Uncertainty & Information Topics

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EC327 Game Theory

Outline

Semiseparating Equilibria

Semiseparating Equilibria

Equilibria in 2-Player Signaling Games

- So far we have covered the general concepts of incomplete information.
- We saw how adverse selection can arise in games with many players.
- But now we will solve for equilibria in the case of a simpler 2-player game.

Semiseparating Equilibria

- We saw Pooling Equilibria in which all types take the same action
  - aka *'babbling equilibria'*
- And we also saw Separating Equilibria in which different types take *completely different* actions
  - sometimes called *'cheap talk equilibria'*

Market Entry Game

- **Players:** competing auto manufacturers: Tudor and Fordor
- Tudor is a current monopolist in the auto industry
- Fordor is a potential entrant in the market
- Tudor has **private information** on how tough they will be able to compete against a Fordor entrant.

Market Entry Game

Sequential Game

- **Stage 1:** Tudor sets price $\in \{low,\ high\}$
- **Stage 2:** Fordor makes entry decision $\in \{in,\ out\}$

Payouts:

- Profits for each firm are market price - production costs
  - **Market Demand:** $P = 25 - Q$

Market Entry Game

Costs:

- Fordor's upfront cost of entry: 40
- Fordor's per-unit cost: 10
- Tudor's costs:
  - If **high-cost**: 15
  - If **low-cost**: 5

Market Entry Game

Payouts:

- If Tudor is high-cost:
  - and Fordor stays out: $\Pi_{T1} = 5 * (20 - 15) = 25$ and $\Pi_{T2} = 25$, $\Pi_F = 0$
  - and Fordor enters: $\Pi_T = 25 + 3$, $\Pi_F = 45$ - startup cost of 40
- If Tudor is low-cost:
  - and Fordor stays out: $\Pi_{T1} = 100$ and $\Pi_{T2} = 100$, $\Pi_F = 0$
  - Fordor enters: $\Pi_T = 100 + 69$, $\Pi_F = 11$ - startup cost of 40

Market Entry - Extensive Form Game

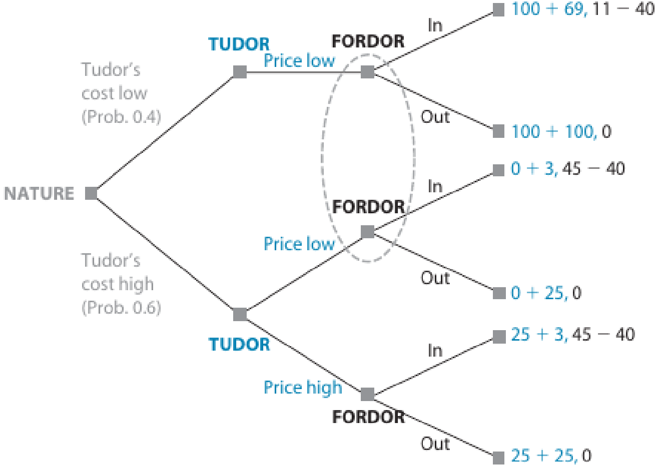

FIGURE 8.7 Extensive Form of Entry Game: Tudor's Low Cost Is 5

Signaling Strategies

- Tudor might use its price as a signal of its cost.
- A *low-cost* firm would charge a lower price, so Tudor might hope to keep its price low to show Fordor that they are a low-cost firm and therefore more difficult to fight.
- However, Tudor might also try to **bluff** Fordor into staying out.

Checking for Separating Equilibrium:

1. **Step 1:** Prune strategies using rollback:
    - What should Fordor do if they see a **high price**?

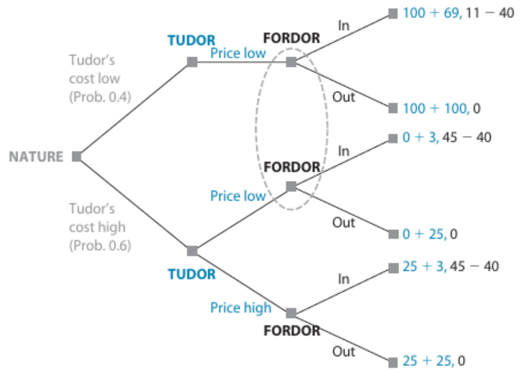

FIGURE 8.7 Extensive Form of Entry Game: Tudor's Low Cost Is 5

Market Entry Game - Separating Equilibrium

Checking for Separating Equilibrium:

- How many Strategies does each player have?
  - (After pruning *Out if Price High* for Fordor)

Checking for Separating Equilibrium:

Step 2: Represent game in *Strategic Form:*

		FORDOR	
		Regardless (II)	Conditional (OI)
TUDOR	Bluff (LL)	$169 \times 0.4 + 3 \times 0.6 = 69.4$, $-29 \times 0.4 + 5 \times 0.6 = -8.6$	$200 \times 0.4 + 25 \times 0.6 = 95$, 0
	Honest (LH)	$169 \times 0.4 + 28 \times 0.6 = 84.4$, $-29 \times 0.4 + 5 \times 0.6 = -8.6$	$200 \times 0.4 + 28 \times 0.6 = 96.8$, $5 \times 0.6 = 3$

FIGURE 8.8 Strategic Form of Entry Game: Tudor's Low Cost Is 5

Checking for Separating Equilibrium:

Step 3: Look for NE in the *Strategic Form*

		FORDOR	
		Regardless (II)	Conditional (OI)
TUDOR	Bluff (LL)	$169 \times 0.4 + 3 \times 0.6 = 69.4$, $-29 \times 0.4 + 5 \times 0.6 = -8.6$	$200 \times 0.4 + 25 \times 0.6 = 95$, 0
	Honest (LH)	$169 \times 0.4 + 28 \times 0.6 = 84.4$, $-29 \times 0.4 + 5 \times 0.6 = -8.6$	$200 \times 0.4 + 28 \times 0.6 = 96.8$, $5 \times 0.6 = 3$

FIGURE 8.8 Strategic Form of Entry Game: Tudor's Low Cost Is 5

Market Entry Game - Separating Equilibrium

Checking for Separating Equilibrium:

- So when Tudor's Low Cost is 5, the Nash Equilibrium is (Honest, Conditional)
- This is a *Separating* equilibrium, because the Tudor's action of *Price High* or *Price Low* completely reveals their type to Fordor.

Is it guaranteed that this game will *always* result in complete separation of types?

- What if we change the Tudor's Low Cost to 10 instead of 5?

Market Entry Game - Pooling Equilibrium

Can you prune any strategies?

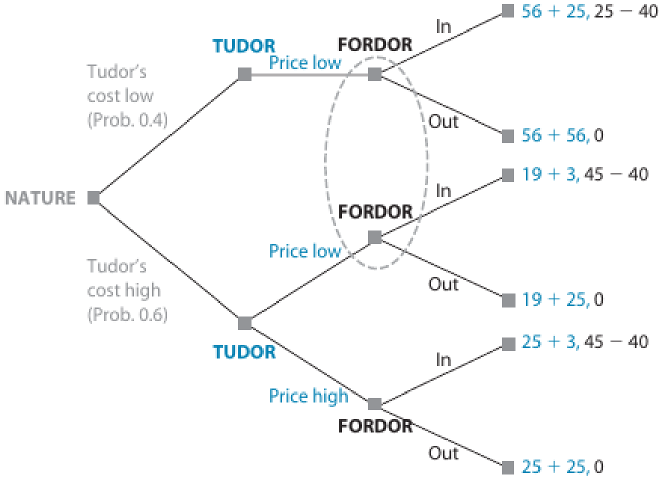

FIGURE 8.9 Extensive Form of Entry Game: Tudor's Low Cost Is 10

Market Entry Game - Pooling Equilibrium

Now what is the Nash Equilibrium of this game?

		FORDOR	
		Regardless (II)	Conditional (OI)
TUDOR	Bluff (LL)	$81 \times 0.4 + 22 \times 0.6 = 45.6$, $-15 \times 0.4 + 5 \times 0.6 = -3$	$112 \times 0.4 + 44 \times 0.6 = 71.2$, 0
	Honest (LH)	$81 \times 0.4 + 28 \times 0.6 = 49.2$, $-15 \times 0.4 + 5 \times 0.6 = -3$	$112 \times 0.4 + 28 \times 0.6 = 61.6$, $5 \times 0.6 = 3$

FIGURE 8.10 Strategic Form of Entry Game: Tudor's Low Cost Is 10

Market Entry Game - Pooling Equilibrium

- So when Tudor's Low Cost is 10, the Nash Equilibrium is (Bluff, Conditional)
- This is a *Pooling* equilibrium, because Tudor always takes the same action of *Price Low*.
  This gives Fordor no signal of their type, but Fordor still doesn't have any incentive to change their strategy.

Market Entry Game

- So far, we found that depending on the relative difference between a *low-cost* Tudor and a *high-cost* Tudor, there may either be a **Pooling** or **Separating** equilibrium.
- But there might also be an equilibrium somewhere in between: where there is *partial* sorting of types
- We call this type of equilibrium **Semiseparating**

Market Entry Game - Semiseparating

Now let's change the original probability that a Tudor is low cost from .4 to .1

- (But keep all of the payoffs the same as in the last case)

Market Entry Game - Semiseparating

Can you find a Nash Equilibrium with the new expected utilities?

		FORDOR	
		Regardless (II)	Conditional (OI)
TUDOR	Bluff (LL)	$81 \times 0.1 + 22 \times 0.9 = 27.9$, $-15 \times 0.1 + 5 \times 0.9 = 3$	$112 \times 0.1 + 44 \times 0.9 = 50.8$, 0
	Honest (LH)	$81 \times 0.1 + 28 \times 0.9 = 33.3$, $-15 \times 0.1 + 5 \times 0.9 = 3$	$112 \times 0.1 + 28 \times 0.9 = 36.4$, $5 \times 0.9 = 4.5$

FIGURE 8.11 Strategic Form of Entry Game: Tudor's Low Cost Is 10 with Probability 0.1

Looking for Mixed Strategy Nash Equilibrium

- Suppose Tudor plays Bluff with probability p, Honest with $1-p$
- When will Fordor play a mixed strategy?

Looking for Mixed Strategy Nash Equilibrium

- Suppose Fordor plays Regardless with probability q, Conditional with $1 - q$
- When will Tudor play a mixed strategy?

Market Entry Game - Semiseparating

- So this version of the game has the MSNE:
  { (1/3 Bluff, 2/3 Honest), (16/22 Regardless, 6/22 Conditional) }
  - In this equilibrium, instead of *complete separation* or *complete pooling*, we have *semiseparating*
  - A high price conveys full information to Fordor, but a low price could mean that the Tudor is *either* a **low-price** *or* a **high-price** type.

Market Entry Game - Semiseparating

Bayes' Rule

		TUDOR'S PRICE: Low	TUDOR'S PRICE: High	Sum of row
TUDOR'S COST	Low	0.1	0	0.1
	High	$0.9 \times 1/3 = 0.3$	$0.9 \times 2/3 = 0.6$	0.9
Sum of column		0.4	0.6	

FIGURE 8.12 Applying Bayes' Theorem to the Entry Game

The CEO's new clothes

clip

Dinesh has let the power of a CEO position go to his head. His new confidence/vanity has led him to trying out a new hairstyle, but he starts to suspect that there is a non-zero probability that he looks **ridiculous** to other people.

The CEO's new clothes

Suppose that looking **ridiculous** is not something that Dinesh can subjectively observe about himself, but is only observable by the people around him.

Gilfoyle can observe whether or not Dinesh looks **ridiculous** and would like it if Dinesh embarrassed himself by looking **ridiculous** in public.

But he knows that if Dinesh thinks he looks **ridiculous** , he will want to change his look back to the more boring (but less risky) style he had as a nerdy programmer.

The CEO's new clothes

Suppose that Dinesh's preferences (from best to worst) are as follows:

- He wears his *new style* proudly and people think he is **cool**
- He wears his *old style* and people think he looks average
- He wears his *new style* but people think it is **ridiculous**

Suppose that Gilfoyle's preferences are:

- Dinesh looks **ridiculous** with the *new style*, continues to wear it and gets embarrassed in public
- Dinesh goes back to his *old style*
- Dinesh looks **cool** with the *new style*, continues to wear it and people think he's cool.

The CEO's new clothes

Model this as an asymmetric information game where Gilfoyle has the private information of whether Dinesh looks **ridiculous**.