Econ 327: Game Theory

Practice Final Exam

University of Oregon

March 20, 2024

Version 1

Question	1.	2.	3.	4.	Total
Points	8	16	20	16	60
Score					

- Complete *all* questions and parts. All questions will be graded.
- Carefully explain all your answers on short and long answer questions.

  An incorrect answer with clear explanation will earn partial credit, an incorrect answer with no work will get zero points.
- If you do not understand what a question is asking for, ask for clarification.

<u>Allowed Materials:</u>

- A single 5” by 3” note card
- A non-programmable calculator
- Pencils, color pens, eraser, ruler/straight-edge etc.

Name ____

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page or another sheet of paper.

Short Answer

Question 1. (8 P.)

You are the Dean of the Faculty at St. Anford University. You hire Assistant Professors for a probationary period of 7 years, after which they come up for tenure and are either promoted and gain a job for life or turned down, in which case they must find another job elsewhere. Your Assistant Professors come in two types, Good and Brilliant. Any types worse than Good have already been weeded out in the hiring process, but you cannot directly distinguish between Good and Brilliant types. Each individual Assistant Professor knows whether he or she is Brilliant or merely Good. You would like to tenure only the Brilliant types. The payoff from a tenured career at St. Anford is \$6 million; think of this as the expected discounted present value of salaries, consulting fees, and book royalties, plus the monetary equivalent of the pride and joy that the faculty member and his or her family would get from being tenured at St. Anford. Anyone denied tenure at St. Anford will get a fac- ulty position at Boondocks College, and the present value of that career is \$1 million. Your faculty can do research and publish the findings. But each publication requires effort and time and causes strain on the family; all these are costly to the faculty member. The monetary equivalent of this cost is \$25,000 per publication for a Brilliant Assistant Professor and \$50,000 per publication for a Good one. You can set a minimum number, N, of publications that an Assistant Professor must produce in order to achieve tenure.

A) (4 P.) What is the minimum number N you could require so that only *brilliant* professors apply and *good* professors don't apply?

in thousands

$U_G(\text{Apply}) = 6000 - 50N$

$6000 - 50N < 1000$

$U_B(\text{Boondocks}) = 1000$

$N \geq 101$

$5000 < 50N \Rightarrow N > 100$

B) (4 P.) What is the maximum number N that you could require so that *brilliant* professors still want to apply?

$U_b(\text{Apply}) = 6000 - 25N$

$U_B(\text{Boondocks}) = 1000$

Long Answer

Question 2. (16 P.)
Consider the strategic form game below:

		P_2			
		Hall	**Office**	**Library**	**Bathroom**
$\frac{1}{3}$	**Roof**	0 , 2	1 , 1	0 , 2	5, 0
0 P_1	**Mezzanine**	1 , 1	0 , 2	0 , 2	4, 0
2/3	**Ground**	0 , 2	0 , 2	1 , 0	3, -1

A) (4 P.) Find any **pure strategy** Nash equilibria

No PSNE

No intersection of BR in table above

B) (6 P.) Consider the following mixed strategy profile:

$r = 1/3 \quad m = 0 \quad g = \frac{2}{3}$

- Player 1 plays 1/3 **Roof**, 0 **Mezzanine**, and 2/3 **Ground**
- Player 2 plays 0 **Hall**, 1/2 **Office**, 1/2 **Library**, and 0 **Bathroom**

$h = 0 \quad o = \frac{1}{2} \quad l = \frac{1}{2}$

violation

Check whether this is a **mixed strategy Nash equilibrium** and explain why or why not.

$EU_1(\text{Roof}) = \frac{1}{2}(1) + \frac{1}{2}(0) + 0(0) + 0(5) = \frac{1}{2}$

$EU_1(M) = \frac{1}{2}(0) + \frac{1}{2}(0) + 0(1) + 0(4) = 0$ ✓

$EU_1(\text{Ground}) = \frac{1}{2}(0) + \frac{1}{2}(1) + 0(0) + 0(3) = \frac{1}{2}$

$EU_2(\text{Hall}) = \frac{1}{3}(2) + 0(1) + \frac{2}{3}(2) = 2$

$EU_2(O) = \frac{1}{3}(1) + 0(2) + \frac{2}{3}(2) = 5/3$

$EU_2(L) = \frac{1}{3}(2) + 0(2) + \frac{2}{3}(0) = 2/3$

Hall s.D. O, L

Contradiction

~~Not~~ MSNE

C) (6 P.) Now consider the strategy profile:

- Player 1 plays 1/4 **Roof**, 1/2 **Mezzanine**, and 1/4 **Ground**
- Player 2 plays 1/3 **Hall**, 1/3 **Office**, 1/3 **Library**, and 0 **Bathroom**

× violation

Check whether this is a **mixed strategy Nash equilibrium** and explain why or why not.

$EU_1(\text{Roof}) = \frac{1}{3}(1) + \frac{1}{3}(0) + \frac{1}{3}(0) + 0(5) = \frac{1}{3}$

$EU_1(M) = \frac{1}{3}(0) + \frac{1}{3}(0) + \frac{1}{3}(1) + 0(4) = \frac{1}{3}$

$EU_1(\text{Ground}) = \frac{1}{3}(0) - \frac{1}{3}(1) + \frac{1}{3}0) + 0(3) = \frac{1}{3}$

Indiff, no violation

$BR(\frac{1}{4}, \frac{1}{2}, \frac{1}{4}) = O = 1$ only office

These totals were incorrect in the original key, leading to mistakenly finding an MSNE

$EU_2(\text{Hall}) = \frac{1}{4}(2) + \frac{1}{2}(1) + \frac{1}{4}(2) = 3/2$

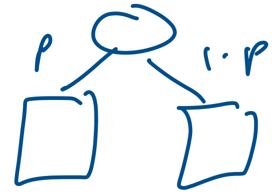
$EU_2(O) = \frac{1}{4}(1) + \frac{1}{2}(2) + \frac{1}{4}(2) = \frac{7}{4}$

$EU_2(L) = \frac{1}{4}(2) + \frac{1}{2}(2) + \frac{1}{4}(0) = 3/2$

H, L are strictly dom. by Office

→ Not a MSNE

Asymm. Info



Question 3. (20 P.)

Consider a Wild West shootout between Earp and the Stranger.

With probability .75, the Stranger is a Gunslinger type and the table shows Earp's and the Stranger's payoffs

		Gunslinger Stranger	
		Draw	Wait
Earp	Draw	2, 3	3, 1
	Wait	1, 4	8, 2

But with probability .25, the Stranger is a Cowpoke type and the table shows Earp's and the Stranger's payoffs

		Cowpoke Stranger	
		Draw	Wait
Earp	Draw	5, 2	4, 1
	Wait	6, 3	8, 4

Perfect Info

A) (4 P.) What is the Nash equilibrium **when ~~Earp~~ stranger is always a Gunslinger**?

(Draw, Draw) is NE

B) (4 P.) What is the Nash equilibrium **when ~~Earp~~ stranger is always a Cowpoke**?

(Wait, Wait) is NE

Earp doesn't know stranger's type

C) (6 P.) Show whether there is a *Separating* equilibrium where a Gunslinger Stranger will choose Draw and a Cowpoke Stranger will choose Wait.

(Wait, Draw if GS, Wait if CP)) → Sep. Nash Eq.

$EU_E(\text{Draw}) = \frac{3}{4}(2) + \frac{1}{4}(4) = 2.5$

$EU_E(\text{Wait}) = \frac{3}{4}(1) + \frac{1}{4}(8) = 2.75$

Earp will wait

D) (6 P.) Consider a strategic move variation where the ~~Gunslinger~~ stranger can commit to only playing Wait before Nature has assigned them a type.

Is this type of commitment *credible*? Why or why not?

(Wait, Wait if GS, Wait if CP)

Not credible for GS because

Draw SD. Wait

$$8 + 8\delta + 8\delta^2 + \cdots = \sum_{t=1}^{\infty} 8\delta^{t-1} = 8\delta^0 + \sum_{t=2}^{\infty} 8\delta^t$$

$$= 8 + 8\delta \underbrace{\sum_{t=2}^{\infty} \delta^{t-1}}_{\to \frac{1}{1-\delta}}$$

Question 4. (16 P.)
Consider the strategic form game below:

		Column	
		Cooperate	Defect
Row	Cooperate	8 , 8	0, 10
	Defect	10 , 0	3 , 3

Defect, Defect

A) (4 P.) What will happen when this game is a *one-shot* game and neither player can make any strategic moves?

(Defect, Defect)

B) (2 P.) Will this outcome be *Pareto optimal*?

(3, 3) is Pareto dom. by (8, 8)

C) (4 P.) What could you change about the structure of this game to ensure that a socially optimal outcome will be reached in equilibrium?

- reputation
- strat. moves: limiting to coop.
- allow for repeated

D) (6 P.) Suppose that both players have a *discount factor* of $\delta = 3/4$. Can a strategy profile of both players using *grim trigger* strategies be sustained in the game where the strategic form game above is repeated infinitely?

Show all calculations and explain your answer.

$$pv(\text{Cooperate}) = 8 + 8\delta + 8\delta^2 + 8\delta^3 + \cdots \to 8 + \delta(8 + 8\delta + \cdots) \to 8 + \frac{8\delta}{1-\delta}$$

$$pv(\text{Defect}) = 10 + 3\delta + 3\delta^2 + 3\delta^3 + \cdots = \underbrace{10 + \delta(\underbrace{3 + 3\delta + 3\delta^2 + \cdots}_{\frac{3}{1-\delta}})}_{\to 10 + \frac{3\delta}{1-\delta}}$$

$$pv(\text{Coop}) \geq pv(\text{Defect})$$

$$8 + \frac{8\delta}{1-\delta} \geq 10 + \frac{3\delta}{1-\delta}$$

$$\frac{5\delta}{1-\delta} \geq 2$$

$$5\delta \geq 2 - 2\delta$$

$$7\delta \geq 2 \Rightarrow \delta \geq \frac{2}{7}$$

as long as $\delta \geq 2/7$ cooperation can be sustained $\to$ so $\delta = \frac{3}{4}$