Econ 327: Game Theory

Homework #4

University of Oregon

Due: Oct. 24^{th}

Question:	**Q1**	**Q2**	**Q3**	**Q4**	Total
Points:	12	10	8	12	42
Score:					

For homework assignments:

- Complete *all* questions and parts.
- You will be graded on not only the content of your work but on how clearly you present your ideas. Make sure that your handwriting is legible. Please use extra pages if you run out of space but make sure that all parts of a question are in the correct order when you submit.
- You may choose to work with others, but everyone must submit to Canvas individually.

  Please include the names of everyone who you worked with below your own name.

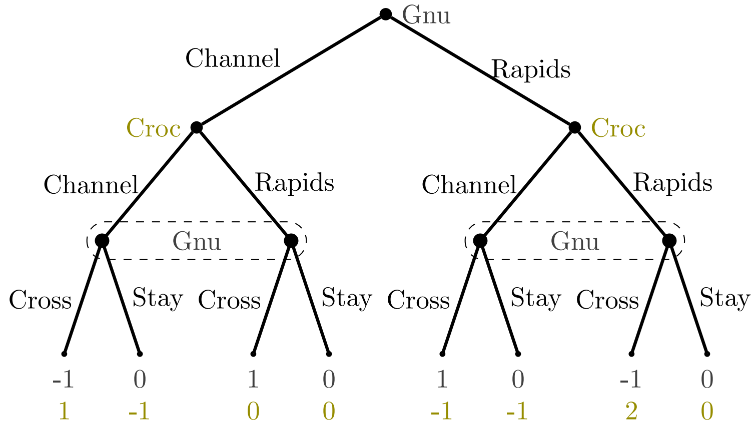
Name ____

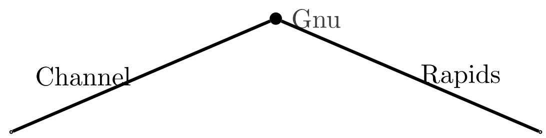
Q1. Recall the gnu and croc river crossing game from Homework 2, Question 3.

Now, consider what happens if the gnu have bad eyesight and can't see where the crocs are choosing to lie in wait.

(a) [4 points] Draw out the new extensive form game.

Solution: Either of the following extensive forms is acceptable:





Gnu		Croc: Channel	Croc: Rapids
	Cross	-1,1	1,0
	Stay	0,-1	0,0

Gnu		Croc: Channel	Croc: Rapids
	Cross	1,-1	-1,2
	Stay	0,-1	0,0

(b) [4 points] First, solve for all Nash equilibria in every proper *subgame* of the extensive form game tree.

Solution:

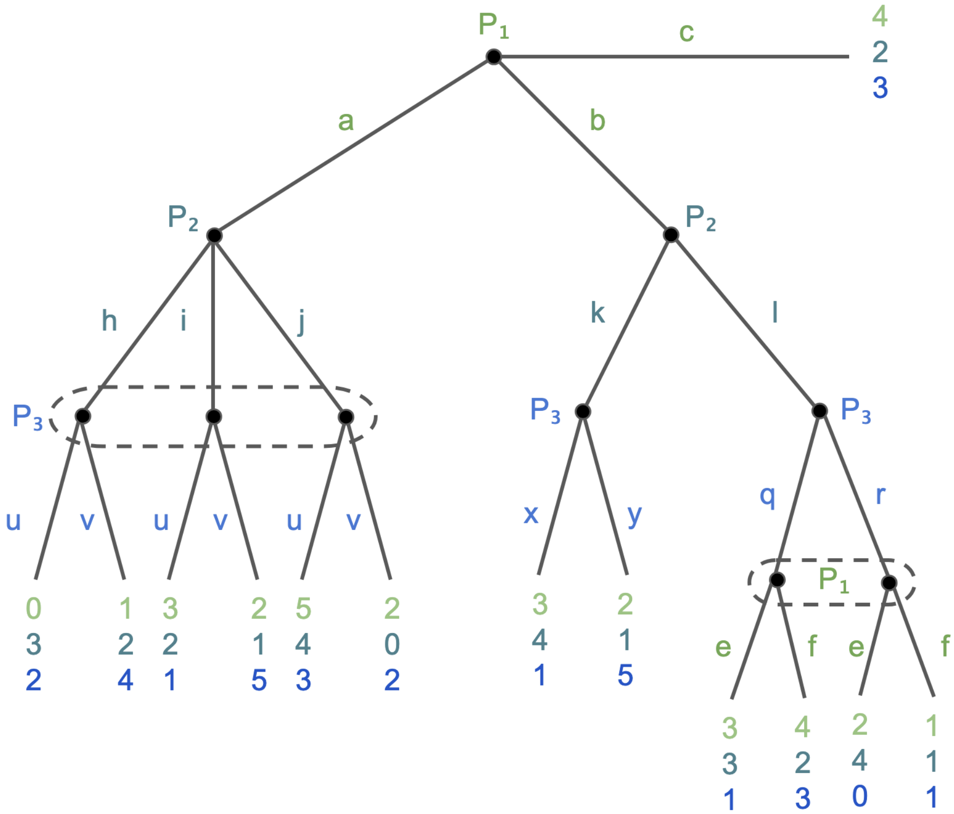
In the subgame following the gnu choosing Channel, the only Nash is in mixed strategies which is when the gnu **Cross 50% of the time** (and stay 50% of the time) and the crocs wait in the **Channel 50% of the time** (and wait in the rapids 50% of the time).

In the subgame following the gnu choosing Rapids, the only Nash is in pure strategies where the gnu **Stay** and the crocs wait in the **Rapids**.

(c) [4 points] How does your prediction change from homework 2? Would you still expect to always see the gnu safely crossing at the channel?

Solution: If the Gnu can't see the Crocs, they can't use a strategy where they only selectively cross the Channel if the Crocs are there. If they choose to Cross the Channel in this case, then the Crocs will decide it's worth it to go to the Channel. So the equilibrium from earlier doesn't apply because the information sets of the extensive form game have changed.

Q2. Consider the extensive form game treee below.



(a) [2 points] What should a complete strategy profile look like?
How many elements will each player have in their *complete* strategy?

(b) [4 points] Find all subgame perfect Nash equilibria in pure strategies.

(c) [4 points] Can you find a Nash equilibrium that is not subgame perfect? Carefully explain.

Q3. A game theorist is walking down the street in his neighborhood and finds \$20. Just as he picks it up, two neighborhood kids, Jane and Tim, run up to him, asking if they can have it. Because game theorists are generous by nature, he says he's willing to let them have the \$20, but only according to the following procedure: Jane and Tim are each to submit a written request as to their share of the \$20. Let t denote the amount that Tim requests for himself and j be the amount that Jane requests for herself. Tim and Jane must choose j and t from the interval $[0, 20]$. If $j + t \leq 20$, then the two receive what they requested, and the remainder, $20 - j - t$, is split equally between them. If, however, $j + t > 20$, then they get nothing, and the game theorist keeps the \$20. Tim and Jane are the players in this game. Assume that each of them has a payoff equal to the amount of money that he or she receives. [1]

(a) [4 points] Describe Tim's and Jane's best response rules.

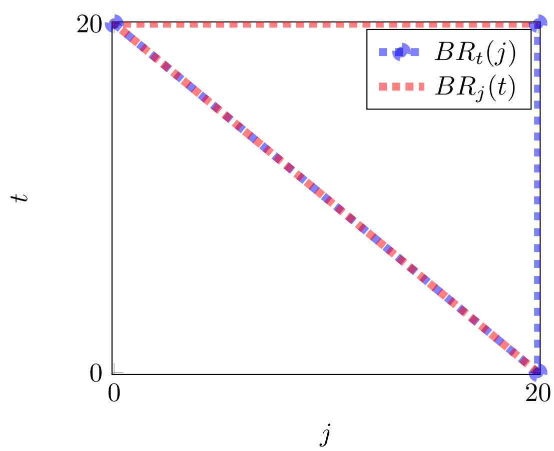
Solution:
Tim's best response rule:

$$BR_t(j) = \begin{cases} 20 - j & \text{if } j < 20 \\ [0, 20] & \text{if } j = 20 \end{cases}$$

Jane's best response rule:

$$BR_j(t) = \begin{cases} 20 - t & \text{if } t < 20 \\ [0, 20] & \text{if } t = 20 \end{cases}$$

(b) [4 points] Find all Nash equilibria. Be careful about corner solutions.



Solution:
The NE are any points on the graph where the $BR_j(t) = t$ and $BR_t(j) = t$. This includes the diagonal line which is the set of all pairs, j and t such that $j + t = 20$ where $j, t < 20$. When $j = 20$, there is one NE when $t = 0$ and another when $t = 20$. When $t = 20$, there is one NE when $j = 0$ (and $j = 20$ is still NE).

[1] Harrington *Games, Strategies, and Decision Making*

Q4. **[n-person game theory]**

Suppose there are two types of music fans: *normies* only like a band if a majority of other people like them too; *hipsters* only like a band if a minority of other people like them.

Suppose that the payoff to *normies* from liking *Wolf Alice* is $100 + 2m$, where m is the number of people who like them. The payoff to a *hipster* from liking *Wolf Alice* is $500 - 5m$. Anyone can choose to like *The National* which has a payoff of 100.

Assume there arre 100 people, 75 are *normies*, and 25 are *hipsters*. [2]

(a) [4 points] Is it a Nash equilibrium for only the *hipsters* to like *Wolf Alice*?

> **Solution:**
> $u(Wolf\ Alice)_{hipsters} = 375 > 100 = u(National)$, so *hipsters* are willing to like WA.
> But *normies* also like WA better because $u(Wolf\ Alice)_{normies} = 150 > 100$.
> So this is not a Nash.

(b) [4 points] Is it a Nash equilibrium for only the *normies* to like *Wolf Alice*?

> **Solution:** $u(Wolf\ Alice)_{hipsters} = 125 > 100 = u(National)$, so *hipsters* would also like *Wolf Alice*. So this is not a Nash.

(c) [4 points] Is there a Nash equilibrium where *Wolf Alice* has fans of both types?

> **Solution:** For *hipsters* to be indifferent between WA and N,
>
> $$\begin{aligned} u(WA)_h &= u(N)_h \\ 500 - 5m &= 100 \\ m &= 80 \end{aligned}$$
>
> So all *normies* and 5 *hipsters* liking *Wolf Alice* is a Nash equilibrium.

[2] Adapted from Cliff Bekar, Lewis & Clark College