Short Answer - 40 points

Answer the following questions to the best of your ability. For full credit, show all of your work and clearly indicate your final solution for each party by circling the answer. Be sure to properly label any and all graphs you make. Partial credit is very much in play, so attempt every question.

1. [10 points] Imagine a firm with the cost function $C(Q) = Q^3 + 100Q + 200$. Currently, the firm can sell its product for a price of $175. What is the short-run profit maximizing quantity? Imagine the government imposes a $60 per-unit tax on the good that this firm sells. What is the new short-run profit maximizing quantity, assuming the price goes up to $208

Solution: Short-run profit max Q:

$$P = MC = 3Q^2 + 100 = 175 \Rightarrow 3Q^2 = 75 \Rightarrow Q^2 = 25 \Rightarrow Q^* = 5$$

With tax, the cost function becomes:

$$C(Q) = Q^3 + 100Q + 200 + 60Q = Q^3 + 160Q + 200$$

$$\begin{aligned} MC &= 3Q^2 + 160 \\ P &= MC \\ 208 &= 3Q^2 + 160 \\ 3Q^2 &= 48 \\ Q^2 &= 16 \\ Q^* &= 4 \end{aligned}$$

2. [10 points] For a firm with the cost function $C(Q) = \frac{1}{4}Q^3 - 3Q^2 + 25Q + 60$, below what price will the firm shut down? What are the firm's profits at that price?

Solution: Recall shut-down is at $P < min(AVC)$

We find AVC and then use it to find our $min(AVC)$ quantity

$$AVC = \frac{1}{4}Q^2 - 3Q + 25$$

$$\frac{\partial AVC}{\partial Q} = \frac{1}{2}Q - 3 = 0 \Rightarrow \frac{1}{2}Q = 3 \Rightarrow Q = 6$$

Plug $Q = 6$ into our $AVC(Q)$ function to determine shutdown price

$$AVC(6) = \frac{1}{4}(6)^2 - 3(6) + 25 = 9 - 18 + 25 = 16$$

The shut-down price is $P = 16$

Profits at shutdown price are just $-FC$:

$$\pi = -FC = -60$$

3. [10 points] Maria is the sole producer of dog outfits in Eugene and faces the following cost function:

$$C(Q) = Q^2 + 9$$

The demand for dog outfits is given by

$$Q_D = 112 - 4P$$

. What price will Maria charge and how much will she produce if she is maximizing profit?

Solution: Rearranging the demand function gives us the demand curve $P = 28 - \frac{1}{4}Q$. We plug this into revenue to find MR:

$$Revenue = \left(28 - \frac{1}{4}Q\right) Q = 28Q - \frac{1}{4}Q^2$$

We use this to find marginal revenue:

$$MR = 28 - \frac{1}{2}Q$$

Firms choose their quantity by setting $MR = MC$, where in this case $MC = 2Q$

$$MC = MR \Rightarrow 2Q = 28 - \frac{1}{2}Q \Rightarrow Q_M = 11.2$$

To find price, we plug in the quantity into the demand curve:

$$P_M = 28 - \frac{1}{4}(11.2) = 25.2$$

4. [10 points] Imagine two firms are competing in Stackelberg competition. Firm A is the follower and has the best response function $Q_A = 100 - Q_B$, where firm B is the leader. Firm B has the cost function $C(Q_B) = \frac{1}{2}Q_B^3$. The market demand curve is $P = 250 - Q$. Write down firm B's profit function and find their profit maximizing quantity.

Solution: Firm B's profit function is:

$$\begin{aligned}
\pi_B &= P \cdot Q_B - \frac{1}{2}Q_B^3 \\
\pi_B &= (250 - Q)Q_B - \frac{1}{2}Q_B^3 \\
\pi_B &= (250 - Q_A - Q_B)Q_B - \frac{1}{2}Q_B^3 \\
\pi_B &= (250 - 100 + Q_B - Q_B)Q_B - \frac{1}{2}Q_B^3 \\
\pi_B &= 150Q_B - \frac{1}{2}Q_B^3
\end{aligned}$$

We can find their profit maximizing quantity by:

$$\begin{aligned}
\frac{\partial \pi_B}{\partial Q_B} &= 150 - \frac{3}{2}Q_B^2 = 0 \\
150 &= \frac{3}{2}Q_B^2 \\
Q_B^* &= 10
\end{aligned}$$

Long Answer - 102 points

1. [24 points] The firm you manage is a monopolist in the internet business in Eugene. You face a cost function $C(Q) = Q^2 + 80Q + 400$. Demand for internet in Eugene is given by the demand curve $P = 200 - Q_D$. *No matter how much you complete this problem, do not forget to try the graph.*

   (a) [4 points] Write down the profit function of your firm, using the fact that it is a monopoly. Find the profit-maximizing quantity for the **monopolist**. find the price the **monopolist** will charge.

   (b) [4 points] If the government forces your firm to be **perfectly competitive** (government always bullying business around), what would the market equilibrium price and quantity be (in the short-run)?

   (c) [8 points] In the (Q, P) plane, graph and label the market demand curve, your firm's marginal cost curve and your firm's marginal revenue curve. Add a label for the perfectly competitive short-run supply curve you use d in part b. Label the P and Q values associated with the monopoly outcome from part a, and the P and Q values associated with the short-run perfectly competitive equilibrium outcome from part b.

   (d) [4 points] Imagine that the barriers to entry that have helped you maintain your monopoly status have disappeared. Now, any other firm that wants to join your market can. Assuming all these new firms have the same cost function as your firm, what will the long-run market equilibrium price and quantity be? How many firms would exist?

   (e) [4 points] You decide you miss the good old days and team up with one of the new firms to invent a technology that changes both of your cost functions to $C(Q) = 20Q$. By forming a cartel with this new technology, you successfully kick every other firm out of the market. However, one day the other firm stabs you in the back and breaks the cartel agreement, and you end up competing in Cournot competition. How much do you produce, what is total market quantity, and the market price? Are consumers happier under true perfect competition in the long-run (as in part d) or do they prefer this imperfectly competitive outcome with better technology?

Solution:

(a) We obtain Q_M^* from the monopolist profit function:

$$\pi = PQ - C(Q) = (200 - Q)Q - Q^2 - 80Q - 400$$
$$\frac{\partial \pi}{\partial Q} = 200 - 2Q - 2Q - 80 = 0$$
$$\Rightarrow 120 = 4Q \Rightarrow Q_M^* = 30$$

We can then obtain P_M^* by plugging Q_M^* into the demand curve:

$$P_M^* = 200 - 30 = 170$$

(b) In a Perfectly Competitive market, we know $P = MC$ where we can obtain Q^*_{PC}:

$$P = MC \Rightarrow P = 2Q + 80 \Rightarrow Q^*_{PC} = \frac{P - 80}{2}$$

Price can be obtained setting Supply equal to Demand:

$$Q_S = \frac{P - 80}{2} = 200 - P = Q_D$$

$$\begin{aligned} \frac{P-80}{2} &= 200 - P \\ P - 80 &= 400 - 2P \\ 3P &= 480 \\ P &= 160 \end{aligned}$$

Plugging this back into our Q^*_{PC} equation:

$$Q^*_{PC} = \frac{P - 80}{2} = \frac{160 - 80}{2} = 40$$

(c) The graph should have the proper labels and look like:

(d) We can find Equilibrium Price through $P = min(AC)$ or $AC = MC$

$$AC = Q + 80 + \frac{400}{Q} \; ; \; MC = 2Q + 80$$

For $P = min(AC)$:

$$\frac{\partial AC}{\partial Q} = 1 - \frac{400}{Q^2} = 0 \Rightarrow 1 = \frac{400}{Q^2} \Rightarrow Q^2 = 400 \Rightarrow Q = 20 \text{ (each firm makes 20)}$$

For $AC = MC$

$$Q + 80 + \frac{400}{Q} = 2Q + 80$$
$$Q^2 + 80Q + 400 = 2Q^2 + 80Q$$
$$Q^2 = 400$$
$$Q = 20$$

Plug the found Q into any function:

$$P = MC = 2(20) + 80 = 120$$

To find then number of firms:

$$Q_D = 200 - P = 200 - 120 = 80$$

We know we can get firms from:

$$N = \frac{Q_D}{Q^*} = \frac{80}{20} = 4$$

(e) We find your firm's profit function:

$$\pi_A = PQ_A - 20Q_A = (200 - Q_A - Q_B)Q_A - 20Q_A$$
$$\frac{\partial \pi_A}{\partial Q_A} = 200 - 2Q_A - Q_B - 20 = 0$$
$$\Rightarrow 180 - Q_B = 2Q_A \quad \Rightarrow \quad Q_A^* = \frac{180 - Q_B}{2}$$

By symmetry, we know $Q_A = Q_B$:

$$Q_A^* = \frac{180 - Q_A^*}{2}$$
$$2Q_A^* = 180 - Q_A^*$$
$$3Q_A^* = 180$$
$$Q_A^* = 60$$

Market quantity is given by $Q_A^* + Q_B^*$ which we know are the same by symmetry:

$$Q_s = 120$$

And Price is:

$$P = 200 - Q_s = 80$$

Because things are cheaper and there is more quantity, consumers prefer this outcome of imperfect competition with better technology.

2. [18 points] You are employed by Ferrari to run their Formula 1 car production factory. The production function of a new part is given by $F(L, K) = L \cdot K$. Currently the wage, w, is \$12.50 and the rental rate, r, is \$1.25.

   As a hint for somewhere in this problem: $\sqrt{40,000} = 200$, $\sqrt{400,000} = 632$, $\sqrt{4,000,000} = 2000$ (you might not need all of these). *Do not forget to attempt the graph regardless of how much of the problem you do.*

   (a) [3 points] Find the MRTS and price ratio. Ferrari's Team Principal, Frédéric Vasseur, rings you up and says you need to produce 400,000 parts for the season. Write down the quantity constraint that represents this.

   (b) [5 points] What are the cost-minimizing demands for labor, L^*, and capital, K^*? What is your minimized cost, C^*?

   (c) [8 points] On a graph in the (L, K) plane, draw and label the isoquant quantity constraint in this problem. Add the isocost line corresponding to the minimized cost: label it the value of C^* from part b. Label the L^* and K^* values at the cost-minimizing point with your answers from part b.

   (d) [2 points] If you tried to construct a minimized cost function, $C^*(Q)$ for this problem, and used it to do profit maximization, you would get an answer that did not make any sense (you would actually find the profit-minimizing quantity rather than the profit-maximizing quantity). Why? **Hint: Look closely at the production function**

Solution:

(a)

$$MRTS = \frac{MP_L}{MP_K} = \frac{K}{L} \quad ; \quad \frac{w}{r} = \frac{12.5}{1.25} = 10$$

Quantity constraint is:

$$400,000 = L \cdot K = F(L, K)$$

(b)

$$MRTS = \frac{w}{r} \quad ; \quad \frac{K}{L} = 10 \Rightarrow K = 10L$$

$$\Rightarrow 400,000 = L \cdot 10L = 10L^2 \quad \Rightarrow \quad L^2 = 40,000 \Rightarrow L^* = \sqrt{40,000} = 200$$

$$K^* = 10L^* = 2,000$$

The minimized cost is:

$$C^* = wL^* + rK^* = 12.5 \cdot 200 + 1.25 \cdot 2,000 = 5,000$$

(c) Graph should look like this:

(d) Because the production function shows Increasing Returns to Scale

3. [30 points] Suppose the market for thrifted clothes in Springfield is perfectly competitive. The thrift firms are identical and have long-run cost functions given by $C(Q) = 10Q^3 - 100Q^2 + 300Q$. Market demand is given by $Q_D = 5,000 - 90P$.
   (a) [3 points] What is the marginal cost curve for each firm in this market?
   (b) [3 points] What is the average total cost curve for each firm in this market?
   (c) [10 points] Find the long-run equilibrium price for this market
   (d) [5 points] What is the equilibrium level of output in this market?
   (e) [5 points] How many firms are in the market in the long-run equilibrium?
   (f) [4 points] Suppose market demand increases. What do you expect will happen to Q^*, P^*, and the number of firms? For each of these, mention if it will increase, decrease, or stay the same and explain your reasoning. Address both the short-run and long-run dynamics. A graph may also help your explanation.

Solution:

(a)

$$MC = 30Q^2 - 200Q + 300$$

(b)

$$AC = 10Q^2 - 100Q + 300$$

(c) We first find Q^* by setting $MC = AC$

$$\begin{aligned} MC &= AC \\ 30Q^2 - 200Q + 300 &= 10Q^2 - 100Q + 300 \\ 20Q^2 &= 100Q \\ Q^* &= 5 \end{aligned}$$

Then, we can plug in our Q^* into either equation to find price:

$$P^* = MC(Q^*) = AC(Q^*) = 50$$

(d) We plug our P^* into the market demand function:

$$Q_D^* = 5,000 - 90(50) = 500$$

(e) The number of firms is given by:

$$N^* = \frac{Q_D^*}{Q^*} = \frac{500}{5} = 100$$

(f) The increased demand will lead to a new short-run equilibrium where price and quantity both increase. As a result, firms will earn positive profit in the short-run. In the long-run, firms will enter the martket to try and capture those positive profits which will push prices downward until it reaches $min(AC)$ again. So, in the long-run equilibrium, N^* will increase, Q^* will increase, but P^* will remain the same because costs did not change.

4. [30 points] Two firms are engaged in Cournot competition. Firm A has costs according to $C_A = 0.5a^2$ and firm B has costs according to $C_B = 0.75b^2$. Market demand is given by $P = 114 - 0.5Q$ where $Q = a + b$.
   (a) [2 points] What is the name of this type of market? (The answer is not Cournot)
   (b) [4 points] What is firm A's best response function?
   (c) [4 points] What is firm B's best response function?
   (d) [6 points] What is the Nash Equilibrium of this game?
   (e) [8 points] Suppose instead of a simultaneous game (Cournot) the two firms engaged in a sequential game (Stackelberg) where firm B is the follower. What is the Nash Equilibrium of this game?
   (f) [6 points] Does firm B earn higher profits in the Stackelberg game or the Cournot game? If so, why? (Show your work)

Solution:

(a) Oligopoly or Duopoly

(b)

$$BR_a(b) = 57 - 0.25b$$

(c)

$$BR_b(a) = 45.6 - 0.2a$$

(d) For firm A:

$$\begin{aligned} a^* &= BR_a(BR_y) \\ a^* &= 57 - 0.25\,(45.6 - 0.2a^*) \\ a^* &= 48 \end{aligned}$$

For a firm B:

$$\begin{aligned} b^* &= BR_b(a^*) \\ b^* &= 45.6 - 0.2(48) \\ b^* &= 36 \end{aligned}$$

(e) Firm B's profit function is based on firm A's best response function:

$$\begin{aligned} \pi_B &= (114 - 0.5BR_a - 0.5b)b - 0.75b^2 \\ &= 85.5b - 1.125b^2 \end{aligned}$$

Then the profit maximizing level of b is:

$$\begin{aligned} \frac{\partial \pi_B}{\partial b} &= 85.5 - 2.25b = 0 \\ b^* &= 38 \end{aligned}$$

Plugging b^* into BR_a:

$$a^* = 57 - 0.25(38) = 47.5$$

(f) To find the market price in both games, plug the equilibrium values of a and b into $P(Q)$:

$$P_c = 114 - 0.5(48 + 36) = 72$$
$$P_s = 114 - 0.5(47.5 + 38) = 71.25$$

Profits are then derived by plugging in the prices into the respective profit equation:

For Cournot:

$$\pi_b^c = 36(72) - 0.75(36^2) = 1,620$$

For Stackelberg:

$$\pi_b^s = 38(71.25) - 0.75(38^2) = 1,624.5$$

So firm B earns higher profits in Stackelberg due to their *mover's advantage*. Firm B can crowd out firm A due to their ability to set production first, hence, they can capture more profit.