## Consumer Behavior

EC 311 - Intermediate Microeconomics

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2024

## **Chapter 4**

#### **Outline**

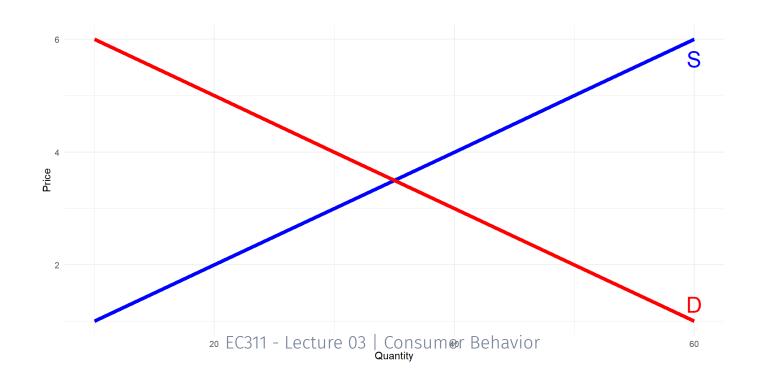
- Topics
  - Preferences and Utility (4.1)
  - Indifference Curves (4.2)
  - Budget Constraints (4.3)
- Other Stuff
  - Remember Office Hours on Thursdays from 12:00 14:00 in PLC 523
  - Also feel free to email me your questions at jrojas2@gmail.com

# Preferences and Utility

## Where Does Demand Come From?

Imagine the market for coffee on campus

- Additionally, picture that the only other good that can be purchased is tea
- On a basic level, **demand for coffee** is derived from individual's choosing how to divide their income between coffee and tea



## Where Does Demand Come From?

This is the problem we will be dealing with through the first half of the course

How does an individual allocate a **finite amount of resources** between **two goods**?

**Note:** There are clearly more than just two goods out there, so how can this be useful?

The main critique I always hear about economics courses is that they're unrealistic. That's mostly true, but we can learn about the aggregate by simplifying and making assumptions

## Where Does Demand Come From?

- We will frame the decisions as a two-good model where you may choose between:
  - Food and durable consumption
  - Leisure (not working) and consumption (paid for by earning a wage)
  - Consumption now and consumption later

The key takeaway here is that we can frame many important choices as "two-good" decisions

• This makes things simpler for us to solve while still maintaining some sense of the real world

## **Determinants of Consumption**

Consumption of any single good has two parts:

- How it **BENEFITS** the consumer
  - We call this **UTILITY**
- What it **COSTS** the consumer
  - What we give up to purchase the good

Let's see what this means through a 1-good example  $\rightarrow$  Beer

- Imagine the following scenario:
  - You just arrived at the bar and have had zero drinks so far
  - Each beer costs the same: \$4

## Beer

<b>Number of Beers</b>	<b>Overall Level of Happiness</b>	<b>Change in Level of Happiness</b>
0	0	-
1	10	10
2	25	15
3	35	10
4	40	5
5	42	3
6	30	-12

If beer were free, how many beers should this person drink?

Now recall that beer costs \$4, how many beers should this person drink?

5

4

## Intuition Behind "Choice" in Economics

You **cannot** simply find the consumption amount that makes you the happiest. **But why?** 

The goal is to maximize your **utility** whilst acknowledging you have **constraints** 

The choice is simple: consume an additional unit until the cost of doing so outweighs the benefit

Commit this idea to memory: it is the crux of economics and drives everything we will be doing

We maximize utility up to the point that it does not make sense to do so

## Back to Beer

<b>Number of Beers</b>	<b>Overall Level of Happiness</b>	<b>Change in Level of Happiness</b>
0	0	-
1	10	10
2	25	15
3	35	10
4	40	5
5	42	3
6	30	-12

#### Some Questions:

- What is the marginal benefit:
  - When you have not consumed any beer?
  - When you have already consumed 3 beers?
  - When you have already consumed 5 beers?

- What is the marginal cost of beer?
  - Does it change as we consume more?

# Being and Thinking at the Margin

We found two important values:

Marginal Benefit (MB)

Marginal Cost (MC)

The additional benefit gained for an added unit of consumption

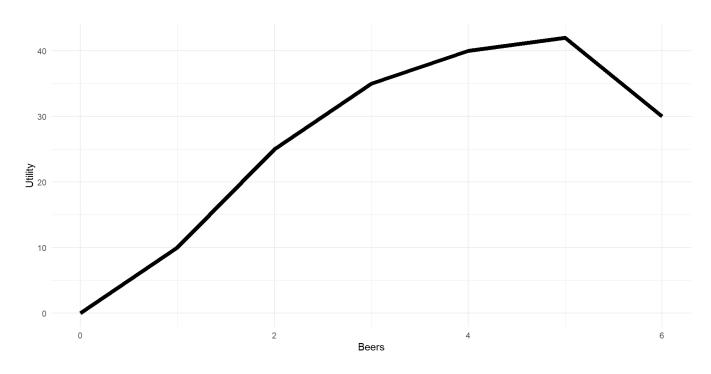
The additional cost paid for an added unit of consumption

We can describe the decision-making process in a more formal manner:

- Initially: MB > MC → Consume more!
- Eventually:  $MB < MC \rightarrow We went too far!$
- At some point:  $MB = MC \rightarrow Just right!$

Ask yourself: Why must they be equal?

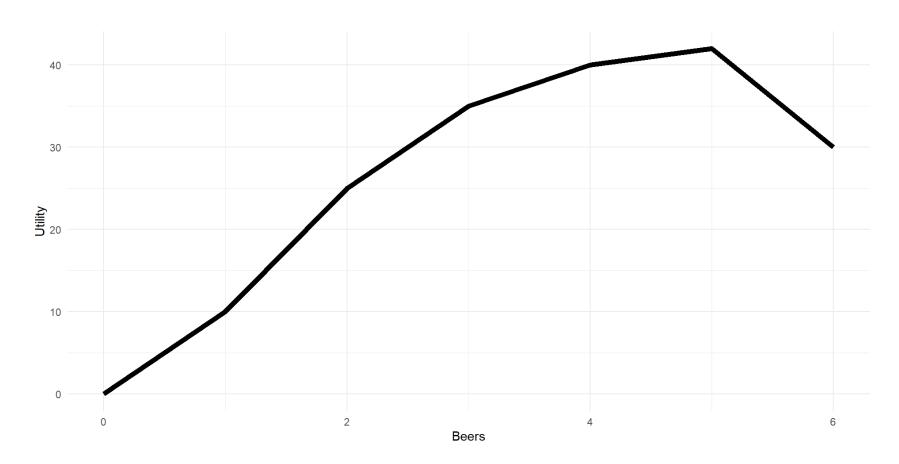
## What's Going On? Graphically



What matters for choice is the **marginal benefit** of an additional beer In other words, what matters is the change in **utility** that occurs as we move to the right on the graph

Recall: A change in y as x increases is the derivative

## What's Going On? Graphically - Derivatives

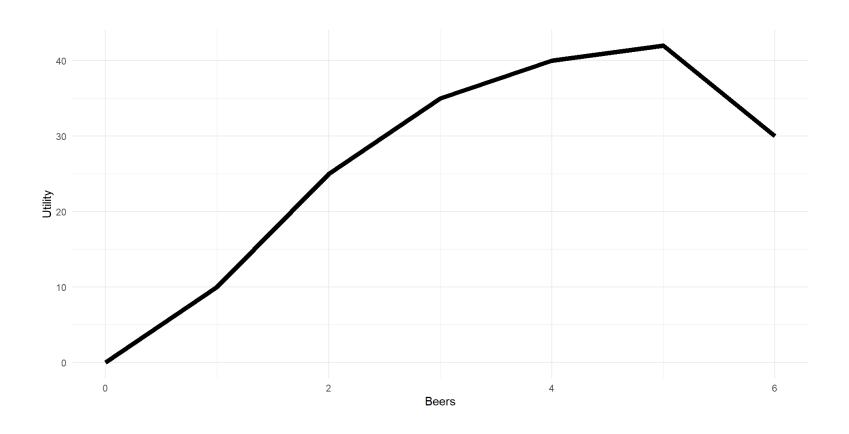


The derivative of this function is the marginal benefit of beer

#### This is why derivatives are important

We can use derivatives to figure out the optimal amount to consume

## What's Going On? Graphically - MB & MC



**Recall:** The optimal choice is the point where MB = MC

The MC = 4. So we would choose the quantity of beers where MB = 4

# What's Going On? Mathematically

The utility function of beer we've been using is:

$$U(x) = -x^2 + 12x$$

**Let's practice:** What would the optimal consumption amount be if the cost of beer is \$2?

## A Two-Good Problem

The beer example was fairly straightforward. But we will be dealing with making choices between two goods.

Before we dive in, a couple of things to consider are:

- When we spend our resources on one good, it cannot be spent on something else
  - Because we are making decisions amongst both things, we need a
     cost relationship between them
- We do not measure our happiness (utility) in dollars
  - We need to find a way to choose without directly comparing costs and benefits across goods

## **Enter Utility Functions**

A **Utility Function** is a function of two variables:

$$U = f(x, y)$$

#### Some facts:

- ullet x and y are the (positive) amounts of goods you consume
- ullet The function converts x and y to happiness (utility) from consuming the two goods

#### For Example:

• Let's say that the utility I receive from **Beer (B)** and **Soda (S)** can be modeled as:

$$U = f(B, S) = 10B + 2S$$

- Before, we were only interested in the slope of Beer
- Now, we are interested in the slopes of both Beer & Soda

# Utility Functions - Beer & Soda

$$U = f(B, S) = 10B + 2S$$

For a function of two variables there are **two slopes** 

- One for each partial derivative which we will call Marginal Utilities
- In the **Beer (B)** and **Soda (S)** example we will have:
  - The Marginal Utility of Beer ( $MU_B$ )
  - ullet The Marginal Utility of Soda ( $MU_S$ )

What are the Marginal Utilities of Beer and Soda?

## Utility Functions - PB&J

Now find the Marginal Utilities for Peanut Butter and Jelly

$$U = f(P, J) = P^2 \cdot J$$

## What do Utility Functions even Mean?

They help us represent how people feel about goods x and y. There are certain properties that help us determine:

- Do I like or dislike x?
- Does how much I like  $oldsymbol{x}$  depend on how much  $oldsymbol{x}$  I already have?
- Does how much I like  $oldsymbol{x}$  depend on how much  $oldsymbol{y}$  I have?

We can understand these properties by looking at the marginal utilities!

## Utility Functions - Do I Like or Dislike x?

### If I consume more x, how does my utility move?

Beyond graphing the utility function, we need to find a way to answer this formally and mathematically

We can look at the sign of the derivative

ullet If the derivative of U with respect to (w.r.t.) x is:

### **Positive**

I like x

## **Negative**

I dislike  $oldsymbol{x}$ 

## Like or Dislike - Example

Given my utility curve for **Beer (B)** and **Soda (S)**, what do I like or dislike?

$$U = f(B, S) = 10B + 2S$$

I like **Beer** 

$$MU_B = 10 > 0$$

I like Soda

$$MU_{S} = 2 > 0$$

# Does How Much I like $\boldsymbol{x}$ Depend on How Much I Already Have?

#### Remember to think at the margin

Is each additional unit of x bring me **more**, **less**, or **equal** happiness as the previous unit?

This is slightly trickier to figure out, but we still use marginal utility logic

- In fact, we will use what is called the Second Derivative
- ullet Mathematically, this is the **derivative** of  $MU_x$  w.r.t. x and we ask:
  - Is this second derivative positive, negative, or zero?

# Depend on How Much I Already Have - Example

My utility for Cookies and Milk:

$$U=f(C,M)=C^{1/2}M^{1/2}$$

#### **Cookies**

$$egin{align} MU_c &= rac{1}{2} \cdot C^{-1/2} \cdot M^{1/2} \ MU_{cc} &= rac{-1}{2} \cdot rac{1}{2} \cdot C^{-1/2-1} \cdot M^{1/2} \ MU_{cc} &= rac{-1}{4} \cdot rac{1}{C^{3/2}} \cdot M^{1/2} \ MU_{cc} &= rac{-M^{1/2}}{4C^{3/2}} \ \end{align}$$

#### Milk

$$U_{mm} = rac{-C^{1/2}}{4M^{3/2}}$$

 $MU_{cc}$  is **negative** so we can say that **Cookies** have a Decreasing Marginal Utility

# Does How Much I Like $\boldsymbol{x}$ Depend on How Much $\boldsymbol{y}$ I Have?

This one is more straightforward: Does the marginal utility of x depend on y?

- ullet Mathematically, we take the derivative of  $MU_x$  w.r.t. to  $\emph{y}$ , and viceversa.
  - This is called the cross-partial derivative
- ullet Notationally, we have:  $MU_{xy}$

Where we can determine the order of derivatives by looking at the subscript:

ullet x is first, and y is the second derivative

# Depend on How Much of the Other Good I Have - Example

How about this utility for Peanut Butter and Jelly

$$U = f(P, J) = P^2 \cdot J$$

#### **Peanut Butter**

**Jelly** 

$$MU_P = 2P^{2-1} \cdot J = 2PJ \qquad \qquad MU_J = P^2 \cdot J^{1-1} = P^2 \ MU_{PJ} = 2P \cdot J^{1-1} = 2P \qquad \qquad MU_{JP} = P^2 = 2P$$

Notice that the cross-partials are the same and this will always be the case for any utility function!

## Meaning of a Utility Function

What are Utility Functions?

- They are a flexible tool that help us describe the relationship between two godos and the utility (happiness) you gain from them
- They allow us to get a good intuition of how we can change function properties so they relate to the choice we are attempting to model
- Let's think about some goods and decide what the utility function should look like

## Modeling with Utility Functions

#### Let's consider **Homework** and **Pizza**

- First, we decide whether the good is desirable **(good)** or undesirable **(bad)** 
  - I'll make the bold assumption that Homework is a bad and that
     Pizza is a good

Note: This implies that the marginal utilities are  $MU_{H}<0$  and  $MU_{P}>0$ 

# Modeling with Utility Functions - Example

Homework is a bad and Pizza is a good

Let's also set the following requirements:

The marginal disutility of homework is larger when I have more of it

 $MU_H$  is decreasing in H
ightarrow We need an H in  $MU_H$ 

The marginal utility of pizza is smaller when I have more of it

 $MU_P$  becomes smaller as I have more  $P 
ightarrow ext{We need an } H$  in  $MU_H$ 

 $MU_P$  does **not** depend on **homework**  $MU_P$  does not have an H

Attempt creating a utility function with the above characteristics

## Modeling our Homework and Pizza

Here's my version:

$$U(H,P) = -H^2 + ln(P)$$

Now let's prove that it meets the requirements

**Homework** is a **bad** and must be worse the more I have of interested

$$MU_H = -2H < 0$$

$$MU_{HH}=-2<0$$

**Pizza** is a **good**, I get less joy from it the more I have, and it does not depend on how much **homework** I have

$$MU_P = \frac{1}{P} > 0$$

$$MU_{PP}=rac{-1}{P^2}<0$$

$$MU_{PH} = 0 = MU_{HP}$$

# Meaning of a Utility function

The single most important property of a utility function is that we can measure **the relative preference for one good over the other** 

- It measures how many units of y would you give up to get one more unit of x?
  - We call this the Marginal Rate of Substitution (MRS)

$$MRS = rac{MU_x}{MU_y}$$

# Marginal Rate of Substition (MRS)

Here we are talking about the relative preference of x over y, but how?

- ullet Consider U=f(x,y)=4x+2y
  - lacktriangle You get 4 units of utility for each x 
    ightarrow 4
  - lacktriangle You get 2 units of utility for each y 
    ightarrow 2
  - lacktriangle We can say that each x is twice as valuable as each y

Using our MRS formula we have:

$$MRS = rac{MU_x}{MU_y} = rac{4}{2} = 2$$

# Types of Utility Functions

In Economics, we mainly deal with 4 typess of functions, each with its set of properties and tricks

- Cobb-Douglas
- Quasi-linear
- Perfect Substitutes
- Perfect Complements

# Cobb-Douglas

$$U(x,y) = x^a y^b$$

#### Find the MRS of this general function

$$MRS = rac{MU_x}{MU_y} = rac{ax^{a-1}y^b}{bx^ay^{b-1}} = rac{a}{b} \cdot rac{x^{a-1-a}}{y^{b-1-b}} = rac{a}{b} \cdot rac{x^{-1}}{y^{-1}} = rac{a}{b} \cdot rac{y}{x}$$

#### The MRS for a Cobb-Douglas will always look like

$$\frac{a}{b} \cdot \frac{y}{x}$$

## Cobb-Douglas: Keys to Remember

$$MRS = \frac{a}{b} \cdot \frac{y}{x}$$

- ullet The MRS is a ratio of y to x, multiplied by a constant
- ullet MRS is your **willingness to trade** y **for** x
- ullet As you get more x, the MRS goes down
- ullet As you get more y, the MRS goes up

## Cobb-Douglas - Example

$$U(x,y)=x^3y^{1/2}$$

### Find the MRS of this utility function

$$MRS = rac{MU_x}{MU_y} = rac{3x^2y^{1/2}}{1/2x^3y^{-1/2}} = rac{3}{1/2} \cdot rac{y^{1/2}y^{1/2}}{x^3x^{-2}} = 6 \cdot rac{y}{x}$$

If we recall that  $MRS = rac{a}{b} \cdot rac{y}{x}$  then we can take a shortcut:

$$MRS = rac{a}{b} \cdot rac{y}{x} 
ightarrow rac{3}{1/2} \cdot rac{y}{x} = 6 \cdot rac{y}{x}$$

### Quasi-Linear

$$U(x,y) = a \cdot ln(x) + b \cdot y$$

Where  $a \cdot ln(x)$  is the "quasi" part and  $b \cdot y$  is the "linear" part

#### Find the MRS of this general function

$$MRS = rac{MU_x}{MU_y} = rac{a/x}{b} = rac{a}{b} \cdot rac{1}{x}$$

# Quasi-Linear: Keys to Remember

$$MRS = \frac{a}{b} \cdot \frac{1}{x}$$

- ullet The MRS is a constant times 1/x
- ullet As you get more x, the MRS decreases
- ullet As you get more y, the MRS remains the same

# Quasi-Linear - Example

$$U(x,y) = 1/3 \cdot ln(x) + y$$

#### Find the MRS of this utility function

$$MRS = rac{MU_x}{MU_y} = rac{1/3\cdot 1/x}{1} = rac{1}{3}\cdot rac{1}{x}$$

Using our shortcut we get:

$$MRS = \frac{a}{b} \cdot \frac{1}{x} = \frac{1/3}{1} \cdot \frac{1}{x} = \frac{1}{3} \cdot \frac{1}{x}$$

### Perfect Substitutes

$$U(x,y) = a \cdot x + b \cdot y$$

#### Find the MRS of this general function

$$MRS = rac{MU_x}{MU_y} = rac{a}{b}$$

### Keys to Remember

MRS is a constant

# Perfect Substitutes - Example

$$U(x,y)=6x+rac{1}{2}y$$

#### Find the MRS of this utility function

$$MRS=rac{MU_x}{MU_y}=rac{6}{1/2}=12$$

And our shortcut shows:

$$MRS = \frac{6}{1/2} = 12$$

### Perfect Complements

$$U(x,y) = \min\{a \cdot x, b \cdot y\}$$

This utility function requires a different form of intuition Let's first think of a simple example:

Imagine we are trying to make some hot chocolate which requires 1 pack of chocolate powder and 12 oz of milk

$$U(x,y) = \min\{1 \, choco, 12 oz \, milk\}$$

You check your kitchen and find that there are 3 packs of chocolate powder and you have 15 oz of milk in your fridge

#### How many hot chocolates can we make?

# Perfect Complements Intuition

These are goods that **have to be consumed together in an exact proportion** in order to produce any utility

There is no MRS, so we use a property called the **No-Waste Condition**:

$$U(x,y) = \min\{a\cdot x, b\cdot y\}$$
  $a\cdot x = b\cdot y$ 

# Perfect Complements: Keys to Remember

- MRS is not defined (We cannot take a derivative)
- We can use another form of logic called **No-Waste Condition**:
  - ullet When  $a \cdot x < b \cdot y$  you will give up any y you can to get more x
  - lacktriangle When  $a \cdot x < b \cdot y$  you will give up any x you can to get y

# Perfect Complements - Example

$$U(x,y) = \min\{\frac{x}{2}, \frac{y}{4}\}$$

#### Find the No-Waste Condition of this utility function

No-Waste Condition is 
$$rac{x}{2}=rac{y}{4}
ightarrow 4x=2y
ightarrow 2x=2$$

We can say:

- ullet If 2x>y o Too much x, so we would trade some x for some y
- ullet If  $2x < y \; o \;$  Too much y, so we would trade some y for some x

### Bundles

When you have a utility function of two goods, any given combination of those two goods is called a **Bundle** 

Every bundle has an associated utility level

Take the following utility function and bundles

$$U(x,y) = x^2 y$$

Bundle 1 is (3,2)

[Math Processing Error]

Bundle 2 is (1,0)

### **Bundles & Axioms of Preferences**

We will use the following axioms about preferences between bundles to ensure logical consistency

- Completeness
- "More is Better"
- Transitivity

These come from economic theory so they will help you think like an economist but do not think this tells us how people behave

We are attempting to successfully predict behavior, so we will simply assume that everyone behaves accordingly

### Completeness

We say that preferences are always **complete**. So when comparing bundles A and B, you can always say:

- ullet I prefer A to B  $(A\succeq B)$
- I prefer B to A  $(A \succeq B)$
- ullet I am indifferent between A and B  $(A\sim B)$

This allows us to compare and order any pair of bundles

### "More is Better"

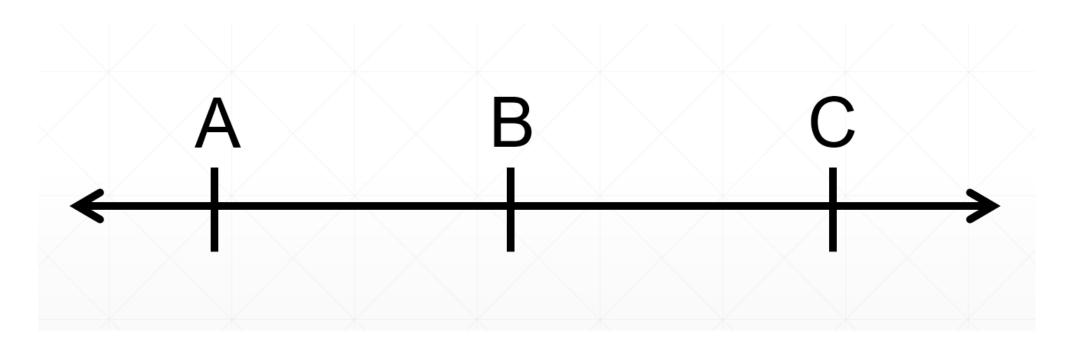
Or at least more is no worse than less In general, if a good is desirable we will want more of it

 However, sometimes products can be bads (instead of goods) and we would, obviously, want less of those

# **Transitivity**

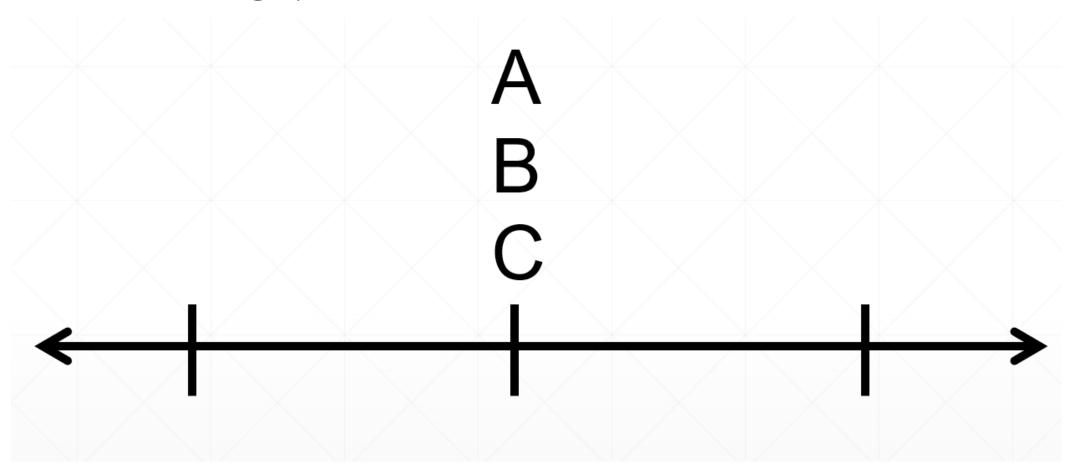
Preferences are transitive. This means that when comparing bundles A, B, and C you can get logical orderings through rankings:

• If you prefer B to A  $(B \succeq A)$  and C to B  $(C \succeq B)$  then you must prefer C to A  $(C \succeq A)$ 



# **Transitivity**

What would this graph mean?



All bundles are preferred equally!

# **Preferences Assumptions**

All of these are necessary to understand utility funcitons and be able to graph them

To do so we first need to learn about **Indifference Curves** 

# Indifference Curves

### What is Indifference?

It is exactly what it sounds like between two bundles

 Any two bundles that give the same utility level makes you indifferent between those two Bundles

For example, for the utility of U(x,y)=2x+3y I am indifferent between bundles (3,2) and (0,4)

### Indifference Set

Given a utility function and a level of utility, you can find a whole set of bundles that you are indifferent between

For example, let U(x,y)=x+y and set U=10. Then we can find an infinite set of x and y that will give us our stated utility level

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### Indifference Curves

We can create a function that helps us find **ALL** possible bundles that make you indifferent at a given utility level

We call this an **Indifference Curve** 

For our previous utility function U(x,y)=x+y where U=10, we solve for y and get:

# Indifference Curve - Example

$$U(x,y) = xy$$
 where  $U = 16$ 

What type of Utility Function is this?

ullet It is a Cobb-Douglas  $x^ay^b$  where a=b=1

What is the associated Indifference Curve?

$$16 = xy \rightarrow y = \frac{16}{x} \rightarrow \text{IC}$$

### Indifference Curves

So why do we care about these curves?

- We can graph them
- Graphing is a key step to figuring out how to solve an individual's choice problem

Each utility function has a unique shape that we will learn

### Let's Draw - Perfect Substitutes

$$U(x,y) = 3x + y$$
 with  $U = 6, 9, 15$ 

First, find the indifference curves for each Utility value

$$U=6$$

$$U=9$$

$$U=15$$

$$y = 6 - 3x$$

$$y = 9 - 3x$$

$$y = 15 - 3x$$

# Let's Draw - Perfect Complements

$$U(x,y) = \min\{x, \frac{y}{2}\} \text{ with } U = 2, 8, 9$$

Find the indifference curves for each utility value

$$U=2$$
  $U=8$   $U=9$   $x=2 ext{ or } y=4$   $x=8 ext{ or } y=16$   $x=9 ext{ or } y=18$ 

# Let's Draw - Cobb-Douglas

$$U(x,y) = x^{1/2}y \ ext{with} \ U = 4, \, 8, \, 10$$

#### Find the indifference curves for each utility value

$$U=4$$

$$U=8$$

$$U = 10$$

$$y=rac{4}{x^{1/2}}$$

$$y=rac{8}{x^{1/2}}$$

$$y=rac{10}{x^{1/2}}$$

### Let's Draw - Quasi-Linear

$$U(x,y) = ln(x) + y \text{ with } U = 5, 15, 20$$

Find the indifference curves for each utility value

$$U=5$$
  $U=15$   $U=20$   $y=5-ln(x)$   $y=15-ln(x)$   $y=20-ln(x)$ 

### Indifference Curve - Rules

It is very important that you understand the **intuition** behind indifference curves

Let's view an example that can help:

Consider Weather Reports:

- On cold days, what the weather feels like is a function of:
  - 1. Temperature
  - 2. Windchill
- An indifference curve represents all of the different combinations of temperature and windchill that cause you to feel the exact same thing
- If the windchill is suddenly lower, what must **intuitively** happen to the temperature to keep you feeling the same outside?

If windchill ↓ then temperature ↑

### Indifference Curve - Rules

We use this exact same logic for utility between two goods

**Intuitively**, if I want to stay at the same level of happiness as I lose some y, what must happen to x?

• I need more of x

This is why we read them from left to right and why they have a negative slope

• I know the perfect complements is odd but the same logic tends to apply

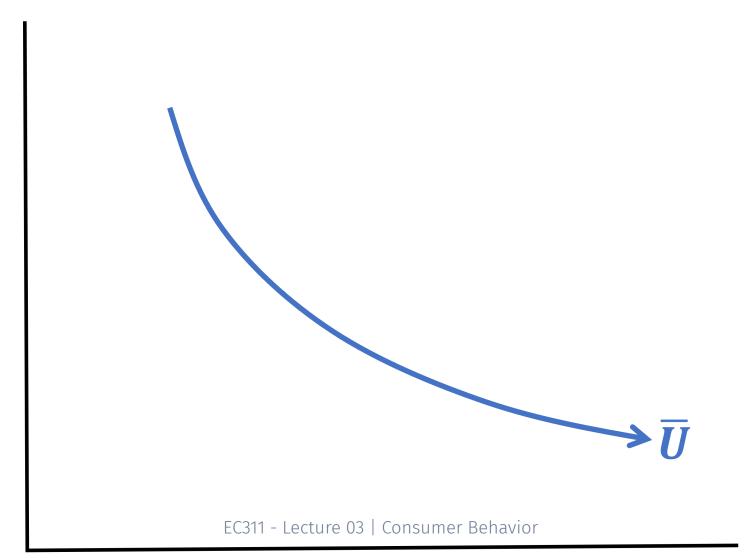
### Formal Indifference Curve Rules

All of the stuff from before can be formalized into the following 3:

- **Monotonicity:** Indifference curves always go from the top left to the bottom right of the graph without changing direction at any point
- Non-Crossing: If at least two curves cross, this leads to logical contradictions
- **Convex:** Balanced combinations of two goods are preferred to extreme outcomes (A lot of one good, little of the other)

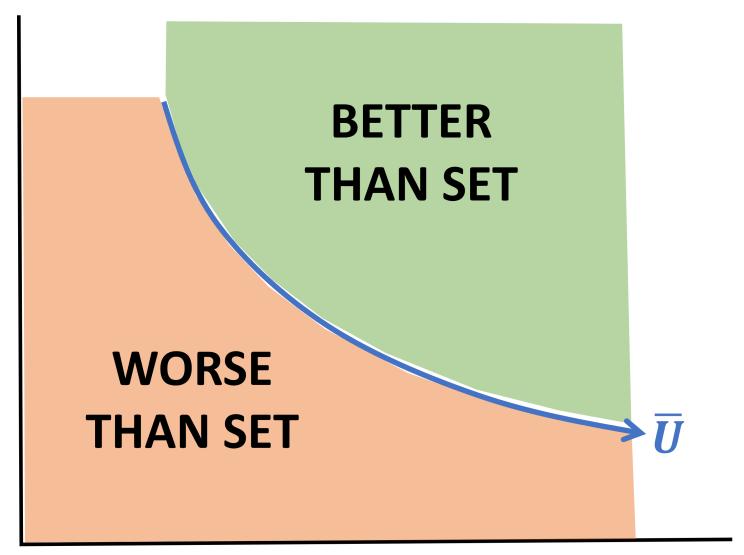
# Monotonicity

ICs always go from the top left to the bottom right without changing direction



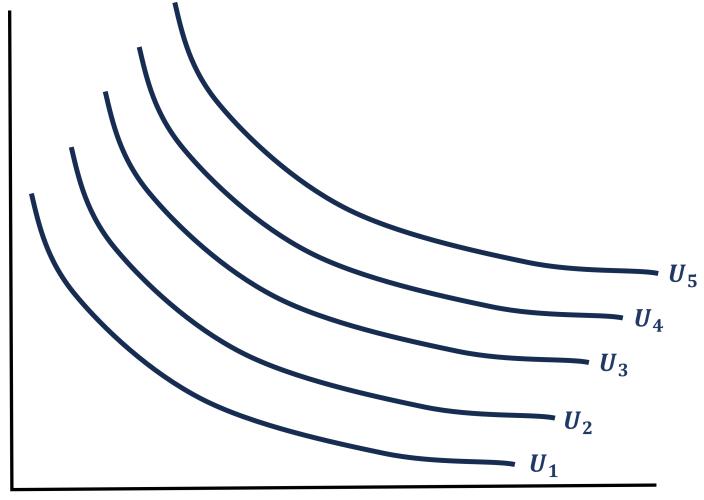
# Monotonicity

Additionally, this helps us visualize two important sets of bundles



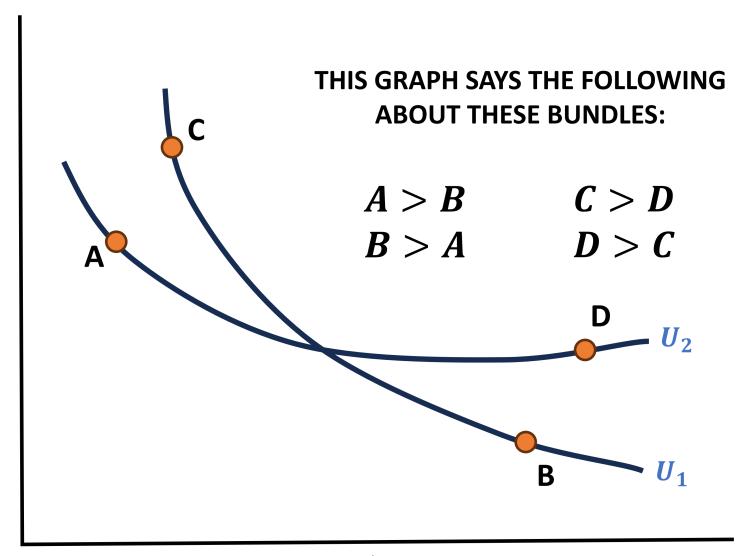
# Non-Crossing

This is the expected behavior of ICs. There are infinitely many, each representing a unique level of Utility



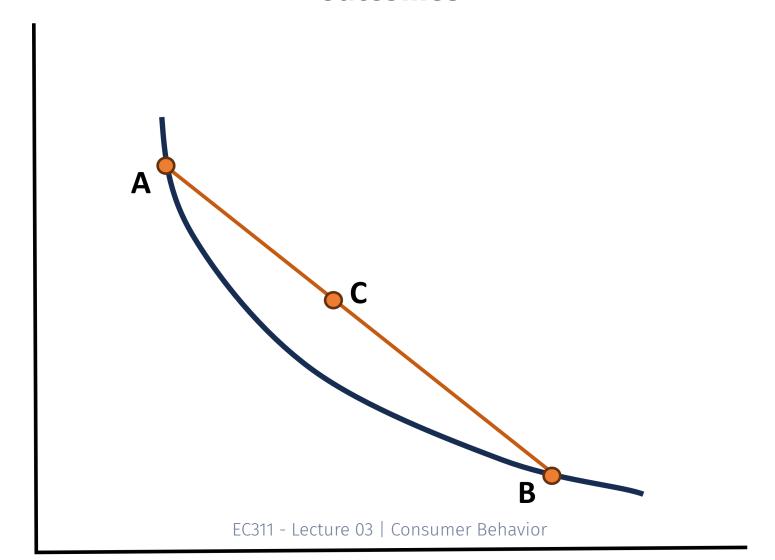
# Non-Crossing - Logical Contradiction

#### If ICs cross, they are contradictions



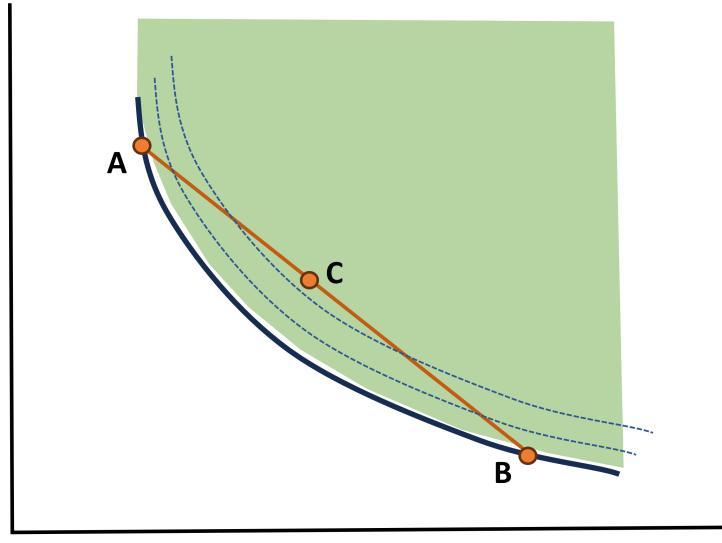
### Convex

# A balanced combination of two goods are preferred to extreme outcomes



### Convex

#### **Recall the Better-than-set**



### What About the Other Functional Forms?

I drew Cobb-Douglas curves (mostly because they're easier to illustrate these properties) but what about Perfect Complements or Perfect Substitutes?

They fulfill all 3 properties, but not strongly

#### **Perfect Complements**

- Never Crosses
- Convex
- Wealky Monotonic

#### **Perfect Substitutes**

- Never Crosses
- Monotonic
- Weakly Convex

Let's look at the board again

#### Most Important Facts of Indifference Curves

Recall the intuition of what a movement along the indifference curve means:

- ullet You are trading y for x, holding **constant** your level of **utility**
- The slope of the indifference curve measures your **willingness to**  ${f tradeoff\ between\ }x\ {f and\ }y$
- This is also known as the Marginal Rate of Substition (times -1)

#### Most Important Facts of Indifference Curves

This **Marginal Rate of Substition (MRS)** thing is pretty important First, why is the slope the **negative MRS**?

ullet The MRS is the **ratio** of the effect of increasing x on your utility and the effect of increasing y on your utility

$$ullet rac{MU_x}{MU_y}$$

- ullet The Indifference Curve slope is all about keeping the utility level constant while we move x and y in opposite directions
- Therefore the IC slope = -MRS

#### Simple Mathematical Proof

For

$$U(x,y) = ax + by$$

1. Find the MRS and times -1

2. Find the IC and it's slope

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Slope is the derivative! 
$$\rightarrow \frac{\partial y}{\partial x} = \frac{-a}{b}$$

#### Indifference Curves

From this lecture you have learned:

- Everything about utility functions and how to use them to find an MRS
- Everything about Indifference Curves and how the IC slope relates to the MRS

The MRS is going to be key to solving utility maximization problems

- Mathematically (Using derivatives)
- Graphically (Drawing ICs)

However, when we maximize utility functions we have constraints, we called these **Budget Constraints** 

### Budget Constraints

#### **Budgets**

**Economic Theory** says that individuals make themselves as happy as they possibly can, after choosing from a set of all bundles they can afford

 We do not just maximize utility functions, but rather we maximize them subject to Budget Constraints (BC)

#### **Budgets - Variables**

Inside a budget constraint we use the exact same variables we use in our utility function, namely  $oldsymbol{x}$  and  $oldsymbol{y}$ 

But we'll need to introduce some new notation and terminology:

- ullet The **price** of good  $x \ o \ P_x$
- ullet The **price** of good  $y \; o \; P_y$
- ullet Your income, budget, or money-on-hand o M
  - ullet The book also calls this I

#### **Budgets - Functional Form**

Putting together our variables we get our **Budget Constraint** 

$$P_x \cdot x + P_y \cdot y \leq M$$

Do not let the inequality leq scare you, it just means we can spend less than our total income

However, in our applications we are going to treat it as a strict equality

$$P_x \cdot x + P_y \cdot y = M$$

And let's think why?

• If we are maximizing utility, it does not make sense to leave any income unspent!

#### Budgets - Spending All Our Income

But do we usually spend **all** of our money?

Of course not, but here is how to think about it in the economics sense:

- When you go to the grocery store you make to choices before even stepping inside:
  - 1. How much to spend at the store
  - 2. What to spend it on
- Once you allocate your budget, you usually spend all of it

The trick is to create the context in which it makes sense that everything gets spent

#### Let's Construct A Budget Constraint

First, we need our two goods from before: Beer (B) and Soda (S) Let's label everything properly:

- B = Beer
- S = Soda
- $P_B = \$4$
- $P_S = \$2$
- M = \$20

What does the Budget Constraint look like?

$$B \cdot P_B + S \cdot P_S = M$$

$$4B+2P=20$$

#### Let's Graph A Budget Constraint

We graph the budget in the same space in as our IC, but what are we graphing?

$$P_x \cdot x + P_y \cdot y = M$$

And thankfully, this is just a straight line

# Bringing Intuition into Budget Constraint Graphs

$$P_x \cdot x + P_y \cdot y = M$$

Thinking intuitively, this graph represents:

- ullet All combinations of (x,y) that cost exactly M
- ullet If we spend nothing on good y, what is the most possible x that we can purchase?

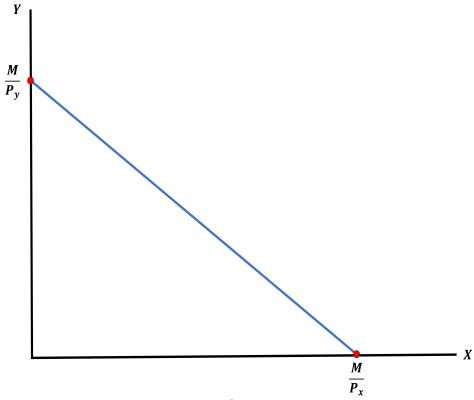
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#### Let's Graph A Budget Constraint

Following the logic we just did, the budget line will connect two points:

One for when we buy only  $oldsymbol{x}$  and one for when we buy only  $oldsymbol{y}$ 

$$(\frac{M}{P_x}, 0)$$
 and  $(0, \frac{M}{P_y})$ 



## Interpreting the Slope of the Budget Constraint

The Budget Constraint is the line:

$$y = rac{M}{P_y} - rac{P_x}{P_y} \cdot x \;\; ext{ where the slope is } \; -rac{P_x}{P_y}$$

Now, let's define

$$P_x = \frac{\$}{x} \text{ and } P_y = \frac{\$}{y}$$

We get this result in terms of units!

$$rac{P_x}{P_y} = rac{\$/x}{\$/y} = rac{\$}{x} \cdot rac{y}{\$} = rac{y}{x}$$

#### The Slope of the Budget Constraint

As we just saw, the price ratio  $P_x/P_y$  can be measured in units of x per units of y

We had already seen something that is measured in units  $\rightarrow$  the MRS This leads us to understand the differences between the two:

- ullet The MRS represents how much  $oldsymbol{y}$  you would be willing to give up in order to get a unit of  $oldsymbol{x}$
- ullet The **Price Ratio** represents how much  $oldsymbol{y}$  you **would have to** give up in order to get a unit of  $oldsymbol{x}$

#### What Happens When Things Change?

Budgets are not static. Prices and income can change based on market conditions so it is important to understand the effects when factors change

- We can have changes in income (increase or decrease)
- ullet Price of x can change (increase or decrease)
- ullet Price of y can change (increase or decrease)

Note - We normally consider what happens when only one of the possible factors change, and hold all others fixed

#### Changes in Income

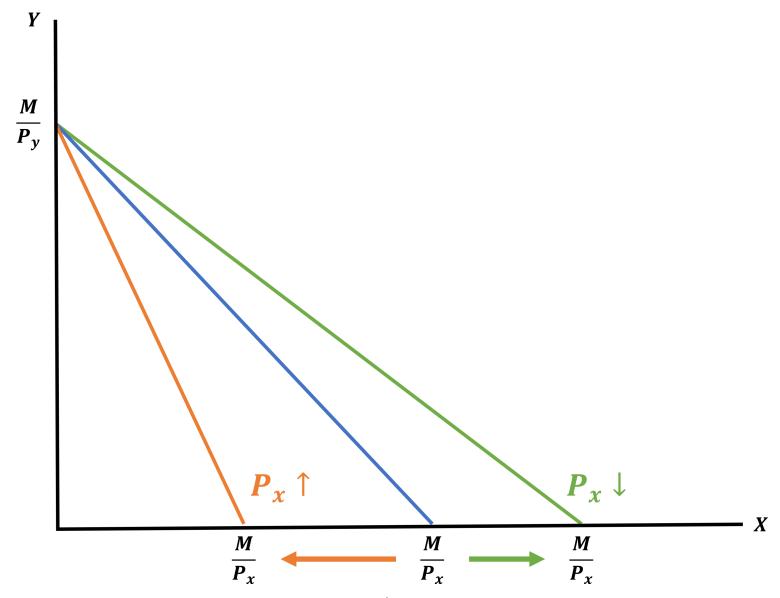
What happens to the budget if income (M) increases?

- ullet  $\dfrac{M}{P_x}$  , the maximum amount of x that can be purchased goes up
- ullet  $\dfrac{M}{P_y}$  , the maximum amount of y that can be purchased goes up
- ullet The slope of the BC  $(-P_x/P_y)$  stays the same

Income changes affect the **overall amount** an individual can consume, but has **no effect on the relative cost of the goods** 

Intuitively, the opposite is true if income  $\left(M\right)$  decreases

#### Changes in Income - Graph



#### Changes in Prices

What happens to the budget if the price of good  $x\left(P_{x}\right)$  increases?

- ullet The maximum amount of x that can be consumed goes down  $(rac{M}{P_x})\downarrow$
- ullet The maximum amount of y that can be consumed stays the same  $(rac{M}{P_x})$
- ullet The price ratio  $(-rac{P_x}{P_y})$  becomes steeper

Once more, the opposite happens with a decrease

How about a shift in  $P_y$  ?

### Changes in Prices - Graph

