Production

EC 311 - Intermediate Microeconomics

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Outline

Chapter 6

- Topics
  - Producer Theory Basics (6.1)
  - Short-run vs Long-run (6.2, 6.3)
  - Cost Minimization Problems (6.4)
  - Returns to Scale (6.5)
  - Technological Change (6.6)

Big Picture

As economists, we want a model of **market economies** based in **rational choice theory**.

Why?

- Make *positive* predictions about economics consequences of future world-events
- Guide **efficient** production of market goods. I.e., how to make the most of the limited resources we have
- Guide policy makers to craft **socially-efficient** policies. E.g., How to make public funds benefit the maximum amount of people in need?

Big Picture

Our first step was to construct a model of a **rational consumer**.

- This allows us to predict what *will* people do when things change:
  - how income changes affect consumption
  - how price changes affect consumption and substitution
- …without telling making judgements about what they *should* do

Big Picture

But markets are not only influenced by *consumers*, so now we will consider the other side; *firms*, or **producers** of goods.

- If *consumers* care about maximising their happines, what do we think **producers** care about?
  - **Profit!** 💰💰💰
  - Meeting the needs of their consumers
    - could apply even to non-profits like government agencies or NGO's

Big Picture

Why should we care about what big corporations do with their inputs?

- Because as consumers we are affected by the choices they make
- Our models here can be applied to sujects like anti-trust policy, environmental regulations, and other consumer welfare issues!
- We'll start off with assuming perfect competition, full information, no externalities, etc. but by relaxing these assumptions we can apply our simple models to tackle big issues

Why care about industrial production?


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The Guardian

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Climate crisis

Graham Readfearn

Wed 23 Apr 2025 00.00 EDT

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More than 80% of the world's reefs hit by bleaching after worst global event on record

An ashen pallor and an eerie stillness all that remains where there should be fluttering fish and vibrant colours in the reefscape, one conservationist says

2:03

Tracing the worst coral bleaching event in recorded history – video

The world's coral reefs have been pushed into "uncharted territory" by the worst global bleaching event on record that has now hit more than 80% of

Most viewed

- Irish woman living in US for decades detained by immigration officials
- Mob of Orthodox Jewish men chases woman after protest at Brooklyn synagogue
- Final autopsy results on Gene Hackman and his wife, Betsy Arakawa, reveal complex health issues
- Tyrants like Trump always fall – and we can already predict how he will be dethroned Simon Tisdall
- Tens of millions across Spain and Portugal hit by huge power outage

Big Picture

Before we deal with the more interesting examples of **market failures**, we'll need to understand how the market works under optimal conditions:

- **perfect information**
- **many producers**
- **consumers internalize all costs/benefits privately** (no externalities)

Producer Theory Basics

Firms: What do They Make? How Do They Make It? Let's Find Out!

Let's determine the role of the firm

- Firms produce **goods (output)** to sell consumers
- To produce **output**, firms need **factors of production (inputs)** such as **labor and capital**
- Firms use a **production process (technology)** to transform **inputs** into **output**

Firm Productivity

How firms choose to produce things is guided by a similar principle to how individuals consume things

Marginal Productivity

- **The impact of one additional factor of production on the total amount produced**
- Recall that utility is usually diminishing as consumption increases
- Productivity also diminishes as a firm uses more inputs

Production Assumptions

Just like we made simplifying assumptions about *consumers*, we'll need some ground rules for **producers**:

1. Single production good
2. Product choice is fixed
3. Cost minimization
4. Capital is fixed in short-run
5. More inputs $\longrightarrow$ more outputs
6. Diminishing marginal returns
7. Firm's aren't budget constrained

I'll tackle each one in a slightly different order, but this is how they appear in the textbook

Assumption 1: Single Output

In our models, each type of firm only produces one type of good.

- I.e., Shoe companies just produce generic shoes, we don't care about different styles

Production Functions

In mathematical language: a **production function** takes inputs to create a quantity of output:

$$f(x_1, \ldots, x_n) : \mathbb{R}^n \to \mathbb{R}$$

- Just like *utility functions*, **production functions** can take multiple inputs

Production Functions

We will only consieder production with **two** inputs at a time:

$$Q = f(K, L)$$

This production function means that a firm produces Q units of it's output good using:

- K: **Captial** (machines, buildings, computers, etc.)
- L: **Labor** - the human workers who use the capital
    - Units could be in number of workers, labor hours, etc.

Assumption 2: Product Choice is Fixed

We will assume that all potential producing firms in a market have already decided on their single product.

- We don't consider e.g., Amazon going from ecommerce to cloud services

Assumption 5: More Inputs -> More Output

For simplicity, we assume that firms can always achieve higher production levels by using more inputs

- $(f(K,L)$ is **monitonically increasing** in K, L ($\frac{df}{dK}, \frac{df}{dL} > 0$)
- similar to our "more is better" assumption of consumer utility

Assumption 6: Diminishing Returns

More capital and labor always leads to more production, *but* those gains will fade out the more you have already used

- $\frac{df}{dK^2}, \frac{df}{dL^2} < 0$
- If graphed on the *input-product* axes, the production function will be **concave**.

The Production Problem

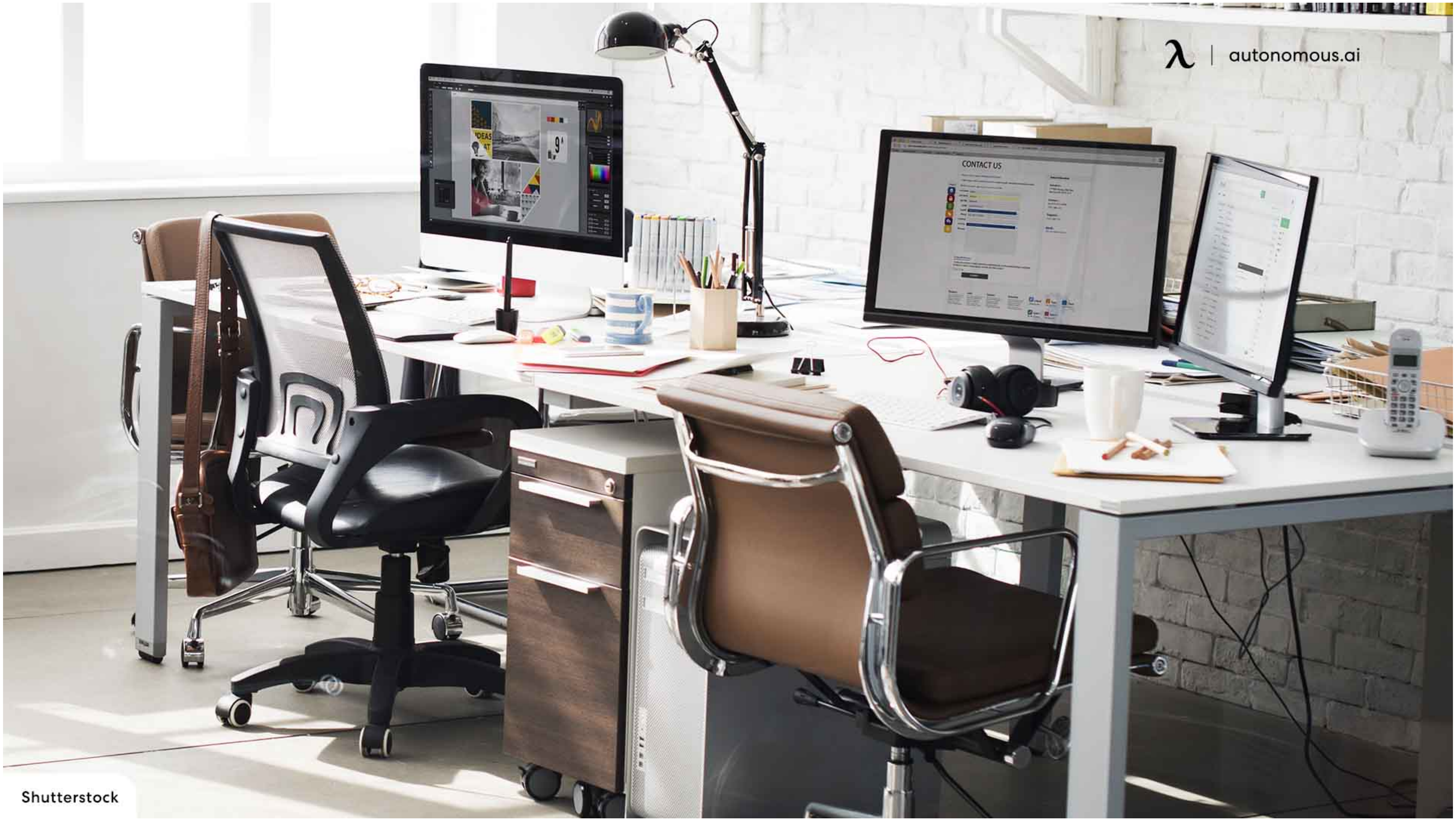
We will begin by making a simplifying assumption to make our lives easier

Firms will have **2 typical inputs**

- **Labor (L)** $\Rightarrow$ Workers
  - The cost of a unit of labor is the **wage** (w) paid
- **Capital (K)** $\Rightarrow$ Factory space, equipment, hardware, etc.
  - The cost of a unit of capital is the **rental rate or interest rate** (r)

Labor Inputs for an Office

Capital Inputs for an Office



Formalizing the Production Problem

Similar to the Budget Constraint from Consumer Theory, we can formalize the firm's costs:

$$\text{Costs: } \; w \cdot L + r \cdot K$$

Write the costs of a firm that faces a wage of \$10 and a rental rate of \$12:

$$\text{Costs: } \; 10L + 12K$$

Long vs Short Run Production

Assumption 4: Capital is Fixed in the Short-Run

We implicitly define the **short-run** as the time-span where firms don't have enough time to buy more capital.

- As opposed to the **long-run** where capital is assumed to be variable

Assumption 7: Firms aren't Budget Constrained

For the short-run, capital is only constrained by time-frame, not by how much money the firm has.

- In the long-run, the firm can choose any combination of capital *and* labor it wants
- We assume that as long as there is profit to be made, there is a complete credit market that will provide funds

Example: Nike Waffle Trainers

Nike founding story: Bill Bowerman and Phil Knight

$$Q_s^{\text{shoes}} = f(\text{waffle irons, workers}) = \sqrt{KL}$$

Example: Nike Waffle Trainers

Suppose they only have 4 irons to start out with $\bar{K} = 4$, but can hire as many workers as I need.

Irons	Workers	Shoes Produced
4	1	2
4	2	$\sqrt{8} \approx 2.8$
4	3	$\sqrt{12} \approx 3.5$
4	4	4

- What do you notice as the number of workers increases?

Example: Nike Waffle Trainers

The **marginal product of labor** is decreasing!

Irons	**Workers**	**Shoes Produced**	**Marginal Product**
4	0	0	-
4	1	2	2
4	2	$\sqrt{8} \approx 2.8$	0.8
4	3	$\sqrt{12} \approx 3.5$	0.7
4	4	4	0.5

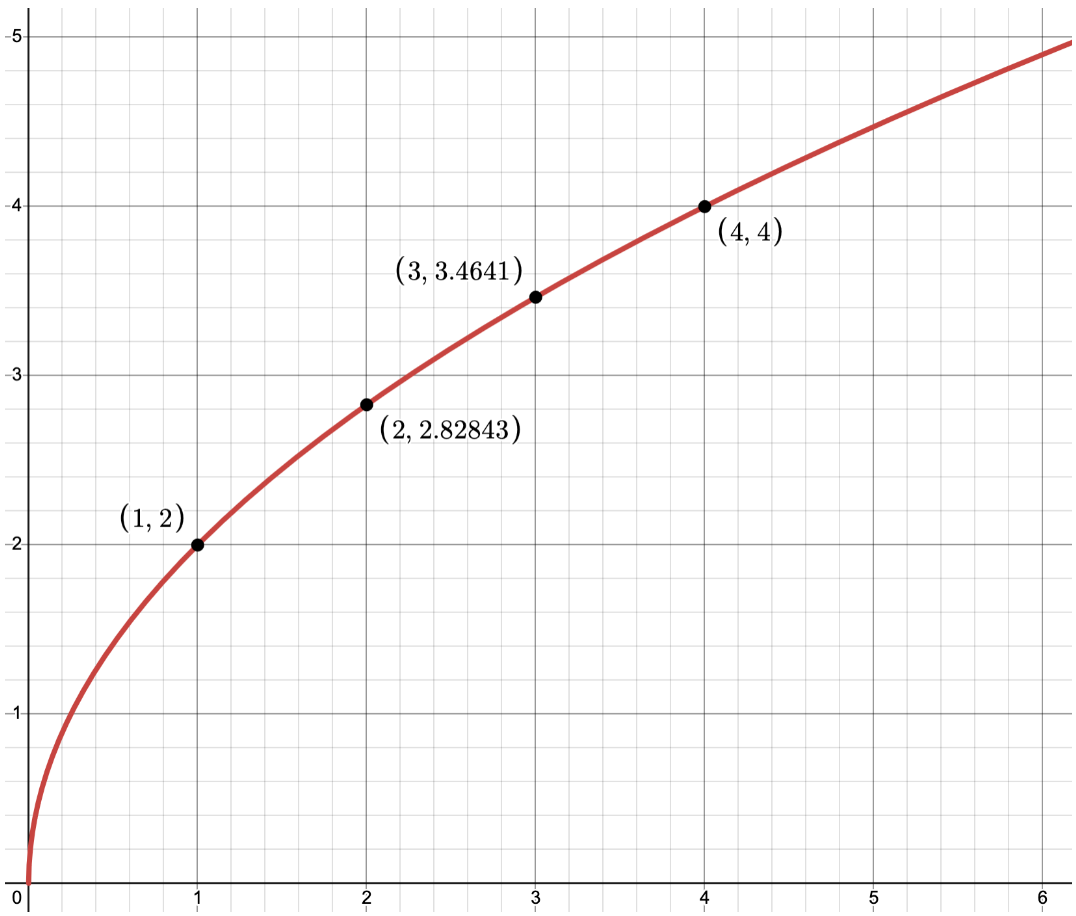
The next worker adds less and less to the total production the more workers we already have in the kitchen

Marginal Product of Labor

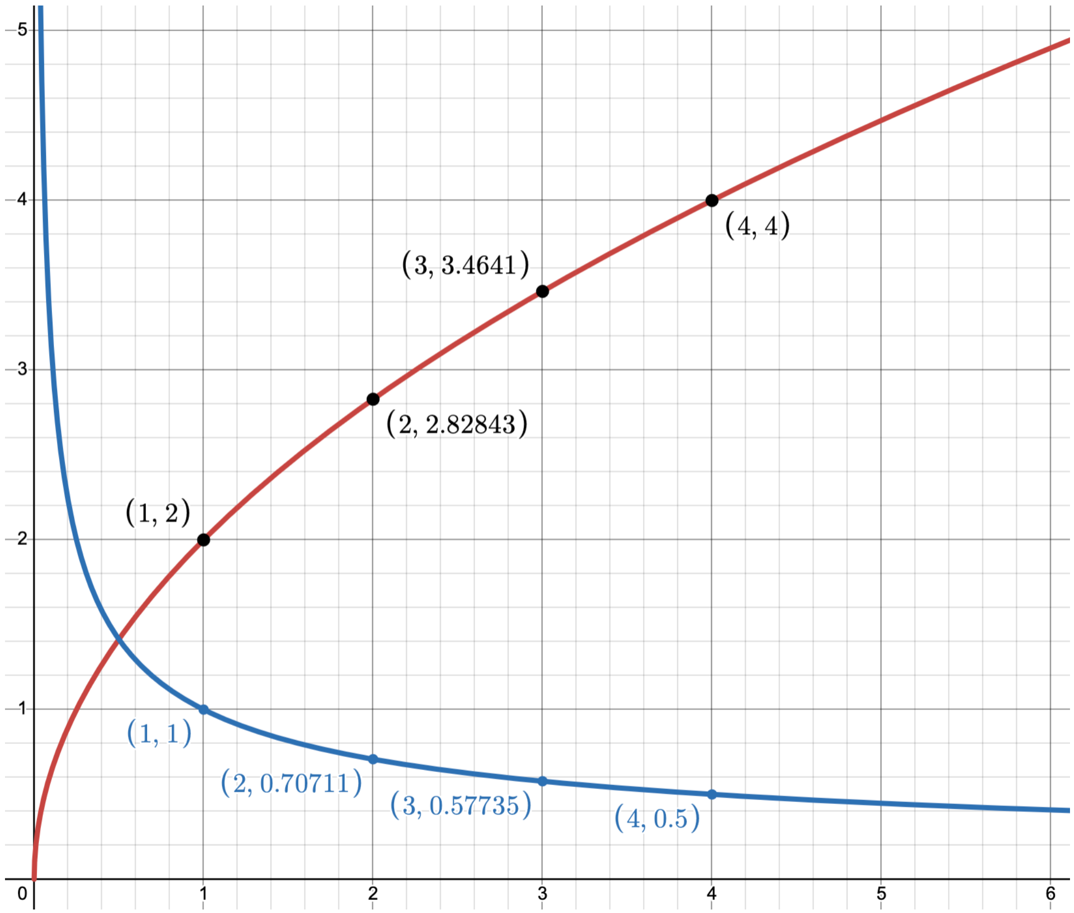
The **Marginal Product of Labor** is the change in the quantity of production caused by a change in the quantity of labor as an input.

- Given a production function $f(K, L)$, how could you find an expression which represents this change *for any quantity change* of labor?
- Take the partial derivative $\frac{df}{dL}$ holding capital fixed!
- This is the slope of our **short-run production function**!

Graphing the Short-Run Production Function and Marginal Product



Graphing the Short-Run Production Function and Marginal Product



Average Product of Labor

Average Product is the quantity of output produced *per unit of input*.

$$AP_L = Q/L$$

- The **average product** is related to the **marginal product**, but not the same

$$AP_L = Q/L$$

Average Product of Labor

Irons	Workers	Shoes Produced	Marginal Product	Average Product
4	0	0	-	-
4	1	2	2	2
4	2	$\sqrt{8} \approx 2.8$	0.8	≈ 1.4
4	3	$\sqrt{12} \approx 3.5$	0.7	≈ 1.2
4	4	4	0.5	1

Assumption 4: Capital is Variable in the Long-Run

The flip side of assumption 4: the **Long-Run** is the time when Capital is not fixed

- Our Long-Run Production Function:

$$Q = f(K, L)$$

- The same as the SR version, but now K is treated as a *variable*, not a *constant*

Long-Run Production Functions

	$L = 1$	$L = 2$	$L = 3$	$L = 4$
$K = 1$	1.00	1.41	1.73	2.00
$K = 2$	1.41	2.00	2.45	2.83
$K = 3$	1.73	2.45	3.00	3.46
$K = 4$	2.00	2.83	3.46	4.00

What are some tools we have already learned that will help us analyze *multi-variable functions*?

Cost Minimization Problems

Assumption 3: Cost Minimization

Instead of trying to *maximize production*, firms try to *minimize costs* while trying to meet a specific level of production.

$$\text{minimize} W \cdot L + R \cdot K$$
$$\text{subject to } f(K, L) = Q$$

- Notice how this is the flip side of the constrained *maximization* problem?

Setting up Cost-Minimization

What are some questions we could ask using the math tools we have learned?

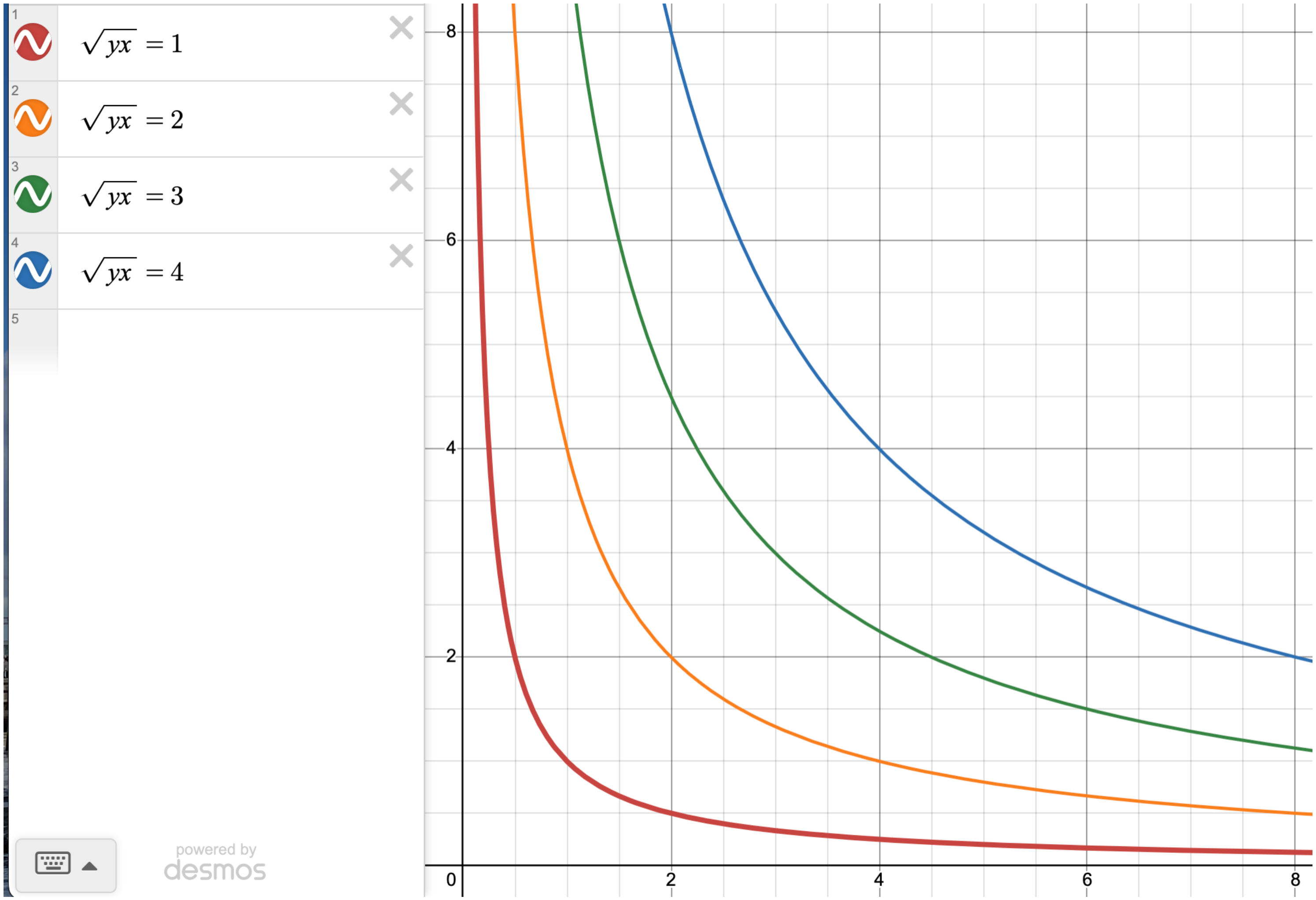
- What combinations of *inputs* produce the same quantity of output
- How do firm's choose to *substitute* one input for another?
- How do changes in *input costs* affect production?

Isoquants

What do these production functions look like graphically?

- **isoquants** are sets of bundles of inputs that produce the same level of output
- Visually similar to *indifference curves*, but conceptually different

Isoquants



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Marginal Products

How much does one more worker (or unit of input) contribute towards production?

$$MP_L = \frac{\Delta f}{\Delta L}$$

$$MP_K = \frac{\Delta f}{\Delta K}$$

Marginal Rate of Technical Substitution

How many units of capital would the firm have to give up to use one more worker, **when holding output constant**

$$MRTS_{LK} = \frac{MP_L}{MP_K}$$

Production Function Example

Identify and find the MRTS for the following Production Function

$$F(L,K) = L^{1/3}K^{2/3}$$

$$\text{MRTS} = \frac{MP_L}{MP_K} = \frac{1/3 \cdot L^{-2/3}K^{2/3}}{2/3 \cdot L^{1/3}K^{-1/3}} = \frac{1/3}{2/3} \cdot \frac{K^{2/3}K^{1/3}}{L^{1/3}L^{2/3}} = \frac{1}{2} \cdot \frac{K}{L}$$

Relationships Between Inputs

Apply the concepts of **substitute** and **complement** goods from *consumer* theory to production:

- Can you name two inputs which could be **perfect substitutes** in production?
- What about **perfect complements**?
- Some inputs with both substitution and complementarities?

Production Function Forms

We have the exact same functional forms that we used for utility functions

- Cobb-Douglas
- Quasi-Linear
- Perfect Substitutes
- Perfect Complements

This is good news! It means that mathematically nothing should be suprising

We are just relabeling variables and using the same processes we already know

Icocosts

How do we describe all input combinations that cost the same to the firm?

$$C = RK + WL$$

- R is the **rental rate** of capital
- W is the **wage rate** paid to workers

Firm Optimization Story Time

Imagine you are the manager of a clothing factory that produces Ducks football jerseys

It is almost Fall and Mr. Nike himself calls you. They tell you **"We need 20,000 jerseys made for the start of the season"**

Your goal is to choose how many **workers** (L) and how much **capital** (L) to use to produce the 20,000 jerseys as cheaply as possible

> How do you figure out how to use L and K to make 20,000 jerseys?
>
> **With a Production Function**

Production Functions

These are a function of how a firm can transform **inputs** into **outputs**

It will work just like a utility function

- In our the jersey example, we have:

$$F(L, K) = Q$$
$$F(L, K) = 20,000$$

Putting it Together

The problem the factory manager solves can be written as

$$\min \quad w \cdot L + r \cdot K \quad s.t. \quad F(L, K) = Q = 20,000$$

We can read this as:

- The firm minimizes their **costs** $(w \cdot L + r \cdot K)$ such that you produce a given **quantity** (Q) using **labor** (L) **and capital** (K) with **Production Technology** $F(L, K)$

Let's Give Things Some Values

Now let's say we have the following values:

- Wages are $w = 10$
- Rental rates are $r = 5$
- The factory's Production Function is $F(L, K) = L \cdot K$

The problem becomes:

$$\min \quad 10L + 5K \quad s.t. \quad F(L, K) = K \cdot K = 20,000$$

Cost Minimization Example

$$\min \quad 10L + 5K \quad s.t. \quad F(L,K) = L \cdot K = 20,000$$

Find the Optimal Values of Labor and Capital

Find MRTS and set it equal to Price Ratio

$$\text{MRTS} = \frac{w}{r}$$
$$\frac{K}{L} = \frac{10}{5} = 2$$
$$K^* = 2L$$

Plug Optimality Condition into Production Function constraint

$$L \cdot K^* = 20,000$$
$$L \cdot 2L = 20,000$$
$$2L^2 = 20,000$$
$$L^2 = 10,000$$
$$L^* = \sqrt{10,000}$$
$$L^* = 100$$

Find Optimal Capital

$$K^* = 2 \cdot L^* = 2 \cdot 100 = 200$$

Perfect Substitutes Example

Let the firm's production function be $F(L,K) = L + 2K$. What are the cost minimizing L and K to produce 100 goods, when they face $w = 10$ and $r = 10$

Find MRTS and Compare to Price Ratio

$$\begin{aligned} \text{MRTS} &= \frac{w}{r} \\ \frac{MP_L}{MP_K} &= \frac{10}{10} \\ \frac{1}{2} &\lesseqgtr 1 \end{aligned}$$

Determine which is greater

$$\frac{1}{2} < 1$$

Use only Capital→ $L^* = 0$

Plug into Production Function to Determine K^*

$$\begin{aligned} F(L,K) &= Q \\ L^* + 2K^* &= 100 \\ 0 + 2K^* &= 100 \\ K^* &= 50 \end{aligned}$$

What's Actually Different Then?

Although the problem we are solving is essentially the same, the levers we are pulling are not

- Production Functions have **Isoquants** instead of Indifference Curves
- Fortunately, they have the same shape as their Indifference Curves but instead of a level of Utility, they represent a level of quantity produced
- Unlike utility, *production* is a **cardinal** value, with meaningful units

What’s Actually Different? The Process

The key difference is a conceptual one:

- For consumers, we would **maximize the Utility Function, where the costs acted as our constraint**
- For producers, we **minimize the cost function and the production function is the constraint**
  - Additionally, we will call this cost function an **Isocost line**

> **We are looking for the lowest possible Isocost line that touches the production contraint exactly once**

What is the Same?

Some things have not changed

- The slope of the isoquant is the negative MRTS (-MRTS)
- The MRTS tells us the firm's willingness to trade away capital to get another unit of labor
- We still have a price ratio: $\frac{w}{r}$

Returns to Scale

Extra Property of the Production Function

The largest mathematical difference between production and utility are **Returns to Scale**

With utility we were “measuring” units of happiness or utility

- But what is 1 unit of utility? No clue

Production, however, is more easily measured:

One unit of production or Q can be:

- A Ducks jersey
- A Chocolate Bar
- A car
- Etc.

What Are Returns to Scale?

Returns to Scale will measure the following:

If I **increase my inputs by equal amounts (such that labor and capital increase by some constant z)**, how much does my **output increase by**?

There are three possible outcomes:

- Decreasing Returns to Scale (DRS)
- Constant Returns to Scale (CRS)
- Increasing Returns to Scale (IRS)

Returns to Scale Example

Let's say you run a small business where you make corndogs. You are currently employing 10 **labor hours** and 100 **units of capital**

All together, these inputs help you produce 20 Corndogs

Now you double your inputs, such that:

- Labor Hours $10 \Rightarrow 20$
- Units of Capital $100 \Rightarrow 200$
  - Now you produce 30 Corndogs

What type of Returns to Scale do you experience?

Decreasing Returns to Scale

Returns to Scale: Mathematically

As usual, we can show these concepts mathematically

- **Decreasing Returns to Scale**

$$F(zL, zK) > z \cdot F(L, K)$$

- **Constant Returns to Scale**

$$F(zL, zK) = z \cdot F(L, K)$$

- **Increasing Returns to Scale**

$$F(zL, zK) < z \cdot F(L, K)$$

Let's Prove Returns to Scale

Let your Production Function be $F(L,K) = L^2 K$ and you increase your inputs by some constant z

$$\begin{aligned} F(zL, zK) &= (zL)^2 \cdot zK \\ &= z^2 L^2 \cdot zK \\ &= z^3 \cdot L^2 K \end{aligned}$$

Compare this to what scaling your production function by z looks like

$$z^3 \cdot L^2 K > z \cdot L^2 K$$

We have Increasing Returs to Scale (IRS)

Returns to Scale Example

Let the Production Function be $F(L, K) = L^{1/4}K^{3/4}$ and you increase your inputs by some constant z

Show what type of Returns to Scale you have

$$\begin{aligned} F(zL, zK) &= (zL)^{1/4}(zK)^{3/4} \\ &= z^{1/4}L^{1/4} \cdot z^{3/4}K^{3/4} \\ &= z^{1/4}z^{3/4}L^{1/4}K^{3/4} \\ &= z \cdot L^{1/4}K^{3/4} \end{aligned}$$

Compare this to what scaling your production function by z looks like

$$z \cdot L^{1/4}K^{3/4} = z \cdot L^{1/4}K^{3/4}$$

We have Constant Returns to Scale (CRS)

Returns to Scale Example

Let the Production Function be $F(L, K) = L^{1/3}K^{1/2}$ and you increase your inputs by some constant z

Show what type of Returns to Scale you have

$$\begin{aligned} F(zL, zK) &= (zL)^{1/3}(zK)^{1/2} \\ &= z^{1/3}L^{1/3} \cdot z^{1/2}K^{1/2} \\ &= z^{5/6} \cdot L^{1/3}K^{1/2} \end{aligned}$$

Compare this to what scaling your production function by z looks like

$$z^{5/6} \cdot L^{1/3}K^{1/2} < z \cdot L^{1/3}K^{1/2}$$

We have Decreasing Returs to Scale (DRS)

Goal of Cost Minimization

Producers have a target quantity they must achieve

Their goal is to do so in the cheapest form possible using their inputs and their production technology

When choosing quantities, they are also aware of how much their inputs will cost them

All this together means that they will find the **minimum cost line** to achieve a target quantity of goods

- Later we will find the supply function much like we found the demand function for consumers