Mixed Strategies

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EC327 Game Theory

Advanced Mixed Strategies

MSNE in a Larger Game

- Suppose that we have this 3×2 game:

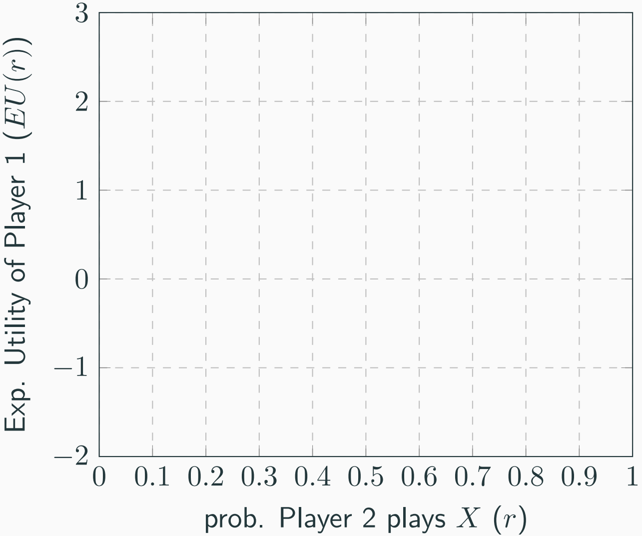
		Player 2	
		X (r)	Y (1 - r)
Player 1	A (p)	2, 1	0, 1
	B (q)	1, 2	2, 0
	C (1 - p - q)	0, 0	3, 2

- Player 1's mixed strategy uses probabilities p, q, and 1 - p - q, since they have three pure strategies.

MSNE in a Larger Game

- Algebraically:
  - $U_1(A) = 2r + 0 = 2r$.
  - $U_1(B) = 1r + 2(1 - r) = 2 - r$.
  - $U_1(C) = 0 + 3(1 - r) = 3 - 3r$.

Graph Player 1's expected utilities



When will Player 1 mix?

- What it would take to get Player 1 to mix different pairs of strategies:
    - A and B: $2r = 2 - r \implies r = \frac{2}{3}$.
    - A and C: $2r = 3 - 3r \implies r = \frac{3}{5}$.
    - B and C: $2 - r = 3 - 3r \implies r = \frac{1}{2}$.
- Note that there is no intersection between all three lines simultaneously
- This means that Player 1 will never mix between all three strategies

MSNE in a Larger Game

- Let's check Player 2's expected payoffs next:
  - $U_2(X) = 1p + 2q + 0$.
  - $U_2(Y) = 1p + 0 + 2(1 - p - q)$.
- So Player 2 will play a mixed strategy if

$$p + 2q = p + 2(1 - p - q)$$

$$\implies q = 1 - p - q$$

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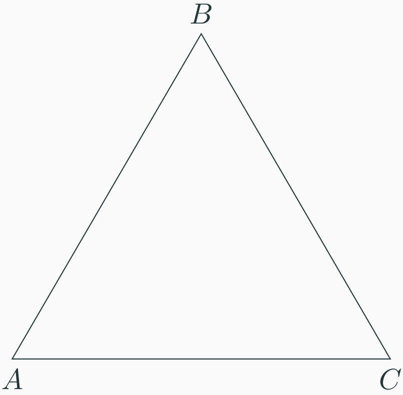
  - Recall that q was the probability we put on Player 2 playing B,
  - and $1 - p - q$ was the probability they play C.

visualizing Player 2's Best Responses



When will Player 2 mix?

- We found they are indifferent between X and Y when

$$q = 1 - p - d$$

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- There are two ways that this can be true:
  - Either Player 1 plays B and C with equal probability (and we know from earlier that they would **only** be playing these two, not A),
  - or Player 1 plays A only, and B and C not at all.

MSNE in a Larger Game

Case 1: Player 1 only plays A:

- this requires $2r \geq 2 - r$ and $2r \geq 3 - 3r$,
- which imply that $r \geq \frac{2}{3}$ and $r \geq \frac{3}{5}$.
- **MSNE 1:** $\{$(1, 0, 0), (r, 1 - r)$\}$, where $r \geq \frac{2}{3}$.

MSNE in a Larger Game

Case 2: Player 1 plays B and C with equal probability

- then Player 2 plays X and Y with equal (1/2) probability.
- **MSNE**: $\{(0,\ 1/2,\ 1/2),\ (1/2,\ 1/2)\}$

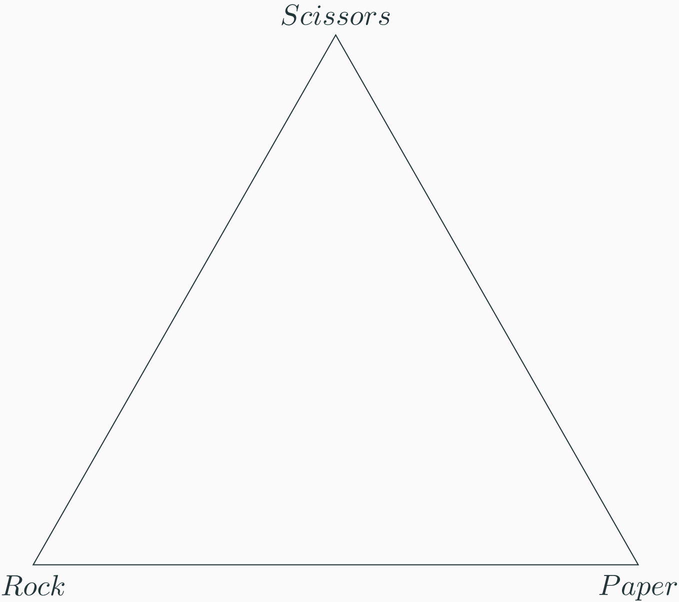
What about a 3x3 game?

		Player 2		
		Rock (r_2)	Paper (p_2)	Scissors ($1 - r_2 - p_2$)
	Rock (r_1)	0, 0	-1, 1	1, -1
Player 1	Paper (p_1)	1, -1	0,0	-1, 1
	Scissors	-1, 1	1, -1	0, 0

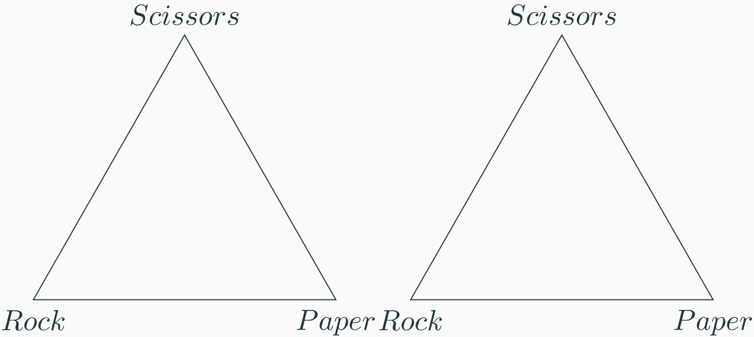
What about a 3x3 game?

- Rock, Paper, Scissors is a *symmetric* game, so let's just pay attention to Player 1's utility
- $EU_1(Rock|r_2, p_2) =$
- $EU_1(Paper|r_2, p_2) =$
- $EU_1(Scissors|r_2, p_2) =$

visualizing Player 1's Best Responses



Finding MSNE in 3x3 game



Finding MSNE in 3x3 game

So the results from our math confirm our intuition that the stable strategies in equilibrium are:

- Player 1 plays Rock with $r = 1/3$, Paper with $p = 1/3$, and Scissors with $1 - p - r = 1/3$
- Player 1 plays Rock with $r = 1/3$, Paper with $p = 1/3$, and Scissors with $1 - p - r = 1/3$

Another 3x3 game

		Player 2		
		Left	Center	Right
	Top	2, 1	3, 0	3, 0
Player 1	Middle	3, 0	0, 1	3, 0
	Bottom	3, 0	3, 0	2, 1

Solving for 3-strategy MSNE

Step 1: Define Mixed Strategies

- Player 1's mixed strategy: Let $\sigma_1 = (t, m, b)$
- Player 2's mixed strategy: Let $\sigma_2 = (\ell, c, r)$

Note that the lowercase letters represent the probabilities played on the uppercase pure strategies.

Solving for 3-strategy MSNE

Step 2: Solve for Expected Utilities

- Player 1:
  - $EU_1(T, \sigma_2) =$
  - $EU_1(M, \sigma_2) =$
  - $EU_1(B, \sigma_2) =$
- Player 2:
  - $EU_2(L, \sigma_1) =$
  - $EU_2(C, \sigma_1) =$
  - $EU_2(R, \sigma_1) =$

Solving for 3-strategy MSNE

Step 3: Find Indifference Conditions

- When will Player 1 mix between 2 pure strategies?
    - When does $EU_1(Top, \sigma_2) = EU_1(Middle, \sigma_2)$:
    - When does $EU_1(Top, \sigma_2) = EU_1(Bottom, \sigma_2)$:
    - When does $EU_1(Middle, \sigma_2) = EU_1(Bottom, \sigma_2)$:

Solving for 3-strategy MSNE

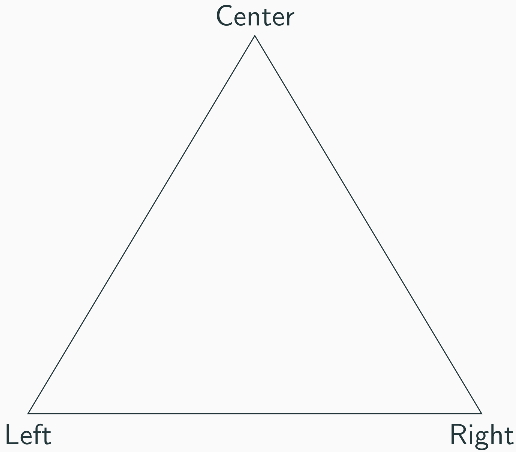
Step 3: Find Indifference Conditions

- When will Player 2 mix between 2 pure strategies?
  - When does $EU_2(Left, \sigma_1) = EU_2(Center, \sigma_1)$:
  - When does $EU_2(Left, \sigma_1) = EU_2(Right, \sigma_1)$:
  - When does $EU_2(Center, \sigma_1) = EU_2(Right, \sigma_1)$:

Step 4.a: Graph Indifference Points on Number Lines for Player 1

Solving for 3-strategy MSNE

Step 4.b: Combine Number Lines into Player 1's BR Triangle

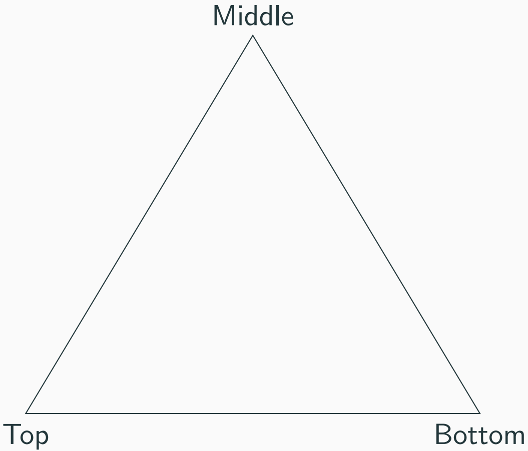


Solving for 3-strategy MSNE

Step 4.c: Graph Indifference Points on Number Lines for Player 2

Solving for 3-strategy MSNE

Step 4.d: Combine Number Lines into Player 2's BR Triangle



Solving for 3-strategy MSNE

Step 5: Check Cases for possible Nash Equilibria: