Production

EC 311 - Intermediate Microeconomics

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Outline

Chapter 6

- Topics
  - Producer Theory Basics (6.1)
  - Cost Minimization Problems (6.4)
  - Returns to Scale (6.5)

Demand & Now Supply

This course boils down to two main things:

Consumers buy things

- We call that **Demand**
- We just slayed that beast

Firms make things

- We call this **Supply**
- We are about to meet this beast

Producer Theory Basics

Firms: What do They Make? How Do They Make It? Let's Find Out!

Let's determine the role of the firm

- Firms produce **goods (output)** to sell consumers
- To produce **output**, firms need **factors of production (inputs)** such as **labor and capital**
- Firms use a **production process (technology)** to transform **inputs** into **output**

Firm Productivity

How firms choose to produce things is guided by a similar principle to how individuals consume things

Marginal Productivity

- **The impact of one additional factor of production on the total amount produced**
- Recall that utility is usually diminishing as consumption increases
- Productivity also diminishes as a firm uses more inputs

The Production Problem

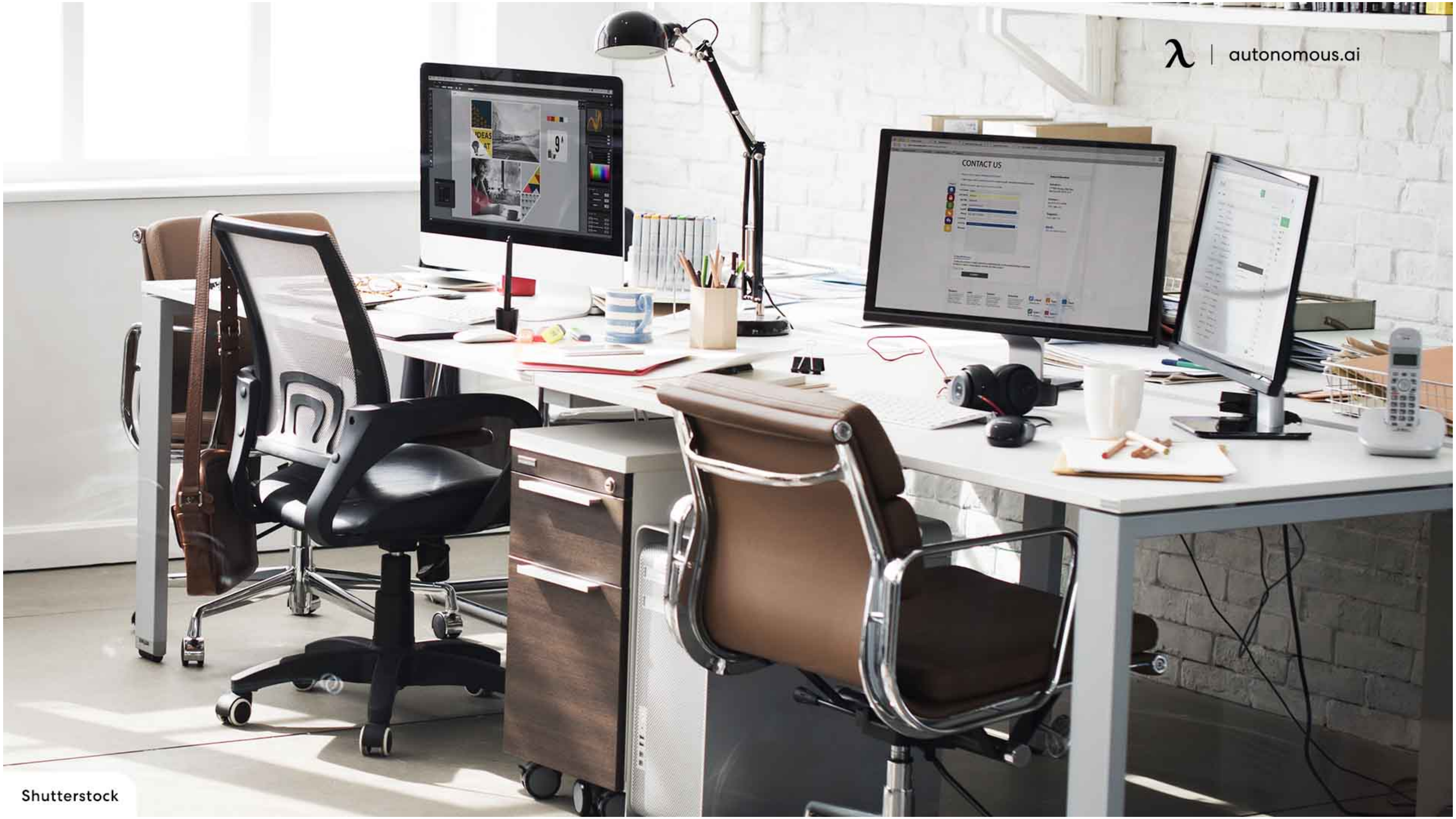
We will begin by making a simplifying assumption to make our lives easier

Firms will have **2 typical inputs**

- **Labor (L)** $\Rightarrow$ Workers
  - The cost of a unit of labor is the **wage** (w) paid
- **Capital (K)** $\Rightarrow$ Factory space, equipment, hardware, etc.
  - The cost of a unit of capital is the **rental rate or interest rate** (r)

Labor Inputs for an Office

Capital Inputs for an Office



Formalizing the Production Problem

Similar to the Budget Constraint from Consumer Theory, we can formalize the firm's costs:

$$\text{Costs:} \;\; w \cdot L + r \cdot K$$

Write the costs of a firm that faces a wage of $10 and a rental rate of $12:

$$\text{Costs:} \;\; 10L + 12K$$

Cost Minimization Problems

How Do Firms Optimize?

Instead of maximizing utility, as consumers do, firms will

MINIMIZE THEIR COSTS OF PRODUCTION

These decisions are also done under a constraint, but what constraints do firms face?

Let's tell a story to see the logic before we jump into the math

Firm Optimization Story Time

Imagine you are the manager of a clothing factory that produces Ducks football jerseys

It is almost Fall and Mr. Nike himself calls you. They tell you **"We need 20,000 jerseys made for the start of the season"**

Your goal is to choose how many **workers** (L) and how much **capital** (L) to use to produce the 20,000 jerseys as cheaply as possible

> How do you figure out how to use L and K to make 20,000 jerseys?
>
> **With a Production Function**

Production Functions

These are a function of how a firm can transform **inputs** into **outputs**

It will work just like a utility function

- In our the jersey example, we have:

$$F(L, K) = Q$$
$$F(L, K) = 20,000$$

Putting it Together

The problem the factory manager solves can be written as

$$\min \quad w \cdot L + r \cdot K \quad s.t. \quad F(L, K) = Q = 20,000$$

We can read this as:

- The firm minimizes their **costs ($w \cdot L + r \cdot K$)** such that you produce a given **quantity** (Q) using **labor** (L) **and capital** (K) with **Production Technology** $F(L, K)$

Let's Give Things Some Values

Now let's say we have the following values:

- Wages are $w = 10$
- Rental rates are $r = 5$
- The factory's Production Function is $F(L, K) = 5L + 2K$

The problem becomes:

$$\min \quad 10L + 5K \quad s.t. \quad F(L, K) = 5L + 2K = 20,000$$

Solving Cost Minimization Problems - New Terms

The methods to solve these problems are practically the same as we saw in Consumer Theory

However, we need to re-label some things:

- The Marginal Rate of Substitution will not be named **Marginal Rate of Technological Substitution (MRTS)**
  - This has a similar interpretation as before:
    - **What is the firm's willingness to trade Labor for Capital**
- Now, instead of using the ratio of marginal utilities, we will use the **ratio of marginal productivities**

$$\text{MRTS} = \frac{MP_L}{MP_K} = \frac{\frac{\partial F}{\partial L}}{\frac{\partial F}{\partial K}} = \frac{\text{Marginal Productivity of Labor}}{\text{Marginal Productivity of Capital}}$$

Let's Solve the Jersey Problem

$$\min \quad 10L + 5K \quad s.t. \quad F(L,K) = 5L + 2K = 20,000$$

The solution should look very familiar. We are trying to choose L^* and K^* that optimizes our productivity.

What type of production function is this?

Perfect Substitutes

Find MRTS and Set Equal to Price Ratio

$$\begin{aligned} \text{MRTS} \; &? \; \text{Price Ratio} \\ \frac{MP_L}{MP_K} \; &? \; \frac{w}{r} \\ \frac{5}{2} \; &? \; \frac{10}{5} \end{aligned}$$

Interpret the MRTS and Price Ratio Relationship

$$\text{MRTS} > \text{Price Ratio}$$

Use only Labor $\rightarrow \; K^* = 0$

Plug into Production Function

$$K^* = 0 \;\Rightarrow\; F(L,K) = 20,000 \;\rightarrow\; 5L + 2(0) = 20,000 \;\rightarrow\; L^* = \frac{20,000}{5} = 4,000$$

Jersey Problem 2

$$\min \quad 10L + 5K \quad s.t. \quad F(L,K) = 5L + 2K = 20,000$$

What if $w = 20$ and $r = 5$?

Find MRTS and Set Equal to Price Ratio

$$\begin{aligned} \text{MRTS} \quad &? \quad \text{Price Ratio} \\ \frac{MP_L}{MP_K} \quad &? \quad \frac{w}{r} \\ \frac{5}{2} \quad &? \quad \frac{20}{5} \end{aligned}$$

Interpret the MRTS and Price Ratio Relationship

$$\text{MRTS} < \text{Price Ratio}$$

Use only Capital $\rightarrow \quad L^* = 0$

Plug into Production Function

$$K^* = 0 \;\Rightarrow\; F(L,K) = 20,000 \;\rightarrow\; 5(0) + 2K = 20,000 \;\rightarrow\; K^* = \frac{20,000}{2} = 10,000$$

Production Function Forms

We have the exact same functional forms that we used for utility functions

- Cobb-Douglas
- Quasi-Linear
- Perfect Substitutes
- Perfect Complements

This is good news! It means that mathematically nothing should be suprising

We are just relabeling variables and using the same processes we already know

Production Function Example

Identify and find the MRTS for the following Production Function

$$F(L,K) = L^{1/3}K^{2/3}$$

$$\text{MRTS} = \frac{MP_L}{MP_K} = \frac{1/3 \cdot L^{-2/3}K^{2/3}}{2/3 \cdot L^{1/3}K^{-1/3}} = \frac{1/3}{2/3} \cdot \frac{K^{2/3}K^{1/3}}{L^{1/3}L^{2/3}} = \frac{1}{2} \cdot \frac{K}{L}$$

Solving Cost Minimization Problems

We will deal with each functional form the same way as we did before.

Cobb-Douglas

- Set MRTS equal to Price Ratio
- This tells us the relationship that must hold between L and K (Optimality Conditions)
- Plug Optimality into Production Function constraint

Quasi-linear

- Set MRTS equal to Price Ratio
- This tells you exactly how much of the input that is inside the $ln()$ function to use
- Plug Optimality into Production Function constraint

Perfect Complements

- Enforce the No-Waste Condition

Perfect Substitutes

Compare the MRTS to the Price Ratio

- If MRTS is larger, use only labor (L)
- If MRTS is smaller, use only capital (K)

Cost Minimization Example

$$\min \quad 10L + 5K \quad s.t. \quad F(L, K) = L \cdot K = 20,000$$

Find the Optimal Values of Labor and Capital

Find MRTS and set it equal to Price Ratio

$$\text{MRTS} = \frac{w}{r}$$
$$\frac{K}{L} = \frac{10}{5} = 2$$
$$K^* = 2L$$

Plug Optimality Condition into Production Function constraint

$$L \cdot K^* = 20,000$$
$$L \cdot 2L = 20,000$$
$$2L^2 = 20,000$$
$$L^2 = 10,000$$
$$L^* = \sqrt{10,000}$$
$$L^* = 100$$

Find Optimal Capital

$$K^* = 2 \cdot L^* = 2 \cdot 100 = 200$$

Perfect Substitutes Example

Let the firm's production function be $F(L, K) = L + 2K$. What are the cost minimizing L and K to produce 100 goods, when they face $w = 10$ and $r = 10$

Find MRTS and Compare to Price Ratio

$$\begin{aligned} \text{MRTS} &= \frac{w}{r} \\ \frac{MP_L}{MP_K} &= \frac{10}{10} \\ \frac{1}{2} &\lesseqgtr 1 \end{aligned}$$

Determine which is greater

$$\frac{1}{2} < 1$$

Use only Capital$\rightarrow$ $L^* = 0$

Plug into Production Function to Determine K^*

$$\begin{aligned} F(L, K) &= Q \\ L^* + 2K^* &= 100 \\ 0 + 2K^* &= 100 \\ K^* &= 50 \end{aligned}$$

What's Actually Different Then?

Although the problem we are solving is essentially the same, the levers we are pulling are not

Let's introduce some new (but familiar) concepts:

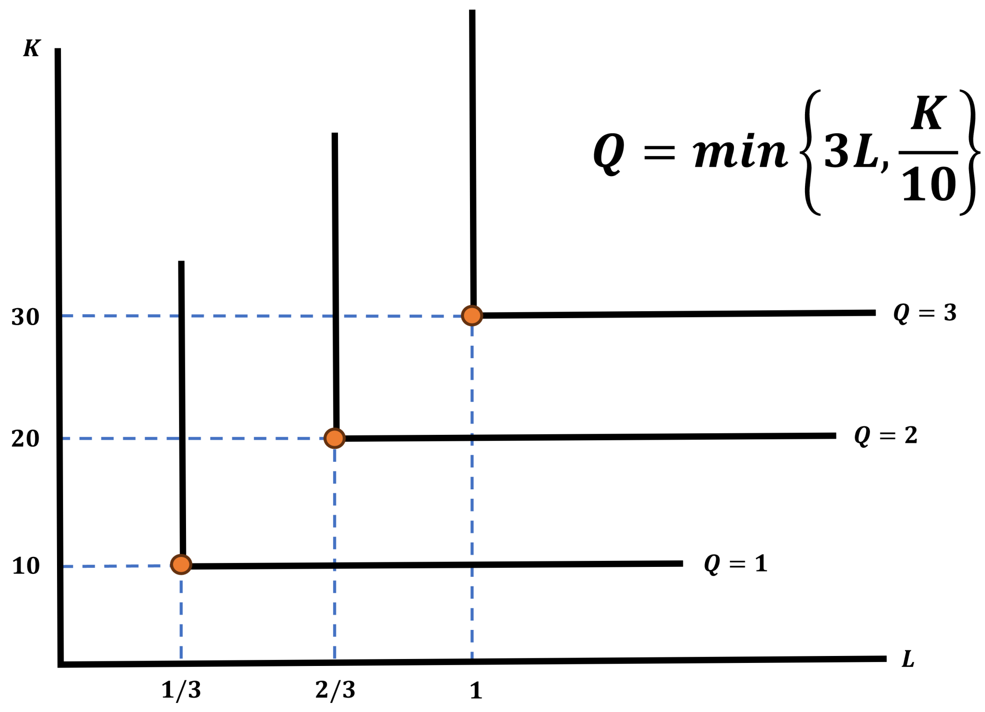
- Production Functions have **Isoquants** instead of Indifference Curves
- **Isoquants** are all the possible combinations of **labor and capital** that produce a certain level of output
- Fortunately, they have the same shape as their Indifference Curves but instead of a level of Utility, they represent a level of quantity produced

Isoquants

Imagine that it takes exactly 20 minutes of labor (1/3 of an hour) **AND** 10 units of capital to make one Ducks Jersey

What form does this Production Function take?

Perfect Complements



What's Actually Different? The Process

The key difference is a conceptual one:

- For consumers, we would **maximize the Utility Function, where the costs acted as our constraint**
- For producers, we **minimize the cost function and the production function is the constraint**
    - Additionally, we will call this cost function an **Isocost line**

> **We are looking for the lowest possible Isocost line that touches the production contraint exactly once**

Understanding the Differences Visually

What is the Same?

Some things have not changed

- The slope of the isoquant is the negative MRTS (-MRTS)
- The MRTS tells us the firm's willingness to trade away capital to get another unit of labor
- We still have a price ratio: $\frac{w}{r}$

Returns to Scale

Extra Property of the Production Function

The largest mathematical difference between production and utility are **Returns to Scale**

With utility we were “measuring” units of happiness or utility

- But what is 1 unit of utility? No clue

Production, however, is more easily measured:

One unit of production or Q can be:

- A Ducks jersey
- A Chocolate Bar
- A car
- Etc.

What Are Returns to Scale?

Returns to Scale will measure the following:

If I **increase my inputs by equal amounts (such that labor and capital increase by some constant z)**, how much does my **output increase by**?

There are three possible outcomes:

- Decreasing Returns to Scale (DRS)
- Constant Returns to Scale (CRS)
- Increasing Returns to Scale (IRS)

Returns to Scale Example

Let's say you run a small business where you make corndogs. You are currently employing 10 **labor hours** and 100 **units of capital**

All together, these inputs help you produce 20 Corndogs

Now you double your inputs, such that:

- Labor Hours $10 \Rightarrow 20$
- Units of Capital $100 \Rightarrow 200$
  - Now you produce 30 Corndogs

What type of Returns to Scale do you experience?

Decreasing Returns to Scale

Returns to Scale: Mathematically

As usual, we can show these concepts mathematically

- **Decreasing Returns to Scale**

$$F(zL, zK) > z \cdot F(L, K)$$

- **Constant Returns to Scale**

$$F(zL, zK) = z \cdot F(L, K)$$

- **Increasing Returns to Scale**

$$F(zL, zK) < z \cdot F(L, K)$$

Let's Prove Returns to Scale

Let your Production Function be $F(L, K) = L^2K$ and you increase your inputs by some constant z

$$
\begin{aligned}
F(zL, zK) &= (zL)^2 \cdot zK \\
&= \color{red}{z^2} L^2 \cdot \color{red}{z}K \\
&= \color{red}{z^3} \cdot L^2K
\end{aligned}
$$

Compare this to what scaling your production function by z looks like

$$z^3 \cdot L^2K > z \cdot L^2K$$

We have Increasing Returs to Scale (IRS)

Returns to Scale Example

Let the Production Function be $F(L, K) = L^{1/4}K^{3/4}$ and you increase your inputs by some constant z

Show what type of Returns to Scale you have

$$\begin{aligned} F(zL, zK) &= (zL)^{1/4}(zK)^{3/4} \\ &= z^{1/4}L^{1/4} \cdot z^{3/4}K^{3/4} \\ &= z^{1/4}z^{3/4}L^{1/4}K^{3/4} \\ &= z \cdot L^{1/4}K^{3/4} \end{aligned}$$

Compare this to what scaling your production function by z looks like

$$z \cdot L^{1/4}K^{3/4} = z \cdot L^{1/4}K^{3/4}$$

We have Constant Returns to Scale (CRS)

Returns to Scale Example

Let the Production Function be $F(L,K) = L^{1/3}K^{1/2}$ and you increase your inputs by some constant z

Show what type of Returns to Scale you have

$$\begin{aligned} F(zL, zK) &= (zL)^{1/3}(zK)^{1/2} \\ &= z^{1/3}L^{1/3} \cdot z^{1/2}K^{1/2} \\ &= z^{5/6} \cdot L^{1/3}K^{1/2} \end{aligned}$$

Compare this to what scaling your production function by z looks like

$$z^{5/6} \cdot L^{1/3}K^{1/2} < z \cdot L^{1/3}K^{1/2}$$

We have Decreasing Returs to Scale (DRS)

Goal of Cost Minimization

Producers have a target quantity they must achieve

Their goal is to do so in the cheapest form possible using their inputs and their production technology

When choosing quantities, they are also aware of how much their inputs will cost them

All this together means that they will find the **minimum cost line** to achieve a target quantity of goods

- Later we will find the supply function much like we found the demand function for consumers