Econ 327: Game Theory

Practice Exam

University of Oregon

February 4, 2024

Version 1

Question:	**Question 1**	**Question 2**	**Question 3**	Total
Points:	20	40	40	100
Score:				

For Exams:

- Complete *all* questions and parts. All questions will be graded.
- Carefully explain all your answers on short and long answer questions.

  An incorrect answer with clear explanation will earn partial credit, an incorrect answer with no work will get zero points.
- If you do not understand what a question is asking for, ask for clarification.

Allowed Materials:

- A single 5” by 3” note card
- A non-programmable calculator
- Pencils, color pens, eraser, ruler/straight-edge etc.

Name Key

Answer the questions in the spaces provided on the question sheets. If you run out of room for an answer, continue on the back of the page or another sheet of paper.

Question 1. [20 points] **Multiple Choice**

See the Quizizz from Tuesday for more practice.
They will look like the multiple choice questions from homework, but there will be 10 total instead of 5.

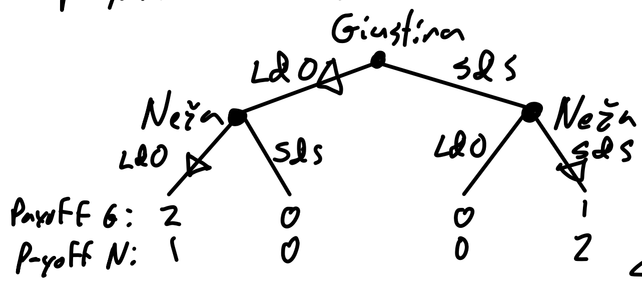
Long Answer

Question 2. Giustina and Neža can each either go to dinner at *Lion & Owl* or *Spice N Steam*. They both would prefer to go to a restaurant together than to go alone. Giustina prefers *Lion & Owl* to *Spice N Steam*, but Neža prefers *Spice N Steam* to *Lion & Owl*. Giustina is the more decisive of the two, so she chooses a restaurant first and then Než decides which restaurant she will go to after seeing where Giustina is going. Let Giustina's strategy be first element of strat profile

(a) [10 points] Draw an extensive form game to go with this story and solve for all subgame perfect Nash equilibria.

Giustina: (L&O, L&O) ≻ (S&S, S&S) ≻ (S&S, L&O) ~ (L&O, S&S)
payoffs: 2 1 0 0

Neža: (S&S, S&S) ≻ (L&O, L&O) ≻ (S&S, L&O) ~ (L&O, S&S)
payoffs: 2 1 0 0



Backwards induction:
If G picks L&O, N prefers L&O
If G picks S&S, N prefers S&S
Knowing this, G would rather choose L&O initially if N will follow

SPNE: {(L&O), (L&O, S&S)}

(b) [10 points] Now represent this game in strategic form and solve for all pure strategy Nash Equilibria. Can you find any Nash equilibria which are not subgame perfect?

Neža

Giustina	(L&O, L&O)	(L&O, S&S)	(S&S, L&O)	(S&S, S&S)
L&O	2, 1	2, 1	0, 0	0, 0
S&S	0, 0	1, 2	0, 0	1, 2

PSNE:
- {(L&O), (L&O, L&O)}
- {(L&O), (L&O, S&S)}
- {(S&S), (S&S, S&S)}

{(L&O), (L&O, L&O)} is a NE but not subgame perfect because Neža is not acting rationally in the right-hand subgame.

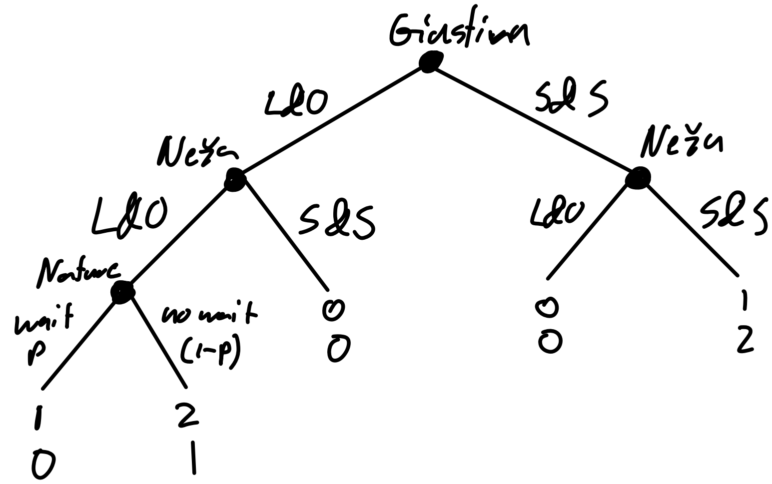
{(S&S), (S&S, S&S)} is another NE that is not SPNE.
↳ this one involves Neža threatening to only go to S&S when Giustina should know she wouldn't follow through

(L&O, L&O)

Now suppose that if Giustina and Neža show up to *Lion & Owl* together, there is a chance that they will have to wait up to an hour to get a table. If they have to wait, Giustina would be equally happy going to *Spice N Steam* together where they wouldn't have to wait. Neža would be equally waiting to go to *Lion & Owl* with Giustina or *Spice N Steam*

(c) [10 points] Draw a new extensive form game to match the updated story. Make sure to define any variables you include.

Let p be the probability of having to wait together at L&O.



(d) [10 points] Find all subgame perfect Nash equilibrium which results in both Giustina and Neža going to *Spice N Steam*.

Your answer should be a function of the probability of waiting in line at *Lion & Owl*.

Represent the payoffs in (L&O, L&O) case as lottery over p.

$$EU_G(L\&O, L\&O) = 1p + 2(1-p) = 2-p$$

$$EU_N(L\&O, L\&O) = 0p + 1(1-p) = 1-p$$

- when would Neža prefer to go S&S when Giustina goes L&O?

when $0 \geq 1-p \Rightarrow p \geq 1$

So if Neža knows for sure they will have to wait, she will be indifferent between following G to L&O or staying at home.

- when would Giustina prefer to go to S&S?

Suppose Neža's strat is $(\delta\, L\&O, (1-\delta)\, S\&S, S\&S)$

$$\left.\begin{array}{l} EU_G(L\&O) = 1\delta + 0(1-\delta) = \delta \\ EU_G(S\&S) = 1 \end{array}\right\}$$ so if $\delta = 1$, G would be indifferent between L&O, S&S

Only way for Giustina to pick S&S in eqm is if $p = 1$

SPNE $= \{(S\&S), (S\&S, S\&S)\}$ when $p = 1$

Question 3. Consider the strategic form game below:

step1: C SD by A so cross out C
step2: Z is now SD by Y
step3: B is now SD by A
step4: Y is now SD by W
step 5: no more SD strats, so stop IDSDS

P_1 \ P_2	A	~~B~~ (3)	~~C~~ (1)	D
W	15, -7	~~8, 2~~	~~18, -7~~	11, 5
X	-3, 18	~~6, -7~~	~~8, -7~~	17, 18
(4) ~~Y~~	~~9, 19~~	~~20, 4~~	~~13, 6~~	~~10, 16~~
(2) ~~Z~~	~~9, 20~~	~~14, 16~~	~~15, 5~~	~~3, 4~~

(a) [8 points] Use Iterated Deletion of Strictly Dominated Strategies and write out a simplified game table with any remaining cells.

P_1 \ P_2	A	D
W	15, -7	11, 5
X	-3, 18	17, 18

(b) [10 points] Find all Nash equilibria in *pure strategies.* Explain why you know they are Nash equilibria.

$BR_1(A) = W$ $BR_2(W) = D$

$BR_1(D) = X$ $BR_2(X) = \{A, D\}$

PSNE: $\{X, D\}$

X is BR to D, D is BR to X, so no incentive for either player to unilaterally deviate

(c) [6 points] Define mixed strategies for each player using any pure strategies left after IDSDS. Make sure to define all variables you introduce.

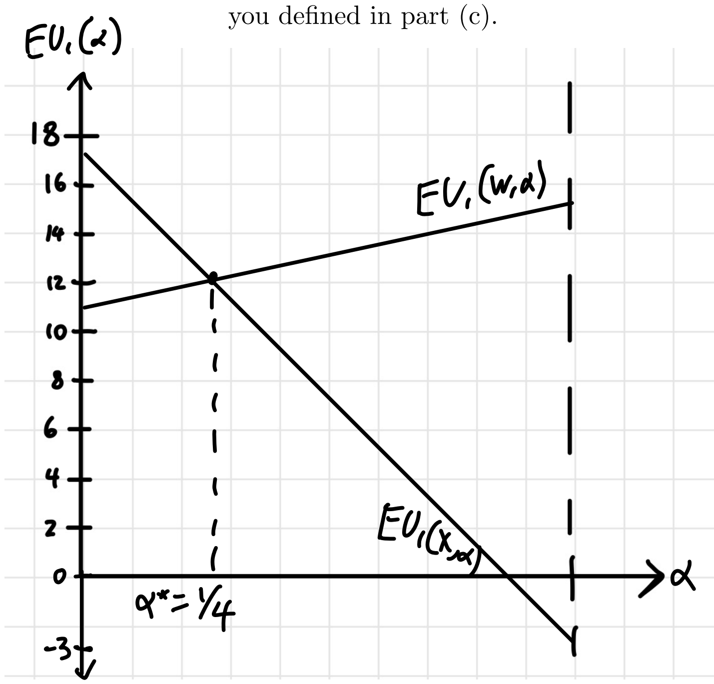
Suppose P_1 plays W w/ prob β, X w/ prob $(1-\beta)$ (Y and Z played w/ 0 prob)

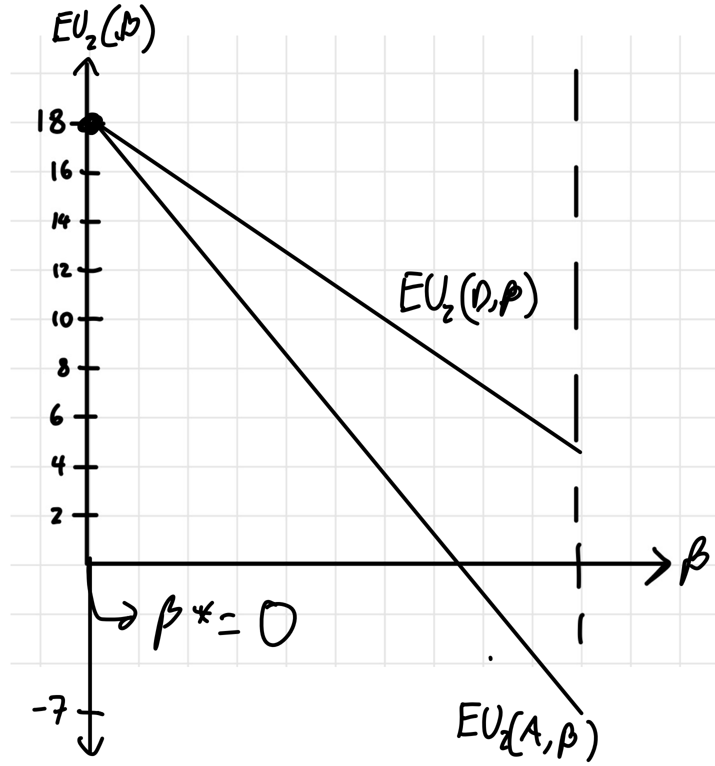
Suppose P_2 plays A w/ prob α, D w/ prob $(1-\alpha)$ (B and C played w/ 0 prob)

P_1's mixed strat: $(\beta W, (1-\beta) X)$

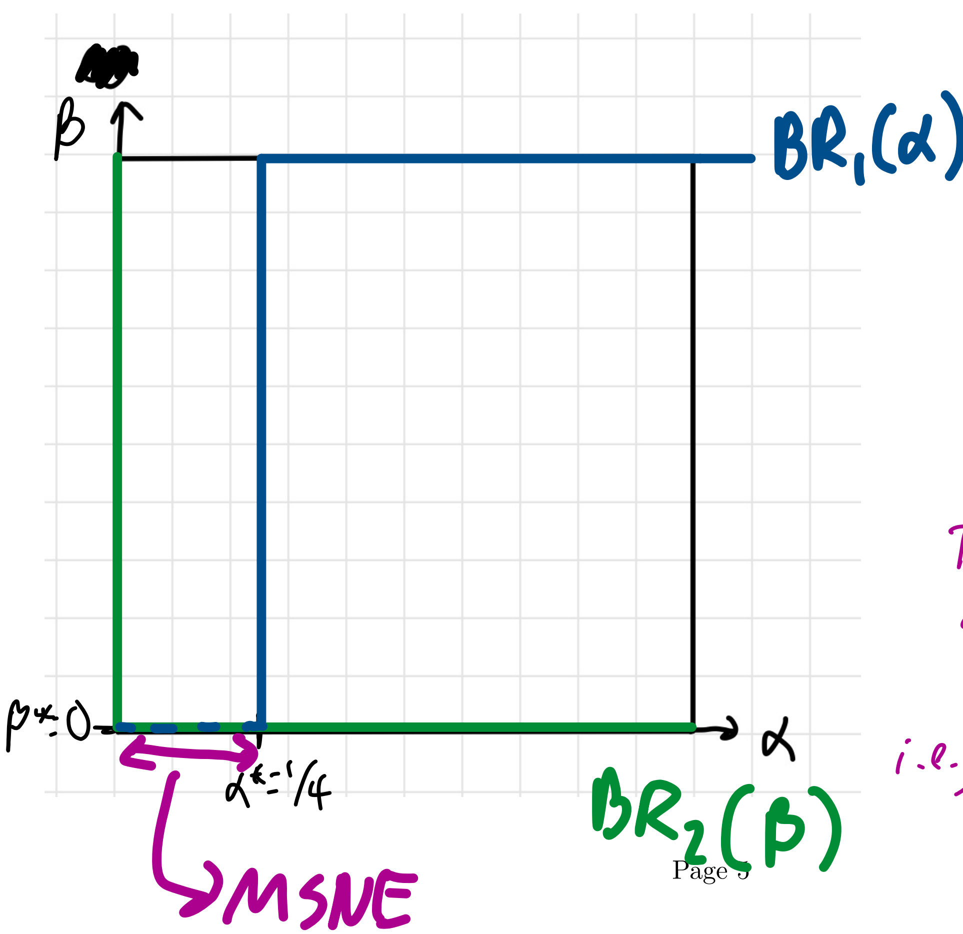
P_2's mixed strat: $(\alpha A, (1-\alpha) D)$

(d) [8 points] Graph each player's expected utilities as functions of the other players' mixed strategy you defined in part (c).





(e) [8 points] Solve for all Mixed Strategy Nash equilibria in this game. A complete answer will include all calculations used and a graph of best response functions.



$$\boxed{\begin{array}{l}\underline{\text{MSNE:}} \\ \{X, (\alpha A, (1-\alpha) D)\} \\ \text{when } 0 \le \alpha \le 1/4\end{array}}$$

no other intersections of BR's, so this includes <u>all</u> Nash eq. of this game.

The pure strat NE is a subcase when $\alpha^* = 0$ and $\beta^* = 0$

i.e., $\{X, D\}$ as in part (b)

For part d:

$EU_1(W,\alpha) = 15\alpha + 11(1-\alpha) = 11 + 4\alpha$

$EU_1(X,\alpha) = -3\alpha + 17(1-\alpha) = 17 - 20\alpha$

$EU_2(A,\beta) = -7\beta + 18(1-\beta) = 18 - 25\beta$

$EU_2(D,\beta) = 5\beta + 18(1-\beta) = 18 - 13\beta$

For part e:

P_1 will mix strategies when:

$11 + 4\alpha = 17 - 20\alpha \Rightarrow 24\alpha = 6$
$\Rightarrow \alpha^* = 1/4$

P_2 will mix strategies when:

$18 - 25\beta = 18 - 13\beta$

$\Rightarrow \beta^* = 0$

Short Answer

These questions were cut for time on the actual midterm exam, but they are still good practice. Just don't count how much time you spend on them when you're gauging how long the real exam will take you.

(a) Consider the strategic form game below:

		Aslanbek		
		Low	*Moderate*	*High*
Hagano	*Low*	0,0	3,2	7,3
	Moderate	2,3	5,5	6,4
	High	3,7	4,6	4,5

Find all pure Nash strategy profiles.

$BR_H(L) = H$
$BR_H(M) = M$
$BR_H(H) = L$

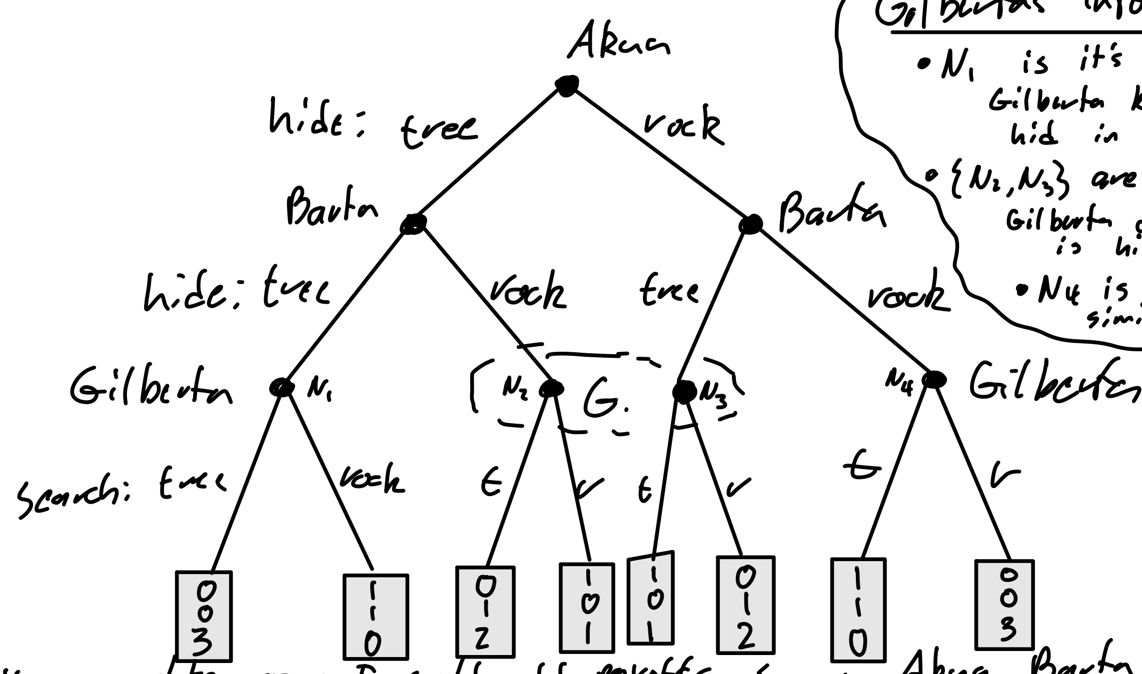
$BR_A(L) = H$
$BR_A(M) = M$
$BR_A(H) = L$

3 PSNE: {H, L}, {M, M}, {L, H}

(b) Akua, Barta, and Gilberta are playing a version of hide and seek. There are only two good hiding spots; up a tree, or behind a rock. Akua gets to hide first. Barta also hides, but she gets to see which spot Akua is hiding before she picks. Once Akua and Barta are hidden, Gilberta has to choose one and only one place to look. If there are two people hiding in the same spot, they crowd each other and Gilberta can see them. If there is only one person in a spot, Gilberta can't see who's hiding there.

Create an extensive form game tree and clearly specify Gilberta's information set.

Gilberta's info sets:
- N_1 is it's own info set: Gilberta knows when both hid in tree
- $\{N_2, N_3\}$ are in one info set. Gilberta doesn't know who is hidden where
- N_4 is one info set similar to N_1



To make it a complete game I could add payoffs. Suppose Akua, Barta get 0 if found, 1 if Gilberta doesn't find them.
Maybe Gilberta would prefer catching both players and would get 3 utility.
Gilberta gets 2 from only finding Akua and 1 from only finding Barta.
She gets 0 if she doesn't find anyone.

(c) Suppose that two fishing boats are selling to the same market. Let V be the tons of fish caught by Vlatislav's boat, and J be the tons of fish caught by Jeren's boat. People in this town only want to buy so many fish, so the price P of fish is given by the inverse demand function:

$$P = 60 - (R + S)$$

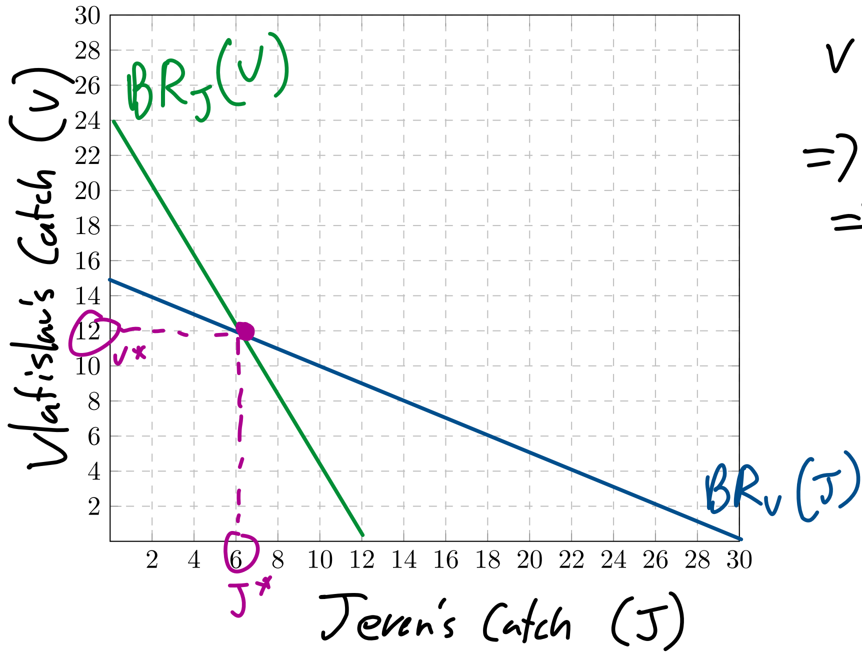
Assuming both boat owners only care about profit, we get that Vlatislav's best response function is

$$\underline{V = 15 - \frac{J}{2}}$$

and that Jeren's best response function is

$$\underline{J = 12 - \frac{V}{2}}$$

Graph both players' best response functions and find all Nash Equilibria. Label your graph appropriately.



Find intersection: $BR_J = J$ in BR_V

$V = 15 - \frac{(12 - V/2)}{2}$

$\Rightarrow V = 15 - 6 + V/4$

$\Rightarrow 3/4 V = 9$

$\Rightarrow V^* = \frac{36}{3} = 12$

plug back into BR_J:

$J^* = 12 - \frac{12}{2}$

$\Rightarrow J^* = 6$

NE: $\{V = 12, J = 6\}$

only one NE because only one intersection point