

# Consumer Behavior

## EC 311 - Intermediate Microeconomics

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# Chapter 4

## Outline

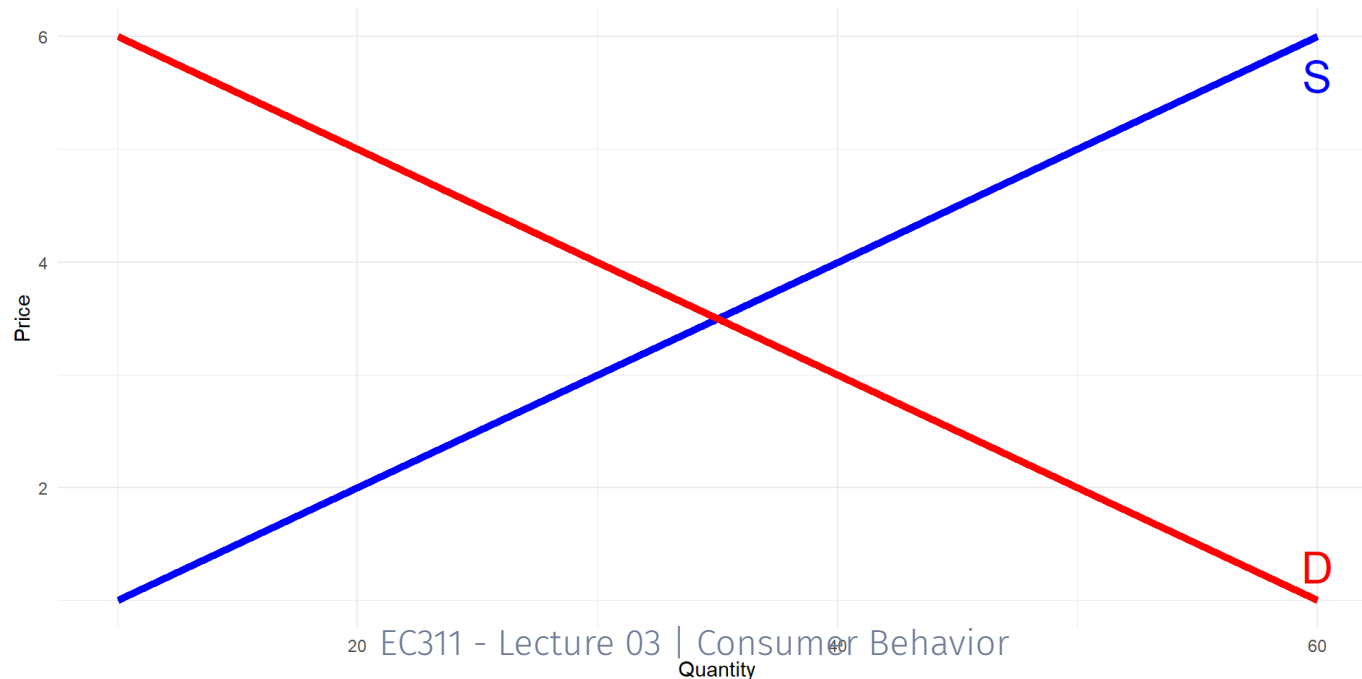
- Topics
  - Preferences and Utility (4.1)
  - Indifference Curves (4.2)
  - Budget Constraints (4.3)
- Other Stuff
  - Remember Office Hours on **Thursdays from 12:00 - 14:00 in PLC 523**
  - Also feel free to email me your questions at **[jrojas2@uoregon.com](mailto:jrojas2@uoregon.com)**

# Preferences and Utility

# Where Does Demand Come From?

Imagine the market for coffee on campus

- Additionally, picture that the only other good that can be purchased is tea
- On a basic level, **demand for coffee** is derived from individual's choosing how to divide their income between coffee and tea



# Where Does Demand Come From?

This is the problem we will be dealing with through the first half of the course

How does an individual allocate a **finite amount of resources** between **two goods**?

**Note:** There are clearly more than just two goods out there, so how can this be useful?

**The main critique I always hear about economics courses is that they're unrealistic. That's mostly true, but we can learn about the aggregate by simplifying and making assumptions**

# Where Does Demand Come From?

- We will frame the decisions as a two-good model where you may choose between:
  - Food and durable consumption
  - Leisure (not working) and consumption (paid for by earning a wage)
  - Consumption now and consumption later

**The key takeaway here is that we can frame many important choices as "two-good" decisions**

- This makes things simpler for us to solve while still maintaining some sense of the real world

# Determinants of Consumption

Consumption of any single good has two parts:

- How it **BENEFITS** the consumer
  - We call this **UTILITY**
- What it **COSTS** the consumer
  - What we give up to purchase the good

Let's see what this means through a 1-good example → Beer

- Imagine the following scenario:
  - You just arrived at the bar and have had zero drinks so far
  - Each beer costs the same: \$4

# Beer

Number of Beers	Overall Level of Happiness	Change in Level of Happiness
0	0	-
1	10	10
2	25	15
3	35	10
4	40	5
5	42	3
6	30	-12

If beer were free, how many beers should this person drink?

5

Now recall that beer costs \$4, how many beers should this person drink?

4



# Intuition Behind “Choice” in Economics

You **cannot** simply find the consumption amount that makes you the happiest. **But why?**

The goal is to maximize your **utility** whilst acknowledging you have **constraints**

The choice is simple: **consume an additional unit until the cost of doing so outweighs the benefit**

**Commit this idea to memory: it is the crux of economics and drives everything we will be doing**

**We maximize utility up to the point that it does not make sense to do so**

# Back to Beer

Number of Beers	Overall Level of Happiness	Change in Level of Happiness
0	0	-
1	10	10
2	25	15
3	35	10
4	40	5
5	42	3
6	30	-12

Some Questions:

- What is the marginal benefit:
  - When you have not consumed any beer?
  - When you have already consumed 3 beers?
  - When you have already consumed 5 beers?
- What is the marginal cost of beer?
  - Does it change as we consume more?

# Being and Thinking at the Margin

We found two important values:

- **Marginal Benefit (MB)**

The additional benefit gained for an added unit of consumption

- **Marginal Cost (MC)**

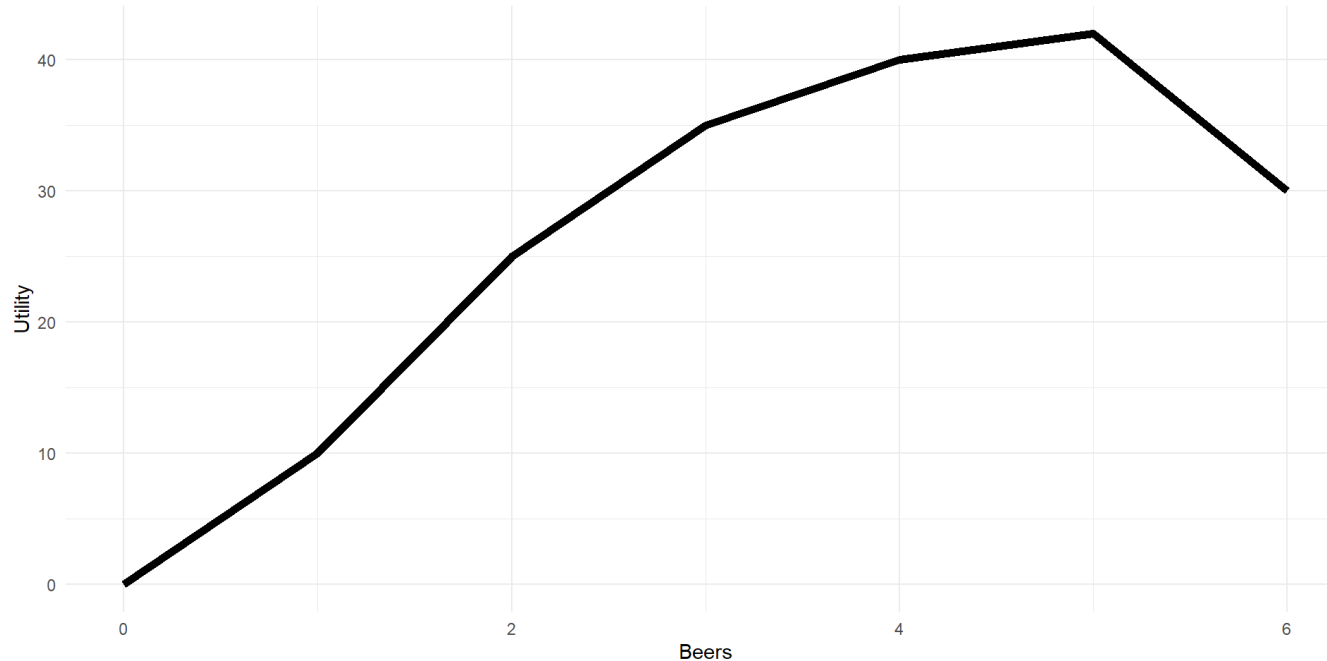
The additional cost paid for an added unit of consumption

We can describe the decision-making process in a more formal manner:

- Initially:  $MB > MC \rightarrow$  Consume more!
- Eventually:  $MB < MC \rightarrow$  We went too far!
- At some point:  $MB = MC \rightarrow$  Just right!

**Ask yourself: Why must they be equal?**

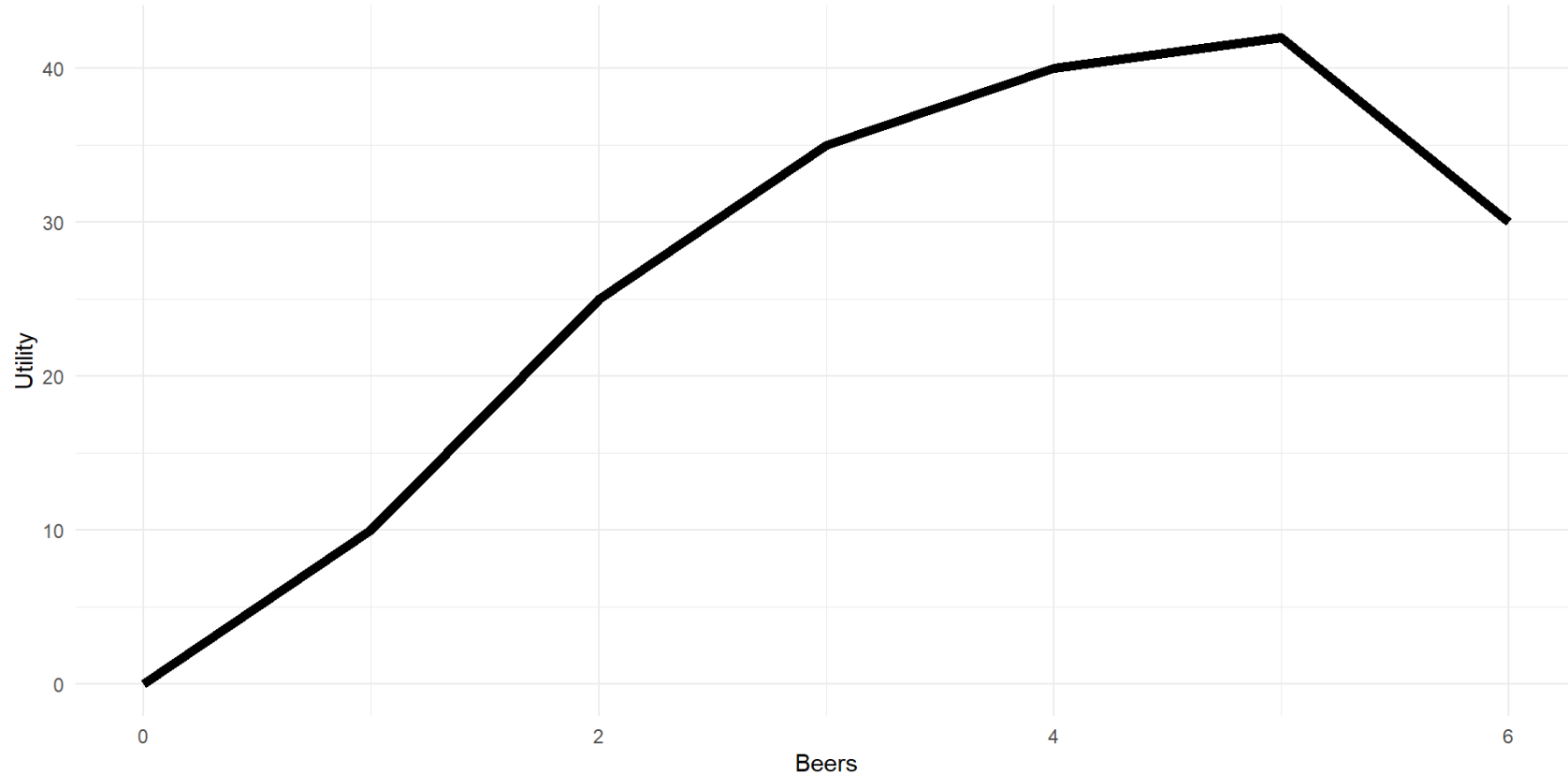
# What's Going On? Graphically



What matters for choice is the **marginal benefit** of an additional beer  
In other words, what matters is the change in **utility** that occurs as we move to the right on the graph

**Recall: A change in  $y$  as  $x$  increases is the derivative**

# What's Going On? Graphically - Derivatives

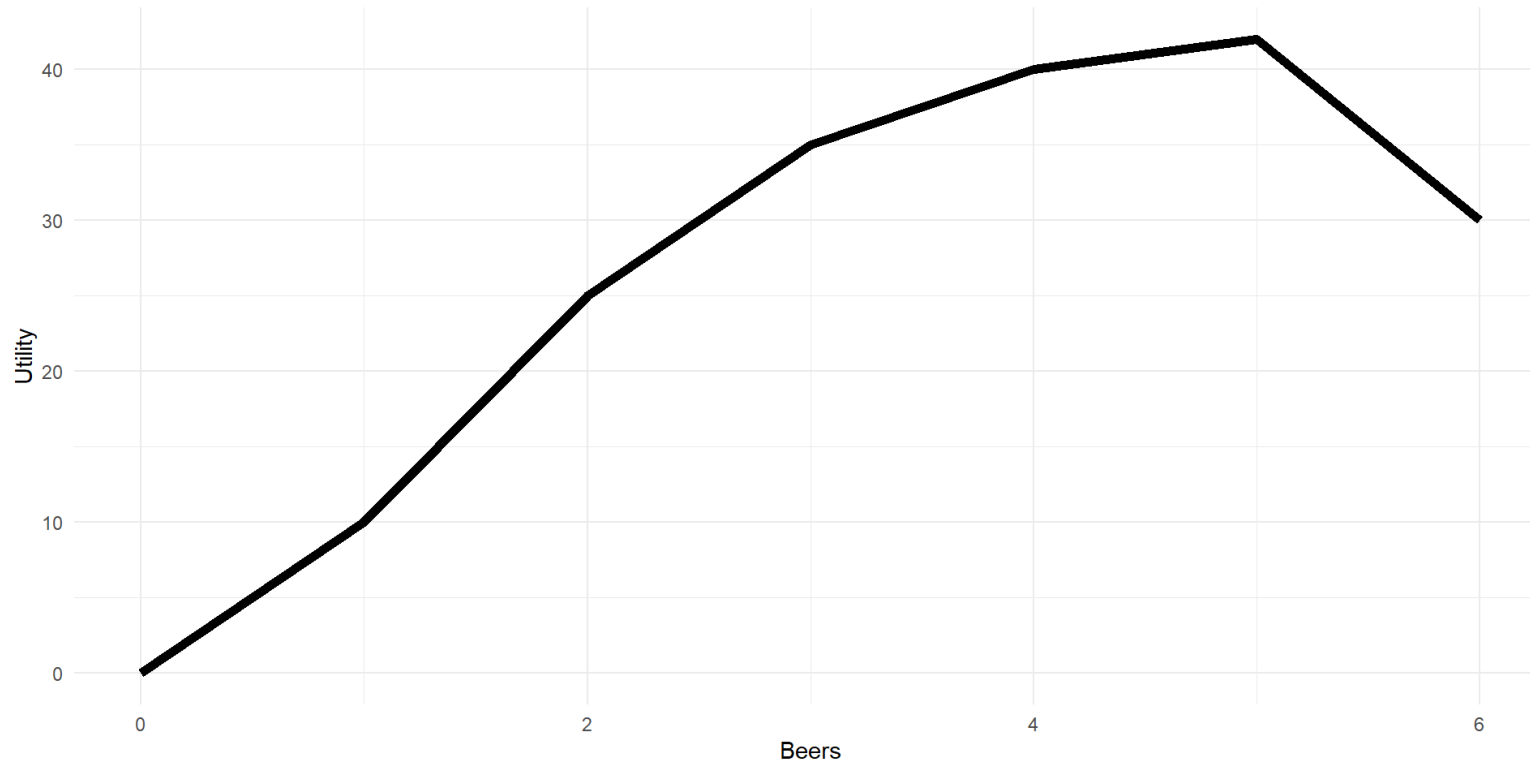


The derivative of this function is the **marginal benefit** of beer

**This is why derivatives are important**

We can use derivatives to figure out the optimal amount to consume

# What's Going On? Graphically - MB & MC



**Recall:** The optimal choice is the point where **MB** = **MC**

The **MC** = 4. So we would choose the quantity of beers where **MB** = 4

# What's Going On? Mathematically

The utility function of beer we've been using is:

$$U(x) = -x^2 + 12x$$

**Let's practice:** What would the optimal consumption amount be if the cost of beer is \$2?

**Find the Marginal Benefit**

$$MU_x = \frac{\partial U}{\partial x} = -2x + 12$$

**Set the MB equal to MC**

$$\begin{aligned} MB &= MC \\ -2x + 12 &= 2 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

# A Two-Good Problem

The beer example was fairly straightforward. But we will be dealing with making choices between two goods.

Before we dive in, a couple of things to consider are:

- When we spend our resources on one good, it cannot be spent on something else
  - Because we are making decisions amongst both things, we need a **cost relationship** between them
- We do not measure our happiness (utility) in dollars
  - We need to find a way to choose without directly comparing costs and benefits across goods



# Enter Utility Functions

A **Utility Function** is a function of two variables:

$$U = f(x, y)$$

## Some facts:

- $x$  and  $y$  are the (positive) amounts of goods you consume
- The function converts  $x$  and  $y$  to happiness (utility) from consuming the two goods

## For Example:

- Let's say that the utility I receive from **Beer (B)** and **Soda (S)** can be modeled as:

$$U = f(B, S) = 10B + 2S$$

- Before, we were only interested in the slope of Beer
- Now, we are interested in the **slopes** of both Beer & Soda

# Utility Functions - Beer & Soda

$$U = f(B, S) = 10B + 2S$$

For a function of two variables there are **two slopes**

- One for each **partial derivative** which we will call **Marginal Utilities**
- In the **Beer (B)** and **Soda (S)** example we will have:
  - The **Marginal Utility of Beer** ( $MU_B$ )
  - The **Marginal Utility of Soda** ( $MU_S$ )

What are the Marginal Utilities of Beer and Soda?

# Utility Functions - PB&J

Now find the Marginal Utilities for Peanut Butter and Jelly

$$U = f(P, J) = P^2 \cdot J$$

**Peanut Butter**

$$MU_P = 2P^{2-1} \cdot J = 2PJ$$

**Jelly**

$$MU_J = P^2 \cdot J^{1-1} = P^2$$

# What do Utility Functions even Mean?

They help us represent how people feel about goods  $x$  and  $y$

There are certain properties that help us determine:

- Do I **like or dislike**  $x$ ?
- Does **how much I like**  $x$  depend on **how much  $x$  I already have**?
- Does **how much I like**  $x$  depend on **how much  $y$  I have**?

**We can understand these properties by looking at the marginal utilities!**

# Utility Functions - Do I Like or Dislike $x$ ?

**If I consume more  $x$ , how does my utility move?**

Beyond graphing the utility function, we need to find a way to answer this formally and mathematically

We can look at the **sign of the derivative**

- If the derivative of  $U$  with respect to (w.r.t.)  $x$  is:

**Positive**

I like  $x$

**Negative**

I dislike  $x$

# Like or Dislike - Example

Given my utility curve for **Beer (B)** and **Soda (S)**, what do I like or dislike?

$$U = f(B, S) = 10B + 2S$$

I like **Beer**

$$MU_B = 10 > 0$$

I like **Soda**

$$MU_S = 2 > 0$$

# Does How Much I like $x$ Depend on How Much I Already Have?

**Remember to think at the margin**

Is each additional unit of  $x$  bring me **more**, **less**, or **equal** happiness as the previous unit?

This is slightly trickier to figure out, but we still use marginal utility logic

- In fact, we will use what is called the **Second Derivative**
- Mathematically, this is the **derivative** of  $MU_x$  w.r.t.  $x$  and we ask:
  - Is this **second derivative** **positive**, **negative**, or **zero**?

# Depend on How Much I Already Have - Example

My utility for **Cookies** and **Milk**:

$$U = f(C, M) = C^{1/2} M^{1/2}$$

## Cookies

$$MU_c = \frac{1}{2} \cdot C^{-1/2} \cdot M^{1/2}$$

$$MU_{cc} = \frac{-1}{2} \cdot \frac{1}{2} \cdot C^{-1/2-1} \cdot M^{1/2}$$

$$MU_{cc} = \frac{-1}{4} \cdot \frac{1}{C^{3/2}} \cdot M^{1/2}$$

$$MU_{cc} = \frac{-M^{1/2}}{4C^{3/2}}$$

## Milk

$$U_{mm} = \frac{-C^{1/2}}{4M^{3/2}}$$

$MU_{cc}$  is **negative** so we can say that **Cookies** have a Decreasing Marginal Utility



# Does How Much I Like $x$ Depend on How Much $y$ I Have?

This one is more straightforward: Does the marginal utility of  $x$  depend on  $y$ ?

- Mathematically, we take the derivative of  $MU_x$  w.r.t. to  $y$ , and vice-versa.
  - This is called the **cross-partial derivative**
- Notationally, we have:  $MU_{xy}$

Where we can determine the order of derivatives by looking at the subscript:

- $x$  is first, and  $y$  is the second derivative

# Depend on How Much of the Other Good I Have - Example

How about this utility for Peanut Butter and Jelly

$$U = f(P, J) = P^2 \cdot J$$

**Peanut Butter**

**Jelly**

$$MU_P = 2P^{2-1} \cdot J = 2PJ$$

$$MU_J = P^2 \cdot J^{1-1} = P^2$$

$$MU_{PJ} = 2P \cdot J^{1-1} = 2P$$

$$MU_{JP} = P^2 = 2P$$

**Notice that the cross-partials are the same and this will always be the case for any utility function!**

# Meaning of a Utility Function

What are Utility Functions?

- They are a flexible tool that help us describe the relationship between two goods and the utility (happiness) you gain from them
- They allow us to get a good intuition of how we can change function properties so they relate to the choice we are attempting to model
- Let's think about some goods and decide what the utility function should look like

# Modeling with Utility Functions

Let's consider **Homework** and **Pizza**

- First, we decide whether the good is desirable (**good**) or undesirable (**bad**)
  - I'll make the bold assumption that **Homework** is a bad and that **Pizza** is a good

**Note:** This implies that the marginal utilities are  $MU_H < 0$  and  $MU_P > 0$

# Modeling with Utility Functions - Example

**Homework** is a **bad** and **Pizza** is a **good**

Let's also set the following requirements:

The **marginal disutility** of **homework** is larger when I have more of it

$MU_H$  is decreasing in  $H \rightarrow$  We need an  $H$  in  $MU_H$

The **marginal utility** of **pizza** is smaller when I have more of it

$MU_P$  becomes smaller as I have more  $P \rightarrow$  We need an  $H$  in  $MU_H$

$MU_P$  does **not** depend on **homework**  $MU_P$  does not have an  $H$

**Attempt creating a utility function with the above characteristics**

# Modeling our Homework and Pizza

Here's my version:

$$U(H, P) = -H^2 + \ln(P)$$

Now let's prove that it meets the requirements

**Homework** is a **bad** and must be worse the more I have of interested

$$MU_H = -2H < 0$$

$$MU_{HH} = -2 < 0$$

**Pizza** is a **good**, I get less joy from it the more I have, and it does not depend on how much **homework** I have

$$MU_P = \frac{1}{P} > 0$$

$$MU_{PP} = \frac{-1}{P^2} < 0$$

$$MU_{PH} = 0 = MU_{HP}$$

# Meaning of a Utility function

The single most important property of a utility function is that we can measure **the relative preference for one good over the other**

- We can measure **how many units of  $y$  would you give up to get one more unit of  $x$ ?**
  - We call this the **Marginal Rate of Substitution (MRS)**

$$MRS = \frac{MU_x}{MU_y}$$

# Marginal Rate of Substitution (MRS)

Here we are talking about the relative preference of  $x$  over  $y$ , but how?

- Consider  $U = f(x, y) = 4x + 2y$ 
  - You get 4 units of utility for each  $x \rightarrow 4$
  - You get 2 units of utility for each  $y \rightarrow 2$
  - We can say that each  $x$  is twice as valuable as each  $y$

Using our MRS formula we have:

$$MRS = \frac{MU_x}{MU_y} = \frac{4}{2} = 2$$



# Types of Utility Functions

In Economics, we mainly deal with 4 types of functions, each with its set of properties and tricks

- Cobb-Douglas
- Quasi-linear
- Perfect Substitutes
- Perfect Complements

# Cobb-Douglas

$$U(x, y) = x^a y^b$$

**Find the MRS of this general function**

$$MRS = \frac{MU_x}{MU_y} = \frac{ax^{a-1}y^b}{bx^ay^{b-1}} = \frac{a}{b} \cdot \frac{x^{a-1-a}}{y^{b-1-b}} = \frac{a}{b} \cdot \frac{x^{-1}}{y^{-1}} = \frac{a}{b} \cdot \frac{y}{x}$$

**The MRS for a Cobb-Douglas will always look like**

$$\frac{a}{b} \cdot \frac{y}{x}$$

# Cobb-Douglas: Keys to Remember

$$MRS = \frac{a}{b} \cdot \frac{y}{x}$$

- The MRS is a ratio of  $y$  to  $x$ , multiplied by a constant
- MRS is your **willingness to trade  $y$  for  $x$**
- As you get more  $x$ , the MRS goes down
- As you get more  $y$ , the MRS goes up

# Cobb-Douglas - Example

$$U(x, y) = x^3 y^{1/2}$$

**Find the MRS of this utility function**

$$MRS = \frac{MU_x}{MU_y} = \frac{3x^2 y^{1/2}}{1/2 x^3 y^{-1/2}} = \frac{3}{1/2} \cdot \frac{y^{1/2} y^{1/2}}{x^3 x^{-2}} = 6 \cdot \frac{y}{x}$$

If we recall that  $MRS = \frac{a}{b} \cdot \frac{y}{x}$  then we can take a shortcut:

$$MRS = \frac{a}{b} \cdot \frac{y}{x} \rightarrow \frac{3}{1/2} \cdot \frac{y}{x} = 6 \cdot \frac{y}{x}$$

# Quasi-Linear

$$U(x, y) = a \cdot \ln(x) + b \cdot y$$

Where  $a \cdot \ln(x)$  is the “**quasi**” part and  $b \cdot y$  is the “**linear**” part

**Find the MRS of this general function**

$$MRS = \frac{MU_x}{MU_y} = \frac{a/x}{b} = \frac{a}{b} \cdot \frac{1}{x}$$

# Quasi-Linear: Keys to Remember

$$MRS = \frac{a}{b} \cdot \frac{1}{x}$$

- The MRS is a constant times  $1/x$
- As you get more  $x$ , the MRS decreases
- As you get more  $y$ , the MRS remains the same

# Quasi-Linear - Example

$$U(x, y) = 1/3 \cdot \ln(x) + y$$

**Find the MRS of this utility function**

$$MRS = \frac{MU_x}{MU_y} = \frac{1/3 \cdot 1/x}{1} = \frac{1}{3} \cdot \frac{1}{x}$$

Using our shortcut we get:

$$MRS = \frac{a}{b} \cdot \frac{1}{x} = \frac{1/3}{1} \cdot \frac{1}{x} = \frac{1}{3} \cdot \frac{1}{x}$$

# Perfect Substitutes

$$U(x, y) = a \cdot x + b \cdot y$$

**Find the MRS of this general function**

$$MRS = \frac{MU_x}{MU_y} = \frac{a}{b}$$

## Keys to Remember

- MRS is a constant



# Perfect Substitutes - Example

$$U(x, y) = 6x + \frac{1}{2}y$$

**Find the MRS of this utility function**

$$MRS = \frac{MU_x}{MU_y} = \frac{6}{1/2} = 12$$

And our shortcut shows:

$$MRS = \frac{6}{1/2} = 12$$

# Perfect Complements

$$U(x, y) = \min\{a \cdot x, b \cdot y\}$$

This utility function requires a different form of intuition

Let's first think of a simple example:

Imagine we are trying to make some hot chocolate which requires 1 pack of chocolate powder and 12 oz of milk

$$U(x, y) = \min\{1 \text{ choco}, 12\text{oz milk}\}$$

You check your kitchen and find that there are 3 packs of chocolate powder and you have 15 oz of milk in your fridge

**How many hot chocolates can we make?**

# Perfect Complements Intuition

These are goods that **have to be consumed together in an exact proportion** in order to produce any utility

There is no MRS, so we use a property called the **No-Waste Condition**:

$$U(x, y) = \min\{a \cdot x, b \cdot y\}$$

$$a \cdot x = b \cdot y$$

# Perfect Complements: Keys to Remember

- MRS is not defined (We cannot take a derivative)
- We can use the **No-Waste Condition**:
  - When  $a \cdot x < b \cdot y$  you will give up any  $y$  you can to get more  $x$
  - When  $a \cdot x > b \cdot y$  you will give up any  $x$  you can to get  $y$

# Perfect Complements - Example

$$U(x, y) = \min\left\{\frac{x}{2}, \frac{y}{4}\right\}$$

**Find the No-Waste Condition of this utility function**

No-Waste Condition is  $\frac{x}{2} = \frac{y}{4} \rightarrow 4x = 2y \rightarrow 2x = y$

We can say:

- If  $2x > y \rightarrow$  Too much  $x$ , so we would trade some  $x$  for some  $y$
- If  $2x < y \rightarrow$  Too much  $y$ , so we would trade some  $y$  for some  $x$

# Bundles

When you have a utility function of two goods, any given combination of those two goods is called a **Bundle**

- Every **bundle** has an associated utility level

Take the following utility function and bundles

$$U(x, y) = x^2 y$$

Bundle 1 is (3,2)

$$\begin{aligned} U(3, 2) &= 3^2 \cdot 2 \\ &= 9 \cdot 2 = 18 \end{aligned}$$

Bundle 2 is (1,0)

$$\begin{aligned} U(1, 0) &= 1^2 \cdot 0 \\ &= 1 \cdot 0 = 0 \end{aligned}$$

# Bundles & Axioms of Preferences

We will use the following axioms about preferences between bundles to ensure logical consistency

- Completeness
- “More is Better”
- Transitivity

**These come from economic theory so they will help you think like an economist but do not think this tells us how people behave**

**We are attempting to successfully predict behavior, so we will simply assume that everyone behaves accordingly**

# Completeness

We say that preferences are always **complete**. So when comparing bundles A and B, you can always say one of the following:

- I prefer A to B ( $A \succeq B$ )
- I prefer B to A ( $B \succeq A$ )
- I am indifferent between A and B ( $A \sim B$ )

This allows us to compare and order any pair of bundles



# “More is Better”

Or at least more is no worse than less

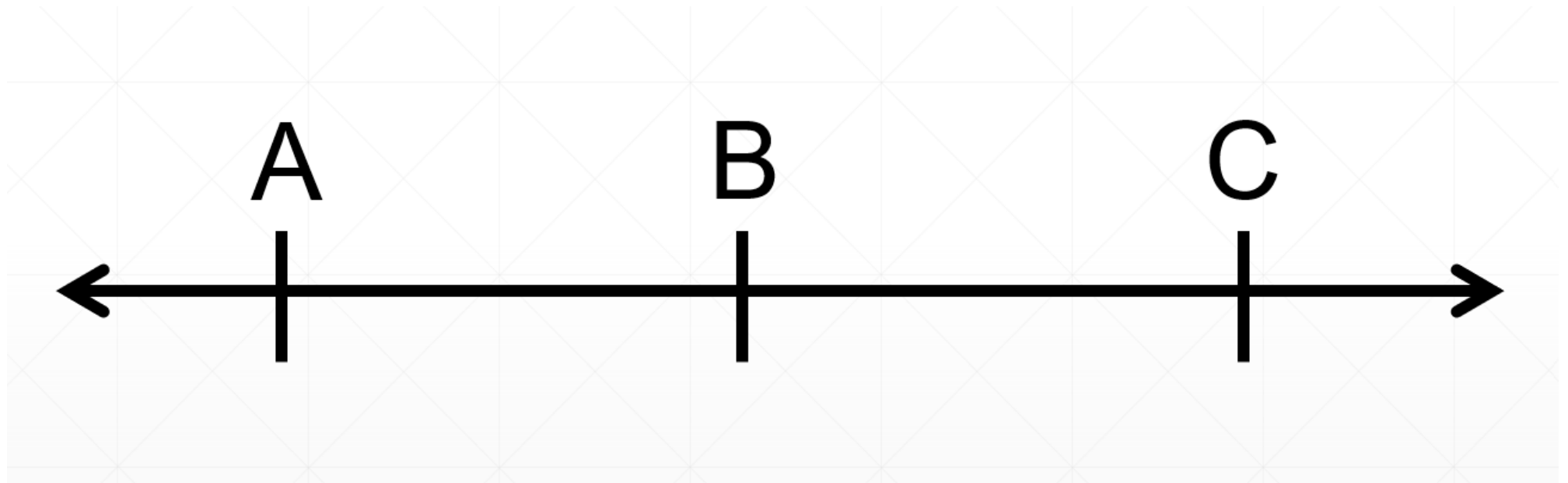
In general, if a good is desirable we will want more of it

- However, sometimes products can be bads (instead of goods) and we would, obviously, want less of those

# Transitivity

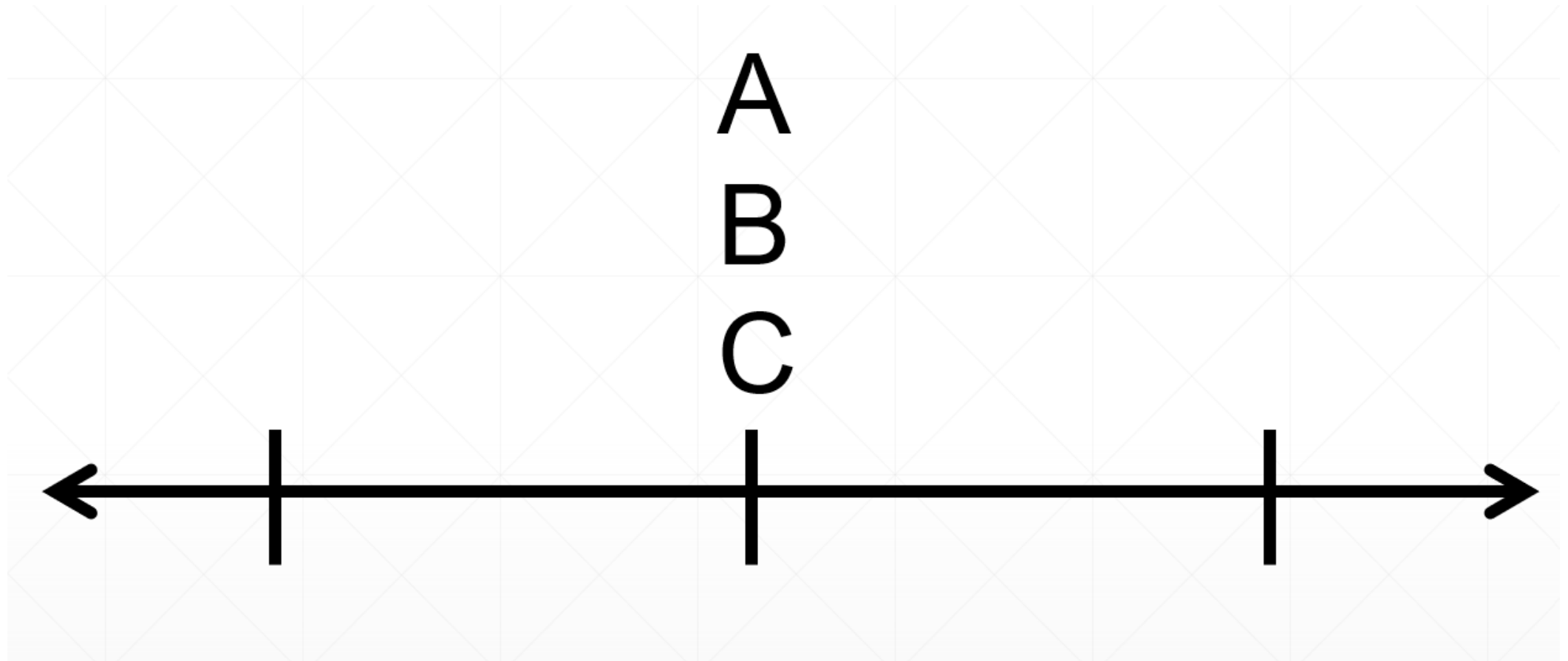
Preferences are transitive. This means that when comparing bundles A, B, and C you can get logical orderings through rankings:

- If you prefer B to A ( $B \succeq A$ ) and C to B ( $C \succeq B$ ) then you must prefer C to A ( $C \succeq A$ )



# Transitivity

What would this graph mean?



All bundles are preferred equally!

# Preferences Assumptions

All of these are necessary to understand utility functions and be able to graph them

To do so we first need to learn about **Indifference Curves**

# Indifference Curves

# What is Indifference?

It is exactly what it sounds like

- Any two bundles that give the same utility level makes you indifferent between those two Bundles

For example, for the utility of  $U(x, y) = 2x + 3y$  I am indifferent between bundles (3,2) and (0,4)

$$U(3, 2) = 2(3) + 3(2) = 6 + 6 = 12$$

$$U(0, 4) = 2(0) + 3(4) = 12$$

# Indifference Set

Given a utility function and a level of utility, you can find a whole set of bundles that you are indifferent between

For example, let  $U(x, y) = x + y$  and set  $U = 10$ . Then we can find an infinite set of  $x$  and  $y$  that will give us our stated utility level

$$x = 10$$

$$y = 0$$

$$x = 9$$

$$y = 1$$

$$x = 8$$

$$y = 2$$

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# Indifference Curves

We can create a function that helps us find **ALL** possible bundles that make you indifferent at a given utility level

We call this an **Indifference Curve**

For our previous utility function  $U(x, y) = x + y$  where  $U = 10$ , we solve for  $y$  and get:

$$U(x, y) = x + y$$

$$10 = x + y$$

$$10 - x = y \rightarrow \text{Indifference Curve (IC)}$$



# Indifference Curve - Example

$$U(x, y) = xy \text{ where } U = 16$$

What type of Utility Function is this?

- It is a Cobb-Douglas  $x^a y^b$  where  $a = b = 1$

What is the associated Indifference Curve?

$$16 = xy \rightarrow y = \frac{16}{x} \rightarrow \text{IC}$$

# Indifference Curves

So why do we care about these curves?

- We can graph them
- Graphing is a key step to figuring out how to solve an individual's choice problem

**Each utility function has a unique shape that we will learn**

# Let's Draw - Perfect Substitutes

$$U(x, y) = 3x + y \text{ with } U = 6, 9, 15$$

**First, find the indifference curves for each Utility value**

$$U = 6$$

$$y = 6 - 3x$$

$$U = 9$$

$$y = 9 - 3x$$

$$U = 15$$

$$y = 15 - 3x$$

**Next, we graph these functions**

# Let's Draw - Perfect Complements

$$U(x, y) = \min\left\{x, \frac{y}{2}\right\} \quad \text{with } U = 2, 8, 9$$

**Find the indifference curves for each utility value**

$$U = 2$$

$$x = 2 \text{ or } y = 4$$

$$U = 8$$

$$x = 8 \text{ or } y = 16$$

$$U = 9$$

$$x = 9 \text{ or } y = 18$$

**Next, we graph these functions**

# Let's Draw - Cobb-Douglas

$$U(x, y) = x^{1/2}y \text{ with } U = 4, 8, 10$$

**Find the indifference curves for each utility value**

$$U = 4$$

$$y = \frac{4}{x^{1/2}}$$

$$U = 8$$

$$y = \frac{8}{x^{1/2}}$$

$$U = 10$$

$$y = \frac{10}{x^{1/2}}$$

**Next, we graph these functions**

# Let's Draw - Quasi-Linear

$$U(x, y) = \ln(x) + y \text{ with } U = 5, 15, 20$$

**Find the indifference curves for each utility value**

$$U = 5$$

$$y = 5 - \ln(x)$$

$$U = 15$$

$$y = 15 - \ln(x)$$

$$U = 20$$

$$y = 20 - \ln(x)$$

**Next, we graph these functions**

# Indifference Curve - Rules

It is very important that you understand the **intuition** behind indifference curves

Let's view an example that can help:

Consider Weather Reports:

- On cold days, what the weather feels like is a function of:
  1. Temperature
  2. Windchill
- An indifference curve represents **all of the different combinations of temperature and windchill that cause you to feel the exact same thing**
- If the windchill is suddenly lower, what must **intuitively** happen to the temperature to keep you feeling the same outside?

If windchill ↓ then temperature ↑

# Indifference Curve - Rules

We use this exact same logic for utility between two goods

**Intuitively**, if I want to stay at the same level of happiness as I lose some  $y$ , what must happen to  $x$ ?

- I need more of  $x$

This is why we read them from left to right and why they have a negative slope

- I know the perfect complements is odd but the same logic tends to apply



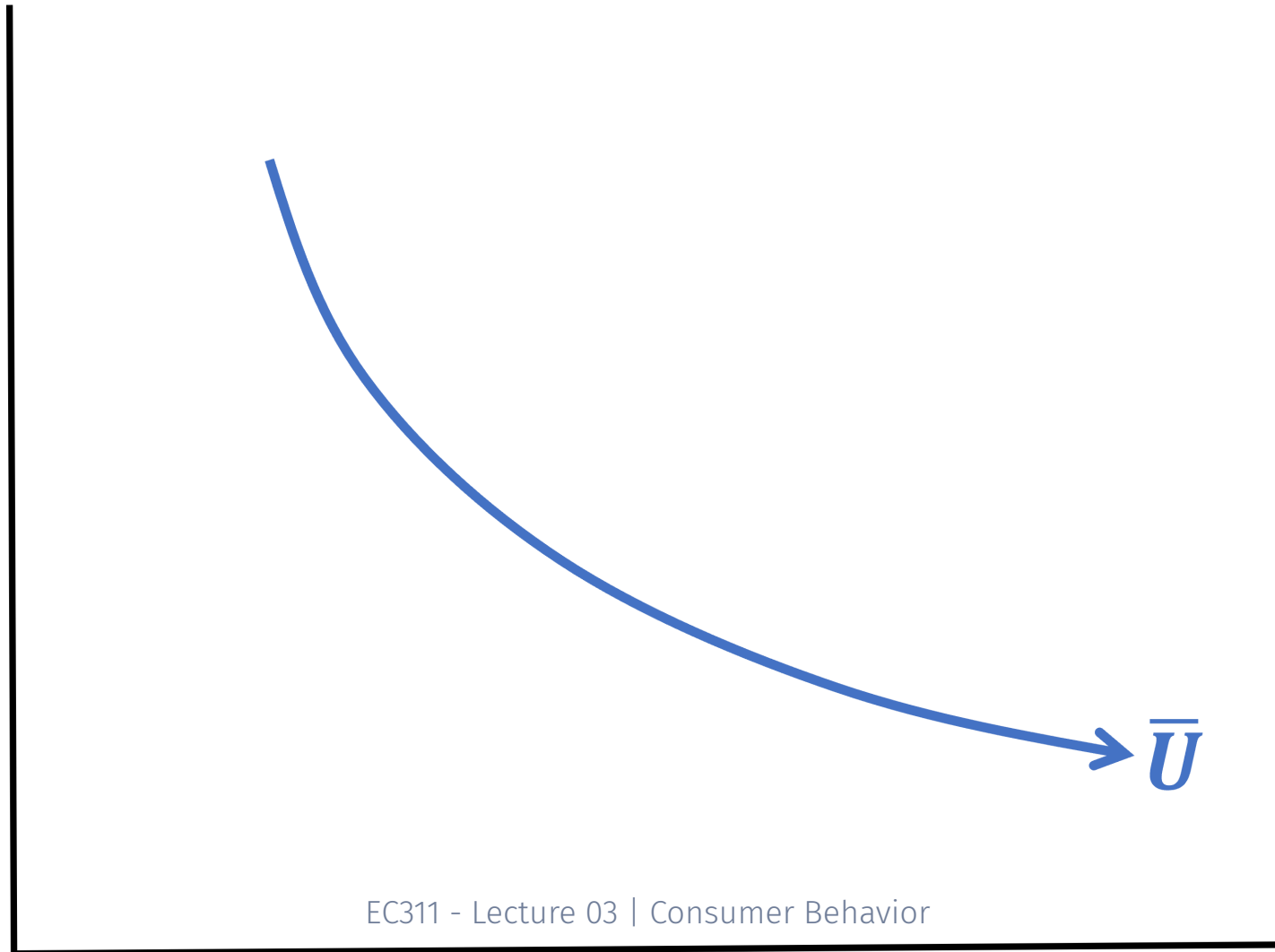
# Formal Indifference Curve Rules

All of the stuff from before can be formalized into the following 3:

- **Monotonicity:** Indifference curves always go from the top left to the bottom right of the graph without changing direction at any point
- **Non-Crossing:** If at least two curves cross, this leads to logical contradictions
- **Convex:** Balanced combinations of two goods are preferred to extreme outcomes (A lot of one good, little of the other)

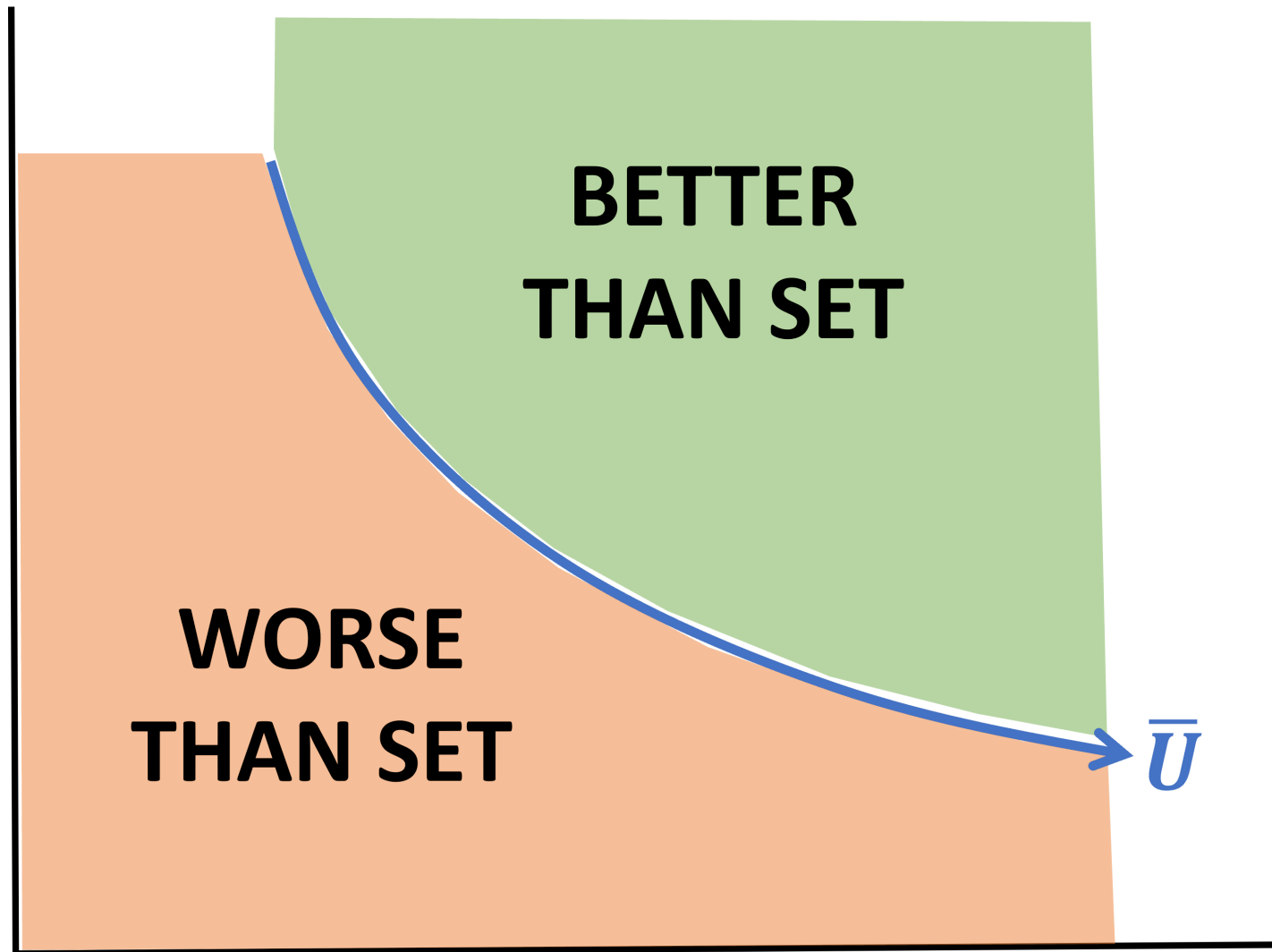
# Monotonicity

**ICs always go from the top left to the bottom right without changing direction**



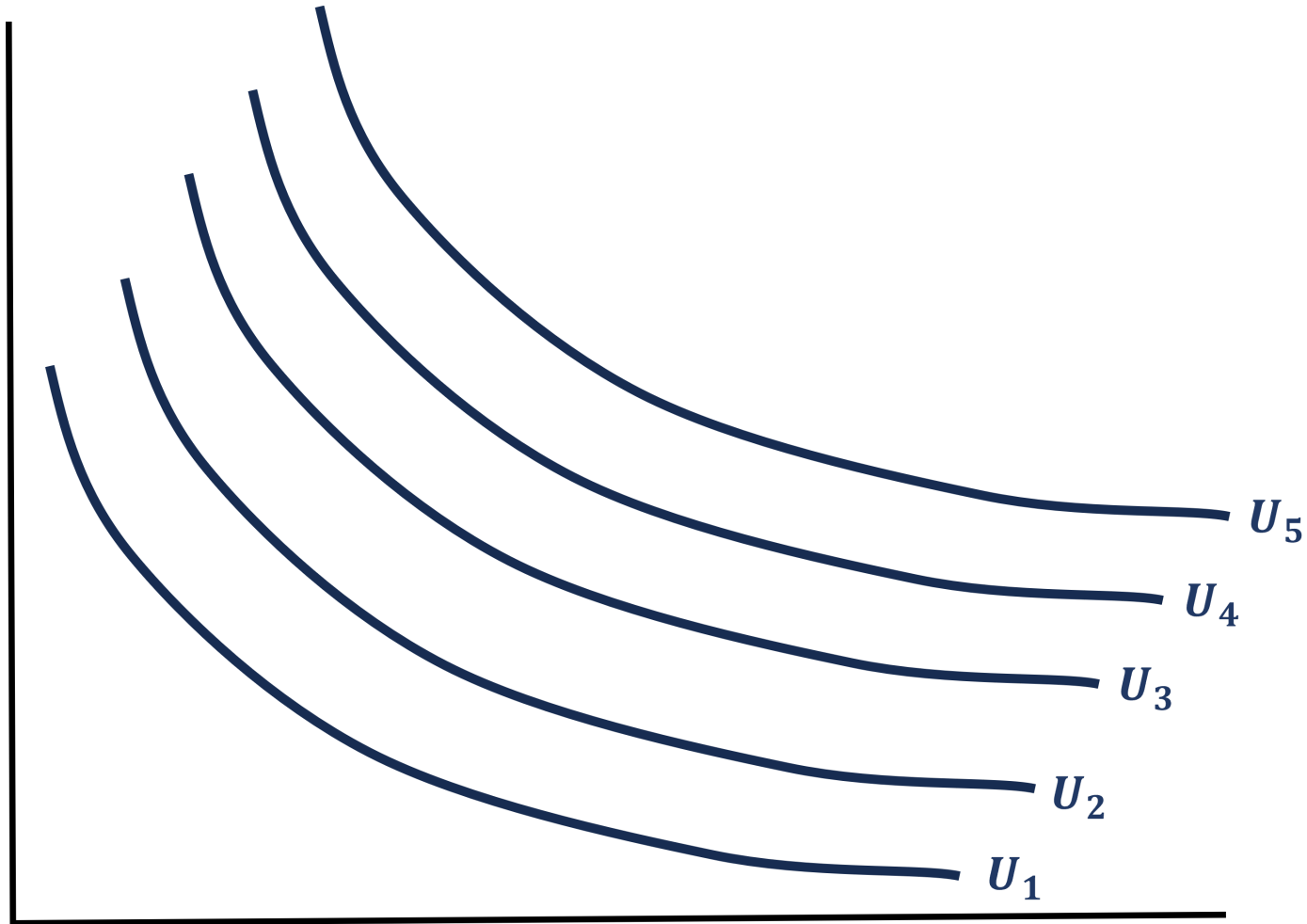
# Monotonicity

Additionally, this helps us visualize two important sets of bundles



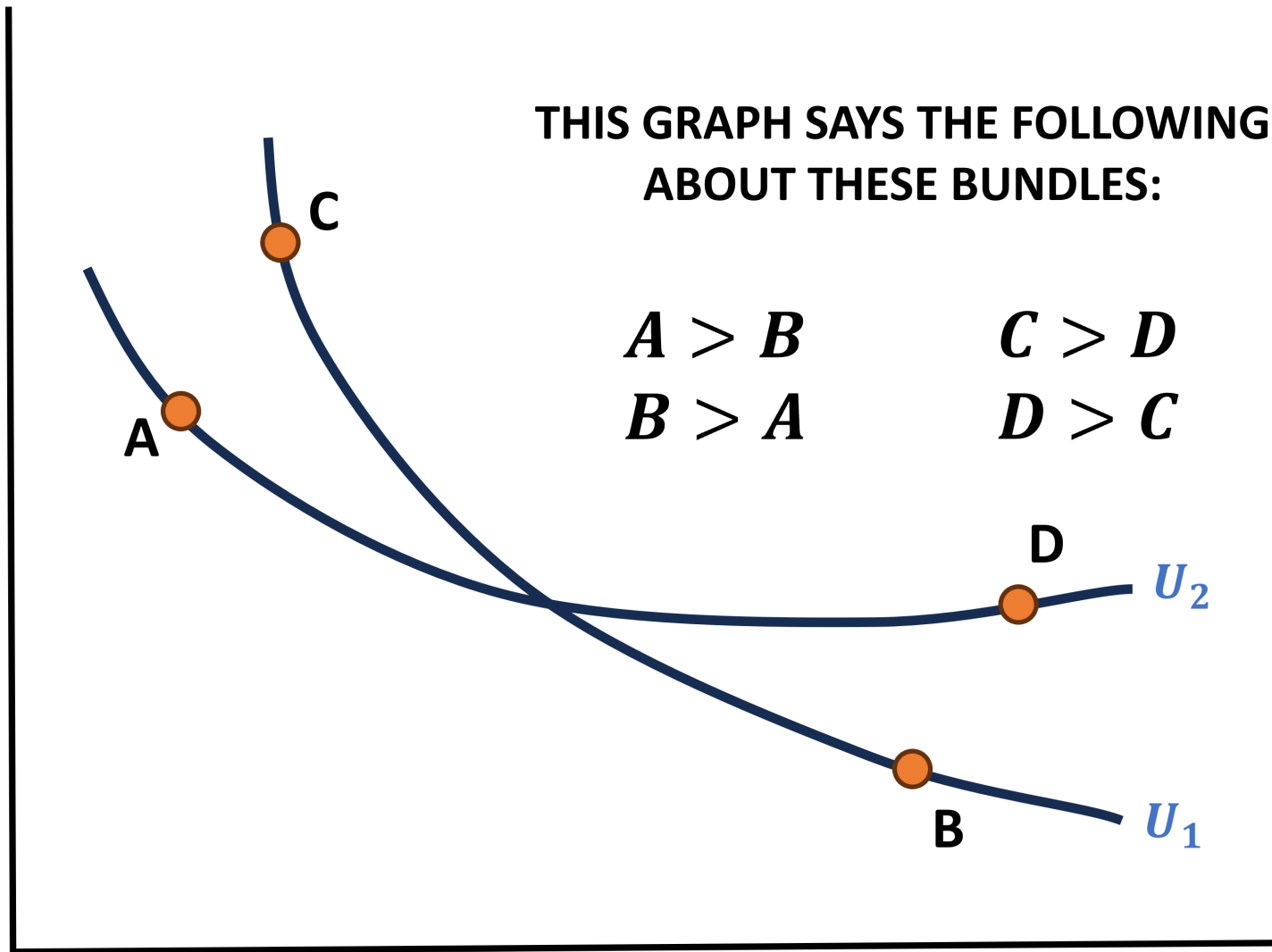
# Non-Crossing

**This is the expected behavior of ICs. There are infinitely many, each representing a unique level of Utility**



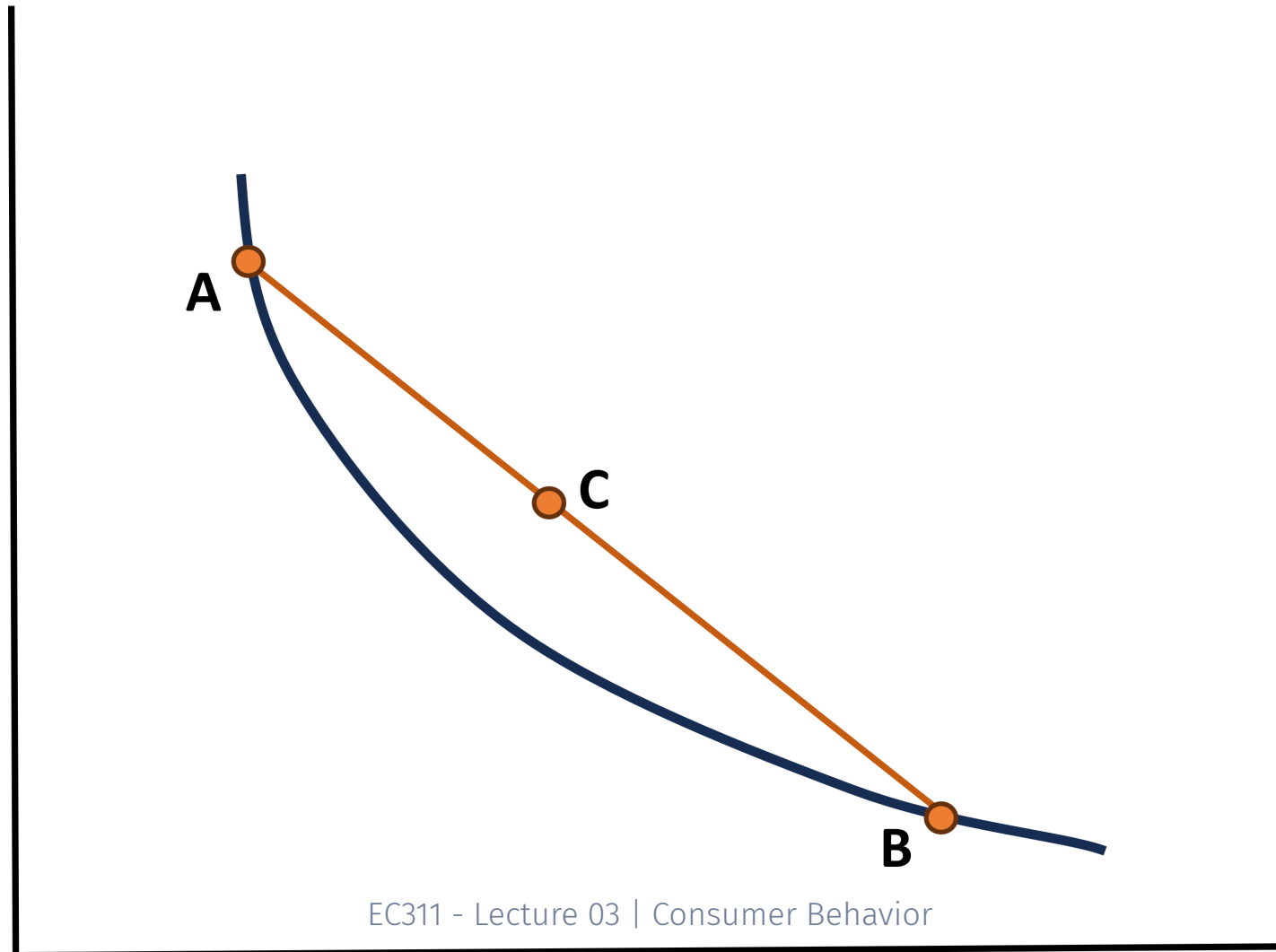
# Non-Crossing - Logical Contradiction

If ICs cross, they are contradictions



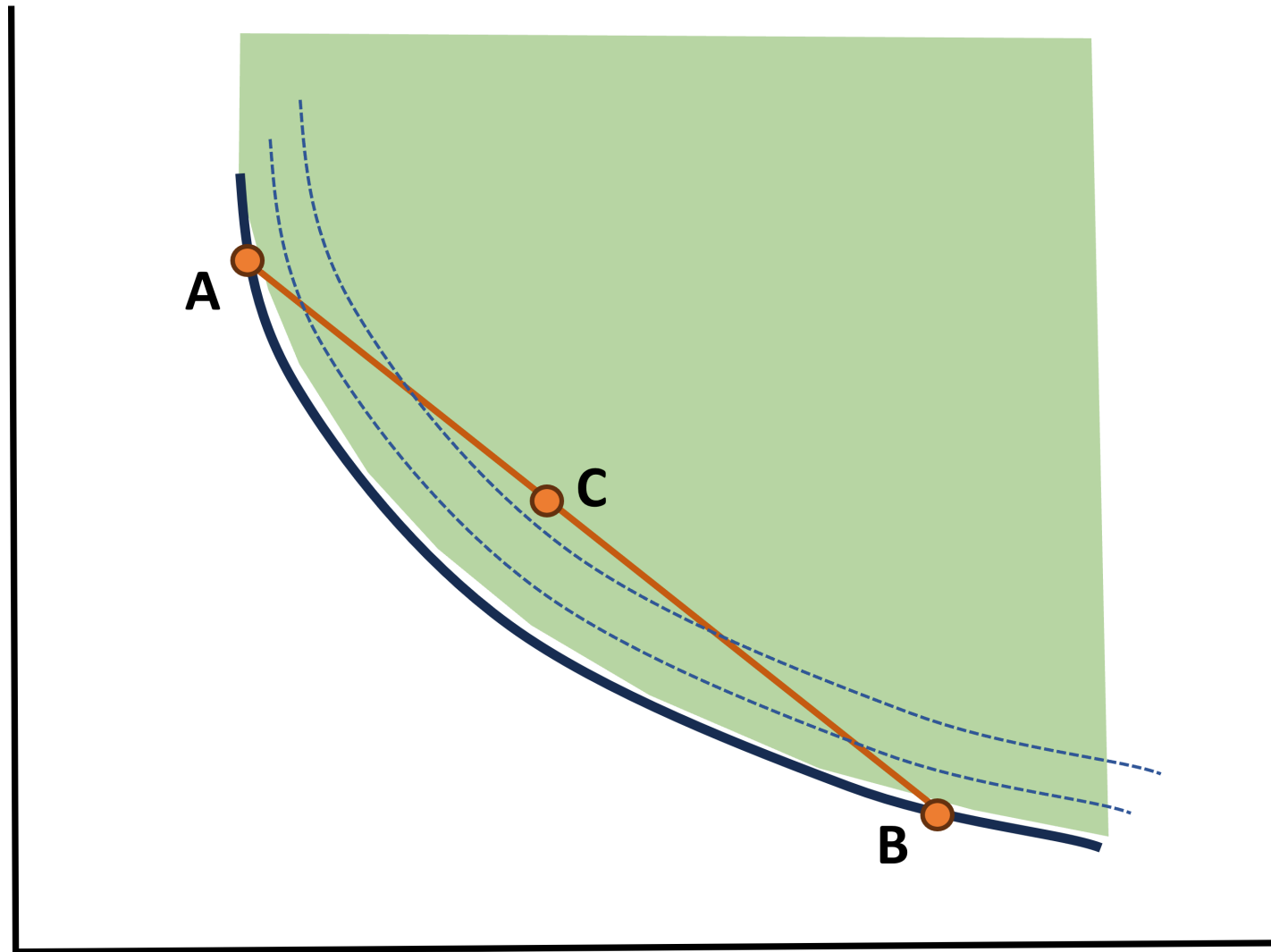
# Convex

**A balanced combination of two goods are preferred to extreme outcomes**



# Convex

Recall the Better-than-set



# What About the Other Functional Forms?

I drew Cobb-Douglas curves (mostly because they're easier to illustrate these properties) but what about Perfect Complements or Perfect Substitutes?

- They fulfill all 3 properties, **but not strongly**

## Perfect Complements

- Never Crosses
- Convex
- **Weakly Monotonic**

## Perfect Substitutes

- Never Crosses
- Monotonic
- **Weakly Convex**

Let's look at the board again



# Most Important Facts of Indifference Curves

Recall the intuition of what a movement along the indifference curve means:

- You are trading  $y$  for  $x$ , holding **constant** your level of **utility**
- The slope of the indifference curve measures your **willingness to tradeoff between  $x$  and  $y$**
- This is also known as the **Marginal Rate of Substitution** (times  $-1$ )

# Most Important Facts of Indifference Curves

This **Marginal Rate of Substitution (MRS)** thing is pretty important

First, why is the slope the **negative MRS**?

- The MRS is the **ratio** of the effect of increasing  $x$  on your utility and the effect of increasing  $y$  on your utility

- $$\frac{MU_x}{MU_y}$$

- The Indifference Curve slope is all about keeping the utility level constant while we move  $x$  and  $y$  in opposite directions
- Therefore the IC slope = -MRS

# Simple Mathematical Proof

For

$$U(x, y) = ax + by$$

1. Find the MRS and times -1

2. Find the IC and it's slope

$$MRS = \frac{MU_x}{MU_y} = \frac{a}{b}$$

$$-MRS = \frac{-a}{b}$$

$$\bar{U} = ax + by$$

$$\bar{U} - ax = by$$

$$\frac{\bar{U}}{b} - \frac{ax}{b} = y$$

$$\text{Slope is the derivative!} \rightarrow \frac{\partial y}{\partial x} = \frac{-a}{b}$$

# Indifference Curves

From this lecture you have learned:

- Everything about utility functions and how to use them to find an MRS
- Everything about Indifference Curves and how the IC slope relates to the MRS

The MRS is going to be key to solving utility maximization problems

- Mathematically (Using derivatives)
- Graphically (Drawing ICs)

However, when we maximize utility functions we have constraints, we called these **Budget Constraints**

# Budget Constraints

# Budgets

**Economic Theory** says that individuals make themselves as happy as they possibly can, after choosing from a set of all bundles they can afford

- We do not just maximize utility functions, but rather we maximize them subject to **Budget Constraints (BC)**

# Budgets - Variables

Inside a budget constraint we use the exact same variables we use in our utility function, namely  $x$  and  $y$

But we'll need to introduce some new notation and terminology:

- The **price** of good  $x \rightarrow P_x$
- The **price** of good  $y \rightarrow P_y$
- Your income, budget, or money-on-hand  $\rightarrow M$ 
  - The book also calls this  $I$

# Budgets - Functional Form

Putting together our variables we get our **Budget Constraint**

$$P_x \cdot x + P_y \cdot y \leq M$$

**Do not let the inequality  $\leq$  scare you, it just means we can spend less than our total income**

However, in our applications we are going to treat it as a strict equality

$$P_x \cdot x + P_y \cdot y = M$$

And let's think why?

- If we are maximizing utility, it does not make sense to leave any income unspent!



# Budgets - Spending All Our Income

But do we usually spend **all** of our money?

Of course not, but here is how to think about it in the economics sense:

- When you go to the grocery store you make choices before even stepping inside:
  1. How much to spend at the store
  2. What to spend it on
- Once you allocate your budget, you usually spend all of it

# Let's Construct A Budget Constraint

First, we need our two goods from before: Beer ( $B$ ) and Soda ( $S$ )

**Let's label everything properly:**

- $B$  = Beer
- $S$  = Soda
- $P_B$  = \$4
- $P_S$  = \$2
- $M$  = \$20

**What does the Budget Constraint look like?**

$$B \cdot P_B + S \cdot P_S = M$$

$$4B + 2P = 20$$

# Let's Graph A Budget Constraint

We graph the budget in the same space in as our IC, but what are we graphing?

$$P_x \cdot x + P_y \cdot y = M$$

And thankfully, this is just a straight line

# Bringing Intuition into Budget Constraint Graphs

$$P_x \cdot x + P_y \cdot y = M$$

Thinking intuitively, this graph represents:

- All combinations of  $(x, y)$  that cost exactly  $M$
- If we spend nothing on good  $y$ , what is the most possible  $x$  that we can purchase?

$$P_x \cdot x + P_y \cdot 0 = M$$

$$P_x \cdot x = M$$

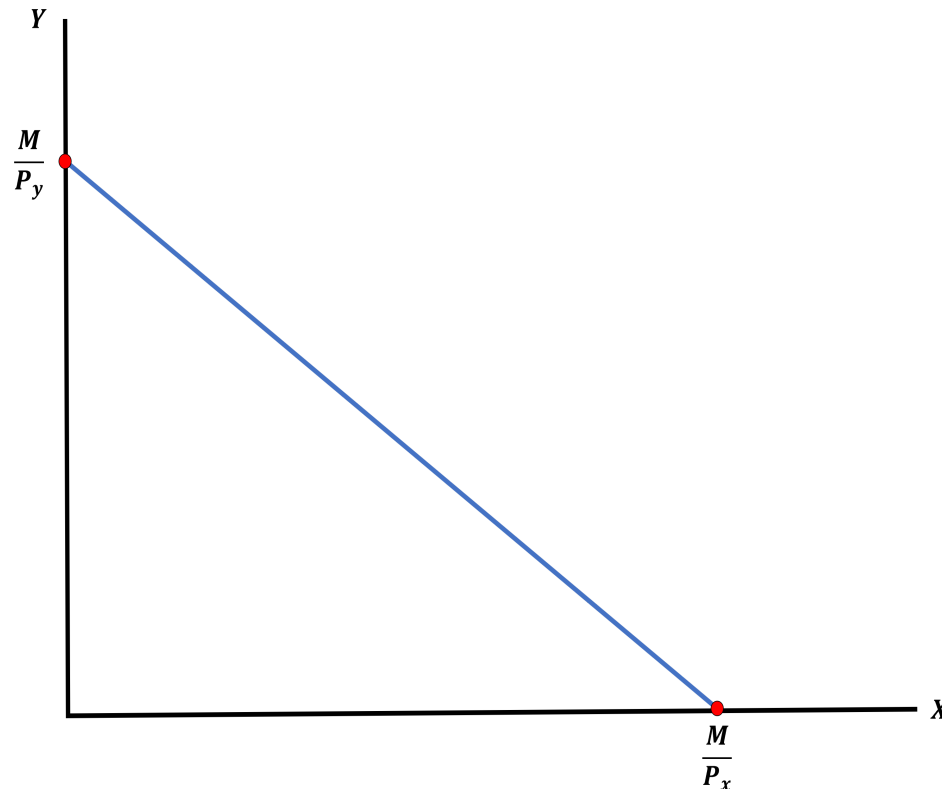
$$x = \frac{m}{P_x}$$

# Let's Graph A Budget Constraint

Following the logic we just did, the budget line will connect two points:

One for when we buy only  $x$  and one for when we buy only  $y$

$$\left(\frac{M}{P_x}, 0\right) \text{ and } \left(0, \frac{M}{P_y}\right)$$



# Interpreting the Slope of the Budget Constraint

The Budget Constraint is the line:

$$y = \frac{M}{P_y} - \frac{P_x}{P_y} \cdot x \quad \text{where the slope is} \quad -\frac{P_x}{P_y}$$

Now, let's define

$$P_x = \frac{\$}{x} \text{ and } P_y = \frac{\$}{y}$$

We get this result in terms of units!

$$\frac{P_x}{P_y} = \frac{\$/x}{\$/y} = \frac{\$}{x} \cdot \frac{y}{\$} = \frac{y}{x}$$

# The Slope of the Budget Constraint

As we just saw, the price ratio  $P_x/P_y$  can be measured in units of  $x$  per units of  $y$

We had already seen something that is measured in units → the MRS

This leads us to understand the differences between the two:

- The **MRS** represents how much  $y$  you **would be willing to** give up in order to get a unit of  $x$
- The **Price Ratio** represents how much  $y$  you **would have to** give up in order to get a unit of  $x$

# What Happens When Things Change?

Budgets are not static. Prices and income can change based on market conditions so it is important to understand the effects when factors change

- We can have changes in income (increase or decrease)
- Price of  $x$  can change (increase or decrease)
- Price of  $y$  can change (increase or decrease)

**Note - We normally consider what happens when only one of the possible factors change, and hold all others fixed**



# Changes in Income

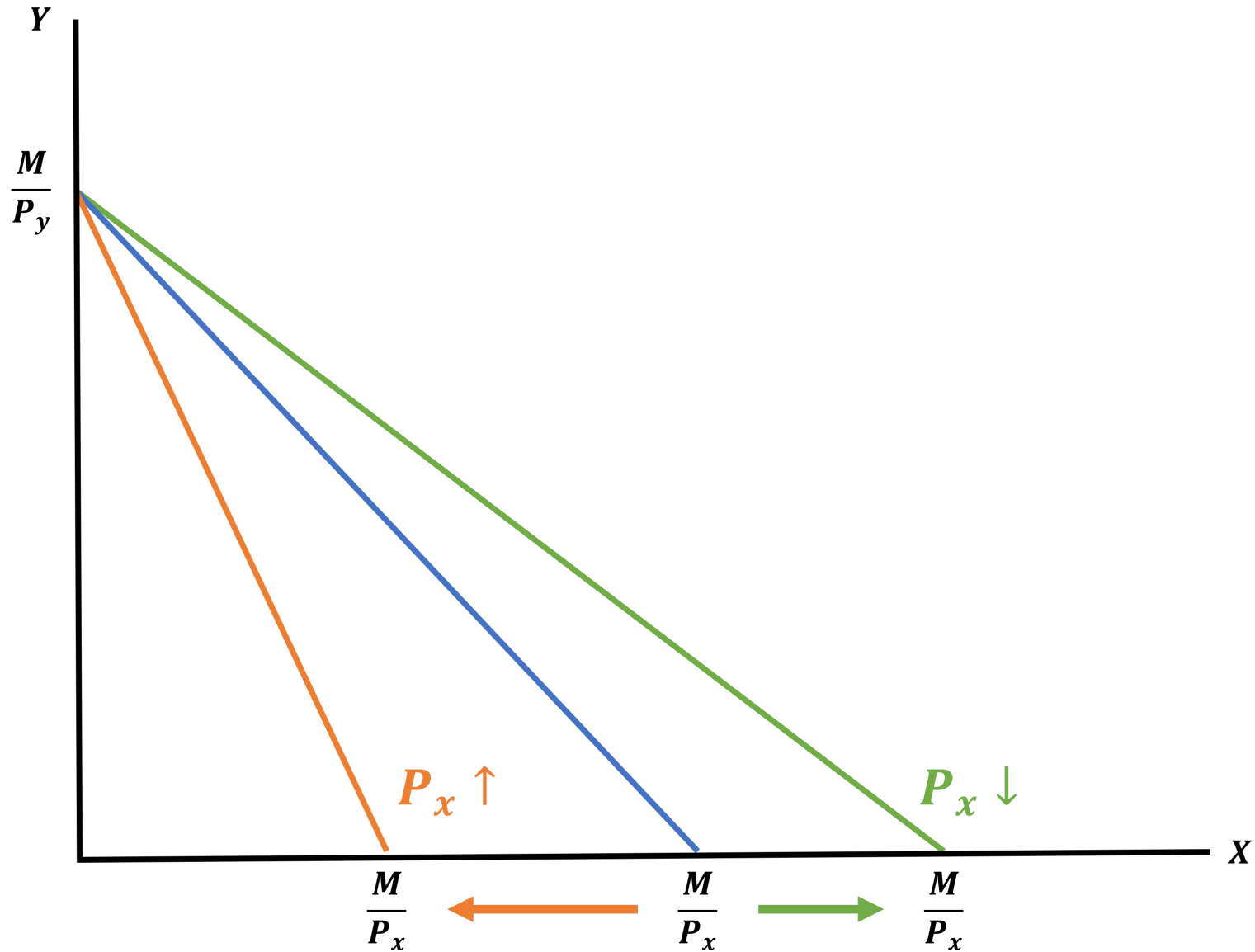
What happens to the budget if income ( $M$ ) increases?

- $\frac{M}{P_x}$ , the maximum amount of  $x$  that can be purchased goes up
- $\frac{M}{P_y}$ , the maximum amount of  $y$  that can be purchased goes up
- The slope of the BC ( $-P_x/P_y$ ) stays the same

Income changes affect the **overall amount** an individual can consume, but has **no effect on the relative cost of the goods**

**Intuitively, the opposite is true if income ( $M$ ) decreases**

# Changes in Income - Graph



# Changes in Prices

What happens to the budget if the price of good  $x$  ( $P_x$ ) increases?

- The maximum amount of  $x$  that can be consumed goes down ( $\frac{M}{P_x}$ )  $\downarrow$
- The maximum amount of  $y$  that can be consumed stays the same ( $\frac{M}{P_y}$ )
- The price ratio ( $-\frac{P_x}{P_y}$ ) becomes steeper

**Once more, the opposite happens with a decrease**

**How about a shift in  $P_y$  ?**

# Changes in Prices - Graph

