EC 311
Production and Costs
Winter 2024

1. Do the following production functions have increasing, decreasing, or constant returns to scale?

   a.) $F(K,L) = K^{.25}L^{.75}$

   Constant

   b.) $F(K,L) = 10K^{.25}L^{.25}$

   Decreasing

   c.) $F(K,L) = \frac{1}{4}K^{0.5}L^{0.5}$

   Constant

   d.) $F(K,L) = 0.5\,K^2L^2$

   Increasing

2. What is the average product of labor for the following production functions?

   a.) $Y = F(K,L) = 4K^2L^4$

$$\frac{Y}{L} = \frac{4K^2L^4}{L} = 4K^2L^3$$

   b.) $Y = F(K,L) = K^{.25}L^{.75}$

$$\frac{Y}{L} = \frac{K^{.25}L^{.75}}{L} = K^{.25}L^{-.25}$$

3. For the following production function, find the cost function when the rental rate of capital is \$5 and the wage rate for labor is \$5. **Hint:** Find L* and K* first.

$$Q = F(K,L) = L^{\frac{1}{3}}K^{\frac{1}{3}}$$

$$MRTS = \frac{w}{r}$$

$$\frac{{}^{1}\!/_{3}\,L^{\frac{-2}{3}}K^{\frac{1}{3}}}{{}^{1}\!/_{3}\,K^{\frac{-2}{3}}L^{\frac{1}{3}}} = \frac{5}{5}$$

$$\frac{K}{L} = 1 \longrightarrow K = L$$

$$Q = L^{\frac{1}{3}}K^{\frac{1}{3}} = K^{\frac{1}{3}}K^{\frac{1}{3}} = K^{\frac{2}{3}} = Q$$

$$K^{\frac{2}{3}} = Q \longrightarrow K^* = Q^{\frac{3}{2}} = L^*$$

$$C(Q) = wL(Q) + rK(Q) = 5Q^{\frac{3}{2}} + 5Q^{\frac{3}{2}} = 10Q^{\frac{3}{2}}$$

4. Show that a quadratic cost function has increasing and linear marginal costs.

$$C(Q) = 3Q^2 + 35$$

$$MC(Q) = \frac{\partial C(Q)}{\partial Q} = 6Q > 0 \Longrightarrow Increasing\ MC$$

$$\frac{\partial MC(Q)}{\partial Q} = 6 \ \Rightarrow Linear$$

5. Show that a cubic cost function will have an initial decreasing and eventual increasing marginal cost. For what range of Q is MC increasing? Decreasing?

$$C(Q) = 3Q^3 - 10Q^2 + 5Q + 10$$

$$MC(Q) = \frac{\partial C(Q)}{\partial Q} = 9Q^2 - 20Q + 5$$

$$\frac{\partial MC(Q)}{\partial Q} = 18Q - 20$$

$$18Q - 20 > 0\ when\ Q > \frac{10}{9} \Longrightarrow Increasing\ MC$$

$$18Q - 20 > 0\ when\ Q < \frac{10}{9} \Longrightarrow Decreasing\ MC$$