1. Why must budget constraints be binding?

   A. We do not model savings so we would never save

   B. We maximize utility and more goods bought = more utility

   C. Money has no value

   D. Money loses value so it will be worthless tomorrow

2. Barry's income decreases from \$10,000 to \$5,000, so he increases his weekly consumption of light beer from 5 to 6. Based on his income elasticity of demand, what type of good is light beer?

   A. Inferior

   B. We do not model savings so we would never save

   C. Money has no value

   D. Money loses value so it will be worthless tomorrow

3. Find the utility maximizing amount of each good for the following utility functions subject to budgets $M = P_x X + P_y Y$:

   (a) $U(x, y) = x^{1/2}y^{1/2} \quad s.t. \quad 120 = 4x + y$

   (b) $U(x, y) = \alpha ln(x) + y \quad s.t. \quad M = P_x x + P_y y$

   (c) $U(x, y) = min\{2x, y\} \quad s.t. \quad 16 = 2x + y$

   (d) $U(x, y) = 4x + 5y \quad s.t. \quad 10 = 2x + 3y$

Solution:

(a) $x^* = \frac{1}{2} \cdot \frac{120}{4} = 15 \qquad \& \qquad y^* = \frac{1}{2} \cdot \frac{120}{1} = 60$

(b) $x^* = \frac{\alpha}{x} = \frac{4}{1} \to 4x = \alpha \to x^* = \frac{\alpha}{4} \qquad \& \qquad 120 = 4\left(\frac{\alpha}{4}\right) + y \to y^* = 120 - \alpha$

(c) $2x = y \to 16 = 2 \cdot x + 2x \to x^* = 4 \qquad \& \qquad y^* = 2(4) = 8$

(d) $MRS = \frac{P_x}{P_y} \to \frac{4}{5} = \frac{2}{3} \to MRS > Price\, Ratio \to y^* = 0 \to x^* = 5$

4. Harvey's utility is given by $U(x, y) = 10x^{0.35}y^{1.3}$. Does Harvey exhibit diminishing marginal utility in x? What about y? Show your work

Solution: Diminishing marignal utility implies the second derivative must be negative. For x:

$$MU_x = 3.5x^{-0.65}y^{1.3}$$
$$\frac{\partial MU_x}{\partial x} = -2.275x^{-}1.65y^{1.3} < 0$$

The second derivative with respect to x is negative, hence, it exhibits diminishing marginal utility. Looking at y:

$$MU_y = 13x^{0.35}y^{0.3}$$
$$\frac{\partial MU_y}{\partial y} = 3.9x^{0.35}y^{-0.7} > 0$$

Second derivative with respect to y is positive, hence, it does NOT exhibit diminishing marginal utility.

5. Suppose you only consume two goods: x and y. If y is an inferior good, what type of good must x be? Explain why.

Solution: If y is an inferior good, that means y^* will decrease when income increases. This means that x^* must be a normal good as it would have to increase to accommodate for the larger income available. If x^* were to decreases as well (being an inferior good), then we would not be spending our entire budget, hence, we are not optimizing/maximizing utility.

6. Consider the demand function $x^* = M - P_x^2 + P_y^{0.5}$
   (a) Is X a normal or inferior good? Use a derivative and an inequality to show it.
   (b) Is X a substitute or a complement for Y? Use a derivative and an inequality to show it
   (c) Assume that $M = 10$ and $P_y = 4$. Graph the demand curve for X by plotting the points where $P_x = 1, 2$ and 3 and connecting them. Label this curve x^*. I recommend giving yourself lots of extra room on the horizontal axis so that you can add the next part clearly.
   Add to the graph a market demand curve, assuming that there are 3 total consumers in the market. There's no need to derive the demand curve, just remember the right way to add up demand in the graph. Label this curve Q_D.

Solution:

(a)

$$\frac{\partial x^x}{\partial M} = 1 > 0 \Rightarrow Normal$$

(b)

$$\frac{\partial x^*}{\partial P_y} = 0.5 \cdot P_y^{-0.5} > 0 \Rightarrow Substitute$$