1. Recall Returns to Scale. This tells us how much output is increased relative to how much we increased inputs. For the following scenarios, draw a line to match the expression to the corresponding Returns to Scale expression:

   (a) $F(zL, zK) = z \cdot F(L, K)$ — Increasing

   (b) $F(zL, zK) > z \cdot F(L, K)$ — Decreasing

   (c) $F(zL, zK) < z \cdot F(L, K)$ — Constant

2. Imagine you finally graduate, find a job in your field, and generally life is going great. Your boss, after having read your transcript, sees you took some econ classes. Through some spreadsheets you are able to derive the following production functions:

$$wL + rK = 5L + 5K \qquad s.t. \quad Q = f(L, K) = L^{1/3}K^{1/3}$$

Find your new employer's cost function

Hint: Find L^* and K^* and cost function looks like $C(Q) = wL(Q) + rK(Q)$

I hope this never happens to you but industry is weird

Solution:

$$\text{MRTS} = \frac{1/3\ L^{-2/3}K^{1/3}}{1/3\ L^{1/3}K^{-2/3}} = \frac{w}{r} = \frac{5}{5}$$

$$\frac{K}{L} = 1 \rightarrow K = L$$

Use the Production Function:

$$Q = L^{1/3}K^{1/3} = L^{1/3}L^{1/3} = L^{2/3}$$
$$Q^{3/2} = L = K$$

Now use the cost function equation to find:

$$C(Q) = wL(Q) + rK(Q) = 5Q^{3/2} + 5Q^{3/2} = 10\ Q^{3/2}$$

3. **Cost Functions:** Show why a quadratic cost function will always have increasing and linear marginal costs.

$$C(Q) = 3Q^2 + 35$$

Solution:

$$MC(Q) = \frac{\partial C}{\partial Q} = 6Q$$

So the marginal cost is increasing. To show linear:

$$\frac{\partial MC}{\partial Q} = 6 > 0$$

4. **Cost Functions:** Show why a cubic cost function will have an initial decreasing and eventual increasing marginal cost.

$$C(Q) = 3Q^3 - 10Q^2 + 5Q + 10$$

Solution:

$$MC(Q) = \frac{\partial C}{\partial Q} = 9Q^2 - 20Q + 5$$

Take the derivative of the marginal cost w.r.t quantity:

$$\frac{\partial MC}{\partial Q} = 18Q - 20$$

Then for:

$$\begin{aligned} For\ Q &< \frac{10}{9} \quad \text{Decreasing MC} \\ For\ Q &> \frac{10}{9} \quad \text{Increasing MC} \end{aligned}$$