Rules

Rule	Formula	Derivative	Example
Constant	$y = c$	$\frac{dy}{dx} = 0$	$y = 2 \rightarrow \frac{dy}{dx} = 0$
Power	$y = x^n$	$\frac{dy}{dx} = nx^{n-1}$	$y = 2x^2 \rightarrow \frac{dy}{dx} = 4$
Sum (Difference)	$y = u + v$	$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$	$y = 2x + 2x^3 \rightarrow \frac{dy}{dx} = 2 + 6x$
Product	$y = uv$	$\frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$	$y = (x+1)x^2 \rightarrow \frac{dy}{dx} = (x+1)2x + (1)x^2$
Quotient	$y = \frac{u}{v}$	$\frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$	$y = \frac{x+1}{x^2} \rightarrow \frac{dy}{dx} = \frac{x^2(1) - (x+1)(2x)}{x^4}$
Chain	$y = f\left(g(x)\right)$	$\frac{dy}{dx} = f'\left(g(x)\right) \cdot g'(x)$	$y = (2x+1)^3 \rightarrow \frac{dy}{dx} = 3(2x+1)^2 \cdot 2$

Practice

1. A partial derivative is the derivative of a function with two or more variables with respect to one variable. Treat all other variables as constant.

   Find the partial derivative of the following function:

   $$f(x, y) = 3x^3y + y^4 + 2xy^5$$

   (a) With respect to x

   (b) With respect to y

Solution:

(a)

$$f'(x, y) = 3 * 3x^{3-1}y + 0 + 2x^{1-1}y^5$$
$$f'(x, y) = 9x^2y + 2y^5$$

(b)

$$f'(x, y) = 3x^3y^{1-1} + 4y^{4-1} + 5 * 2xy^{5-1}$$
$$f'(x, y) = 3x^3 + 4y^3 + 10xy^4$$

2. The chain rule is used when dealing with a composite function $f(g(x))$ and the derivative is calculated as $f'(g(x)) * g'(x)$.

   Find the derivative of the following functions:

   (a)

   $$f(x) = (3x + 4y)^3$$

   (b)

   $$f(x) = ln(x^2)$$

Solution:

(a) Outside: $(\ )^3$ and Inside: $3x + 4y$

$$f'(x, y) = 3 * (3x + 4y)^{3-1} \cdot 3x^{1-1}$$
$$f'(x, y) = 3(3x + 4y)^2 \cdot 3$$
$$f'(x, y) = 9(3x + 4y)^2$$

(b) Outside: $ln(\ )$ and Inside: x^2

$$f'(x) = \frac{1}{x^2} \cdot 2x^{2-1}$$
$$f'(x) = \frac{1}{x^2} \cdot 2x$$

3. What is the marginal utility of x for the following utility function?

$$U(x, y) = 5x^2y^3$$

Solution:

$$MU_x = 10xy^3$$

4. What is the marginal utility of y for the following utility function?

$$U(x, y) = x^{0.25}y^{0.75}$$

Solution:

$$MU_x = 0.75x^{0.25}y^{-0.25}$$

5. What is the marginal utility of x and y for the following utility function?

$$U(x, y) = 2x^{0.1}y^{0.4}$$

Solution:

$$MU_x = 0.2x^{-0.9}y^{0.4}$$
$$MU_y = 0.8x^{0.1}y^{-0.6}$$