1. A firm has the cost function $C(Q) = 4Q^2 + 12Q + 36$. It operates in a perfectly competitive market.
   (a) At what price will this firm make exactly zero profit?
   (b) What is the firm's profit maximizing quantity if price is \$40? How much profit does the firm make?

Solution:

(a) To find price, we need to find MC and ATC Set them equal to each other

$$MC = \frac{\partial C(Q)}{\partial Q} = 8Q + 12 \qquad ATC = \frac{C(Q)}{Q} = 4Q + 12 + \frac{36}{Q}$$

Set them equal to each other

$$\begin{aligned} MC = 8Q + 12 &= 4Q + 12 + \frac{36}{Q} = ATC \\ 4Q &= \frac{36}{Q} \\ 4Q^2 &= 36 \\ Q &= 3 \end{aligned}$$

We use the fact that MC = P in perfect competition to find P

$$P = MC = 8Q + 12 = 8(3) + 12 = 36$$

(b) Now we know P = 40. We can use the MC equation to find Q

$$\begin{aligned} 40 &= 8Q + 12 \\ 28 &= 8Q \\ \frac{28}{8} = 3.5 &= Q \end{aligned}$$

To find the firm's profit at this price, we use the profit equation

$$\begin{aligned} \pi &= Revenue - Cost \\ &= P \cdot Q - C(Q) \\ &= 40Q - (4Q^2 + 12Q + 36) \\ &= 40(3.5) - 4(3.5)^2 - 12(3.5) - 36 \\ &= 140 - 49 - 42 - 36 \\ &= 13 \end{aligned}$$

2. For the Cost Function $C(Q) = 3Q^2 + 35$ find the Marginal Cost, Average Fixed Cost, Average Variable Cost, and Average Total Cost. What price and quantity meet the zero profit condition?

Solution:

$$MC = 9Q, \quad AFC = \frac{35}{Q}, \quad AVC = 3Q^2, \quad ATC = 3Q + \frac{35}{Q}$$

Zero profit condition means that $MC = ATC = min\{ATC\} = P$

$$\begin{aligned} MC &= ATC \\ 9Q &= 3Q + \frac{35}{Q} \\ 6Q &= \frac{35}{Q} \\ 6Q^2 &= 35 \\ Q &= \sqrt{\frac{35}{6}} \end{aligned}$$

Using the fact that $MC = P$, we can "back out" our price

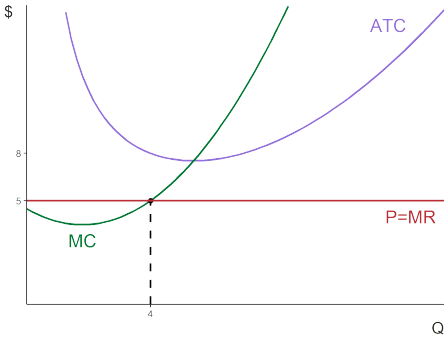
$$P = MC = 9Q = 9\left(\sqrt{\frac{35}{6}}\right)$$

3. For the following Marginal Product of Capital $MP_K = 5K^{1/2}$ what is the amount of capital the firm should rent if the rental rate is 10 and the output good is sold at a price of 20?

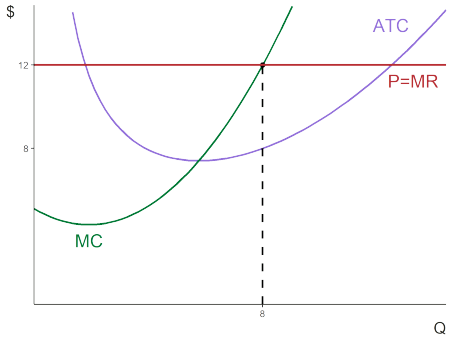
Solution: We first recall that each input will be paid the value of their marginal productivity $\rightarrow P \cdot MP_K = r$

$$\begin{aligned} P \cdot MP_K &= w \\ 20 \cdot 5K^{-1/2} &= 10 \\ 100K^{-1/2} &= 10 \\ K^{-1/2} &= \frac{1}{10} \\ K^{1/2} &= 10 \\ K &= 100 \end{aligned}$$

4. For each graph below, write whether the firm will be receiving positive, negative or zero profits.



(a) Negative



(b) Positive