Econ 327: Game Theory

Homework #2

University of Oregon

Due: Oct. 25^{th}

Question:	**Question 1**	**Question 2**	**Question 3**	**Question 4**	**Question 5**	Total
Points:	28	28	26	10	10	102
Score:						

For homework assignments:

- You will be graded on not only the content of your work but on how clearly you present your ideas. Make sure that your handwriting is legible. Please use extra pages if you run out of space but make sure that all parts of a question are in the correct order when you submit.
- You may choose to work with others, but everyone must submit to Canvas individually.

  Please include the names of everyone who you worked with below your own name.

Name ______________________________

Question 1. Multiple Choice

(a) [4 points] Consider the strategic form game below:

		P_2		
		x	y	z
	a	1,3	2,2	3,2
P_1	b	2,2	2,2	4,3
	c	1,1	0,2	1,1

In the game above, which strategy is strictly dominated?

A. a
B. b
C. c
D. x

(b) [4 points] Perform Iterative Deletion of Strictly Dominated Strategies for the same game as above all the way to completion. What does IDSDS tell you about the Nash equilibrium of this game?

A. The NE is (a, x)
B. The NE is (a, y)
C. The NE is (Y, z)
D. IESDS by itself does not reveal the NE of this game.

(c) [4 points] Consider the strategic form game below:

		OD	
		$Swerve$	$Straight$
CD	$Swerve$	-1,-1	1,1
	$Straight$	1,1	-1,-1

What type of game is this?

A. A zero-sum game
B. A coordination game
C. An anti-coordination game
D. A prisoners' dilemma

(d) [4 points] Consider the strategic form game below:

		Navratilova	
		DL	CC
Evert	DL	50, 50	80, 20
	CC	90,10	20, 80

What is the *pure strategy* Nash equilibrium?

A. (DL, DL)
B. (CC, DL)
C. (DL, CC)
D. There are no Nash equilibria in pure strategies for this game.

(e) [4 points] Cosider the same game as above. Suppose that Navratilova plays DL with probability p and CC with probability $(1-p)$. What are Evert's expected payoffs?

A. $U_{Evret}(DL) = 30 - 80p$, $U_{Evret}(CC) = 70 - 20p$

B. $U_{Evret}(DL) = 80 - 30p$**,** $U_{Evret}(CC) = 20 + 70p$

C. $U_{Evret}(DL) = -60p$, $U_{Evret}(CC) = 100 + 100p$

D. $U_{Evret}(DL) = 90 - 40p$, $U_{Evret}(CC) = 20 + 60p$

(f) [4 points] The difference between a regular Nash equilibrium and a Subgame Perfect Nash equilibrium is that:

A. A Subgame Perfect Nash equilibrium assumes perfect information

B. Mixed strategies cannot be used in Subgame Perfect Nash equilibria

C. Subgame Perfect Nash equilibria assume that players won't fall for non-credible threats

D. There is no difference, they are the same

(g) [4 points] Which of the following are examples of *continuous* strategies?

A. Taylor Swift's choice of which cities to go on tour in

B. How much time Owen waits in line for Taylor Swift tickets

C. How much money TicketMaster charges for a ticket

D. Jose is at home and will only go if the stadium is less than 50% full

E. Both B and C are continuous strategies

F. None of the above are continous strategies

Solution: Note: Give full credit on this question if only B is chosen or only C is chosen or both. I meant for the amount of time Owen waits in line to be a continuous strategy, but some people argued that his decision to wait or not could be discrete.

Question 2. Here's a little ditty, about Jack and Diane, two American kids growing up in the heartland. The game is below. [1]

		Diane		
		x	y	z
Jack	a	1,1	2,1	2,0
	b	2,3	0,2	2,1
	c	2,1	1,2	3,0

(a) [12 points] Find all pure Nash strategy profiles and outcomes if Jack and Diane move simultaneously. Carefully detail and explain the strategy profiles and how they map onto your Nash outcomes.

(b) [16 points] Find all pure Nash strategy profiles and outcomes *if Jack moves first.* Carefully detail and explain your strategy profiles and how they map onto your Nash outcomes.

Solution:

(a) Players: {Jack, Diane}

Strategy sets: $S_{\text{Jack}} = \{a, b, c\}$ $S_{\text{Diane}} = \{x, y, z\}$

For Jack, b and c are best responses to x, a is BR to y, and c is BR to z.

For Diane, x and y are BR to a, x is BR to b, and z is BR to c.

There are two strategy profiles where each player's best responses intersect:

- $N_1 = (b, x)$ results in payoffs (2,3)
  Jack's strategy is to choose b, Diane's strategy is to choose x. Neither have regrets about their strategy choice; given that Jack is choosing b, Diane can't get a higher payoff by deviating. Given that Diane is playing x, Jack is indifferent between playing b and c but he can't get a strictly higher payoff by deviating. The resulting outcome is that Jack gets 2, Diane gets 3.
- $N_2 = (a, y)$ results in payoffs (2,1)
  Jack's strategy is a, Diane's is y. When Jack plays a, Dianne is indifferent between x and y, but still cannot deviate to a strictly higher payoff. When Diane plays y, Jack's best response is a because $2 > \{0, 1\}$. The outcome is that Jack gets 2 and Diane gets 1.

These are the only two *pure strategy* Nash equilibria because there are no other intersections of pure strategy best responses. Note that even though N_1 Pareto dominates N_2 (Jack is indifferent; $2 = 2$, and Diane is better off; $3 > 1$), there is no *unilateral* deviation that would reach N_1 from N_2.

(b) See the extensive form game tree below.

Now the strategy sets are:

$S_{\text{Jack}} = \{a, b, c\}$ because Jack still has only one decision node.

$S_{\text{Diane}} =$ any combination $\{s_1 s_2 s_3\}$; $\forall s \in \{x, y, z\}$; where s can be either x, y, or z at each of her information sets (labelled 1,2,3 on the game tree below).

I represented each of Diane's strategies as a triple where the first letter represents her choice at node 1, second letter at node 2, and third letter at node 3. So for example, xyz would be the strategy where she chooses x in node 1, y in node 2, and z in node 3.

- $\mathbf{N_1} = (\mathbf{a},\ \mathbf{y_1} x_2 s_3)$
  where:

[1] Cliff Bekar, Lewis and Clark College

- s_3 is **x** or **y** or **z**

Any set of strategies in which Jack chooses a and Diane chooses y in node 1 but doesn't choose z at node 3 would result in an equilibrium outcome of $(2, 1)$ where Jack cannot deviate to a higher payoff than 2, and Diane also has no regrets with choosing y.

The payoffs obtained in this equilibrium are $(2, 1)$

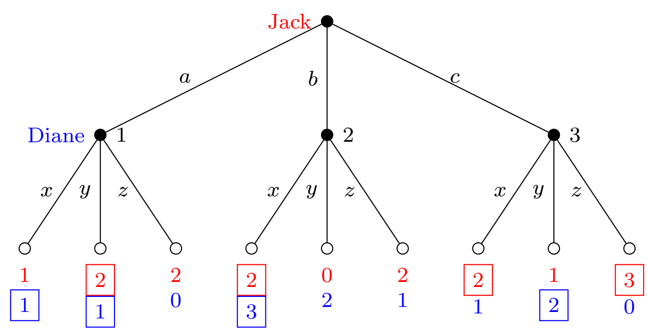
- $\mathbf{N_2} = (\mathbf{b},\ s_1\mathbf{x_2}s_3)$

  where:

  - s_1 could be **x**, **y**;
  - s_3 could be either **x**, **y**, or **z**.

  Any set of strategies in which Jack chooses b, Diane is indifferent between x and y at node 1 which is not on the equilibrium path of play, and Diane chooses x_2 given that jack chooses b.

  The equilibrium outcome of any of these Nash strategy profiles would be $(2, 3)$



Question 3. Consider the strategic form game below:

P_1 \ P_2	A	B	C	D
H	10, 1	-3, 1	0, 1	3, 1
J	16, -2	6, 6	1, -1	4, 0
K	11, 1	0, 3	2, 2	10, 15
L	13, 10	-1, 16	4, 12	5, 20

(a) [12 points] Use Iterated Deletion of Strictly Dominated Strategies and write out a simplified game table with any remaining cells.

Solution:

- Step 1: H is strictly dominated by J, eliminate H
- Step 2: A and C are strictly dominated by B, eliminate A and C
- Step 3: L is strictly dominated by K, eliminate L

P_1 \ P_2	B	D
J	$\underline{6},\underline{6}$	4, 0
K	0,3	$\underline{10},\underline{15}$

(b) [8 points] Find all Nash equilibria in *pure strategies*.

Solution:
The best response to B is J, and the best response to J is B so $(\mathbf{J}, \mathbf{B})$ is one Nash equilibrium. The best response to D is K, and the best response to K is D so $(\mathbf{K}, \mathbf{D})$ is the other PSNE.

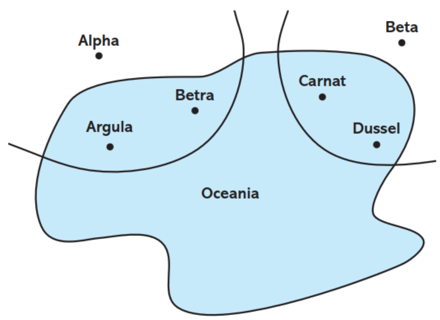
Partial credit may be awarded for an answer that is consistent with mistakes made in eliminating strictly dominated strategies in part *a*.

(c) [6 points] Explain why you know that the strategies you found in part b are Nash equilibria.

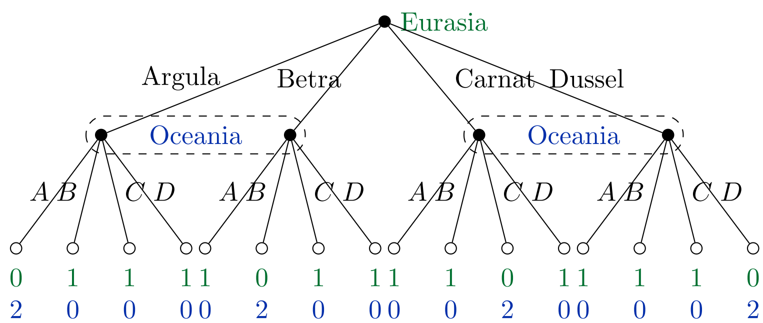
Solution:
These are they only pure strategy NE because we eliminated all strategies in part (a) that will never be played in any NE. We also found the intersection of either players best responses in the table from (a) which is the definition of a NE.

Question 4. [10 points] The countries of Oceania and Eurasia are at war. As depicted in the figure, Oceania has four cities — Argula, Betra, Carnat, and Dussel — and it is concerned that one of them is to be bombed by Eurasia. The bombers could come from either base Alpha, which can reach the cities of Argula and Betra; or from base Beta, which can reach either Carnat or Dussel. Eurasia decides which one of these four cities to attack. Oceania doesn't know which one has been selected, but does observe the base from which the bombers are flying. After making that observation, Oceania decides which one (and only one) of its four cities to evacuate.

Assign a payoff of 2 to Oceania if it succeeds in evacuating the city that is to be bombed and a payoff of 1 otherwise. Assign Eurasia a payoff of 1 if the city it bombs was not evacuated and a zero payoff otherwise. Write down the extensive form game. [2]



Solution:



Note that A, B, C, and D in the last row are short for the city names. Eurasia acts first, so the initial node is labelled accordingly. Oceania has only two info sets which are represented with the dashed ovals. The $(0, 2)$ or $(1, 0)$ payoff sets correspond to Oceania choosing the same city that is bombed, or choosing a different city respectively.

[2]Harrington *Games, Strategies, and Decision Making*

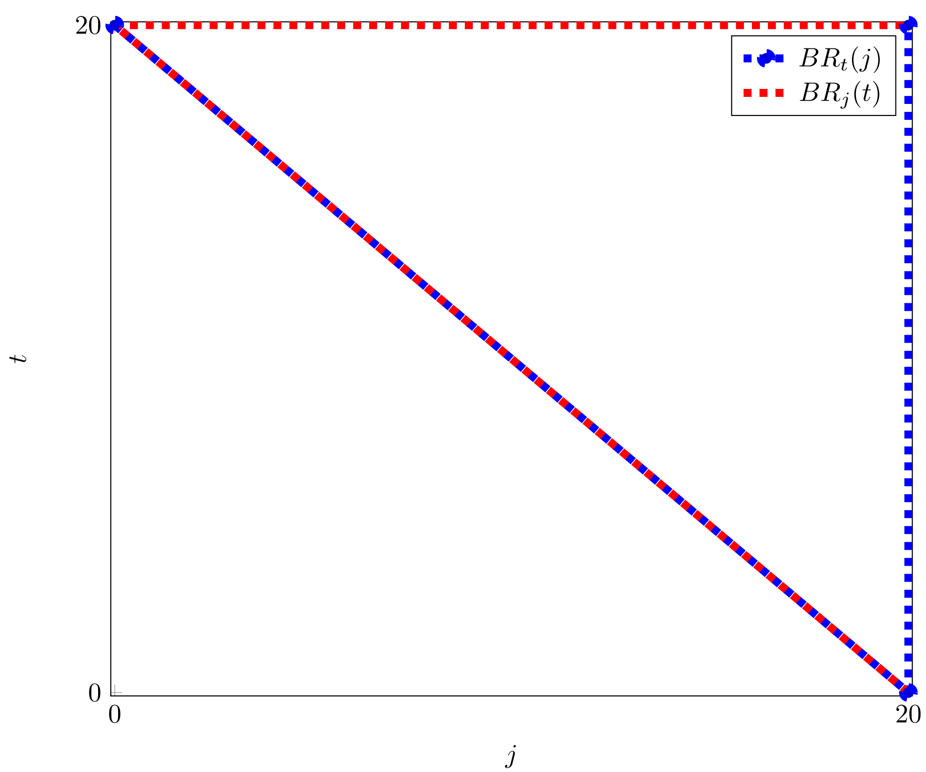
Question 5. [10 points] A game theorist is walking down the street in his neighborhood and finds \$20. Just as he picks it up, two neighborhood kids, Jane and Tim, run up to him, asking if they can have it. Because game theorists are generous by nature, he says he's willing to let them have the \$20, but only according to the following procedure: Jane and Tim are each to submit a written request as to their share of the \$20. Let t denote the amount that Tim requests for himself and j be the amount that Jane requests for herself. Tim and Jane must choose j and t from the interval $[0, 20]$. If $j + t \leq 20$, then the two receive what they requested, and the remainder, $20 - j - t$, is split equally between them. If, however, $j + t > 20$, then they get nothing, and the game theorist keeps the \$20. Tim and Jane are the players in this game. Assume that each of them has a payoff equal to the amount of money that he or she receives. Find all Nash equilibria. [3]

Solution: Tim's best response rule:

$$BR_t(j) = \begin{cases} 20 - j & \text{if } j < 20 \\ [0, 20] & \text{if } j = 20 \end{cases}$$

Jane's best response rule:

$$BR_j(t) = \begin{cases} 20 - t & \text{if } t < 20 \\ [0, 20] & \text{if } t = 20 \end{cases}$$



The NE are any points on the graph where the $BR_j(t) = t$ and $BR_t(j) = t$. This includes the diagonal line which is the set of all pairs, j and t such that $j + t = 20$ where $j, t < 20$. When $j = 20$, there is one NE when $t = 0$ and another when $t = 20$. When $t = 20$, there is one NE when $j = 0$ (and $j = 20$ is still NE).

[3] Harrington *Games, Strategies, and Decision Making*