

Econ 327: Game Theory

Homework #2

University of Oregon

Due: Oct. 10th

Question:	Q1	Q2	Q3	Q4	Total
Points:	12	10	8	8	38
Score:					

For homework assignments:

- Complete *all* questions and parts.
- You will be graded on not only the content of your work but on how clearly you present your ideas. Make sure that your handwriting is legible. Please use extra pages if you run out of space but make sure that all parts of a question are in the correct order when you submit.
- You may choose to work with others, but everyone must submit to Canvas individually. Please include the names of everyone who you worked with below your own name.

Name _____

- Q1.** Imagine a sequential moves version of rock-paper-scissors where player 2 gets to pick what they will do after player 1 picks. Please model the game in its extensive form (as a game tree). Assume both player 1 and player 2 only care about the result of the game and have the following preferences over the result of the game: win \succ tie \succ loss. ¹
- (a) [2 points] How many decisions does **Player 1** have to make in their *complete strategy*? In other words, how many *decision nodes* will Player 1 have in the game tree?
 - (b) [2 points] How many decision nodes does **Player 2** have? Why?
 - (c) [4 points] Draw out the extensive form game. Make sure to clearly label all branches, nodes, and choose any payoff values that reflect the preference ranking above.
 - (d) [4 points] Use **backwards induction** (rollback) to prune branches which are not sequentially rational. Does this game have more than one equilibrium? Why or why not?

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Q2. Use the same sequential Rock, Paper, Scissors setup, but now imagine player 1's preferences change because they want to be seen as a 'tough guy'. Given that what they want to play remains the same, they still have the following preferences over the result of the game: $\text{win} \succ \text{tie} \succ \text{loss}$. However, they now would prefer to lose playing rock than win playing paper or scissors.

- (a) [4 points] Please create a new game tree so the payoffs reflect these new preferences.
- (b) [4 points] Now apply backwards induction to the modified game tree. Clearly show on your game tree which branches survive the pruning process.
- (c) [2 points] What is the rollback equilibrium now?

Q3. At the beginning of the wet season, the herds of gnu which live in the Serengeti migrate South to follow greener grasses. This is also a prime feeding time for the Nile crocodiles which inhabit the Mara river which the gnu must cross on their way.

To simplify the situation, suppose that there are only two crossings which the gnu can initially approach; the shallow *rapids*, or the deeper *channel*. After the gnu arrive at a crossing, suppose the crocodiles can observe where the gnu are gathered and choose which crossing to wait in ambush.

If the gnu cross the river at the same crossing where the crocs are waiting, they are eaten and receive a payoff of -1. However, if they cross at the opposite crossing from the crocs, they successfully cross the river and receive a payoff of 1.

The crocs want to eat gnu, but they also prefer to wait in the rapids where they can sun themselves rather than the channel. If the crocs wait in the rapids and they catch the gnu crossing there, the crocs get a payoff of 2. If the crocs wait in the channel and catch the gnu crossing there, they only get a payoff of 1. If the crocs wait in the rapids but they don't catch any gnu, the crocs earn a payoff of 0. If the crocs wait in the channel but they don't catch any gnu, the crocs get a payoff of -1.

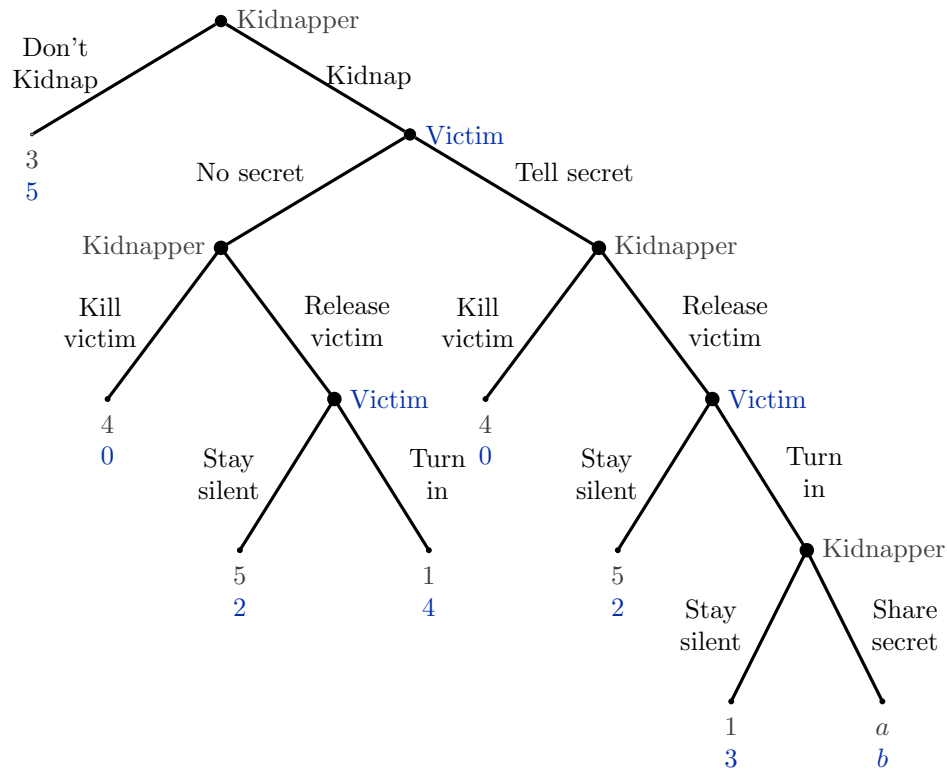
After the gnu have scoped out their choice of crossing, they can choose to either *cross* the river, or *stay* on the banks depending on whether they can see crocs waiting for them. If the gnu decide to stay on the banks, they get a payoff of 0 (and the crocs don't catch them).

- (a) [4 points] Draw out the extensive form game. Clearly label all nodes and branches. ²
- (b) [4 points] Use backwards induction to solve for the equilibrium.

²Hint: there should be eight possible outcomes.

Q4. Suppose that a kidnapper first makes a decision whether to kidnap a high-value victim. The problem is that once the ransom has been paid, the kidnapper has an incentive to kill the victim to prevent them from turning them into the police once released.

Consider the solution proposed by Thomas Schelling whereby the victim reveals his darkest secret to the kidnapper which he would hate to be revealed to the world.³



- [4 points] What values of a and b ensure that the subgame perfect Nash equilibrium results in the victim being released by the kidnapper?
- [4 points] Do you think that the victim revealing an embarrassing secret to their kidnapper could be an effective strategy in real life? Why or why not?

³Adapted from Harrington, *Games, Strategies, and Decision Making*, 2015, page 297.