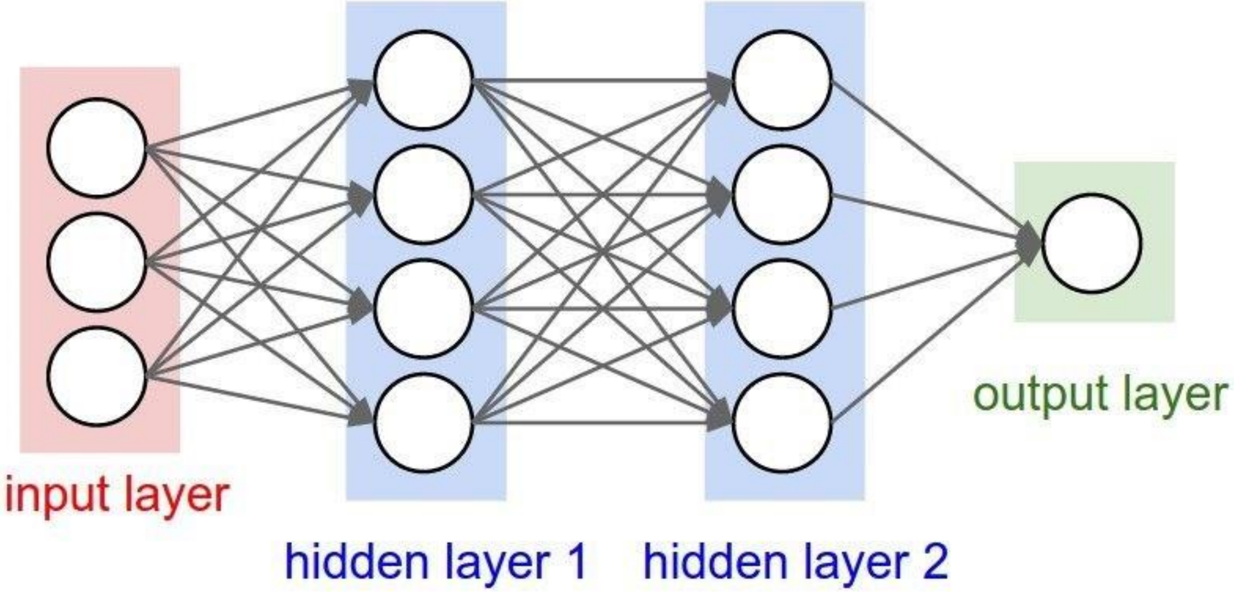


“2-layer Neural Net”, or
“1-hidden-layer Neural Net”

“3-layer Neural Net”, or
“2-hidden-layer Neural Net”

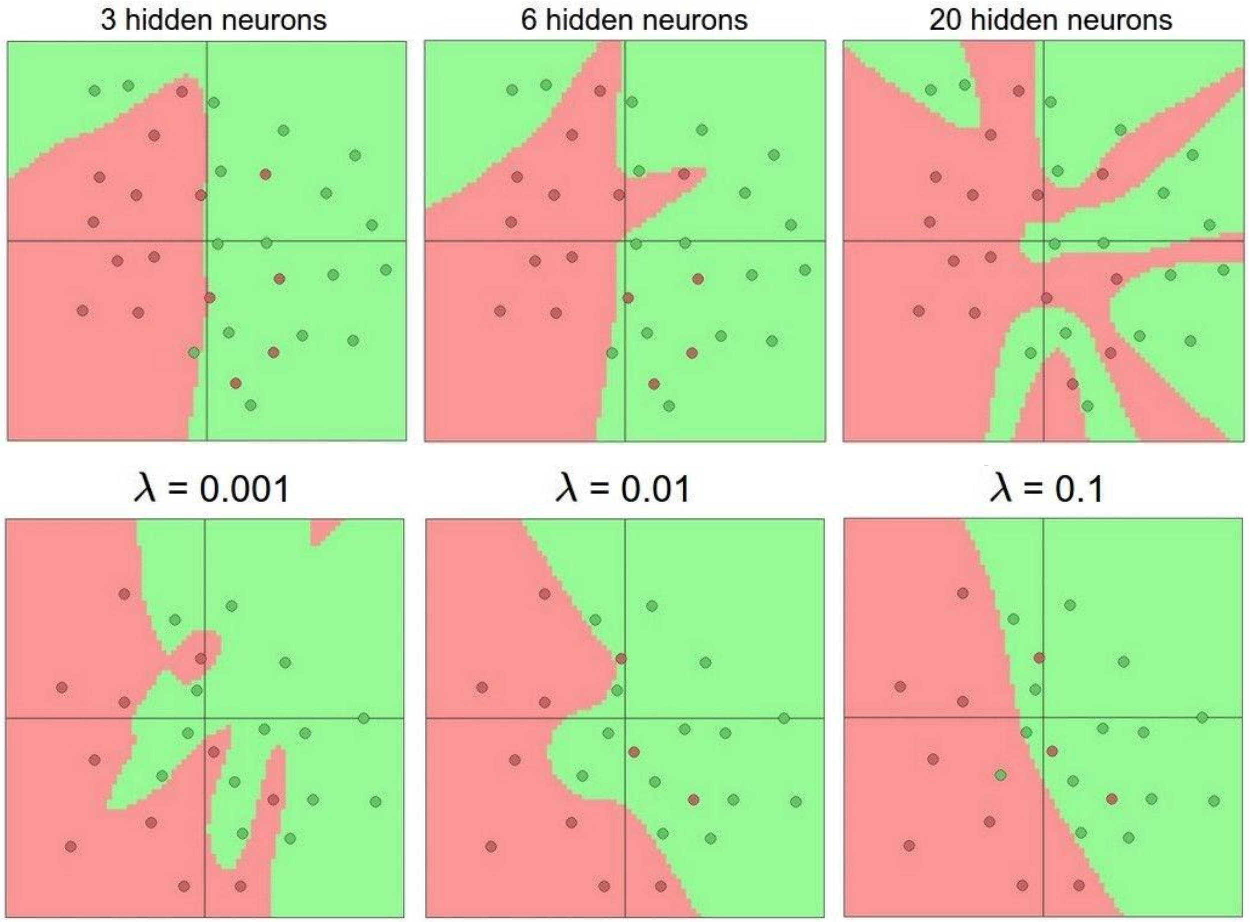
“Fully-connected” layers

Example feed-forward computation of a neural network




```python
# forward-pass of a 3-layer neural network:
f = lambda x: 1.0/(1.0 + np.exp(-x)) # activation function (use sigmoid)
x = np.random.randn(3, 1) # random input vector of three numbers (3x1)
h1 = f(np.dot(W1, x) + b1) # calculate first hidden layer activations (4x1)
h2 = f(np.dot(W2, h1) + b2) # calculate second hidden layer activations (4x1)
out = np.dot(W3, h2) + b3 # output neuron (1x1)
```

Демо онлайн



Setting the number of layers and their sizes

Setting regularization

http://cs.stanford.edu/people/karpathy/convnetjs/demo/classify2d.html

В следующий раз

- Сверточные нейронные сети –
  Convolutional Neural Networks (CNN)