Econ 327: Game Theory

Final Exam

University of Oregon

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Version 2

ANSWER KEY

Multiple Choice

Question 1. (4 P.)
A player using a **mixed strategy** means that:

a) **they are internally uncertain about which action they will choose because they are acting randomly**
b) some parts of their strategy are played simultaneously and other parts are played sequentially
c) they will regret not having chosen their a pure strategy instead
d) they are confused about what action their opponent is taking

Question 2. (4 P.)
A game featuring **asymmetric information**:

a) is repeated multiple times by the same players
b) has Nature acting as a player even though she doesn't have any preference over outcomes
c) means that one player has a strategy with no equivalent strategy avaibable to any other player
d) **has some players who have access to private information which is not directly observable to others**

Question 3. (4 P.)
By **signaling**:

a) only players with the 'bad' condition sort into a market
b) **a player can reveal their own private information through their actions**
c) a player can reveal the private information held by others by designing an incentive mechanism
d) only mixed strategies will be played in equilibrium

Question 4. (4 P.)
An **information set**:

a) holds all pieces of information which are publically observable to all players
b) is used by game theorists to signal how they want their games to be played
c) tells a player what action to take
d) **contains all decision nodes which a player cannot tell the difference between when they reach that part of the game**

Question 5. (4 P.)
Consider the following lottery:

- with probability 1/2 you will receive $1,600.
- with probability 1/2 you only receive $16.

Suppose someone has a risk-averse utility function of $u(\$x) = \sqrt{\$x}$. For what certain amount of dollars, x, will this person be indifferent between taking the certain payment with probability of 1 and taking the lottery defined above?

a) $169
b) $412
c) **$484**
d) $808

Question 6. (4 P.)
In the **Prisoner's Dilemma**, mutual cooperation:

a) **Pareto dominates the outcome of mutual defection**
b) is stable
c) is a credible threat
d) is a dominant strategy equilibrium

Question 7. (4 P.)
The Folk Theorem states that:

a) The Prisoners' Dilemma is the only game with a unique Nash equilibrium.
b) **Any individually rational and feasible outcome can be reached in a repeated game for some sufficiently high enough discount factor.**
c) All Pareto optimal outcomes can always be reached in a Nash equilibrium.
d) No matter how hard you try, some folks will just never cooperate

Question 8. (4 P.)
Consider the strategic form game below:

		P_2		
		Left	Middle	Right
P_1	Up	0,1	9,0	2,3
	Straight	5,9	7,3	1,7
	Down	7,5	10,10	3,5

How many Nash equilibria exist in this simultaneous game, including both **pure** and **mixed** strategies?

a) **One equilibrium**
b) Two equilibria
c) Three equilibria
d) An infinite number of equilibria

P_2

		Split	Steal
P_1	Split	6, 6	-6,10
	Steal	10,-6	0, 0

Question 9. (4 P.)
Consider the Prisoners' Dilemma game with payoffs as shown in the strategic form table below:

Suppose Player 2 is utilizing a **Tit-for-Tat** strategy in which they will start off choosing Split, and after that they will play whatever strategy their oponent used in the previous round.
Which of the following represents Player 1's present value of Stealing in the first period and then Splitting in all following periods?
δ is the per-period discount rate.

a) $10 + 0\delta + 0\delta^2 + 0\delta^3 + ... = 10$

b) $6 + 6\delta + 6\delta^2 + 6\delta^3 + ... = \frac{6}{1-\delta}$

c) $10 + 6\delta + 6\delta^2 + 6\delta^3 + ... = 10 + 6\frac{\delta}{1-\delta}$

d) $10 - 6\delta + 6\delta^2 + 6\delta^3 + ... = 10 - 6\delta + 6\frac{\delta^2}{1-\delta}$ **Correct**

Question 10. (4 P.)
Consider the Prisoners' Dilemma game with payoffs as shown in the strategic form table below:

P_2

		Split	Steal
P_1	Split	9, 9	5,15
	Steal	15,5	7, 7

Suppose Player 2 is utilizing a **Grim Trigger** strategy in which they will start off by Splitting, and continue to split unless their opponent has ever played Steal, in which case they will play Steal in all periods following.
Which of the following represents Player 1's present value of Stealing in the first period (and in all following periods)?

a) $15 + 7\delta + 7\delta^2 + 7\delta^3 + ... = 15 + 7\frac{\delta}{1-\delta}$ **Correct**

b) $9 + 9\delta + 9\delta^2 + 9\delta^3 + ... = \frac{9}{1-\delta}$

c) $15 + 9\delta + 9\delta^2 + 9\delta^3 + ... = 15 + 9\frac{\delta}{1-\delta}$

d) $15 + 5\delta + 9\delta^2 + 9\delta^3 + ... = 15 + 5\delta + 9\frac{\delta^2}{1-\delta}$

Long Answer

Grading:

- Each sub-part is graded out of 4 points. Specific scoring is up to the grader's discretion, but here is how I think about my expectations for students' work:
  - Correct (4/4 points) if the final answer given matches the key and all questions are clearly addressed. The explanation should broadly look correct, but there is room for some error as long as they get the general conclusion.
  - Mostly correct (≈ 3/4 points) if the rest of the answer follows logically from a setup that seems more or less on the right track, but might have some payoff values in the wrong places or has a minor algebra errors.
  - Incorrect (≈ 2/4 points) if an answer is given that doesn't match the key and does not clearly follow the methods from class.
  - Incomplete (≈ 1/4 point) if an answer does not have a clearly stated solution that fits with what the question is asking for and it is not clear from the work shown that a correct answer would be reached given more time.
  - 0 points if the part is left completeley blank.

Question 11. (12 P.)
Consider the strategic form game below:

P_2

P_1		**Left**	**Center**	**Right**
	Top	6 , 6	2 , 0	0 , 4
	Middle	0 , 2	4 , 4	6 , 0
	Bottom	4 , 0	0 , 6	2 , 2

a) (4 P.) Find all **pure strategy** Nash equilibria. If there are none, explain how you know.

$$BR_1(Left) = Top \qquad BR_2(Top) = Left$$
$$BR_1(Center) = Middle \qquad BR_2(Middle) = Center$$
$$BR_1(Right) = Middle \qquad BR_2(Bottom) = Center$$

So the two pure strategy Nash equilibria are **{Top, Left}**, and **{Middle, Center}**

b) (4 P.) Now consider the strategy profile:

- Player 1 plays **Top 1/2** of the time, plays **Middle** the other **1/2**, and never plays Bottom
- Player 2 plays **Left 1/2** of the time, plays **Center** the other **1/2**, and never plays Right

Check whether this is a **mixed strategy Nash equilibrium** and explain why or why not.

What is P_1's best response to $(1/2Left,\ 1/2Center)$?

$$\begin{aligned} EU_1(Top) &= 1/2(6) + 1/2(2) = 4 \\ EU_1(Middle) &= 1/2(0) + 1/2(4) = 2 \\ EU_1(Bottom) &= 1/2(4) + 1/2(0) = 2 \end{aligned}$$

So the best response to $(1/2Left,\ 1/2Center)$ is to play only Top.
***Not* a Nash equilibrium.** Player 1 would prefer to play (Top) over $(1/2Top,\ 1/2Middle)$.

c) (4 P.) Consider the following mixed strategy profile:

- Player 1 plays **Top 1/2** of the time, never plays Middle , and plays **Bottom** the other **1/2**
- Player 2 plays **Left 1/2** of the time, never plays Center, and plays **Right** the other **1/2**

Check whether this is a **mixed strategy Nash equilibrium** and explain why or why not.

What is P_1's best response to $(1/2Left,\ 1/2Right)$?

$$\begin{aligned} EU_1(Top) &= 1/2(6) + 1/2(0) && = 3 \\ EU_1(Middle) &= 1/2(0) + 1/2(6) && = 3 \\ EU_1(Bottom) &= 1/2(4) + 1/2(2) && = 3 \end{aligned}$$

So because Top and $Bottom$ are both best responses, they would be willing to play any mixture of the two.
What is P_2's best response to $(1/2Top,\ 1/2Bottom)$?

$$\begin{aligned} EU_2(Left) &= 1/2(6) + 1/2(0) && = 3 \\ EU_2(Center) &= 1/2(0) + 1/2(6) && = 3 \\ EU_2(Right) &= 1/2(4) + 1/2(2) && = 3 \end{aligned}$$

Because $EU_2(Right) = EU_2(Left) \geq EU_2(Center)$, $(1/2Left,\ 1/2Right)$ is a best response.
This strategy profile is a NE. Neither player could get a strictly higher payoff by unilaterally deviating from these mixed strategies.

Grading:

- For MSNE, a complete answer should use exp. utils. to find the best responses to the mixed strategies given in prompts.
  - If another strategy (usually a pure strategy) would result in a higher exp. util. than original strategy, the answer should be *Not a NE*
  - If all players are indifferent between all pure strategies played with non-zero probability in the original mixed strategy, then there is no unilateral deviation, so the answer should use defn. to say this strat. profile *Is an NE.*
  - If the question asks for explanation, there doesn't have to be full detail given, but it is the student's responsibility to demonstrate that they understand the concepts being assessed.

Question 12. (8 P.)

Suppose that you are in charge of admissions for a prestigious school for elite musicians. You only want to admit the best students. Suppose that there are only two types of young musicians, *prodigies* and *normies*. The chance to attend your school would be life changing for everyone, so both types would receive 3,000 utility from attending your school. If you don't admit a student, they could still get a good (but not as prestigious) training at the state university's music program which they would get 1,000 utility from.
In order to allow you only admit the *prodigies* and not admit any *normies*, you can require a portfolio of N number of pieces that the applicants must submit to be considered. Suppose that *prodigies* have an easier time composing each piece because they are so talented. It costs *normies* 200 utility for each piece they prepare, but only costs 100 per piece for *prodigies*.

A) (4 P.) What is the minimum number N you could require so that only *prodigies* apply and *normies* don't apply? Explain your answer.

To get *normies* to choose the State University over the prestigious school, make the cost of preparing an application higher than their relative benefit of attending (Incentive compatible).

$$\begin{aligned} U_n(State\ School) &\geq U_n(Prestigious) \\ \Rightarrow 1{,}000 &\geq 3{,}000 - 200N \\ 200N &\geq 2{,}000 \\ \Rightarrow N_n^* &\geq 10 \end{aligned}$$

B) (4 P.) What is the maximum number N that you could require so that *prodigies* still want to apply?

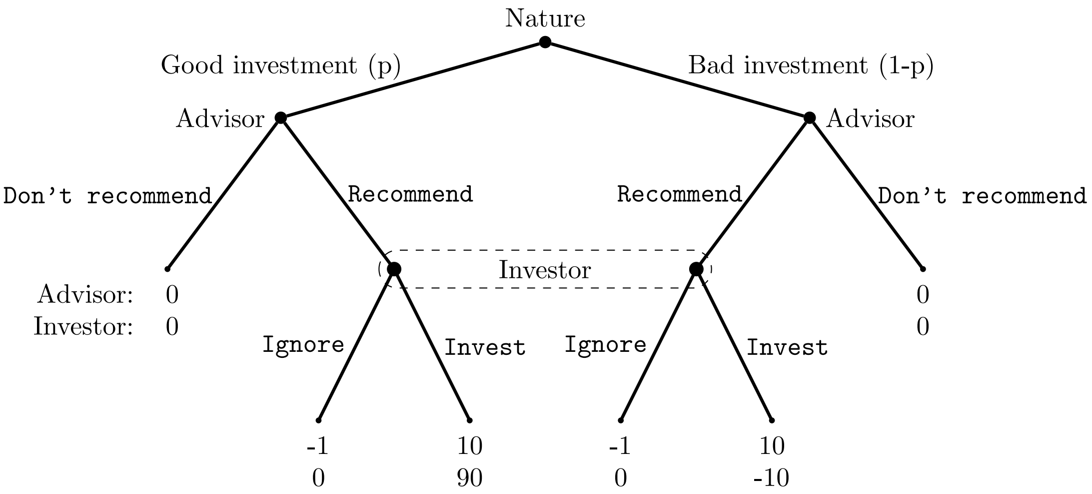
To get *prodigies* to choose the prestigious school over the State University, the benefit to going prestigious must still outweigh the cost of preparing an application (Individually rational).

$$\begin{aligned} U_p(Prestigious) &\geq U_p(State\ School) \\ \Rightarrow 3{,}000 - 100N &\geq 1{,}000 \\ 100N &\leq 2{,}000 \\ \Rightarrow N_p^* &\leq 20 \end{aligned}$$

Question 13. (12 P.)

Consider simplified model of asymmetric information in financial advising with two types of potential investment options; Good investments have positive returns, and Bad investments have negative returns. Assume the Investor does not know which type of investment will turn out to be good or bad, but they have a perfectly-informed advisor who can choose whether to recommend either type of investment. If the investor invests, they have to pay their advisor a 10% fee, but they take 90% of the return for themselves if it is good, and have to pay out of their own pocket if bad. The advisor loses nothing from not recommending either type of investment, but the time spent putting together a recommendation that their client does not invest in costs them an equivalent of 1% of the asset's value in time spent researching. Assume all players are risk-neutral.

The extensive form game is below:



a) (4 P.) Can there be any *separating equilibria* in this game where the Advisor always recommends good investments and never recommends bad investments? Why or why not?

No separating eq: If Advisor only recommends Good investments, Investor should *always invest* with the belief they will earn 90 expected utility instead of zero by ignoring. But if the Advisor knows this, they should also recommend bad investments to earn their 10% commission.

By similar logic, if the Investor always Ignores, then the Advisor shouldn't recommend either type of investment to avoid the cost of effort of -1.

b) (4 P.) Is there a *pooling equilibrium* where the Investor does invest? Solve for a range of p that describes when the Investor will choose to invest based on their expecation that their advisor will always recommend either investment type.

Investor will `Invest` if:

$$
\begin{aligned}
EU(\texttt{Invest}) &\geq EU(\texttt{Ignore}) \\
90p - 10(1-p) &\geq 0 \\
\Rightarrow p &\geq 1/8
\end{aligned}
$$

Pooling SPNE:

- Investor's strategy: `Invest`
  - Investor's belief: $p \geq 1/8$
- Advisor's strategy: `Recommend` if Good, `Recommend` if Bad
  - Advisor's belief: Investor will always `Invest`

c) (4 P.) What is the *signalling* value of a recommendation from the Advisor? Is the Investor's updated belief of the recommended investment being Good any different from the original probability p in equilibrium? Why or why not?

In the pooling equilibrium above, the `Recommend` strategy has no signaling value.
Because the Investor knows that their Advisor will always recommend, ($prob$(`Recommend`|Good = 1)) when they see a recommended investment, they update their beliefs using Bayes' rule:

$$prob(\text{Good}|\texttt{Recommended}) = \frac{prob(\texttt{Recommend}|\text{Good}) \times prob(\text{Good})}{prob(\texttt{Recommend})} = \frac{1 \times p}{p + (1-p)} = p$$

so their updated belief of the probability of a Good investment is the same as the Ex-ante probability, p.
Note: Using Bayes' Rule is not necessary, but a correct answer should demonstrate an understanding of signaling.

Grading:

- Part (a) any answer which correctly argues there cannot be any separating equilibrium should get full credit. Mostly correct if they identify no sep. eqm. but don't give an explanation.
- Part (b) doesn't need to fully state the SPNE strategy profile as long as they describe the strategies they are using somewhere in their work. Full credit if they get the correct range for p. Mostly correct if they get a different range but use some kind of expected utility setup.
- Writing out the full Bayes' Rule is not necessary (for example in part (c)), but a correct answer should use the correct probabilities attached to the correct outcome payoffs in the expected utilities.
- Students might get lost in how to set up this question, but try to give partial credit if they can explain in words something about the broad intuition of signaling, separating, pooling, etc.
  - The broad goal of this question is to test that students understand the strategic incentives that arise from asymmetric info.

Question 14. (16 P.)

Consider the strategic form game below:

		Column	
		Cooperate	Defect
Row	Cooperate	10 , 10	0, 14
	Defect	14 , 0	8 , 8

a) (4 P.) Describe the equilibrium when this game is a *one-shot* game. Is the equilibrium outcome Pareto efficient?

The only Nash Equilibrium is {Defect, Defect} which is Pareto dominated by the outcome of {Coop, Coop}, so not Pareto efficient ((8,8) < (10,10)).

b) (4 P.) Describe how players' strategies might change if this game is repeated *infinitely.* Compare to your answer from part (a).

Potential answers include (not exhaustive):

- Players might start off Cooperative with expectation of future cooperation
- Shared history, reputation, etc. may play a role in coordinating cooperation
- Tit-for-Tat, Grim Trigger strategies, etc. look to common shared history to see when to cooperate or defect.

Compared to the *one-shot* game of part (a), repetition may allow for Pareto optimal outcome to be reached in eqm., assuming players are patient/forward looking enough, believe they will meet again, etc.

c) (4 P.) Suppose that one player is playing a *grim trigger* strategy in which they start by cooperating, but will always and forever Defect if the other player has ever chosen Defect.
Set-up (but do not solve yet) the *present value* of Defecting once and also the present value of Cooperating forever.

If both players cooperate forever, the Grim trigger is never activated, the present value comes from getting the payoff of 10 forever.
If one player deviates to Defect, they will earn the higher payoff of 14 for the first period, but will be punished with the lower payoff of 8 in all periods following:

$$PV(Coop\ Forever) = 10 + 10\delta + 10\delta^2 + ... = \frac{10}{1-\delta}$$

$$PV(Defect\ Forever) = 14 + 8\delta + 8\delta^2 + ... = 14 + \frac{8\delta}{1-\delta}$$

d) (4 P.) Suppose that both players have a *discount factor* of $\delta = 3/4$. Can a strategy profile of both players using *grim trigger* strategies be sustained in the game where the strategic form game above is repeated infinitely?
Show all calculations and explain your answer.

$$PV_2(Coop\ Forever) \geq PV_2(Defect\ Forever)$$
$$10 + \frac{10\delta}{1-\delta} \geq 14 + \frac{8\delta}{1-\delta}$$
$$2\delta \geq 4 - 4\delta$$
$$\Rightarrow \delta \geq 2/3$$

So if $\delta = 3/4 > 2/3$, that is sufficient for both players to be willing to Cooperate in every period rather than ever Defecting when they know they will be punished by the other player's Grim Trigger strategy.

Grading:

- Students answers to part (b) might vary widely. Give full or most of the credit as long as they mention a class concept somewhere.
- for part (c):
  - $PV(Coop\ Forever)$ can also be written as $5 + 5\frac{\delta}{1-\delta}$.
- Part (d):
  - Students might use the shortcut we learned for a general-form prisoners' dilemma. This would look something like:
    - $\delta \geq \frac{H-C}{H-D}$
    - $\frac{\delta}{1-\delta} \geq \frac{H-C}{C-D}$

    where H is the highest payoff from defecting against a cooperative opponent, C is the payoff in the cooperate, cooperate outcome.
  - If they use one of these fraction shortcuts, they should fully explain what their variables mean somewhere. Give partial credit if they skip to an answer without explaining all of the steps along the way.