Econ 327: Game Theory

Midterm Exam

University of Oregon

October 30th, 2024

Version 2

- Complete *all* questions and parts. All questions will be graded.
- Carefully explain all your answers on short and long answer questions.

  An incorrect answer with clear explanation will earn partial credit, an incorrect answer with no work will get zero points.
- If you do not understand what a question is asking for, ask for clarification.

Allowed Materials:

- A single 5" by 3" note card
- A non-programmable calculator
- Pencils, color pens, eraser, ruler/straight-edge etc.

Name _____

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page or another sheet of paper.

Multiple Choice

Question 1. (5 P.)
Two player take turns emptying pebbles from a jar containing 100 pebbles total. Each player can take any number of pebbles between 1 and 5 on their turn. The player who takes the last pebble **loses** the game. Choose the most correct option relating to this game.

a) There is a second-mover advantage
b) There is a first-mover advantage
c) There is no way to predict what will happen
d) Neither player can win

Question 2. (5 P.)
If an outcome is _____, then it is _____

a) not a Nash equilibrium, not Pareto optimal
b) never Pareto dominated, Pareto Optimal
c) strictly dominated for everyone, Pareto optimal
d) Pareto dominated, a Nash equilibrium

Question 3. (5 P.)
Which assumption is needed for finding Subgame Perfect Nash Equilibria?

a) Common Knowledge of Sequential Rationality
b) There can only be two players
c) Transitivity of Preferences
d) Backwards Induction

Question 4. (5 P.)
A choice that is the best for a player **no matter what everyone else is doing** is referred to as

a) the Cardinal's play
b) a non-viable strategy
c) a Pareto optimal strategy
d) a strictly dominant strategy

Question 5. (5 P.)
Iterative Deletion of Strictly Dominated Strategies is useful because

a) it's not useful because you will get different answers depending on which player you start with
b) it can remove strategies which will never be played in a Nash Equilibrium
c) it will always find all Nash Equilibria of any strategic form game
d) it removes non-credible threats

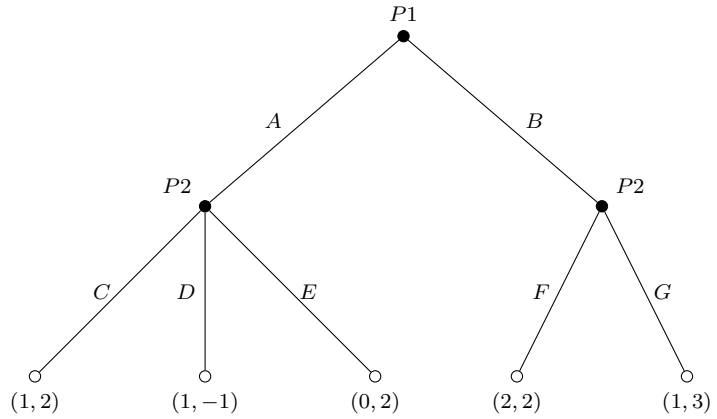
Answers: **1:** b, **2:** b, **3:** a, **4:** d, **5:** b

Question 6. (5 P.)
Consider the strategic form game below:

		P_2		
		Left	Middle	Right
P_1	Up	0,1	9,0	2,3
	Straight	5,9	7,3	1,7
	Down	7,5	10,10	3,5

What is the Nash Equilibrium?
a) Up, Middle
b) Straight, Right
c) Down, Middle
d) Down, Right

Question 7. (5 P.)
Consider the extensive form game below:



Which of the following is a subgame-perfect Nash equilibrium?

a) (A, EF)
b) (B, CG)
c) (B, CF)
d) (B, DG)

Question 8. (5 P.)
I make two bets on separate games. If the UO men's basketball team beats Washington, I win \$6, but if Washington wins I lose \$6. If the UO women's team beats Colorado, I win \$12, but if Colorado wins I lose \$12. Suppose that the probability of the UO men beating Washington is $\frac{1}{3}$ and the probability of the UO women beating Colorado is $\frac{1}{2}$.
What is my expected (dollar) payout across both games?

a) \$0
b) -\$2
c) -\$4
d) \$2

Answers: **6:** c, **7:** b, **8:** b

Long Answer

Question 9. (10 P.)

Consider the restaurant game from class where Xavier and Yvonne restaurant's competing for customers where Xavier's quantity sold is $Q_x = 44 - 2P_x + P_y$ and Yvonne's quantity sold is $Q_y = 44 - 2P_y + P_x$ with Xavier setting his price at P_x, and Yvonne setting hers at P_y.

But now instead of having identical costs, suppose that Yvonne's cost per meal is lowered to \$4 and Xavier's cost per meal is increased to \$12.

Xavier's best response rule is given by:

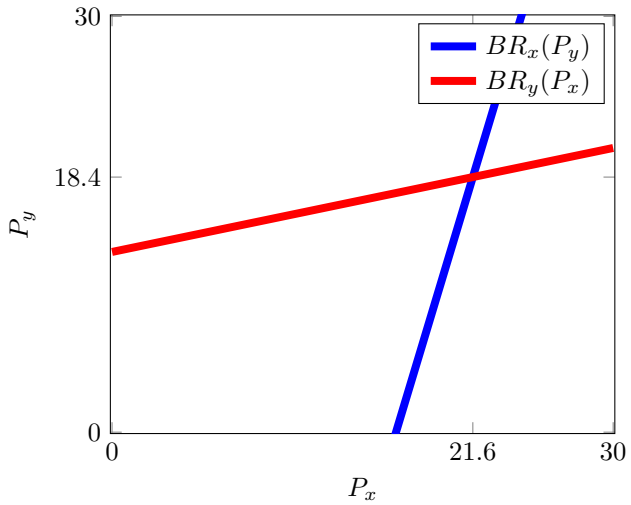
$$BR_x(P_y) = \frac{1}{4}P_y + 17$$

Yvonne's best response rule is given by:

$$BR_y(P_x) = \frac{1}{4}P_x + 13$$

a) Solve for any Nash equilibria in this pricing game. Explain why they are stable.

SOLUTION



In NE, $BR_x(P_y) = P_x$ and $BR_y(P_x) = P_y$.

$$\begin{aligned} P_x &= \frac{1}{4}(\frac{1}{4}P_x + 13) + 17 \\ P_x &= \frac{1}{16}P_x + \frac{13}{4} + 17 \\ \frac{15}{16}P_x &= 20.25 \\ P_x^* &= 21.6 \end{aligned}$$

Plug P_x^* into $BR_y()$:

$$\begin{aligned} P_y &= \frac{1}{4}(21.6) + 13 \\ P_y^* &= 18.4 \end{aligned}$$

($P_x = 21.6$, $P_y = 18.4$) is the only NE in this game because it is the only set of prices both players' best response rules intersect.

Question 10. (20 P.)
Consider the strategic form game below:

		P_2			
		i	j	k	l
P_1	q	8, 2	3, 5	10, 7	0, 8
	r	0, 4	4, 5	11, 6	7, 11
	s	9, 9	1, 10	8, 11	4, 13
	t	8, 7	5, 10	9, 7	4, 9
	v	3, 4	3, 5	6, 6	3, 16

a) (10 P.) Use Iterated Deletion of Strictly Dominated Strategies and write out a simplified game table with any remaining cells.

- Step 1: v is strictly dominated by t, eliminate v.
- Step 2: i and k are strictly dominated by l, eliminate i and k
- Step 3: q and s are strictly dominated by r, eliminate q and s

		P_2	
		j	l
P_1	r	4, 5	$\underline{11}$, $\underline{6}$
	t	$\underline{5}$, $\underline{10}$	9, 7

b) (10 P.) Find all Nash equilibria in this strategic form game. Explain why you know they are Nash equilibria.

The best response to j is t, and the best response to t is j so (j, t) is one Nash equilibrium.
The best response to l is r, and the best response to r is l so (l, r) is the other NE.
These are they only pure strategy NE because we eliminated all strategies in part (a) that will never be played in any NE. We also found the intersection of either players best responses in the table from (a) which is the definition of a NE.

Partial credit may be awarded for an answer that is consistent with mistakes made in eliminating strictly dominated strategies in part *a*.

Question 11. (30 P.)
Consider a group project where both players, **Regina** and **Gretchen** can put in some level of effort, $High$, $Moderate$, or Low.
Both players earn the same grade on the project, but their shared grade depends on the total level of effort:

- If both put in Low effort, they earn an F which both value at 0 utils
- If one puts in Low and the other puts in $High$ effort, they earn a D which they value at 1 util
- If either both put in $Moderate$ effort, or if one puts in $High$ while the other puts Low effort, they earn a B which they value at 3 utils
- If one puts $Moderate$ and the other puts $High$, they earn an A which they value at 4 utils
- And finally, if both put in $High$, they earn an A^+ which they value at 5 utils

Costs to effort: Putting in $High$ effort costs 2.5 util, putting in $Moderate$ effort costs 1 util, putting in Low effort costs 0 utils
Each player's payoff in this game is equal to their payoff from the grade they earn, minus the cost of the level of effort they put in.

a) (10 P.) Write out the extensive form tree when **Gretchen** has to choose her level of effort and then **Regina** chooses her level of effort.

SOLUTION

b) (10 P.) Find all subgame perfect Nash equilibria in the game when Gretchen moves first. A complete answer will consist of a **a complete plan of action** for both players for every decision which could possibly be reached.

SOLUTION

c) (10 P.) Find all Nash equilibria when **Regina** has to choose her level of effort and then **Gretchen** chooses her level of effort. Compare your answer to your answers in parts (a) and (b) and explain the reason for the differences.

Extra Credit

Pick a bid of any number between 0 and 3 bonus points (in half-point increments) which will be matched with another random student's bid. Whoever has the **lowest** bid will be awarded their bonus points, the other student will earn zero points. A tie will result in no bonus points being given.

Your Bid: _____