Short Answer - 40 pts

Answer the following questions to the best of your ability. For full credit, show all of your work and clearly indicate your final solution for each party by circling the answer.

1. [10 points] Create **ONE** utility function that represents **ALL** the characteristics below of good X and good Y. Show how your utility function meets the criteria. Be sure to provide your reasoning as to how it meets the criteria:
   (a) Good Y is something you get tired of: the more you consume, the less joy it brings you each time.
   (b) Good X is something that always impacts you in the same way: you get 3 units of utility from X every time.
   (c) Your marginal utility of X does not depend on the level of consumption of X.

> **Solution:** Anything that is a quasi-linear utility function with diminishing MU_x and a constant MU_y $\rightarrow$ $U(x, y) = ln(y) + 3x$
>
> To show it meets that criteria:
>
> (a) $MU_y = \frac{1}{y}$ so as you get more y your MU_y decreases. Also $\frac{\partial MU_y}{\partial y} = \frac{-1}{y^2} < 0$
>
> (b) $MU_x = 1$ so it is constant. Also $\frac{\partial MU_x}{\partial x} = 0$
>
> (c) $\frac{\partial MU_x}{\partial x} = 0$

2. [10 points] Your utility function over candy, C, and vegetables, V, is

$$U(C, V) = ln(C) + V$$

Explain what the utility function means about your preference for candy and vegetables: Specifically, what does it say about candy that it appears inside the log function and about vegetables that it does not? (Hint: Think at the margin)

> **Solution:** The natural log function ensures that the Marginal Utility of Candy (MU_C) is decreasing each additional candy is less valuable than the last. This is not true for Vegetable: every vegetable delivers the same utility. The function also says both are goods with independent Marginal Utilities.

3. [10 points] Explain why a good cannot be inferior at all income levels. Showing an example Engel curve may be helpful.

Solution: The Engel curve must start at the origin when $x = 0$, hence, it has to be upward sloping for a period before decreasing

4. [10 points] You get utility over hot dogs (H) and mustard (M). If your utility function is $U(H, M) = H \cdot M$, how does the marginal utility of mustard depend on your hot dog consumption? Your answer should include two derivatives and an inequality

Solution:

$$MU_M = \frac{\partial U}{\partial M} = H$$
$$\frac{\partial MU_M}{\partial H} = 1 > 0$$

This means that more hot dogs $\rightarrow$ higher Marginal Utility of Mustard

Long Answer - 60 pts

Answer the following questions to the best of your ability. For full credit, show all of your work and clearly indicate your final solution for each party by circling the answer. Be sure to properly label any and all graphs you make.

1. [15 points] For the utility function $U(x, y) = 12ln(x) + y$:
   (a) [3 points] Find X^* and Y^* demand functions
   (b) [3 points] Draw a graph in the (x, y) plane with 3 Indifference Curves and 3 Budget Constraint such that each Indifference Curve is Utility Maximizing. Identify each of these points.
   (c) [3 points] Draw the Engel Curves for Y^*
   (d) [3 points] Calculate the derivatives of X^* and Y^* with respect to income and classify the good
   (e) [3 points] Calculate the Elasticities of Demand with respect to Income and classify them

Solution:

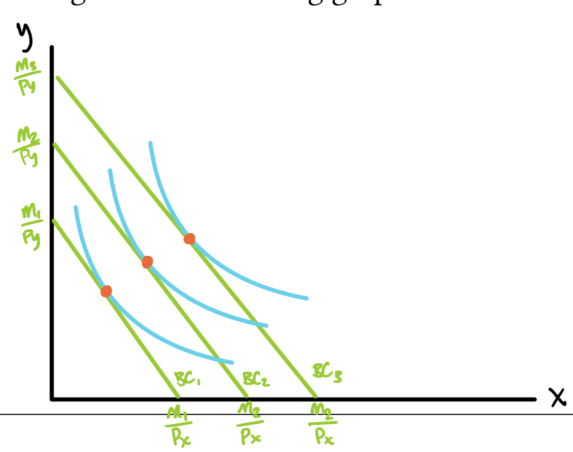
(a)

$$MRS = \frac{P_x}{P_y} \Rightarrow \frac{12}{x} = \frac{P_x}{P_y} \Rightarrow x^* = \frac{12P_y}{P_x}$$

Plug this x^* into the Budget Constraint: $P_x \cdot x + P_y \cdot y = M$

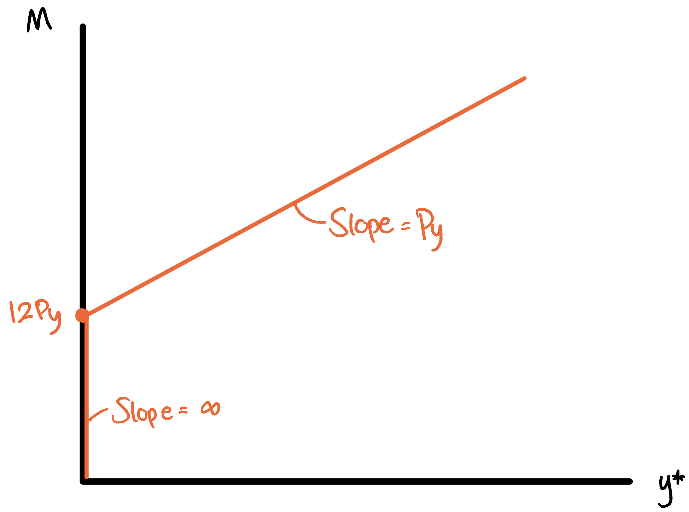
$$\Rightarrow P_x \cdot \frac{12P_y}{P_x} + P_y \cdot y = M$$
$$\Rightarrow 12P_y + P_y \cdot y = M$$
$$\Rightarrow P_y(12 + y) = M$$
$$\Rightarrow 12 + y = \frac{M}{P_y}$$
$$\Rightarrow y^* = \frac{M}{P_y} - 12$$

(b) Should look something like the following graph



(c) To find the Engel curves for a good, we solve the demand function of a good for M

$$y^* = \frac{M}{P_y} - 12$$
$$M = P_y(y^* + 12)$$



(d)

$$\frac{\partial x^*}{\partial M} = 0 \Rightarrow \text{Neither normal or inferior, may answer infinite}$$
$$\frac{\partial y^*}{\partial M} = \frac{1}{P_y} > 0 \Rightarrow \text{Normal}$$

(e)

$$E_{x^*,M} = \frac{\partial x^*}{\partial M} \cdot \frac{M}{x^*} = 0$$
$$E_{y^*,M} = \frac{\partial y^*}{\partial M} \cdot \frac{M}{y^*}$$
$$E_{y^*,M} = \frac{1}{P_y} \cdot \frac{M}{\frac{M}{P_y} - 12} = \frac{M}{M - 12P_y} > 1$$
$$E_{y^*,M} \Rightarrow Elastic$$

2. [15 points] Imagine you are an associate at Pearson Hardman Law Firm, a prestigious law firm in NYC. You are assigned to a class action lawsuit case in which you are tasked to provide empirical background to your partners arguments. To do so, you create a profile of a consumer, with utility over coffee and other food as follows:

$$U(C, F) = C^{1/4}F^{3/4}$$

You make the assumption that the income they will spend on C and F is $\$80$. Since F represents money spent on other food, the price of F is $\$1$

(a) [1 point] What kind of utility function is this? What is a consumer's marginal rate of substitution? (If you use a shortcut, point out why you can use it)

(b) [2 points] Find the utility maximizing demand for coffee, C. Show your work: you can use a shortcut to check your answer, but for full credit you should work from your answer in part a to the solution.

(c) [2 points] Calculate the elasticity of demand for coffee using your answer from part b. Is it inelastic, elastic, or unit elastic?

(d) [2 points] What are the utility-maximizing demands for coffee, C^*, and other food F^*, when the price of coffee is $\$2$? What about when the price is $\$4$?

(e) [8 points] On a graph in the (C, F) plane, draw the Budget Constraint lines when the price of coffee is $\$2$ and when the price of coffee is $\$4$. Label the intercepts. I recommend that you draw a large graph so there will be room to clearly add things to it and details can be more easily viewed.

  i. Add to your graph C^* and F^* for both bundles, the utility maximizing choices from your answers to part d.

  ii. Add to your graph an indifference curve that represents the maximized level of utility from the budget where coffee costs $\$2$. (It does not have to be drawn to scale, it just needs the correct shape and labels)

  iii. Add to your graph a budget that represents the substitution effect of the price of coffee going up from $\$2$ to $\$4$.

Solution:

(a) This is Cobb-Douglas.

$$MRS = \frac{1/4}{3/4} \cdot \frac{F}{C} = \frac{F}{3C}$$

(b) Cobb-Douglas method means:

$$MRS = \frac{P_x}{P_y}$$

$$\frac{F}{3C} = \frac{P_C}{1} \Rightarrow F = 3P_C \cdot C$$

Plug this optimality condition into the Budget Constraint: $P_C \cdot C + F = 80$

$$\Rightarrow P_C \cdot C + 3P_C = 80$$
$$\Rightarrow 4P_C \cdot C = 80$$
$$\Rightarrow C^* = \frac{20}{P_C}$$

(c)

$$E_{C^*,P_C} = \frac{\partial C^*}{\partial P_C} \cdot \frac{P_C}{C^*} \quad ; \quad \frac{\partial C^*}{\partial P_C} = -20P_C^{-2}$$

$$\Rightarrow E_{C^*,P_C} = -20 \cdot P_C^{-2} \cdot \frac{P_C}{20/P_C}$$
$$\Rightarrow E_{C^*,P_C} = -1 \rightarrow \text{Unit Elastic}$$

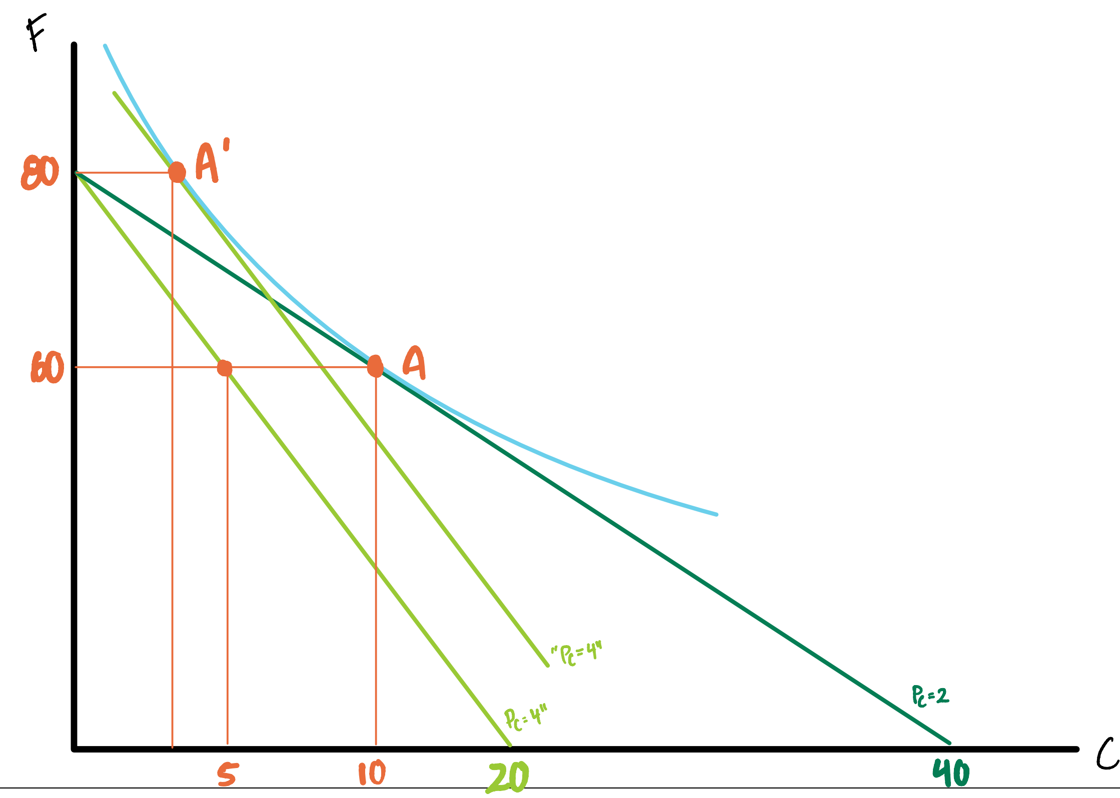
(d) When $P_C = 2 \Rightarrow C^* = 10$

$$\text{Budget: } 2C + F = 80$$
$$\Rightarrow 20 + F = 80 \rightarrow F^* = 60$$

When $P_C = 4 \Rightarrow C^* = 5$

$$\text{Budget: } 4C + F = 80$$
$$\Rightarrow 20 + F = 80 \rightarrow F^* = 60$$

(e) Graph



3. [15 points] There are 150 consumers in a market with identical preferences over goods x and y given by the following utility function

$$U(x,y) = 4x^{1/2}y^{3/2}$$

   (a) [1 point] What is the marginal rate of substitution (MRS) for consumers in this market?
   (b) [2 points] Leaving income in general form (M), find individual demand for x
   (c) [2 points] What is the Engel Curve for x? (Show it mathematically, a graph is not necessary)
   (d) [4 points] Leaving income in general form (M), find individual demand for y
   (e) [6 points] If all 150 consumers have an income of $M = 4000$, what is the market demand for x?

Solution:

(a)

$$MRS = \frac{MU_X}{MU_Y} = \frac{y}{3x}$$

(b) Setting the MRS equal to ratio of prices yields:

$$\frac{y}{3x} = \frac{P_x}{P_y}$$
$$y = \frac{3P_x}{P_y} \cdot x$$

Plugging this into the Budget Constraint:

$$P_x x + P_y y = M$$
$$P_x x + P_y \left(\frac{3P_x}{P_y} \cdot x\right) = M$$
$$P_x x + 3P_x x = M$$
$$x^* = \frac{M}{4P_x}$$

(c) Rearranging demand function for M:

$$x^* = \frac{M}{4P_x}$$
$$M = 4P_x x^*$$

(d) We can plug our demand function for x into our y^* optimality condition:

$$y^* = \frac{3P_x}{P_y} \cdot x^*$$

$$y^* = \frac{3P_x}{P_y}\left(\frac{M}{4P_x}\right)$$

$$y^* = \frac{3M}{4P_y}$$

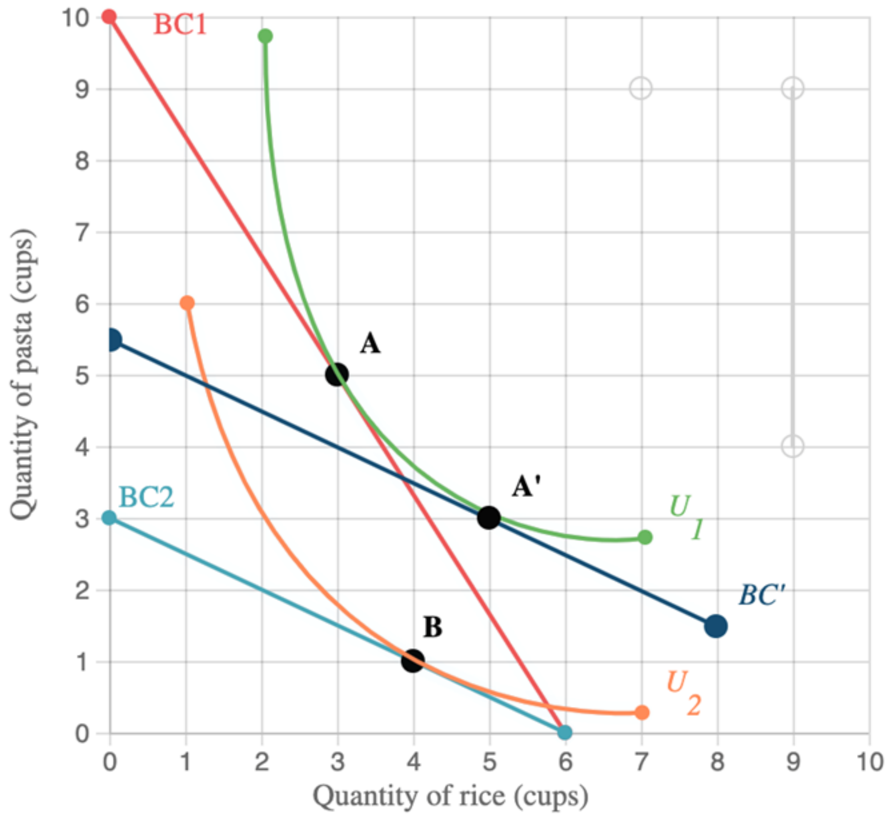
(e) There are 150 identical consumers so we can simply multiply 150 by the individual demand functions and plug $M = 4000$ in:

$$Q_D = 150 \cdot x^*$$

$$Q_D = 150\left(\frac{4000}{4P_x}\right)$$

$$Q_D = \frac{\cancel{600{,}000} \;\; 150{,}000}{P_x}$$

4. [15 points] Suppose that Maria faces an increase in the price of pasta, as depicted below, moving her from an optimum bundle of rice and pasta at A to an optimum bundle at B.



(a) [3 points] What is the total effect of the price change on Maria's demand for pasta? For rice?

(b) [3 points] What is the substitution effect of the price change on pasta? Provide an interpretation of this effect.

(c) [3 points] What is the income effect of the price change on pasta? Provide an interpretation of this effect.

(d) [3 points] Is pasta a normal or an inferior good? Explain your answer.

(e) [3 points] In your own words, why can we only observe the total effect when there is a relative price change? Why is the decomposition into substitution and income effect useful to our understanding of consumer behavior?

Solution:

(a)

$$\Delta Pasta = B - A = 1 - 5 = -4$$
$$\Delta Rice = B - A = 4 - 3 = 1$$

(b)

$$A' - A = 3 - 5$$

The increased price of pasta decreases the *relative* price of rice. Hence, Maria will substitute rice for pasta leading to a decrease in consumption of pasta.

(c)

$$B - A' = 1 - 3 = -2$$

The increased price causes Maria's real income to be lower, so she has lower total income to spend across all goods.

(d) The income effect was negative following a price increase $\rightarrow$ pasta is a normal good

(e) We can only observe the bundles someone purchases, not their reasoning behind the changes. Decomposing this change into substitution and income effects is useful as we can attribute changes to different forms of behaviors by individuals. It also informs us of how individuals are making their choices which may be important to firms when setting prices.