Imperfect Competition

EC 311 - Intermediate Microeconomics

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Outline

Chapter 11

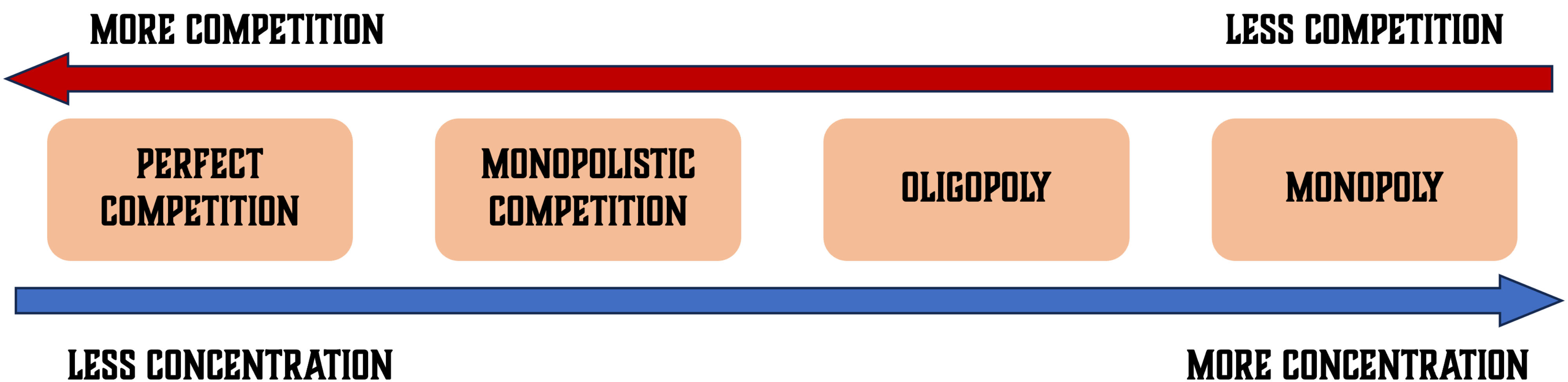
- Topics
  - Oligopoly (11.1)
  - Collusion & Cartels (11.2)
  - Bertrand Competition (11.3)
  - Cournot Competition (11.4)
  - Stackelber Competition (11.5)

Extreme Markets & Imperfect Competition

Up To Now

So far we have talked about **Perfect Competition** and **Monopoly**

- These firms are either **complete price takers** or **complete price setters**
- But these are the extremes, the real world is more nuanced
  - Neither of our models are likely to be truly realistic, but they do a good job at showing us the ends of our **Competitive Spectrum**



Markets

Most markets are somewhere in-between

- Firms cannot set market prices by themselves
- But there are circumstances where there are large firms and are few enough that one single firms choice can impact market prices

This is the basics of **Imperfect Competition** and what we will be tackling until the end

Imperfect Competition

Firms are still competing but they know who their rivals are and will adopt strategies based on what they think their rivals will do

- In order to model firms in these contexts, we need to learn the very basics about **Game Theory**
  - This is the field of decision-making in situations where actions affect others' outcomes and vice-versa
- We will keep things simple by only studying interactions between two firms
  - We call this a **Duopoloy**
- The intuition and approach we will take holds for cases where there are more than 2 firms
  - This is called an **Oligopoly**

Game Theory

Decision-Making Until Now

We have only studied decisions where one person or one firm were making decisions **“in a vacuum”**

- We specifically addressed what is best for the individual or the firm, **but it never depended on what other people or firms do**

Monopolies were slightly different

- They maximized porfits, **while taking into account the behavior of the consumers in their market**
  - Our models of **Imperfect Competition** are going to look similarly, with some modifications

Game Theory Basics

Let's think of a simple example to illustrate **Game Theory**

Imagine that you manage a gas station. Directly across the street from you is a competing gas station. Also assume that you are the only stations in town

Both of you set your gas prices at he start of the day and only have two choices:

Set Low Price **Set High Price**

This means there are four possible outcomes we will consider

Game Theory Basics - Gas Station Example

		COMPETITOR	
		LOW	HIGH
YOU	LOW	10,10	20,0
	HIGH	0,20	15,15

So what should you do?

Game Theory - Nash Equilibrium

Let's formalize the thought process **Game Theory** indicates

Imagine that your competitor chooses to set a **High Price**. How should you respond?

- You can take the entire market and earn 20 by setting a **Low Price**
- This is clearly better than setting a **High Price** and sharing the market earning 15

Now imagine the competitor chose to set a **Low Price**. How should you respond?

- If you do not set a **Low Price**, your competitor will take the entire market and you will earn 0
- Your best response is to also set a **Low Price** and earn 10

Game Theory - Nash Equilibrium

Regardless of what your competitor does, the best thing for you to do is to set a **Low Price**

- Now if you switch perspectives and answer the same questions from your competitor's point of view **You will get the same answer**
- So both of you are responding as you best could, to each other's best decision

This outcome is called a

Nash Equilibrium

Nash Equilibrium

I will abbreviate **Nash Equilibrium** as **NE** Let's define it formally

A set of choices (one for each player) is a **NE** if:

- Each player's choice maximized their payoff **given the choice of the other player**
- Note that the **NE** is not the outcome that maximizes the **total payoffs** earned by each player
    - If both gas stations had set a **High Price** they would have each earned 15 for a total of 30
    - With the outcome we get, both set a **Low Price** and earn 10 each for a total of 20

Nash Equilibrium

The term **Best Responding** is a grat way of thinking about **NE** because it sets us up well for integrating it into our model of firm behavior

- **Firm A** an come up with a **Best Response** to any price that their competitor might set
- In turn, their competitor, **Firm B** can also come up with a **Best Response** strategy to any price that **Firm A** might set
- The prices at which **Both Firms are simultaneously best responding to one another are the Nash Equilibrium Prices**

Nash Equilibrium in Producer Markets

To put this in terms of our models where firms maximize profit based on their choice of **Quantity** Q:

- The **quantities** at which **both firms are simultaneously best responding to one another are the Nash Equilibrium quantities**
- The **Market Price** is determined by **the Sum of the Quantitites**

We are going to be dealing with several models of imperfect competition

- Cornout Competition
- Cartels
- Stackelberg Competition
- Bertrand Competition

Cournot Competition

Cournot Competition = Simultaneous Decisions

This model consists of two firms competing by **Best Responding** to one-another's quantity choices

We call this a **Cournout Competition or Cornout Equilibrium**

- Mathematically, this will look like a modified **Monopoly** with two firms

Price is a function of Market Quantity which is the sum of both firms quantity

$$\pi_A = P \cdot Q_A - C(Q_A)$$
$$\pi_B = P \cdot Q_B - C(Q_B)$$

$$P = f(Q_S) = f(Q_A + Q_B)$$

Which is the Market Demand Curve

Cournot Equilibrium - Example

Let both firms have the following **Cost Functions** and **Demand Curve**

$$C(Q) = Q^2 \quad ; \quad P = 100 - Q_D$$

Demand is the sum of both firms produced quantities

$$Q_D = Q_A + Q_B$$

Find the Demand Curve

$$P = 100 - Q_D$$
$$P = 100 - (Q_A + Q_B)$$
$$P = 100 - Q_A - Q_B$$

Cournot Equilibrium - Profits

We can then write these firms profits using our **updated Demand Curve**

$$(P = 100 - Q_A - Q_B)$$

Remember profits are written as

$$\pi = P \cdot Q - C(Q)$$

Write both Firm A and Firm B's Profit Functions

$$\pi_A = (100 - Q_A - Q_B) \cdot Q_A - Q_A^2$$

$$\pi_B = (100 - Q_A - Q_B) \cdot Q_B - Q_B^2$$

Cournot Equilibrium - Best Response Functions

We find **Best Response Functions** because we want to model it in such a way that we can observe responses to any possible quantity levels

We can find these by finding the profit maximizing quantity of each profit function

$$\frac{\partial \pi_i}{\partial Q_i} = 0$$

Cournot Equilibrium - Example Best Response Functions

$$\pi_A = (100 - Q_A - Q_B) \cdot Q_A - Q_A^2$$

$$\pi_B = (100 - Q_A - Q_B) \cdot Q_B - Q_B^2$$

Find the Best Response Functions for both firms

Hint: You only need to do the work for one, the other is symmetric

Find $\frac{\partial \pi_A}{\partial Q_A} = 0$

$$100 - 2Q_A - Q_B - 2Q_A = 0$$
$$4Q_A = 100 - Q_B$$
$$Q_A^* = \frac{100 - Q_B}{4}$$

By Symmetry, we know Firm B's Best Response

$$Q_B^* = \frac{100 - Q_A}{4}$$

Cournot Equilibrium - Optimal Quantity

Would you believe me if I told you we can get a number for Q_A^* from this?

We have $Q_A^* = \frac{100 - Q_B}{4}$ & $Q_B^* = \frac{100 - Q_A}{4}$

Find Q_A^* by plugging in our Q_B^* function

We solve this by putting one Best Response into the other

$$Q_A^* = \frac{100 - Q_B}{4}$$

$$Q_A^* = \frac{100 - \frac{100 - Q_A}{4}}{4}$$

Cournot Equilibrium - Optimal Quantity & Price

$$Q_A^* = \frac{100 - \frac{100 - Q_A}{4}}{4}$$

Find Optimal Quantity

$$\begin{aligned} 4Q_A^* &= 100 - \frac{100 - Q_A^*}{4} \\ 16Q_A^* &= 400 - (100 - Q_A^*) \\ 15Q_A^* &= 300 \\ Q_A^* &= 20 = Q_B^* \end{aligned}$$

Find Market Supply

$$\begin{aligned} Q_S &= Q_A^* + Q_B^* \\ Q_S &= 20 + 20 \\ Q_S &= 40 \end{aligned}$$

Find Market Price

$$\begin{aligned} P &= 100 - Q_D \\ P &= 100 - 40 \\ P^* &= 60 \end{aligned}$$

Cournot Equilibrium

We just saw that each firm is best responding to the other's decisions

The equilibrium we found is the point where

Both firms are best responding to each other at the same time

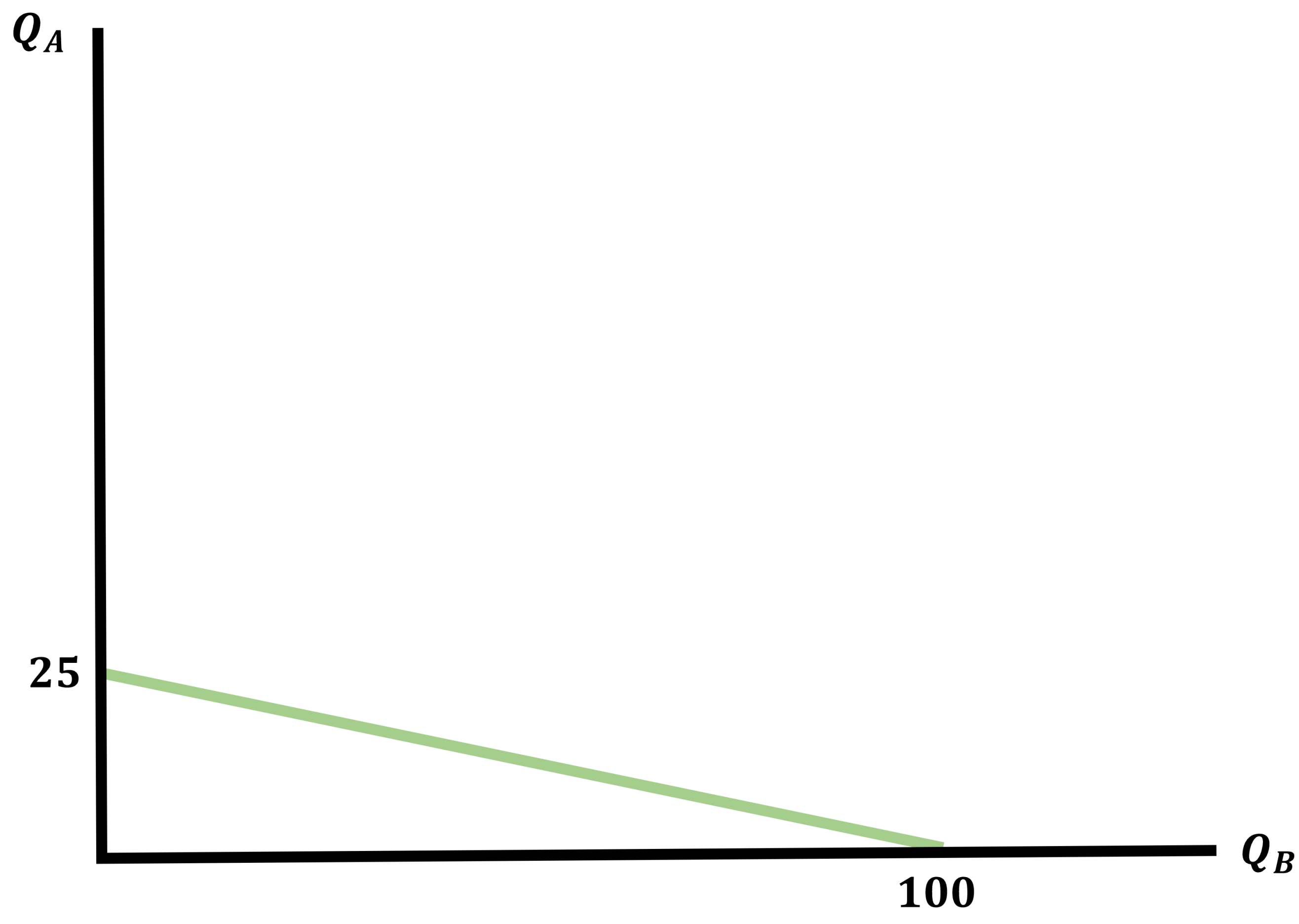
Let's look at a graph of the previous example

Graphing Best Response Functions - Firm A

Firm A's Best Response Function

$$Q_A^* = \frac{100 - Q_B^*}{4}$$

$$Q_A^* = 25 - \frac{Q_B^*}{4}$$

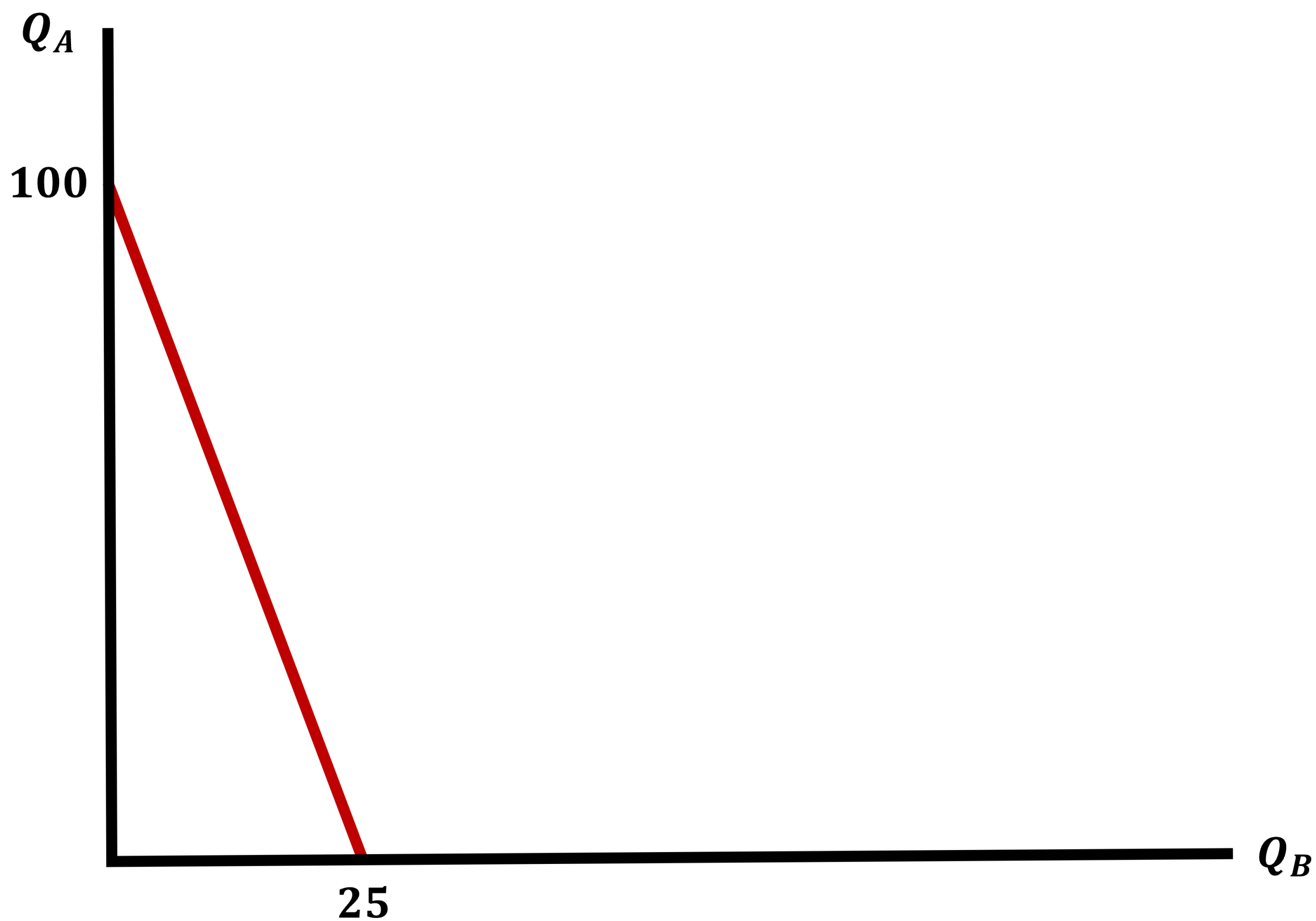


Graphing Best Response Functions - Firm B

Firm B's Best Response Function

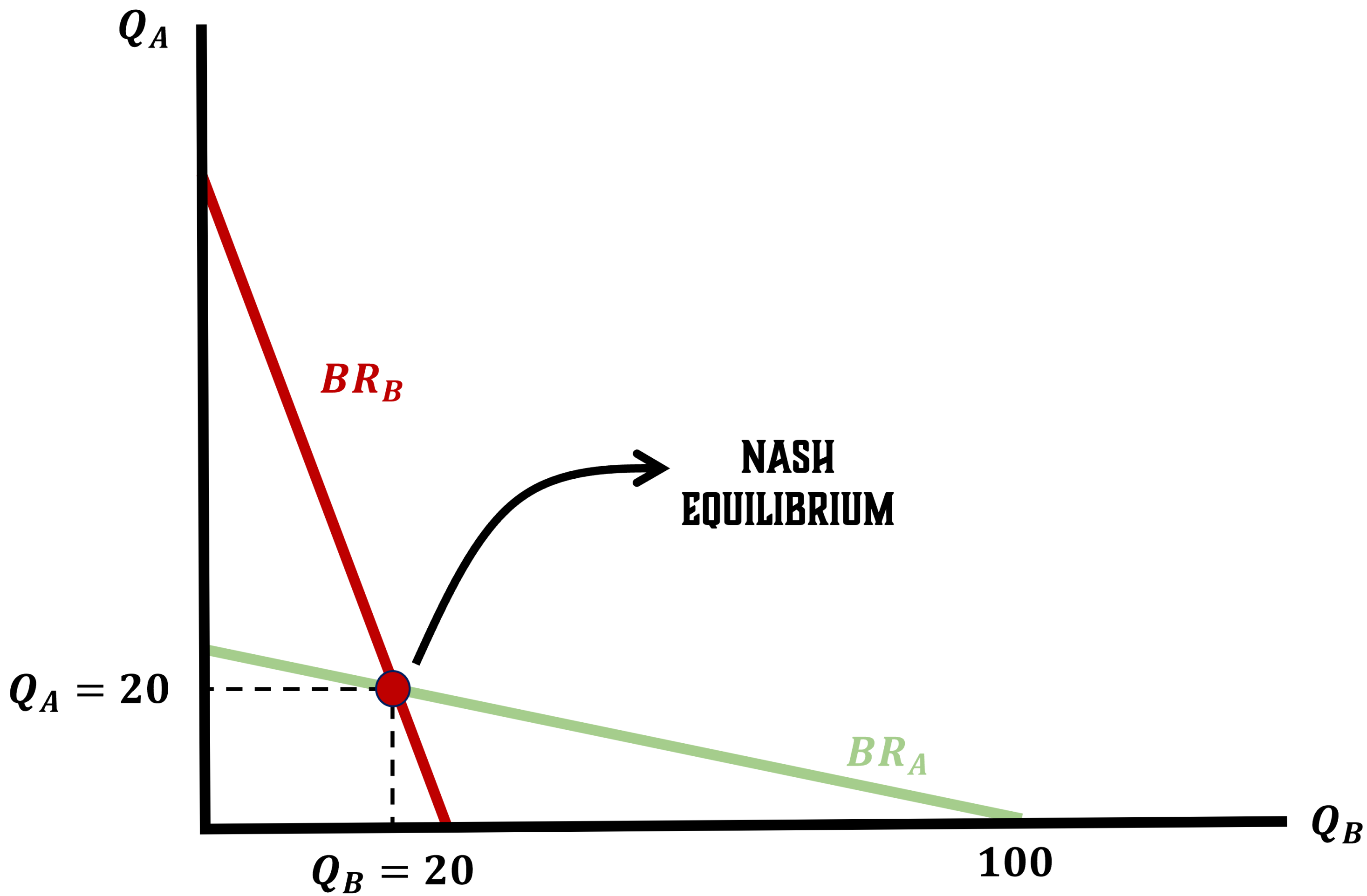
$$Q_B^* = \frac{100 - Q_A^*}{4}$$

$$Q_B^* = 25 - \frac{Q_A^*}{4}$$



Cournot Equilibrium - Graph

The **Cournot Equilibrium Quantities** are where the **Best Response Functions** intersect



Where Does Cournot Place Relative to Other Market Types?

We know how to find **Monopolist Equilibriums** and **Perfectly Competitive Equilibriums** from just the **Cost Functions**

We can rank outcomes intuitively using what we know about each one:

Relative to a **Monopoly**

- Quantity should be higher and price lower because there is **more** competition in a **Cournot Duopoly**

Relative to **Perfect Competition**

- Quantity should be lower and price higher because there is relatively less competition in a **Cournot Duopoly**

Practice Makes Perfect

Let's find **Perfect Competition Equilibrium** and compare our results

$$\text{Profits: } \pi_A = P \cdot Q_A - Q_A^2 \quad \text{and} \quad \pi_B = P \cdot Q_B - Q_B^2$$

$$\text{Demand: } P = 100 - Q_D$$

Find the Perfectly Competitive Equilibrium Quantity and Price
Maximize both profits, find optimal quantities, set supply = demand, and find numbers for Quantity and Price

$$\frac{\partial \pi_A}{\partial Q_A} = 0$$
$$P - 2Q_A = 0$$
$$Q_A^* = \frac{P}{2}$$
$$Q_B^* = \frac{P}{2}$$

$$Q_S = Q_A^* + Q_B^*$$
$$Q_S = \frac{P}{2} + \frac{P}{2} = P$$

$$\text{Supply} = \text{Demand}$$
$$Q_S = 100 - Q_D$$
$$2Q = 100$$
$$Q^* = 50$$

$$P^* = 100 - Q^*$$
$$P^* = 100 - 50$$
$$P^* = 50$$

Cournot Equilibrium vs Monopoly

How does **Cournot Competition** compare to **Monopoly**?

There are now two firms so it is not a **Monopoly** by definition

But we can do some work to make it one

- Imagine that both firms **collude**. In other words, they **cooperatively agree on how much to produce**
- We call this a **Cartel**
- We can find how much they produce by "combining" the firms and maximizing profit

Cartels

Forming a Cartel

Let's stay with our two firms from earlier. Rather than competing with each other, they decide to **collude** and form a **Cartel**

The **Cartel's Profits** is simply both firm's profit functions summed up

$$\pi_{Cartel} = \pi_A + \pi_B$$

Our profits were:

$$\pi_A = P \cdot Q_A - Q_A^2 \quad \& \quad \pi_B = P \cdot Q_B - Q_B^2 \quad ; \quad P = 100 - Q_S$$

Find the Cartel's Profit function (Do not simplify it)

$$\pi_{Cartel} = \pi_A + \pi_B$$
$$\pi_{Cartel} = (100 - Q_A - Q_B) \cdot Q_A + (100 - Q_A - Q_B) \cdot Q_B - Q_A^2 - Q_B^2$$

Cartel - Maximize Profits

We maximize profits by taking the derivative with respect to both quantities (Q_A & Q_B)

$$\pi_{Cartel} = (100 - Q_A - Q_B) \cdot Q_A + (100 - Q_A - Q_B) \cdot Q_B - Q_A^2 -$$

Find the Profit Maximizing Quantities $\frac{\partial \pi_i}{\partial Q_i} = 0$

$$\frac{\partial \pi_A}{\partial Q_A} = 0$$

$$100 - 2Q_A - Q_B - Q_B - 2Q_A = 0$$
$$100 - 4Q_A - 2Q_B = 0$$
$$4Q_A = 100 - 2Q_B$$

$$Q_A^* = 25 - \frac{1}{2}Q_B$$
$$Q_B^* = 25 - \frac{1}{2}Q_A$$

Plug one into the other

$$Q_A = 25 - \frac{1}{2}\left(25 - \frac{1}{2}Q_A\right)$$
$$Q_A = 25 - \frac{25}{2} + \frac{1}{4}Q_A$$
$$\frac{3}{4}Q_A = \frac{25}{2}$$
$$Q_A^* = \frac{100}{6} = Q_B^*$$

Cartel - Maximize Profits (Part 2)

We know

$$Q_A^* = \frac{100}{6} \qquad \& \qquad Q_B^* = \frac{100}{6}$$

Now we can find the **Market Quantity and Market Price**

Find Quantity

$$Q_S = Q_A + Q_B = \frac{100}{6} + \frac{100}{6} = \frac{100}{3} \approx 16.6$$

Find Price

$$P = 100 - Q_S = 100 - \frac{100}{6} = \frac{200}{3} \approx 66.6$$

Compare this to the Cournot Equilibrium

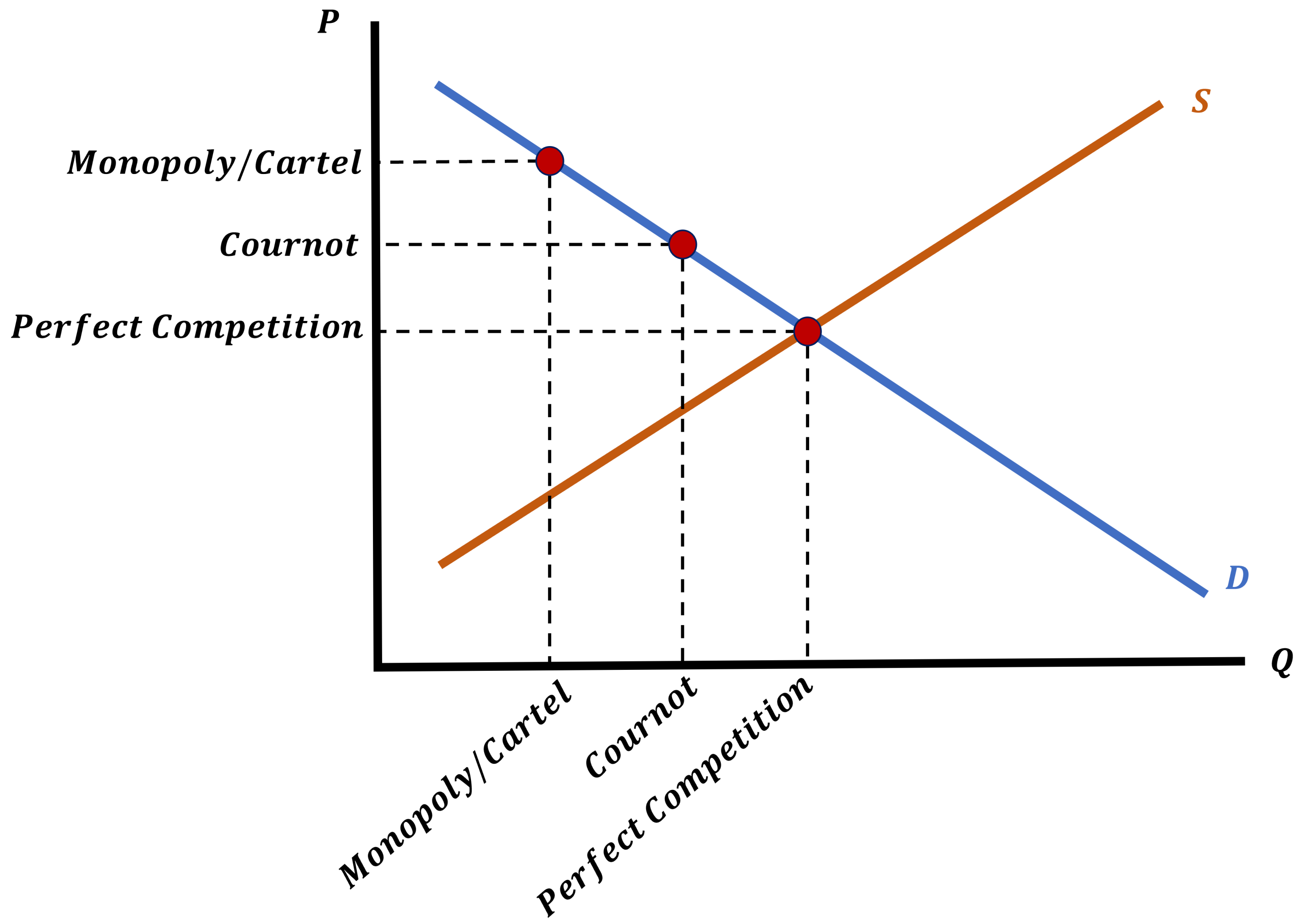
$$Q_C = 40$$

$$P_C = 60$$

Cartel Results

The **Cartel** produces less than the both **Cournot firms** combined and demands a higher price

So basically they managed to create a **Monopoly** by joining the only two firms in the industry



Cartel - Joining Both Firms

There is an exception to this approach of combining two firms when we model **Cartel Profits**

Imagine two firms with the following **Cost Functions**

$$\text{Firm A: } C(Q_A) = 10 \cdot Q_A \quad ; \quad \text{Firm B: } C(Q_B) = 5 \cdot Q_B$$

If these firms combine, should we just add their costs together?

Think about their Marginal Costs

$$\text{Firm A Marginal Costs: } MC = 10$$

$$\text{Firm B Marginal Costs: } MC = 5$$

They have different constant marignal costs

- So it will always be cheaper to produce goods using only Firm B

Cartel - The Exception

If we have two firms, both with **Constant Marginal Costs that are Different**:

- To form a **Cartel**, they will simply shut down the firm with the **Higher Marginal Costs** and the other firm will become a **Monopolist**
- If you were to find profits for both firms in the **Cartel** you would see that they are higher for both firms than in **Cournot Competition**

So why not form Cartels then?

- They are usually illegal
- And more importantly, they are not stable!

Cartels Are Unstable

Let's jump back to our gas station example

- You and your competitor have a backroom agreement to sell your gas at \$5.00/gallon
- The next day you see your "new business partner" posting a price of \$5.00/gallon, what are you tempted to do?
- Post a price of \$4.99/gallon!

In other words, if you follow through with your **Cartel Agreement**

It is not your best response for you to follow through also!

Cartels - Showing Instability

Jumping back to our **Courtot/Cartel** example

The firms split $Q = 100/3$ equally, so $Q_B = 100/6$

What is Firm A's **Best Response** to $Q_B = 100/6$

From the **Cournot Best Response Functions** earlier we have:

$$Q_A^* = \frac{100 - Q_B}{4} = 25 - \frac{Q_B}{4}$$

If we plug in Q_B we get:

$$Q_A^* = 25 - \frac{Q_B}{4}$$

$$Q_A^* = 25 - \frac{\frac{100}{6}}{4}$$

$$Q_A^* = 25 - \frac{25}{6}$$

$$Q_A^* \approx 21 > \frac{100}{6}$$

Cartel - Instability

Each firm is tempted to deviate from their **Cartel Agreement** and produce more than they agreed to

The firm that deviates will increase their profits

The firm that sticks to the agreement will have lower profits

If both firms deviate from the **Cartel Agreement**, then both of them end up with lower profits

Timing Matters

All previous models of quantity competition have firms decide their **Best Responses** at the same time, much like playing rock-paper-scissors

We call these **Simultaneous Games**

Now we will look at a model that introduces timing

Stackelberg Competition

Competing by Quantity & Timing

Imagine a Farmer's Market. Here you have a stand that sells flowers

- The competing flower stand next to you always shows up 30 minutes before you do
- By the time you arrive, they have already got their flower displays set up
  - Their choice can vary on whether they display a large amount of flowers (which depresses market price) or put up a small display (which keeps market price high)

This dynamic introduces timing

Stackelberg Competition - Structure

We remain with only two firms

- We now have a **“Leader”** and a **“Follower”**
- We call this a **Sequential Game**
- To solve a **Sequential Game**, we first figure out how the **Follower** will behave
- We use that information to figure out what the **Leader** does

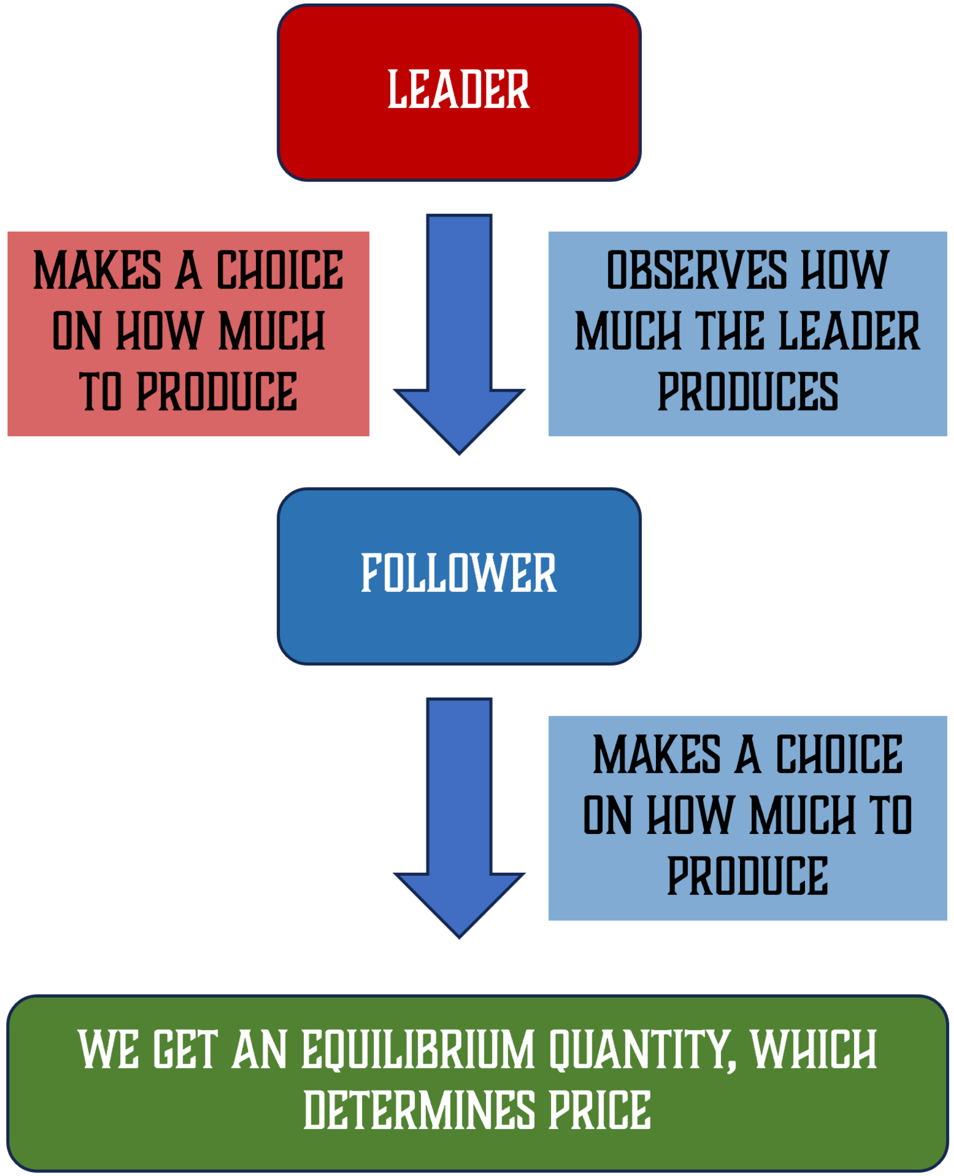
Game Theory Concept - Backwards Induction

This process of working backwards is called **Backwards Induction**

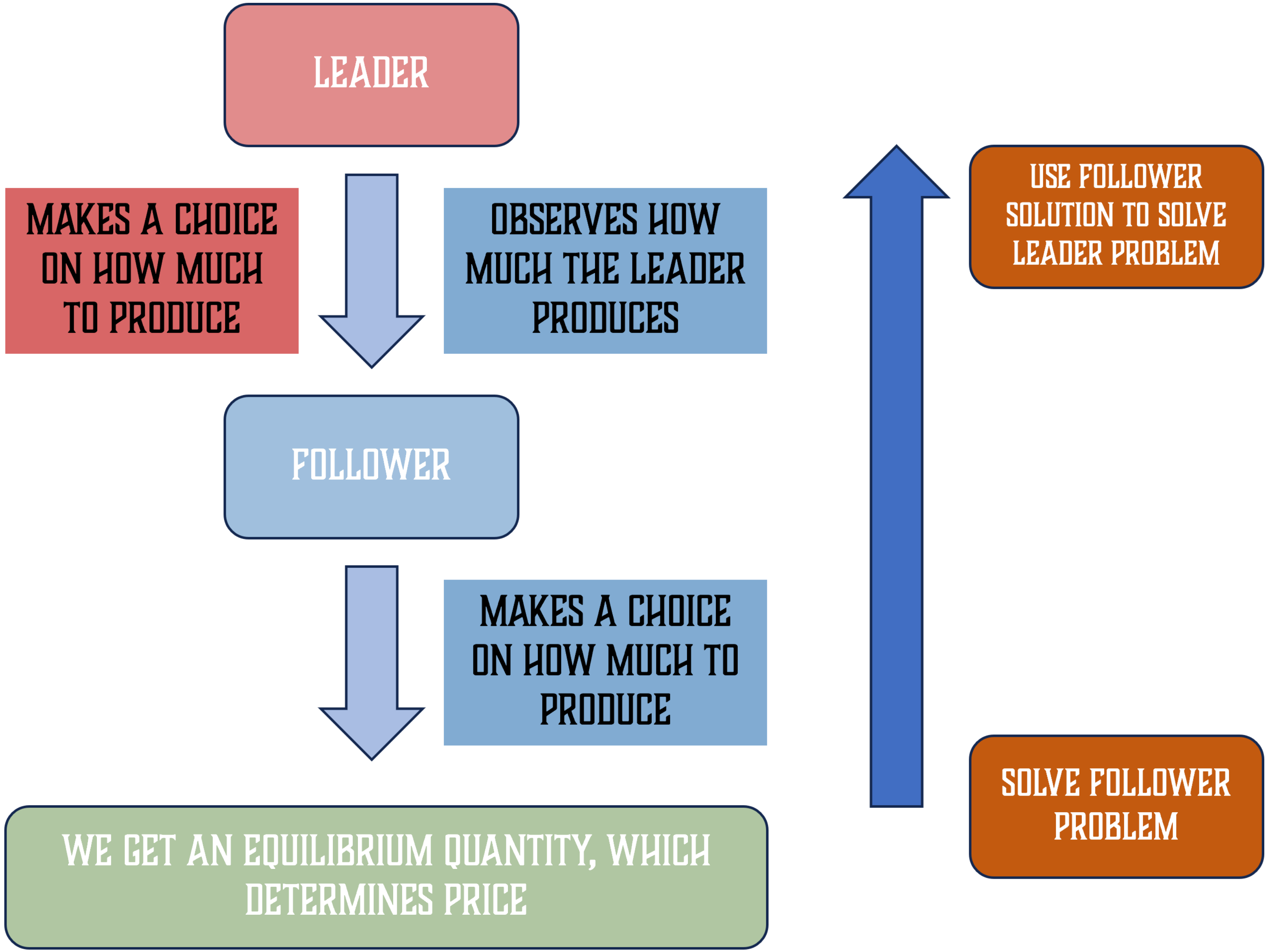
But why solve the Stackelberg Model this way?

- When the **Leader** is considering how much quantity to produce, they are trying to predict how the **Follower** will respond
- The **Follower** is just reacting to an observed choice
- So we start by figuring out how a **Follower** reacts to any choice and then let the **Leader** make a choice in anticipation of this reaction

Backwards Induction in Stackelberg



Backwards Induction in Stackelberg



The Follower

We have looked at **Best Response Functions** before

- **They are the firm's optimal choice of quantity given their competitor's chosen quantity. This is how the follower will behave**

The Leader

If the **Follower** just reacts to what the **Leader** does, then what does the **Leader** do?

- They will maximize their profit with **full knowledge of the follower's Best Response Function**
  - We assume they know it exactly. There are no tricks the **Follower** can pull
- They behave similar to a Monopoly, they will plug in **this knowledge** into their profit function before maximizing

Stackelberg Competition - Example

We will grab the same two firms from before, where we already know their **Best Response Functions**

Let's also say that **Firm B is the Follower** and **Firm A is the Leader**

$$\text{Firm B Best Response: } Q_B^* = \frac{100 - Q_A}{4} \quad ; \quad \pi_A = (100 - Q_A - Q_B)Q_A - Q_A^2$$

Find Firm A's Profit Function

$$\pi_A = \left(100 - Q_A - \frac{100 - Q_A}{4}\right) Q_A - Q_A^2$$

Stackelber - Finding Profit Maximizing Quantity for the Leader

$$\pi_A = \left(100 - Q_A - \frac{100 - Q_A}{4}\right) Q_A - Q_A^2$$

Find $\frac{\partial \pi_A}{\partial Q_A} = 0$

$$100 - 2Q_A - 25 + \frac{Q_A}{2} - 2Q_A = 0$$

$$75 - \frac{3}{2}Q_A = 2Q_A$$

$$75 = \frac{7}{2}Q_A$$

$$150 = 7Q_A$$

$$Q_A^* \approx 21.5$$

Compare this to what we found in Cournot Competition

We had found that $Q_A = 20$

So we have found that the **Leader** will **produce more** than they would have under **Cournot Competition**

Stackelberg - Finding Profit Maximizing Quantity for the Follower

We said that the **Follower** will respond using their **Best Response Function**

$$Q_B^* = \frac{100 - Q_A^*}{4}$$

And we just found that $Q_A^* \approx 21.5$

Find the Follower's Profit Maximizing Quantity

$$Q_B^* = \frac{100 - 21.5}{4}$$

$$Q_B^* = \frac{78.5}{4}$$

$$Q_B^* \approx 19.6$$

Compare this to what we found in Cournot Competition

We had found that $Q_B = 20$

So we have found that the **Follower** will **produce less** than they would have under **Cournot Competition**

Stackelberg - Outcomes

There is a predictable outcome, we can say that relative to the Cournot Competition:

- The **Leader** will earn **higher profits** because they **produce more**
- The **Follower** will earn **lower profits** because they **produce less**

This leads us to the conclusion that

Stackelberg Competition features a First-Mover Advantage

Being able to go first implies that you can set a higher quantity produced and crowd-out your competition from the market

Competition Up To Now

All of these previous forms of competition have modeled firms competing **on choosing quantities**

- Where the sum of the quantities produced determines the **Market Quantity** which determines the **Market Price**

But what if firms can **set prices before they have to produce anything?**

- What if firms can compete based on **their choice of Price**?
- We call these **Bertrand Competition**

Bertrand Competition

Competing by Prices

Forget quantities

In **Bertrand Competition** firms will compete by setting prices **before they begin production**

- Imagine that the competing firms are **bidding on quantities to be delivered later**
    - Think of like an auction with bids for work being submitted
- This changes the nature of competition substantially
    - The firm that announces a **Lower Price** gets 100% of the market

Back to the Gas Station

The story we have been telling about the gas station is a perfect example of this

Instead of you running one, let's just call them **Gas Station A** and **Gas Station B**

- If **Gas Station A** advertises a price of $4.00/gallon, what should the manager of **Gas Station B** do?
  - Advertise a price of $3.99/gallon
- But **Gas Station A** knows this, so how should they respond?
  - Advertise a price of $3.98/gallon
- And the cycle continues

Is Bertrand a Race to the Bottom?

When duopolies compete through **price-bidding**, both firms have an incentive to slightly undercut one another

But how low will the price go?

- The Zero-Profit condition tells us
  - If the **Price** ever falls below the MC of the winning firm, they win **but would not want to fulfill it**

The Price War continues until P = MC

Bertrand Competition - Price Wars

A **Firm's Limit is P = MC**

Under **Bertrand Competition**, firms that compete will both set price equal to their respective **Marginal Cost**

- This is the **Nash Equilibrium**

This gives us two possibilities

One Firm has a lower MC

The firm with the lower Marginal Costs will obtain the entire market by setting a price just lower than their competitor's MC

Both Firms have the same MC

We will see both firms produce in the market and we will get **Price, Quantity and Profit** the same as under **Perfect Competition**

How Does This Work?

It is the proverbial outrun your friend not the bear

If both Firms have different marginal costs, the most efficient will undercut their competition

There is **No Need** for the winner to go any lower than that:

- Recall they also want to make as much profit as possible

Bertrand Competition Example

Let's look at an example that illustrate the previous slide

Let there be two firms in the market. Firm A has cost function $C(Q_A) = 20Q_A$ and Firm B has cost function $(C(Q_B) = Q_B^2)$. Demand in the industry is given by the Demand Curve $P = 120 - Q$

We see that **Firm A** will be able to price **Firm B** out of the market.

If Firm A would like to supply 40 units by themselves, what price should they set? What are profits at that point?

Bertrand Competition Example

If Firm A would like to supply 40 units by themselves, what price should they set? What are profits at that point?

$$\text{From the Demand Curve: } P = 120 - Q = 120 - 40 = 80$$

Finding Price to Set

Firm B has $MC = 2Q_B$

They are willing to to supply 40 units at:

$$MC(Q_B) = 2Q_B$$
$$MC(40) = 2(40) = 80$$

Use fact that P = MC

$$P = MC(40) = 80$$

Firm A's Choice

Firm A knows **Firm B** can only lower their price to $80.

Firm A can set Price equal to $79.99 and **Firm B** becomes unwilling to supply 40 units due to $P < MC$

$$\pi_A = 79.99 \cdot 40 - 40 \cdot 20 \approx 2400$$

Duopoly Example

For duopolies, we are going to take a step that will simplify our lives

We are going to introduce **Cosntant Marginal Cost (Linear) Functions**

Let's practice what we have learned!

Let there be two firms (A & B) with the following **Cost Functions** and facing the **Demand Curve**

$$P = 64 - Q \quad ; \quad C(Q_B) = 10Q_B \;\; ; \;\; C(Q_B) = 4Q_B$$

Duopoly Example - Cournot Competition

$$P = 64 - Q \quad ; \quad C(Q_B) = 10Q_B \;\; ; \;\; C(Q_B) = 4Q_B$$

Find the Best Response Functions for both firms

Recall you find this by finding the profit maximizing quantity

$$\pi_B = P \cdot Q_B - C(Q_B)$$
$$\pi_B = (64 - Q_B - Q_B)Q_B - 10Q_B$$

Find $\frac{\partial \pi_A}{\partial Q_A}$

$$\frac{\partial \pi_A}{\partial Q_A} = 0$$
$$64 - 2Q_A - Q_B - 10 = 0$$
$$2Q_A = 54 - Q_B$$
$$Q_A^* = 27 - \frac{Q_B}{2}$$

$$\pi_B = P \cdot Q_B - C(Q_B)$$
$$\pi_B = (64 - Q_A - Q_B)Q_B - 4Q_B$$

Find $\frac{\partial \pi_B}{\partial Q_B}$

$$\frac{\partial \pi_B}{\partial Q_B} = 0$$
$$64 - Q_A - 2Q_B - 4 = 0$$
$$2Q_B = 60 - Q_A$$
$$Q_B^* = 30 - \frac{Q_A}{2}$$

Duopoly Example - Cournot Competition

$$Q_A^* = 27 - \frac{Q_B}{2} \qquad \& \qquad Q_B^* = 30 - \frac{Q_A}{2}$$

Solve for Q_A^* and Q_B^*

$$Q_A = 27 - \frac{Q_B}{2}$$

$$Q_A = 27 - \left(\frac{30 - \frac{Q_A}{2}}{2}\right)$$

$$2Q_A = 54 - 30 + \frac{Q_A}{2}$$

$$\frac{3}{2}Q_A = 24$$

$$Q_A^* = 16$$

$$Q_B^* = 30 - \frac{Q_A}{2}$$

$$Q_B^* = 30 - \frac{16}{2}$$

$$Q_B^* = 30 - 8$$

$$Q_B^* = 22$$

Duopoly Example - Cournot Competition

$$Q_A^* = 16 \quad \& \quad Q_B^* = 22$$

Find Market Quantity and Price

Market Quantity

$$Q_S = Q_A + Q_B$$
$$Q_S = 16 + 22$$
$$Q_S = 38$$

Market Price

$$P = 64 - Q_S$$
$$P = 64 - 38$$
$$P = 26$$

Duopoly Example - Cournot Competition

$$P = 26 \quad \& \quad Q_A = 16 \quad \& \quad Q_B = 22$$
$$C(Q_A) = 10 \cdot Q_A \quad \& \quad C(Q_B) = 4 \cdot Q_B$$

Find Profits for Each Firm

$$\pi_A = P \cdot Q_A - C(Q_A)$$
$$\pi_A = 26 \cdot 16 - 10 \cdot 16$$
$$\pi_A = 256$$

$$\pi_B = P \cdot Q_B - C(Q_B)$$
$$\pi_B = 26 \cdot 22 - 4 \cdot 22$$
$$\pi_B = 484$$

Duopoly Example - Stackelberg Competition

Let's now say that **Firm A** is the **Leader**

We know that the **Follower's Best Response Function** is given by

$$Q_B = 30 - \frac{Q_A}{2}$$

If **Firm A's Profits** are

$$\pi_A = (64 - Q_A - Q_B)Q_A - 10Q_A$$

Find their profit function in Stackelberg Competition where they are the Leader

$$\pi_A = (64 - Q_A - Q_B)Q_A - 10Q_A$$
$$\pi_A = (64 - Q_A - (30 - \frac{Q_A}{2}))Q_A - 10Q_A$$
$$\pi_A = (34 - Q_A + \frac{Q_A}{2})Q_A - 10 \cdot Q_A$$
$$\pi_A = 34Q_A - Q_A^2 + \frac{Q_A^2}{2} - 10 \cdot Q_A$$

Duopoly Example - Stackelberg Competition

$$\pi_A = 34Q_A - Q_A^2 + \frac{Q_A^2}{2} - 10 \cdot Q_A \qquad \& \qquad Q_B = 30 - \frac{Q_A}{2}$$

Find Profit Maximizing Quantity for Both Firms

$$\frac{\partial \pi_A}{\partial Q_A} = 0$$

$$34 - 2Q_A + Q_A - 10 = 0$$

$$Q_A^* = 24$$

$$Q_B^* = 30 - \frac{Q_A}{2}$$

$$Q_B^* = 30 - \frac{24}{2}$$

$$Q_B^* = 30 - 12$$

$$Q_B^* = 18$$

Duopoly Example - Stackelberg Competition

$$Q_A^* = 24 \quad \& \quad Q_B^* = 18$$
$$C(Q_A) = 10Q_A \quad \& \quad C(Q_B) = 4Q_A$$
$$P = 64 - Q_S$$

Find Market Supply & Market Price

Market Supply

$$Q_S = Q_A + Q_B$$
$$Q_S = 24 + 18$$
$$Q_S = 42$$

Market Price

$$P = 64 - Q_S$$
$$P = 64 - 42$$
$$P = 22$$

Duopoly Example - Stackelberg Competition

Last thing to do is find each firm's profits

$$Q_A^* = 24 \quad \& \quad Q_B^* = 18$$
$$C(Q_A) = 10Q_A \quad \& \quad C(Q_B) = 4Q_A$$
$$P = 22$$

Firm Profits

$$\pi_A = 22 \cdot 24 - 10 \cdot 24 = 288$$
$$\pi_B = 22 \cdot 18 - 4 \cdot 18 = 324$$

Summary of Models

All of the models we have seen are

- Perfect Competition (Price Taking)
- Monopoly (Determines Price Through Quantity Choice - 1 Firm)
- Cournot (Simultaneous Competition By Setting Quantity)
- Cartels (2 Firms unite and behave like a Monopoly)
- Stackelberg (Sequential Competition By Setting Quantity)
- Bertrand (Competing By Setting Price)

Consumer Perception of Markets

We can think how these markets affect consumers

- Bertrand and Perfect Competition yield the same outcome and are the **best for consumers**
  - Lowest price, highest quantity
- Monopoly and Cartel are the **worst for consumers**
  - Highest price, lowest quantity
- Cournot and Stackelberg are somewhere in-between
  - Stackelberg has slightly better outcomes for consumers

Long-Run in Markets

We care about the **Long-Run Equilibriums**

- Firms will only make **Zero-Profits** in the **Long-Run Equilibrium** under **Perfect Competition**
  - We saw that **Bertrand** can also produce Zero-Profits under the correct conditions
- In all forms of **Imperfect Competition**, firms manage to prevent entry of other firms in the **Long-Run**

C'est Fini