# PS01

Due date: Friday, June 28th @ 11:59pm

## Instructions

Complete the following questions below and show all work. You may either type or hand write your answers. However, you must submit your problem set to Canvas as an html or pdf. Meaning any handwritten solutions are to be scanned and uploaded. The onus is yours to deliver an organized, clear, and/or legible submission of the correct file type that loads properly on Canvas. Do not simply change the file type and submit (e.g. edit name of document from .jpg .pdf) Double-check your submissions!

**Integrity:** If you are suspected of cheating, you will receive a zero—for the assignment and possibly for the course. Cheating includes copying work from your classmates, from the internet, and from previous problem sets. You are encouraged to work with your peers, but everyone must submit their own answers. Remember, the problem sets are designed to help you study for the midterm. By cheating, you do yourself a disservice.

## Questions

### Q1. Settlers of Catan 32 pts

*Each part is worth 4 pts*

In the board game *"Settlers of Catan"*, two dice are rolled each turn and summed together. Let $X$ be a random variable that describes the sum of these two dice.

**(a) What is the sample space of $X$?**

**(b) What is the probability density function (PDF) of $X$?**

**(c) What are the three most likely outcomes of $X$?**

**(d) Find the expected value, $\mathbb{E}[X]$, of random variable $X$ from part (a).**

**(e) Find the variance, $Var(X)$, of random variable $X$ from part (a).**

Let $X$ be the same random variable from problem 001 and $Y$ be equal to: $Y = 5X + 1$

**(f) Calculate $\mathbb{E}[Y]$. Is it the same as $5\mathbb{E}[X] + 1$?**

**(g) Calculate $Var[Y]$.**

Suppose we have random variables $W$ and $Q$, and we know $Var(W) = 16, Var(Q) = 36$, and $Cov(W, Q) = 2$. Now, let $A = 2W + 4Q$.

**(h) What is $Var(A)$?**

## Q2. OLS Estimation 36 pts

Suppose we have the following data on a dependent variable (Y) and an independent variable (X):

| Observation Number | Y | X |
|---|---|---|
| 1 | 20 | 12 |
| 2 | 4 | 14 |
| 3 | 12 | 16 |

We wish to estimate the simple linear regression model:

$$Y_i = \beta_0 + \beta_1 X_i + u_i$$

**(a) Calculate the OLS estimates $\hat{\beta}_0$ and $\hat{\beta}_1$. 20 pts**

Show your work.

**(b) Calculate the $R^2$ for this regression. 16 pts**

Show your work.

## Q3. Interpretation of OLS Estimates 32 pts

The `mtcars` dataset in R includes the variables `mpg` which measures a car make's fuel efficiency (in miles per gallon) and `wt` which measures the weight (in 100s of lbs). The following is the output of the regression with the equation:

$$mpg_i = \beta_0 + \beta_1 wt_i + u_i$$

```
# A tibble: 2 x 5
  term        estimate std.error statistic  p.value
  <chr>          <dbl>     <dbl>     <dbl>    <dbl>
1 (Intercept)    37.3      1.88      19.9  8.24e-19
2 wt             -5.34     0.559     -9.56 1.29e-10
```

**(a) Give the interpretation of the two estimates. 16 pts**

**(b) The regression in (a) produced an $R^2$ of 0.75. What does this value tell us? 16 pts**