Consumer Behavior

EC 311 - Intermediate Microeconomics

2025

Chapter 4

Outline

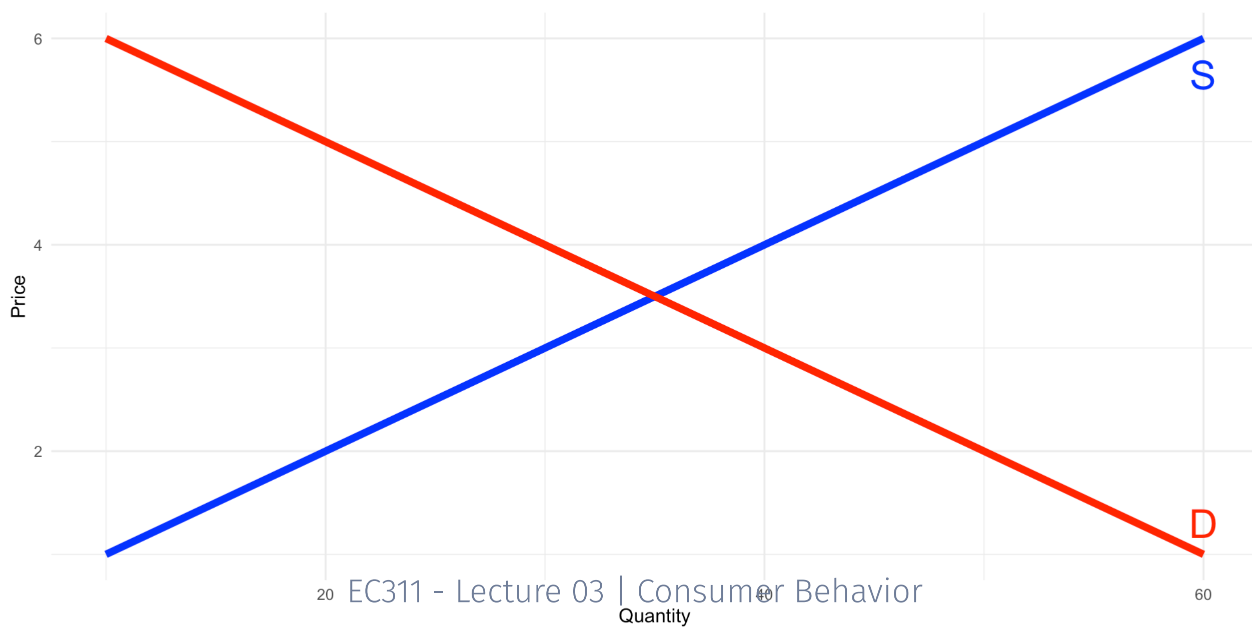
- Topics
  - Preferences and Utility (4.1)
  - Indifference Curves (4.2)
  - Budget Constraints (4.3)
- Other Stuff
  - Remember Office Hours on **Monday 10-11, Wednesday 1-2 in Tykeson 3rd floor**
  - Also feel free to email me your questions at **dyasui@uoregon.edu**

Preferences and Utility

Where Does Demand Come From?

Imagine the market for coffee on campus

- Additionally, imagine that the only other good that can be purchased is boba
- On a basic level, **demand for coffee** is derived from individual's choosing how to divide their income between coffee and boba



Where Does Demand Come From?

This is the problem we will be dealing with through the first half of the course

> How does an individual allocate a **finite amount of resources** between **two goods**?

Note: There are clearly more than just two goods out there, so how can this be useful?

The main critique I always hear about economics courses is that they're unrealistic. That's mostly true, but we can learn about the aggregate by simplifying and making assumptions

Where Does Demand Come From?

- We will frame the decisions as a two-good model where you may choose between:
  - Food and durable consumption
  - Leisure (not working) and consumption (paid for by earning a wage)
  - Consumption now and consumption later

> **The key takeaway here is that we can frame many important choices as "two-good" decisions**

- This makes things simpler for us to solve while still maintaining some sense of the real world

Determinants of Consumption

Consumption of any single good has two parts:

- How it **BENEFITS** the consumer
  - We call this **UTILITY**
- What it **COSTS** the consumer
  - What we give up to purchase the good

Let's see what this means through a 1-good example $\longrightarrow$ Beer

- Imagine the following scenario:
  - You just arrived at the bar and have had zero drinks so far
  - Each beer costs the same: $4

Beer

Number of Beers	Overall Level of Happiness	Change in Level of Happiness
0	0	-
1	10	10
2	25	15
3	35	10
4	40	5
5	42	3
6	30	-12

If beer were free, how many beers should this person drink?

$$5$$

Now recall that beer costs $4, how many beers should this person drink?

$$4$$

Intuition Behind "Choice" in Economics

You **cannot** simply find the consumption amount that makes you the happiest. **But why?**

> The goal is to maximize your **utility** whilst acknowledging you have **constraints**
>
> The choice is simple: **consume an additional unit until the cost of doing so outweighs the benefit**

Commit this idea to memory: it is the crux of economics and drives everything we will be doing

We maximize utility up to the point that it does not make sense to do so

Back to Beer

Number of Beers	Overall Level of Happiness	Change in Level of Happiness
0	0	-
1	10	10
2	25	15
3	35	10
4	40	5
5	42	3
6	30	-12

Some Questions:

- What is the marginal benefit:
  - When you have not consumed any beer?
  - When you have already consumed 3 beers?
  - When you have already consumed 5 beers?
- What is the marginal cost of beer?
  - Does it change as we consume more?

Being and Thinking at the Margin

We found two important values:

- **Marginal Benefit (MB)**

The additional benefit gained for an added unit of consumption

- **Marginal Cost (MC)**

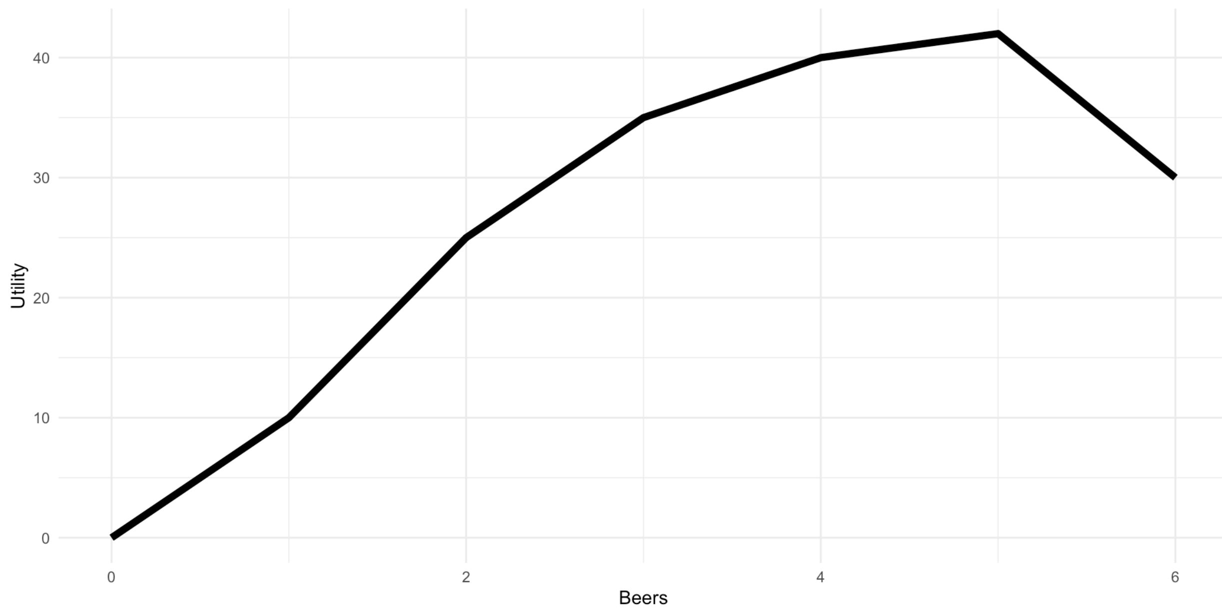
The additional cost paid for an added unit of consumption

We can describe the decision-making process in a more formal manner:

- Initially: **MB** > **MC** $\longrightarrow$ Consume more!
- Eventually: **MB** < **MC** $\longrightarrow$ We went too far!
- At some point: **MB** = **MC** $\longrightarrow$ Just right!

> **Ask yourself: Why must they be equal?**

What’s Going On? Graphically

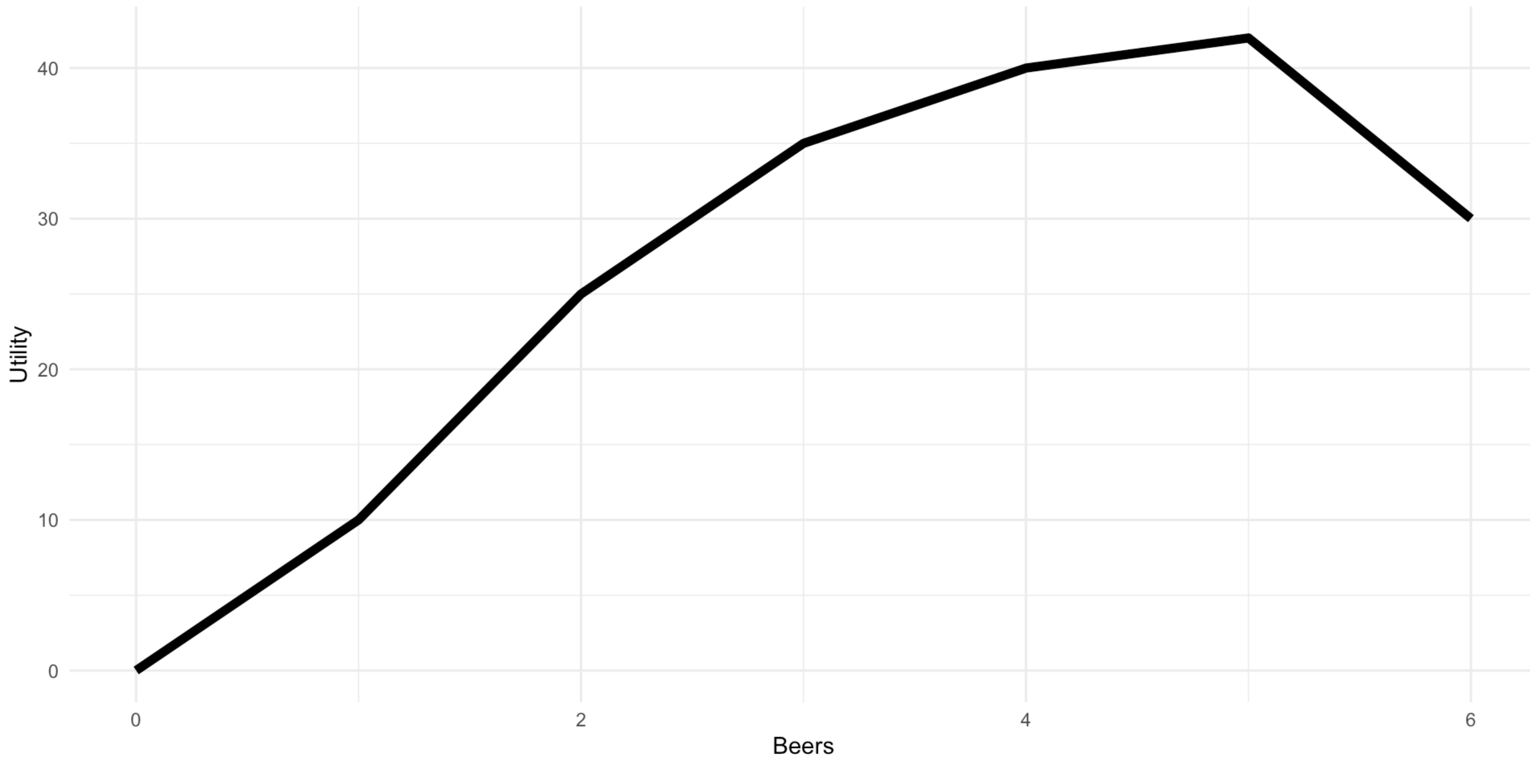


What matters for choice is the **marginal benefit** of an additional beer

In other words, what matters is the change in **utility** that occurs as we move to the right on the graph

> **Recall: A change in y as x increases is the derivative**

What's Going On? Graphically - Derivatives

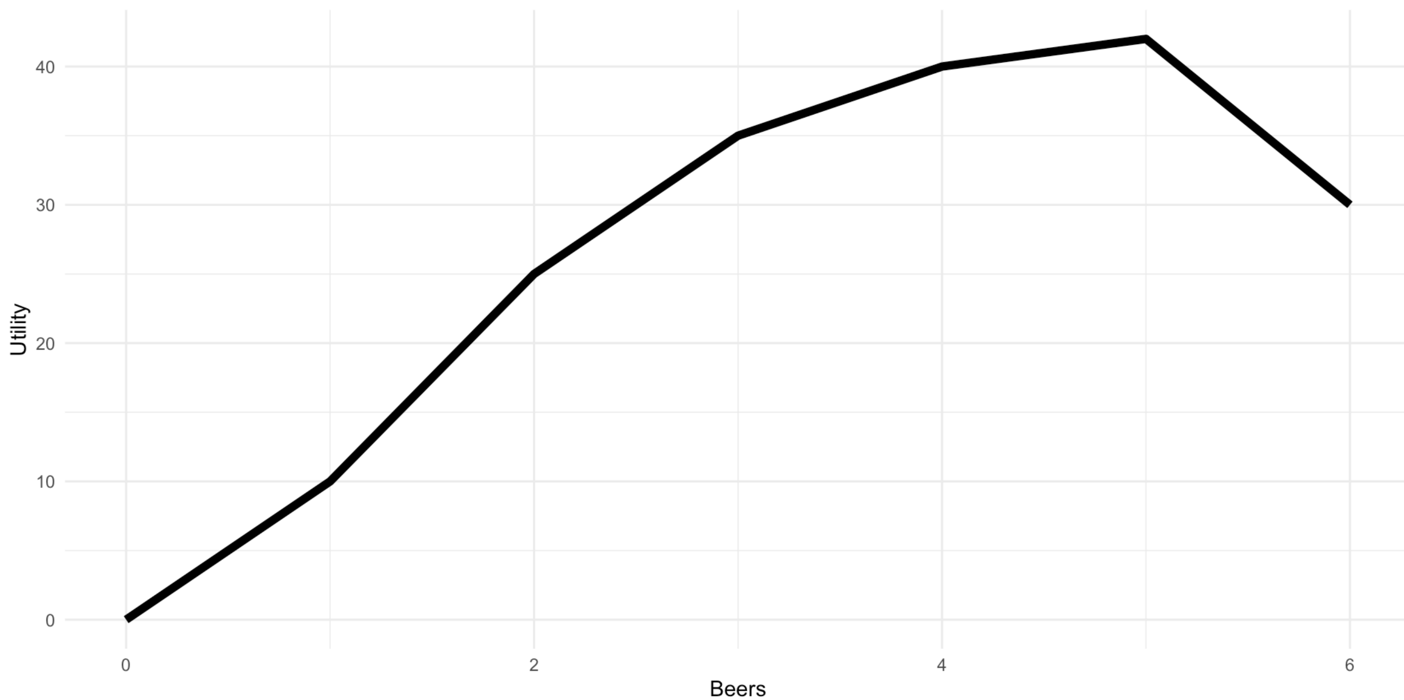


The derivative of this function is the **marginal benefit** of beer

This is why derivatives are important

We can use derivatives to figure out the optimal amount to consume

What's Going On? Graphically - MB & MC



> **Recall:** The optimal choice is the point where **MB** = **MC**

> The **MC** = 4. So we would choose the quantity of beers where **MB** = 4

What's Going On? Mathematically

The utility function of beer we've been using is:

$$U(x) = -x^2 + 12x$$

Let's practice: What would the optimal consumption amount be if the cost of beer is \$2?

Find the Marginal Benefit

$$MU_x = \frac{\partial U}{\partial x} = -2x + 12$$

Set the MB equal to MC

$$\begin{aligned} MB &= MC \\ -2x + 12 &= 2 \\ 2x &= 10 \\ x &= 5 \end{aligned}$$

A Two-Good Problem

The beer example was fairly straightforward. But we will be dealing with making choices between two goods.

Before we dive in, a couple of things to consider are:

- When we spend our resources on one good, it cannot be spent on something else
  - Because we are making decisions amongst both things, we need a **cost relationship** between them
- We do not measure our happiness (utility) in dollars
  - We need to find a way to choose without directly comparing costs and benefits across goods

Enter Utility Functions

A **Utility Function** is a function of two variables:

$$U = f(x, y)$$

Some facts:

- x and y are the (positive) amounts of goods you consume
- The function converts x and y to happiness (utility) from consuming the two goods

For Example:

- Let's say that the utility I receive from **kombucha (K)** and **hop water (H)** can be modeled as:

$$U = f(K, H) = 10K + 2H$$

- Before, we were only interested in the slope of Beer
- Now, we are interested in the **slopes** of both kombucha & hop water

Utility Functions - kombucha & hop water

$$U = f(K, H) = 10K + 2H$$

For a function of two variables there are **two slopes**

- One for each **partial derivative** which we will call **Marginal Utilities**
- In the **kombucha (K)** and **hop water (H)** example we will have:
  - The **Marginal Utility of kombucha** **(MU_B)**
  - The **Marginal Utility of hop water** **(MU_H)**

> What are the Marginal Utilities of kombucha and hop water?

Utility Functions - PB&J

Now find the Marginal Utilities for Peanut Butter and Jelly

$$U = f(P, J) = P^2 \cdot J$$

Peanut Butter

$$MU_P = 2P^{2-1} \cdot J = 2PJ$$

Jelly

$$MU_J = P^2 \cdot J^{1-1} = P^2$$

What do Utility Functions even Mean?

They help us represent how people feel about goods x and y

There are certain properties that help us determine:

- Do I **like or dislike** x?
- Does **how much I like** x depend on **how much** x **I already have**?
- Does **how much I like** x depend on **how much** y **I have**?

> **We can understand these properties by looking at the marginal utilities!**

Utility Functions - Do I Like or Dislike x?

> **If I consume more x, how does my utility move?**

Beyond graphing the utility function, we need to find a way to answer this formally and mathematically

We can look at the **sign of the derivative**

- If the derivative of U with respect to (w.r.t.) x is:

Positive

I like x

Negative

I dislike x

Like or Dislike - Example

Given my utility curve for **kombucha (K)** and **hop water (H)**, what do I like or dislike?

$$U = f(K, H) = 10K + 2H$$

I like **kombucha**

$$MU_K = 10 > 0$$

I like **hop water**

$$MU_H = 2 > 0$$

Does How Much I like x Depend on How Much I Already Have?

> **Remember to think at the margin**

Is each additional unit of x bring me **more**, **less**, or **equal** happiness as the previous unit?

This is slightly trickier to figure out, but we still use marginal utility logic

- In fact, we will use what is called the **Second Derivative**
- Mathematically, this is the **derivative** of MU_x w.r.t. x and we ask:
  - Is this **second derivative** **positive**, **negative**, or **zero**?

Depend on How Much I Already Have - Example

My utility for **Cookies** and **Milk**:

$$U = f(C, M) = C^{1/2} M^{1/2}$$

Cookies

$$MU_c = \frac{1}{2} \cdot C^{-1/2} \cdot M^{1/2}$$
$$MU_{cc} = \frac{-1}{2} \cdot \frac{1}{2} \cdot C^{-1/2-1} \cdot M^{1/2}$$
$$MU_{cc} = \frac{-1}{4} \cdot \frac{1}{C^{3/2}} \cdot M^{1/2}$$
$$MU_{cc} = \frac{-M^{1/2}}{4C^{3/2}}$$

Milk

$$U_{mm} = \frac{-C^{1/2}}{4M^{3/2}}$$

MU_{cc} is **negative** so we can say that **Cookies** have a Decreasing Marginal Utility

Does How Much I Like x Depend on How Much y I Have?

This one is more straightforward: Does the marginal utility of x depend on y?

- Mathematically, we take the derivative of MU_x w.r.t. to y, and vice-versa.
  - This is called the **cross-partial derivative**
- Notationally, we have: MU_{xy}

Where we can determine the order of derivatives by looking at the subscript:

- x is first, and y is the second derivative

Depend on How Much of the Other Good I Have - Example

How about this utility for Peanut Butter and Jelly

$$U = f(P, J) = P^2 \cdot J$$

Peanut Butter

$$MU_P = 2P^{2-1} \cdot J = 2PJ$$
$$MU_{PJ} = 2P \cdot J^{1-1} = 2P$$

Jelly

$$MU_J = P^2 \cdot J^{1-1} = P^2$$
$$MU_{JP} = P^2 = 2P$$

> **Notice that the cross-partials are the same and this will always be the case for any utility function!**

Meaning of a Utility Function

What are Utility Functions?

- They are a flexible tool that help us describe the relationship between two goods and the utility (happiness) you gain from them

- They allow us to get a good intuition of how we can change function properties so they relate to the choice we are attempting to model

- Let's think about some goods and decide what the utility function should look like

Modeling with Utility Functions

Let's consider **Homework** and **Pizza**

- First, we decide whether the good is desirable **(good)** or undesirable **(bad)**
  - I'll make the bold assumption that **Homework** is a bad and that **Pizza** is a good

> **Note: This implies that the marginal utilities are $MU_H < 0$ and $MU_P > 0$**

Modeling with Utility Functions - Example

Homework is a **bad** and **Pizza** is a **good**

Let's also set the following requirements:

The **marginal disutility** of **homework** is larger when I have more of it

MU_H is decreasing in H $\rightarrow$ We need an H in MU_H

The **marginal utility** of **pizza** is smaller when I have more of it

MU_P becomes smaller as I have more P $\rightarrow$ We need an H in MU_H

MU_P does **not** depend on **homework** MU_P does not have an H

> **Attempt creating a utility function with the above characteristics**

Modeling our Homework and Pizza

Here's my version:

$$U(H, P) = -H^2 + ln(P)$$

Now let's prove that it meets the requirements

Homework is a **bad** and must be worse the more I have of interested

$$MU_H = -2H < 0$$

$$MU_{HH} = -2 < 0$$

Pizza is a **good**, I get less joy from it the more I have, and it does not depend on how much **homework** I have

$$MU_P = \frac{1}{P} > 0$$

$$MU_{PP} = \frac{-1}{P^2} < 0$$

$$MU_{PH} = 0 = MU_{HP}$$

Meaning of a Utility function

The single most important property of a utility function is that we can measure **the relative preference for one good over the other**

- We can measure **how many units of y would you give up to get one more unit of x?**
  - We call this the **Marginal Rate of Substitution (MRS)**

$$MRS = \frac{MU_x}{MU_y}$$

Marginal Rate of Substitution (MRS)

Here we are talking about the relative preference of x over y, but how?

- Consider $U = f(x, y) = 4x + 2y$
  - You get 4 units of utility for each $x \ \rightarrow \ 4$
  - You get 2 units of utility for each $y \ \rightarrow \ 2$
  - We can say that each x is twice as valuable as each y

Using our MRS formula we have:

$$MRS = \frac{MU_x}{MU_y} = \frac{4}{2} = 2$$

Types of Utility Functions

In Economics, we mainly deal with 4 types of functions, each with its set of properties and tricks

- Cobb-Douglas
- Quasi-linear
- Perfect Substitutes
- Perfect Complements

Cobb-Douglas

$$U(x, y) = x^a y^b$$

> **Find the MRS of this general function**

$$MRS = \frac{MU_x}{MU_y} = \frac{ax^{a-1}y^b}{bx^a y^{b-1}} = \frac{a}{b} \cdot \frac{x^{a-1-a}}{y^{b-1-b}} = \frac{a}{b} \cdot \frac{x^{-1}}{y^{-1}} = \frac{a}{b} \cdot \frac{y}{x}$$

> **The MRS for a Cobb-Douglas will always look like**
>
> $$\frac{a}{b} \cdot \frac{y}{x}$$

Cobb-Douglas: Keys to Remember

$$MRS = \frac{a}{b} \cdot \frac{y}{x}$$

- The MRS is a ratio of y to x, multiplied by a constant
- MRS is your **willingness to trade y for x**
- As you get more x, the MRS goes down
- As you get more y, the MRS goes up

Cobb-Douglas - Example

$$U(x, y) = x^3 y^{1/2}$$

> **Find the MRS of this utility function**

$$MRS = \frac{MU_x}{MU_y} = \frac{3x^2 y^{1/2}}{1/2 x^3 y^{-1/2}} = \frac{3}{1/2} \cdot \frac{y^{1/2} y^{1/2}}{x^3 x^{-2}} = 6 \cdot \frac{y}{x}$$

If we recall that $MRS = \frac{a}{b} \cdot \frac{y}{x}$ then we can take a shortcut:

$$MRS = \frac{a}{b} \cdot \frac{y}{x} \rightarrow \frac{3}{1/2} \cdot \frac{y}{x} = 6 \cdot \frac{y}{x}$$

Quasi-Linear

$$U(x, y) = a \cdot ln(x) + b \cdot y$$

Where $a \cdot ln(x)$ is the **"quasi"** part and $b \cdot y$ is the **"linear"** part

> **Find the MRS of this general function**

$$MRS = \frac{MU_x}{MU_y} = \frac{a/x}{b} = \frac{a}{b} \cdot \frac{1}{x}$$

Quasi-Linear: Keys to Remember

$$MRS = \frac{a}{b} \cdot \frac{1}{x}$$

- The MRS is a constant times $1/x$
- As you get more x, the MRS decreases
- As you get more y, the MRS remains the same

Quasi-Linear - Example

$$U(x, y) = 1/3 \cdot ln(x) + y$$

> **Find the MRS of this utility function**

$$MRS = \frac{MU_x}{MU_y} = \frac{1/3 \cdot 1/x}{1} = \frac{1}{3} \cdot \frac{1}{x}$$

Using our shortcut we get:

$$MRS = \frac{a}{b} \cdot \frac{1}{x} = \frac{1/3}{1} \cdot \frac{1}{x} = \frac{1}{3} \cdot \frac{1}{x}$$

Perfect Substitutes

$$U(x, y) = a \cdot x + b \cdot y$$

> **Find the MRS of this general function**

$$MRS = \frac{MU_x}{MU_y} = \frac{a}{b}$$

Keys to Remember

- MRS is a constant

Perfect Substitutes - Example

$$U(x, y) = 6x + \frac{1}{2}y$$

> **Find the MRS of this utility function**

$$MRS = \frac{MU_x}{MU_y} = \frac{6}{1/2} = 12$$

And our shortcut shows:

$$MRS = \frac{6}{1/2} = 12$$

Perfect Complements

$$U(x, y) = \min\{a \cdot x, b \cdot y\}$$

This utility function requires a different form of intuition

Let's first think of a simple example:

Imagine we are trying to make some hot chocolate which requires 1 pack of chocolate powder and 12 oz of milk

$$U(x, y) = \min\{1\, choco, 12oz\, milk\}$$

You check your kitchen and find that there are 3 packs of chocolate powder and you have 15 oz of milk in your fridge

> **How many hot chocolates can we make?**

Perfect Complements Intuition

These are goods that **have to be consumed together in an exact proportion** in order to produce any utility

> There is no MRS, so we use a property called the **No-Waste Condition**:
>
> $$U(x, y) = \min\{a \cdot x, b \cdot y\}$$
>
> $$a \cdot x = b \cdot y$$

Perfect Complements: Keys to Remember

- MRS is not defined (We cannot take a derivative)
- We can use the **No-Waste Condition**:
  - When $a \cdot x < b \cdot y$ you will give up any y you can to get more x
  - When $a \cdot x > b \cdot y$ you will give up any x you can to get y

Perfect Complements - Example

$$U(x,y) = \min\{\frac{x}{2}, \frac{y}{4}\}$$

> **Find the No-Waste Condition of this utility function**

No-Waste Condition is $\frac{x}{2} = \frac{y}{4} \rightarrow 4x = 2y \rightarrow 2x = y$

We can say:

- If $2x > y \;\; \rightarrow$ Too much x, so we would trade some x for some y
- If $2x < y \;\; \rightarrow$ Too much y, so we would trade some y for some x

Bundles

When you have a utility function of two goods, any given combination of those two goods is called a **Bundle**

- Every **bundle** has an associated utility level

Take the following utility function and bundles

$$U(x, y) = x^2 y$$

Bundle 1 is (3,2)

$$\begin{aligned} U(3,2) &= 3^2 \cdot 2 \\ &= 9 \cdot 2 = 18 \end{aligned}$$

Bundle 2 is (1,0)

$$\begin{aligned} U(1,0) &= 1^2 \cdot 0 \\ &= 1 \cdot 0 = 0 \end{aligned}$$

Bundles & Axioms of Preferences

We will use the following axioms about preferences between bundles to ensure logical consistency

- Completeness
- "More is Better"
- Transitivity

> **These come from economic theory so they will help you think like an economist but do not think this tells us how people behave**

> **We are attempting to successfully predict behavior, so we will simply assume that everyone behaves accordingly**

Completeness

We say that preferences are always **complete**. So when comparing bundles A and B, you can always say one of the following:

- I prefer A to B ($A \succeq B$)
- I prefer B to A ($B \succeq A$)
- I am indifferent between A and B ($A \sim B$)

This allows us to compare and order any pair of bundles

“More is Better”

Or at least more is no worse than less

In general, if a good is desirable we will want more of it

- However, sometimes products can be bads (instead of goods) and we would, obviously, want less of those

Transitivity

Preferences are transitive. This means that when comparing bundles A, B, and C you can get logical orderings through rankings:

- If you prefer B to A ($B \succeq A$) and C to B ($C \succeq B$) then you must prefer C to A ($C \succeq A$)

Transitivity

What would this graph mean?

All bundles are preferred equally!

Preferences Assumptions

All of these are necessary to understand utility funcitons and be able to graph them

To do so we first need to learn about **Indifference Curves**

Indifference Curves

What is Indifference?

It is exactly what it sounds like

- Any two bundles that give the same utility level makes you indifferent between those two Bundles

For example, for the utility of $U(x, y) = 2x + 3y$ I am indifferent between bundles (3,2) and (0,4)

$$U(3,2) = 2(3) + 3(2) = 6 + 6 = 12$$

$$U(0,4) = 2(0) + 3(4) = 12$$

Indifference Set

Given a utility function and a level of utility, you can find a whole set of bundles that you are indifferent between

For example, let $U(x, y) = x + y$ and set $U = 10$. Then we can find an infinite set of x and y that will give us our stated utility level

$$\begin{array}{cc} x = 10 & y = 0 \\ x = 9 & y = 1 \\ x = 8 & y = 2 \\ \vdots & \vdots \end{array}$$

Indifference Curves

We can create a function that helps us find **ALL** possible bundles that make you indifferent at a given utility level

We call this an **Indifference Curve**

For our previous utility function $U(x, y) = x + y$ where $U = 10$, we solve for y and get:

$$U(x, y) = x + y$$

$$10 = x + y$$

$$10 - x = y \rightarrow \text{Indifference Curve (IC)}$$

Indifference Curve - Example

$$U(x, y) = xy \;\; \text{where } U = 16$$

> What type of Utility Function is this?

- It is a Cobb-Douglas $x^a y^b$ where $a = b = 1$

> What is the associated Indifference Curve?

$$16 = xy \rightarrow y = \frac{16}{x} \rightarrow \text{ IC}$$

Indifference Curves

So why do we care about these curves?

- We can graph them
- Graphing is a key step to figuring out how to solve an individual's choice problem

> **Each utility function has a unique shape that we will learn**

Let's Draw - Perfect Substitutes

$$U(x, y) = 3x + y \text{ with } U = 6,\ 9,\ 15$$

> **First, find the indifference curves for each Utility value**

$$U = 6 \qquad U = 9 \qquad U = 15$$

$$y = 6 - 3x \qquad y = 9 - 3x \qquad y = 15 - 3x$$

> **Next, we graph these functions**

Let’s Draw - Perfect Complements

$$U(x,y) = \min\{x, \frac{y}{2}\} \text{ with } U = 2,\ 8,\ 9$$

> **Find the indifference curves for each utility value**

$U = 2$	$U = 8$	$U = 9$
$x = 2 \text{ or } y = 4$	$x = 8 \text{ or } y = 16$	$x = 9 \text{ or } y = 18$

> **Next, we graph these functions**

Let's Draw - Cobb-Douglas

$$U(x, y) = x^{1/2}y \text{ with } U = 4,\ 8,\ 10$$

> **Find the indifference curves for each utility value**

$$U = 4 \qquad\qquad U = 8 \qquad\qquad U = 10$$

$$y = \frac{4}{x^{1/2}} \qquad\qquad y = \frac{8}{x^{1/2}} \qquad\qquad y = \frac{10}{x^{1/2}}$$

> **Next, we graph these functions**

Let's Draw - Quasi-Linear

$$U(x, y) = ln(x) + y \;\text{ with } U = 5,\ 15,\ 20$$

> **Find the indifference curves for each utility value**

$$U = 5 \qquad\qquad U = 15 \qquad\qquad U = 20$$

$$y = 5 - ln(x) \qquad y = 15 - ln(x) \qquad y = 20 - ln(x)$$

> **Next, we graph these functions**

Indifference Curve - Rules

It is very important that you understand the **intuition** behind indifference curves

Let's view an example that can help:

Consider Weather Reports:

- On cold days, what the weather feels like is a function of:
    1. Temperature
    2. Windchill
- An indifference curve represents **all of the different combinations of temperature and windchill that cause you to feel the exact same thing**
- If the windchill is suddenly lower, what must **intuitively** happen to the temperature to keep you feeling the same outside?

If windchill $\downarrow$ then temperature $\uparrow$

Indifference Curve - Rules

We use this exact same logic for utility between two goods

Intuitively, if I want to stay at the same level of happiness as I lose some y, what must happen to x?

- I need more of x

This is why we read them from left to right and why they have a negative slope

- I know the perfect complements is odd but the same logic tends to apply

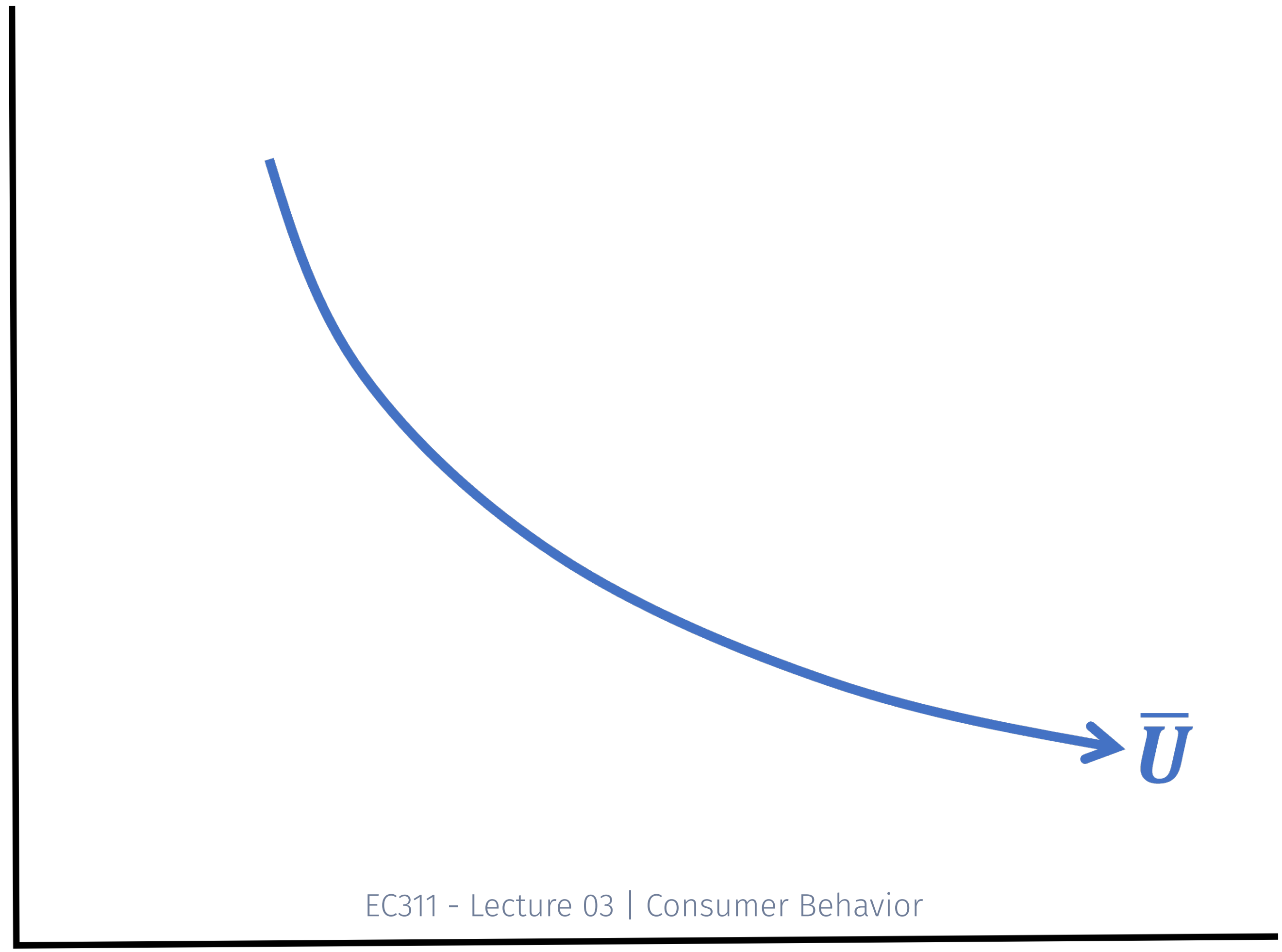
Formal Indifference Curve Rules

All of the stuff from before can be formalized into the following 3:

- **Monotonicity:** Indifference curves always go from the top left to the bottom right of the graph without changing direction at any point
- **Non-Crossing:** If at least two curves cross, this leads to logical contradictions
- **Convex:** Balanced combinations of two goods are preferred to extreme outcomes (A lot of one good, little of the other)

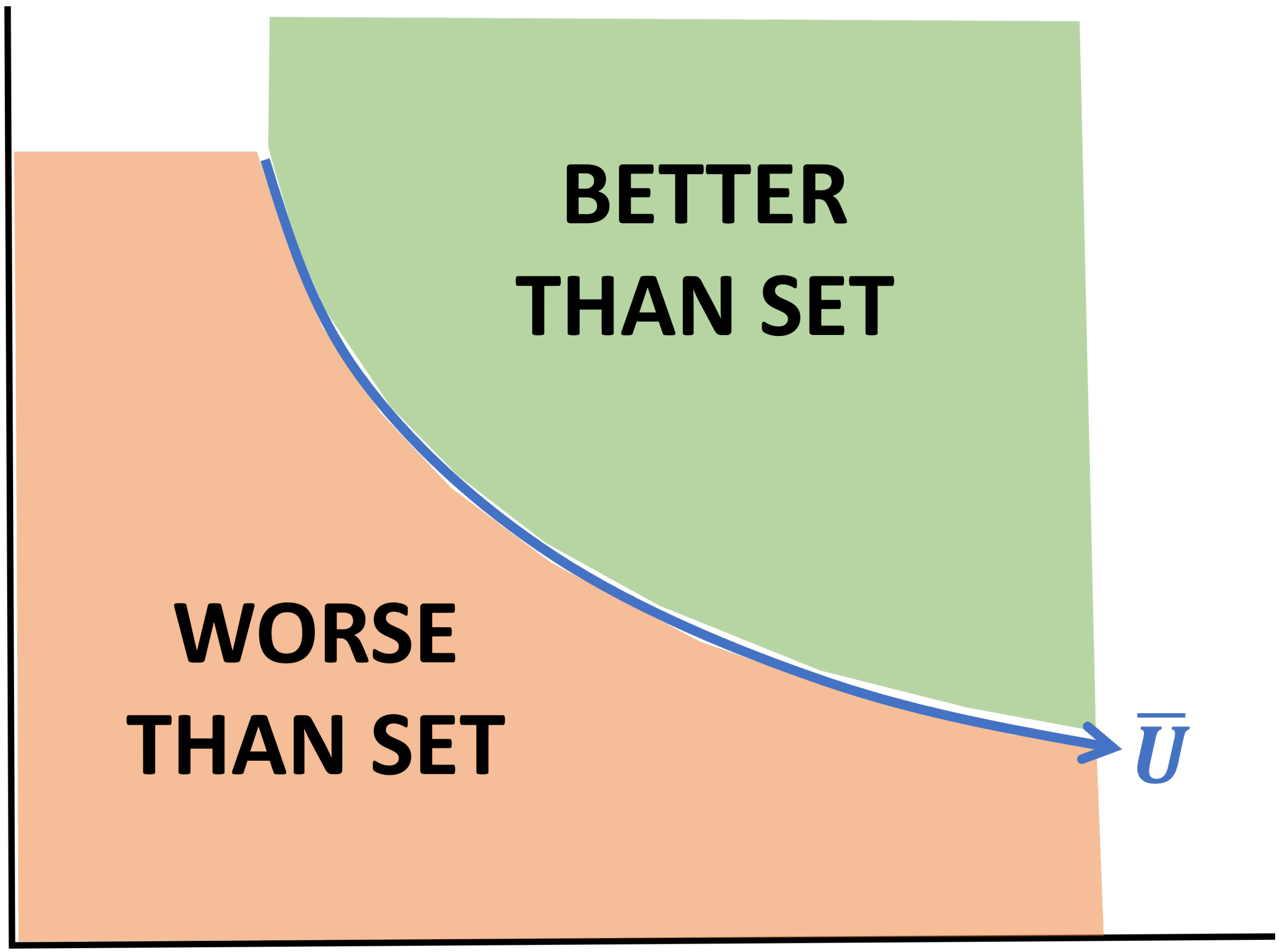
Monotonicity

ICs always go from the top left to the bottom right without changing direction



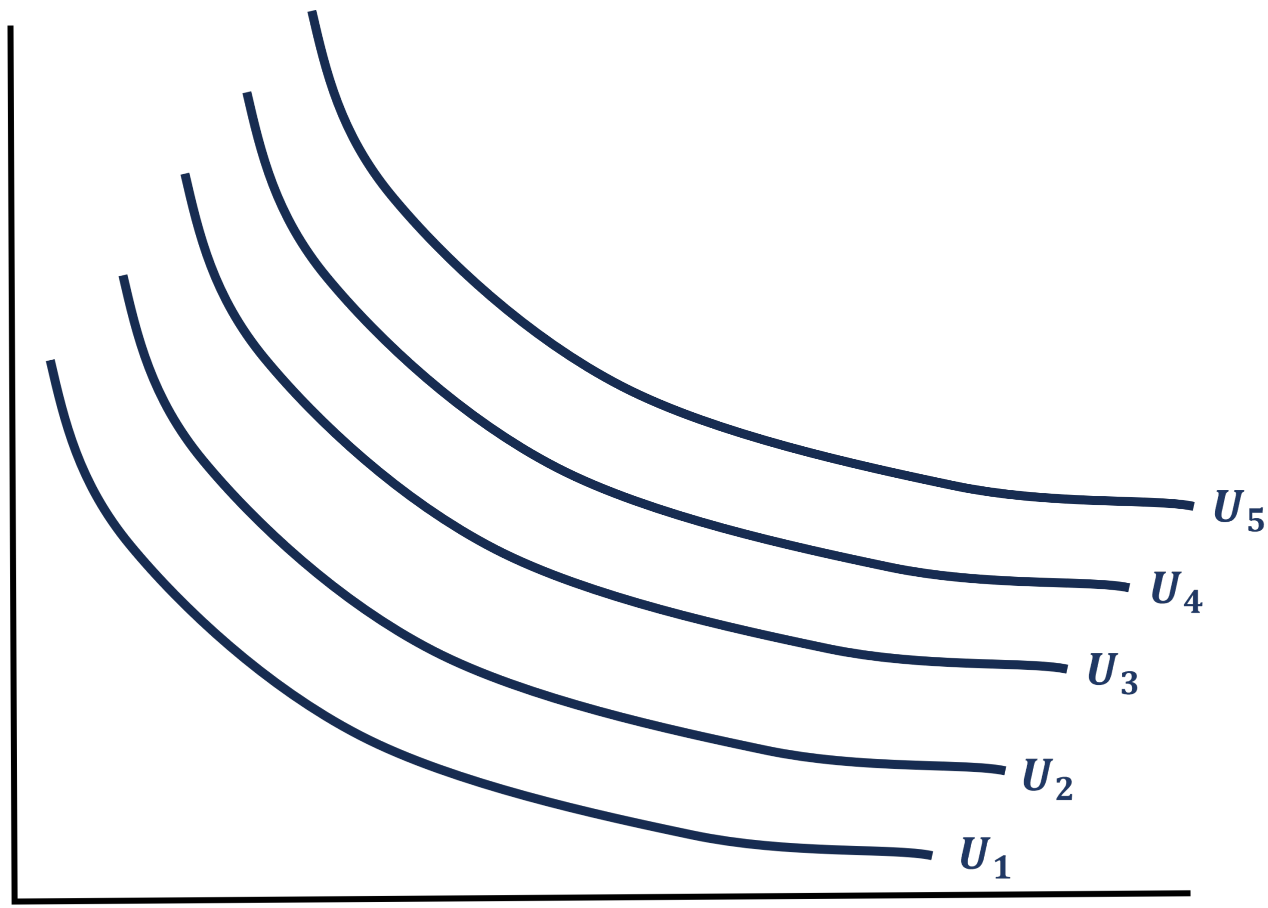
Monotonicity

Additionally, this helps us visualize two important sets of bundles



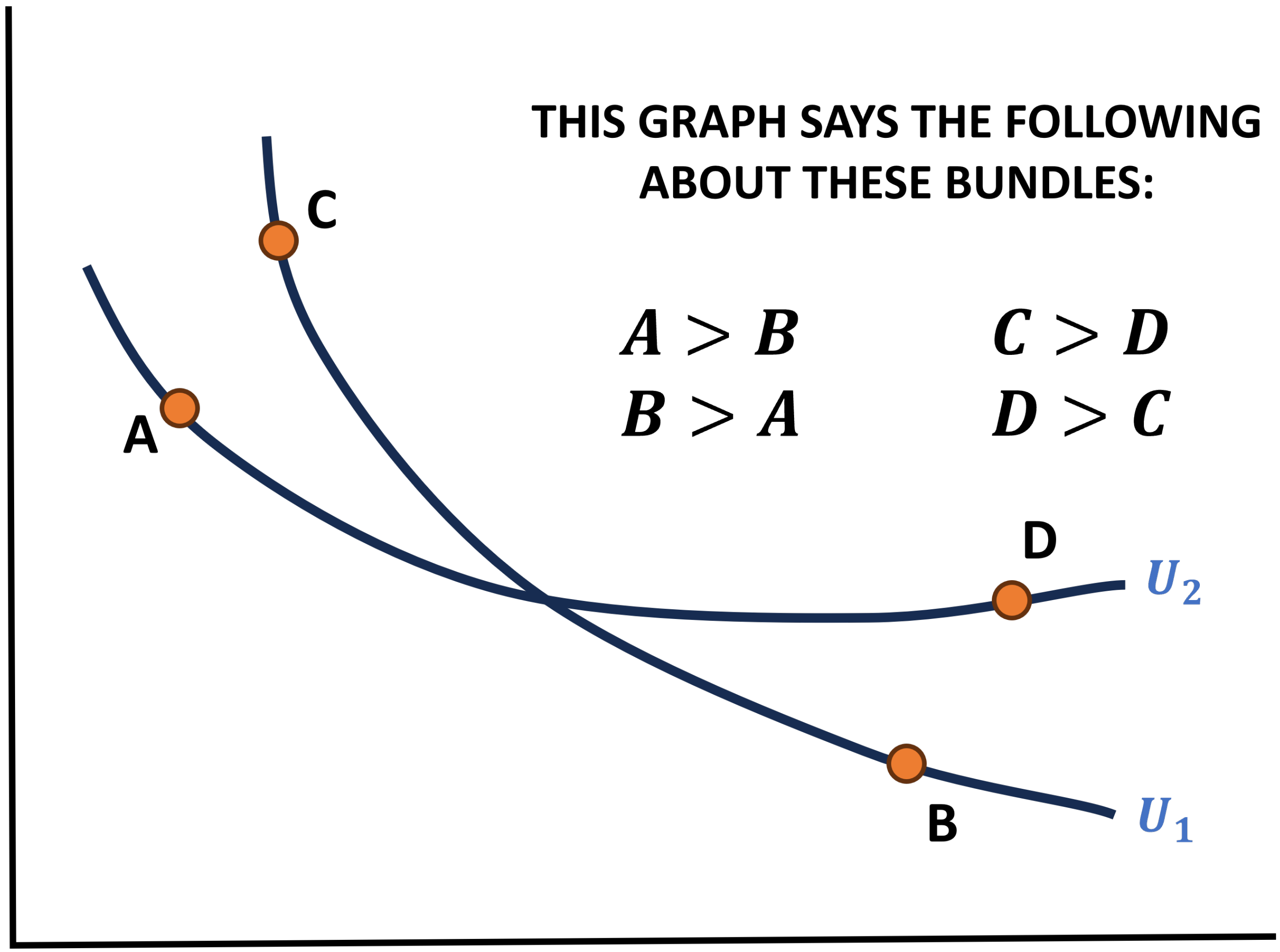
Non-Crossing

This is the expected behavior of ICs. There are infinitely many, each representing a unique level of Utility



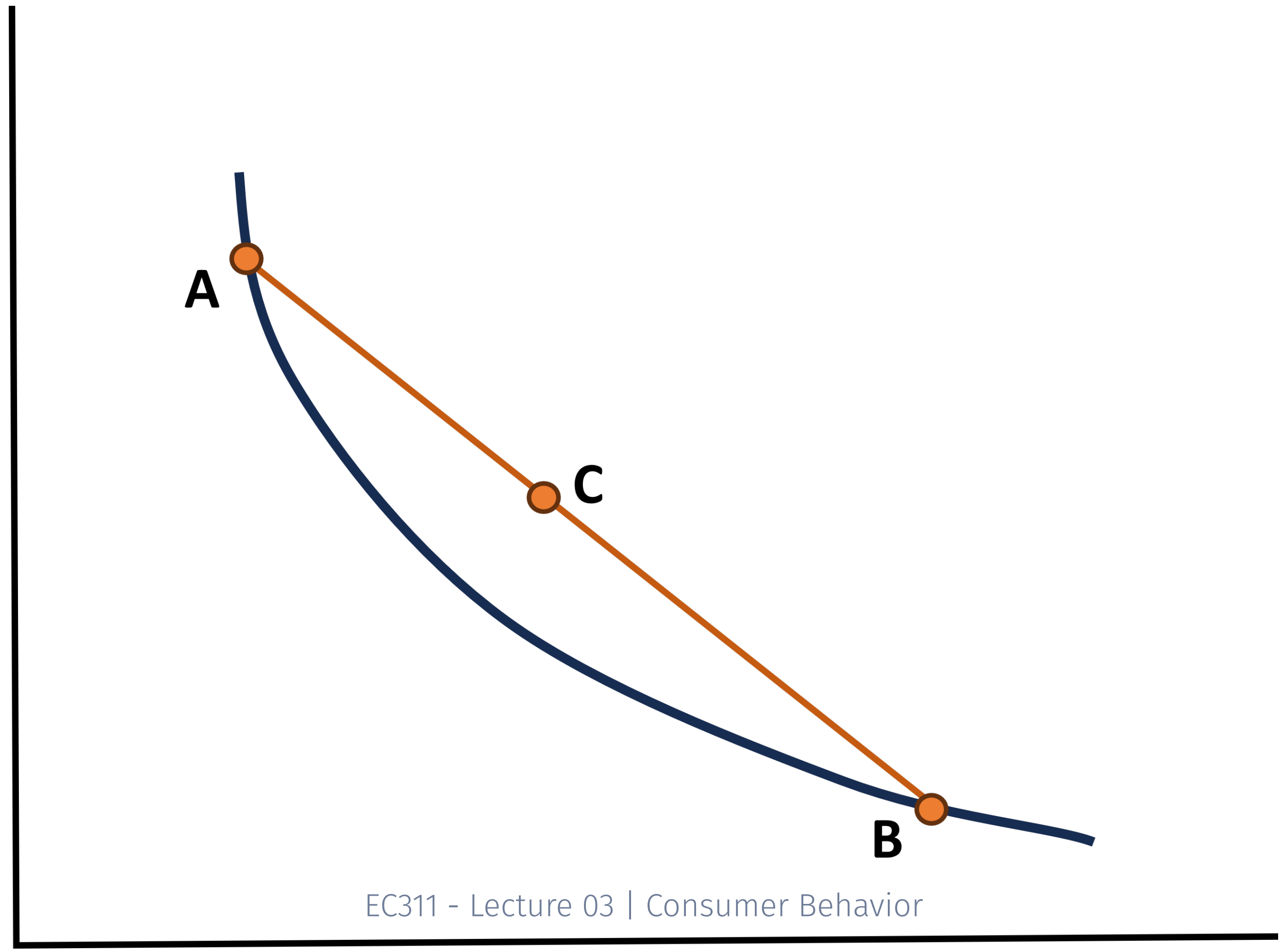
Non-Crossing - Logical Contradiction

If ICs cross, they are contradictions

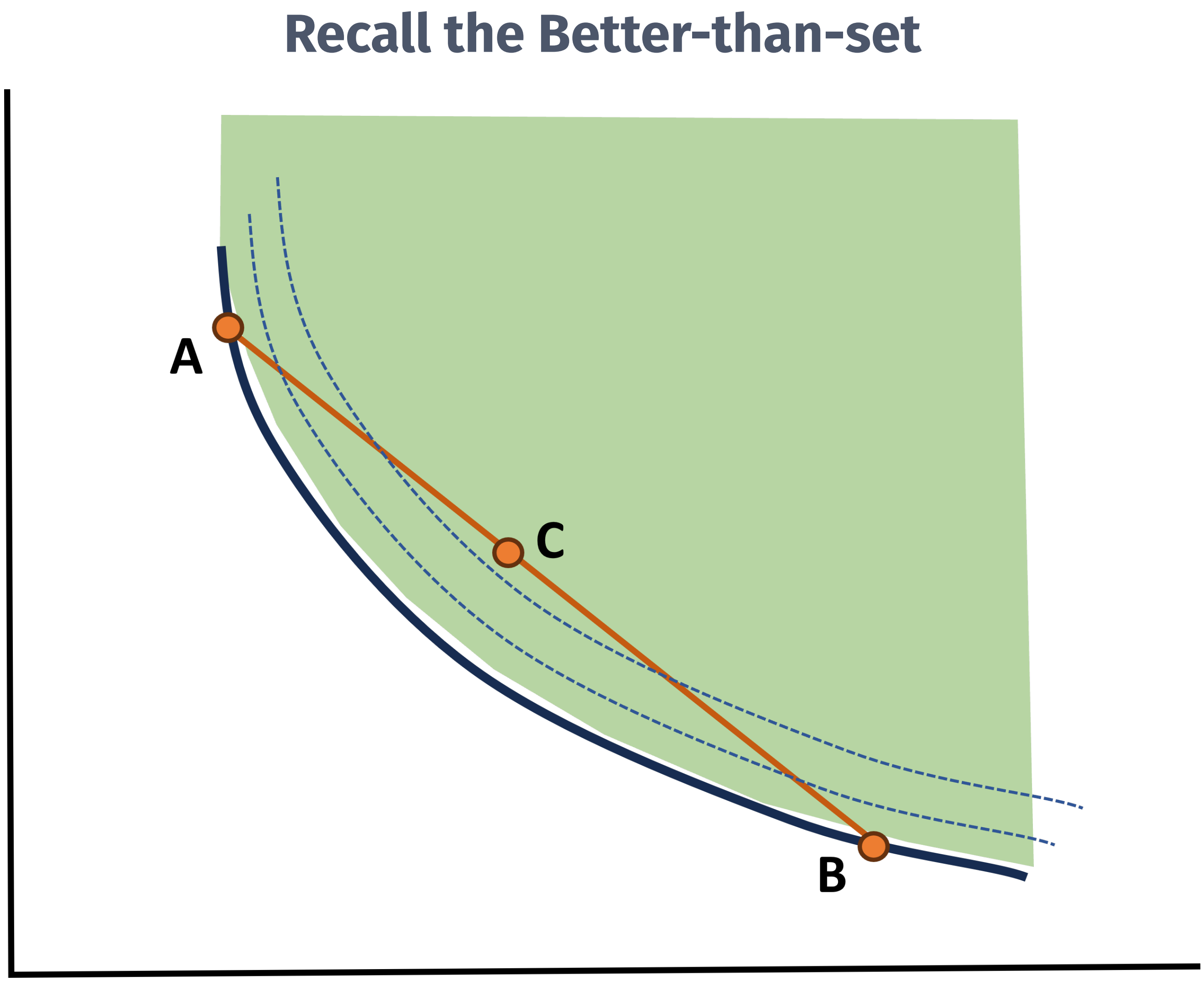


Convex

A balanced combination of two goods are preferred to extreme outcomes



Convex



What About the Other Functional Forms?

I drew Cobb-Douglas curves (mostly because they're easier to illustrate these properties) but what about Perfect Complements or Perfect Substitutes?

- They fulfill all 3 properties, **but not strongly**

Perfect Complements

- Never Crosses
- Convex
- **Wealky Monotonic**

Perfect Substitutes

- Never Crosses
- Monotonic
- **Weakly Convex**

Let's look at the board again

Most Important Facts of Indifference Curves

Recall the intuition of what a movement along the indifference curve means:

- You are trading y for x, holding **constant** your level of **utility**
- The slope of the indifference curve measures your **willingness to tradeoff between x and y**
- This is also known as the **Marginal Rate of Substition** (times -1)

Most Important Facts of Indifference Curves

This **Marginal Rate of Substition (MRS)** thing is pretty important

First, why is the slope the **negative MRS**?

- The MRS is the **ratio** of the effect of increasing x on your utility and the effect of increasing y on your utility
- $\dfrac{MU_x}{MU_y}$
- The Indifference Curve slope is all about keeping the utility level constant while we move x and y in opposite directions
- Therefore the IC slope = -MRS

Simple Mathematical Proof

For

$$U(x, y) = ax + by$$

1. Find the MRS and times -1

$$MRS = \frac{MU_x}{MU_y} = \frac{a}{b}$$

$$-MRS = \frac{-a}{b}$$

2. Find the IC and it's slope

$$\bar{U} = ax + by$$

$$\bar{U} - ax = by$$

$$\frac{\bar{U}}{b} - \frac{ax}{b} = y$$

$$\text{Slope is the derivative!} \rightarrow \quad \frac{\partial y}{\partial x} = \frac{-a}{b}$$

Indifference Curves

From this lecture you have learned:

- Everything about utility functions and how to use them to find an MRS
- Everything about Indifference Curves and how the IC slope relates to the MRS

The MRS is going to be key to solving utility maximization problems

- Mathematically (Using derivatives)
- Graphically (Drawing ICs)

However, when we maximize utility functions we have constraints, we called these **Budget Constraints**

Budget Constraints

Budgets

Economic Theory says that individuals make themselves as happy as they possibly can, after choosing from a set of all bundles they can afford

- We do not just maximize utility functions, but rather we maximize them subject to **Budget Constraints (BC)**

Budgets - Variables

Inside a budget constraint we use the exact same variables we use in our utility function, namely x and y

But we'll need to introduce some new notation and terminology:

- The **price** of good x $\rightarrow$ P_x
- The **price** of good y $\rightarrow$ P_y
- Your income, budget, or money-on-hand $\rightarrow$ M
    - The book also calls this I

Budgets - Functional Form

Putting together our variables we get our **Budget Constraint**

$$P_x \cdot x + P_y \cdot y \leq M$$

> **Do not let the inequality $\leq$ scare you, it just means we can spend less than our total income**

However, in our applications we are going to treat it as a strict equality

$$P_x \cdot x + P_y \cdot y = M$$

And let's think why?

- If we are maximizing utility, it does not make sense to leave any income unspent!

Budgets - Spending All Our Income

But do we usually spend **all** of our money?

Of course not, but here is how to think about it in the economics sense:

- When you go to the grocery store you make to choices before even stepping inside:
  1. How much to spend at the store
  2. What to spend it on
- Once you allocate your budget, you usually spend all of it

Let's Construct A Budget Constraint

First, we need our two goods from before: Beer (B) and Soda (S)

Let's label everything properly:

- $B = \text{Beer}$
- $S = \text{Soda}$
- $P_B = \$4$
- $P_S = \$2$
- $M = \$20$

What does the Budget Constraint look like?

$$B \cdot P_B + S \cdot P_S = M$$

$$4B + 2P = 20$$

Let's Graph A Budget Constraint

We graph the budget in the same space in as our IC, but what are we graphing?

$$P_x \cdot x + P_y \cdot y = M$$

And thankfully, this is just a straight line

Bringing Intuition into Budget Constraint Graphs

$$P_x \cdot x + P_y \cdot y = M$$

Thinking intuitively, this graph represents:

- All combinations of (x, y) that cost exactly M
- If we spend nothing on good y, what is the most possible x that we can purchase?

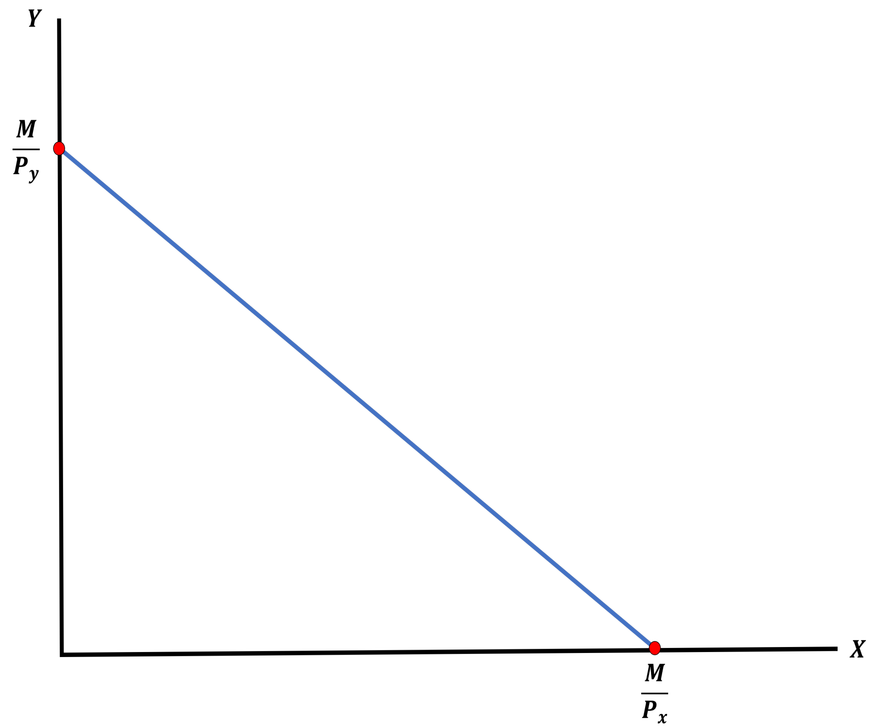
$$\begin{aligned} P_x \cdot x + P_y \cdot 0 &= M \\ P_x \cdot x &= M \\ x &= \frac{m}{P_x} \end{aligned}$$

Let’s Graph A Budget Constraint

Following the logic we just did, the budget line will connect two points:

One for when we buy only $\boldsymbol{x}$ and one for when we buy only $\boldsymbol{y}$

$$(\frac{M}{P_x}, 0) \text{ and } (0, \frac{M}{P_y})$$



Interpreting the Slope of the Budget Constraint

The Budget Constraint is the line:

$$y = \frac{M}{P_y} - \frac{P_x}{P_y} \cdot x \quad \text{where the slope is} \quad -\frac{P_x}{P_y}$$

Now, let's define

$$P_x = \frac{\$}{x} \text{ and } P_y = \frac{\$}{y}$$

We get this result in terms of units!

$$\frac{P_x}{P_y} = \frac{\$/x}{\$/y} = \frac{\$}{x} \cdot \frac{y}{\$} = \frac{y}{x}$$

The Slope of the Budget Constraint

As we just saw, the price ratio P_x/P_y can be measured in units of x per units of y

We had already seen something that is measured in units $\longrightarrow$ the MRS

This leads us to understand the differences between the two:

- The **MRS** represents how much y you **would be willing to** give up in order to get a unit of x

- The **Price Ratio** represents how much y you **would have to** give up in order to get a unit of x

What Happens When Things Change?

Budgets are not static. Prices and income can change based on market conditions so it is important to understand the effects when factors change

- We can have changes in income (increase or decrease)
- Price of x can change (increase or decrease)
- Price of y can change (increase or decrease)

> **Note - We normally consider what happens when only one of the possible factors change, and hold all others fixed**

Changes in Income

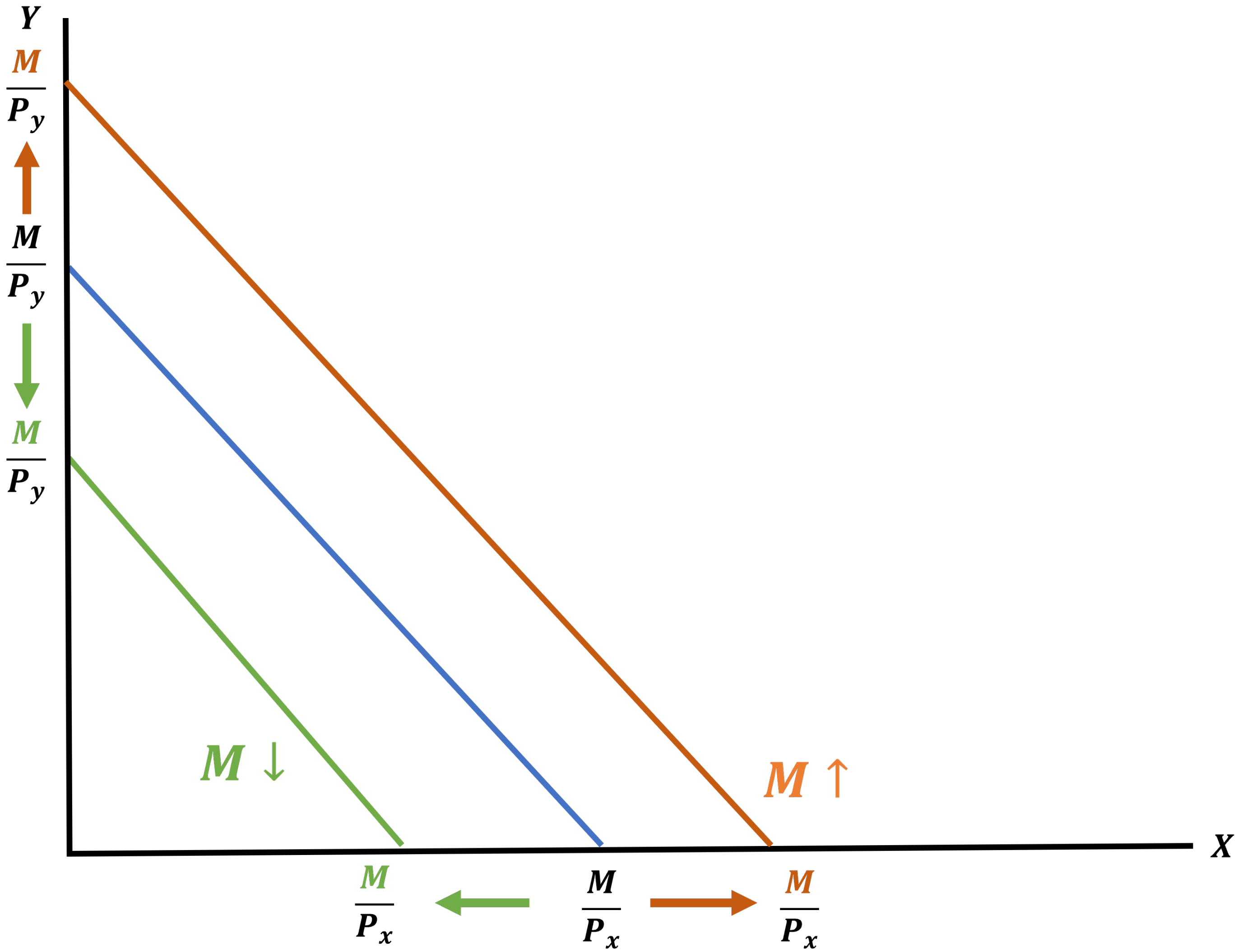
What happens to the budget if income (M) increases?

- $\dfrac{M}{P_x}$, the maximum amount of x that can be purchased goes up
- $\dfrac{M}{P_y}$, the maximum amount of y that can be purchased goes up
- The slope of the BC $(-P_x/P_y)$ stays the same

Income changes affect the **overall amount** an individual can consume, but has **no effect on the relative cost of the goods**

> **Intuitively, the opposite is true if income (M) decreases**

Changes in Income - Graph



Changes in Prices

What happens to the budget if the price of good x (P_x) increases?

- The maximum amount of x that can be consumed goes down $(\frac{M}{P_x}) \downarrow$
- The maximum amount of y that can be consumed stays the same $(\frac{M}{P_x})$
- The price ratio $(-\frac{P_x}{P_y})$ becomes steeper

> **Once more, the opposite happens with a decrease**

> **How about a shift in P_y ?**

Changes in Prices - Graph