Uncertainty & Information Topics

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EC327 Game Theory

Outline

Topics and Definitions

What is Asymmetric Info?

- We already learned about *symmetric* uncertainty in the models where `Nature` makes a play that *neither* player can observe.
- But sometimes one player will know some things that other do not.

Asymmetric Information

describes situations in which some players have private information that is not accessible to other players.

What is Asymmetric Info?

If you are better informed than others:

- You might be able to *conceal* or *reveal misleading* information strategically in order to manipulate the beliefs of others about you
- You might instead want to *selectively reveal* the truth if it helps you.

If you are less informed than other players:

- You might want to *filter out the truth* from lies or misinformation.
- You could instead strategically *remain ignorant* in order to claim "credible deniability".

Cheap Talk

I could let people in on my private info by directly talking to them. But if they know that I have potential incentives to *lie*, they might not believe my *cheap talk*.

Actions Speak Louder Than Words

Behaviors in Asymmetric Info Games

Signaling

When I know something about myself that would benefit me if *others* knew, I might send a signal through my actions

Examples:

- A 4.0 GPA might signal to potential employers that you are hard-working.
- If you're in the market for a product and you're uncertain of its quality, a money-back guarantee might *signal* that it works.

Behaviors in Asymmetric Info Games

Screening

When I want to know something about *someone else's* private info, I might get them to take an action that would screen out people of different *types*.

Examples:

- An employer might not know if a job candidate is a *lazy* or *industrious* type of worker, but they could try to screen out the *lazy* ones by requiring a portfolio of previous work.

Effectiveness of Different Communication Strategies

When are different strategies effective in actually revealing private info?

- Sometimes direct communication works when players' interests align. But trust might break down when there are incentives to send false messages.
- A signal is only effective if not all types take the same action. We'll discuss breakdowns in signaling using the ideas of Separating vs Pooling equilibria

Asymmetric Info in Market Games

- In 201 or 311 you may have learned about the **perfectly competitive** markets model.
- One of the assumptions of that model is perfect information.
- When this assumption breaks, we might see Adverse Selection or other types of market failures.

Cheap Talk

Cheap Talk Equilibrium - When Interests Align

Suppose that I want to meet up with Jose at a coffee shop on campus.

		Jose	
		$Starbucks$	$Roma$
Dante	$Starbucks$	$1, 1$	$0, 0$
	$Roma$	$0, 0$	$2, 2$

We'll also add a first stage to this game where Dante can send Jose a text message saying either *"I'm going to Starbucks"* or *"I'm going to Roma"*.

Cheap Talk Equilibria - When Interests Align

The strategy profile where:

- I send the message *"going to Starbucks"*
- we both go to Starbucks if I send *"going to Starbucks"*
- or both go to Roma if I send *"going to Roma"*

is a **Nash Equilibrium** (specifically a *subgame perfect* NE).

- We'll call this a "cheap talk" equilibrium because it was in my best interest to communicate my actual strategy.
- It cost me nothing to send a *message*.

Cheap Talk vs Babbling Equilibrium

However, this is not the only SPNE of this game. If are strategy profiles in the second stage are:

- Jose will go to Starbucks no matter what message Dante sends
- Dante will go to Starbucks no matter what message he sent

Then Dante will be indifferent between sending either message in the first place.

- We'll call this a "babbling" equilibrium because the initial message sends *no* information about what I will actually do.
- This equilibrium seems unlikely, but if I have an existing *reputation* for always going to Starbucks, this would be plausible and completely rational behavior.

Cheap Talk Equilibria - When Interests are Conflicting

What about a zero-sum game?

		Navratilova DL	Navratilova CC
Evert	DL	$50, 50$	$80, 20$
Evert	CC	$90, 10$	$20, 80$

- Should Navratilova believe what Evert says she will do?
- Should Navratilova believe that Evert will do *exactly the opposite* of what she says she'll do?

What about a zero-sum game?

		Navratilova: DL	Navratilova: CC
Evert	DL	$50, 50$	$80, 20$
	CC	$90, 10$	$20, 80$

- The only equilibrium of this game is a babbling equilibrium.
- There is no message that Evert could send that would give Navratilova any more idea of what she will actually play.

Cheap Talk Equilibria - Partially Aligned Interests

Many real life games have mixtures of conflict and common interest.

- The question of whether direct communication is *credible* or not will depend on the relative degree of each incentive.
- We will use our tools from the first half of the course to make testable predictions based on different ranges of assumptions.

Defensive Medicine

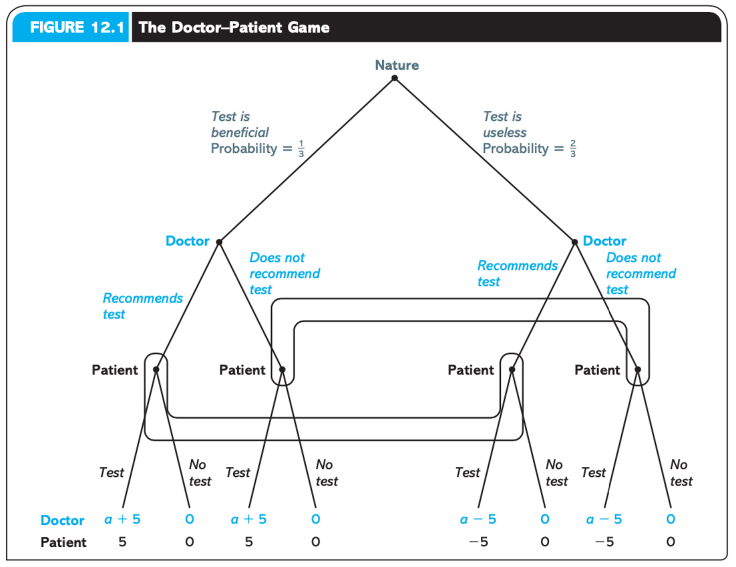
> *In a recent survey of physicians, 93% reported altering their clinical behavior because of the threat of malpractice liability. Of them, 92% used "assurance behavior" such as ordering tests, performing diagnostic procedures, and referring patients for consultation; and 43% reported using imaging technology in clinically unnecessary circumstances.*

Harrington, pg. 461

Defensive Medicine

- Consider a patient who goes to the doctor for an examination.
- The doctor can recommend an expensive test that is not fully covered by the patient's insurance.
- The doctor cares about the patient, but also doesn't want to be sued for malpractice if the patient *does* end up needing the test and the doctor didn't recommend it.
- The patients value v from a beneficial test is 5, and $v = -5$ if the test is useless.
- We'll use a to stand in for the value of a test to a doctor from a malpractice standpoint.

Defensive Medicine



Defensive Medicine - Babbling Strategy

Pooling Equilibrium

- Doctor's Strategy: Recommend the test whether or not it is beneficial.
- Patient's Strategy: Ignore the doctor's recommendation.
- Patient's Beliefs: Ignoring the doctors advice, the probability the test is effective is 1/3.

- This equilibrium is a *babbling equilibrium*.
- The doctor's recommendation contains no real signal to the patient.

Defensive Medicine - Babbling Strategy

Pooling Equilibrium

- Doctor's Strategy: Recommend the test whether or not it is beneficial.
- Patient's Strategy: Ignore the doctor's recommendation.
- Patient's Beliefs: Ignoring the doctors advice, the probability the test is effective is $1/3$.

- The patient's beliefs are consistent, and their expected utility from taking the test is $\frac{1}{3} \cdot 5 + \frac{2}{3} \cdot (-5) = -\frac{5}{3}$.
- Given that the patient will never take the test, the doctor is indifferent between recommending the test or not.
- So this situation in which the doctor always recommends the test and the patient always ignores their advice is *stable*.

Defensive Medicine - Separating Strategies

The previous result was disappointing, but not unexpected.

Insight

For every cheap talk game, there is always a babbling equilibrium.

- But let's now focus on the more interesting question of how to make the doctor's recommendation *meaningful*.

Consider the following strategy profile:

- Doctor's Strategy: Recommend the test if and only if it is beneficial.
- Patient's Strategy: Follow the doctor's recommendation.
- Patient's Beliefs:
  - If the doctor recommends the test, then the test is beneficial with 100% probability.
  - If the doctor does not recommend the test, then the test is beneficial with 0% probability.

Defensive Medicine - Separating Strategies

When will the doctor follow the separating strategy?

1. When $EU_d(\text{Rec. when beneficial}, (T, NT)) \geq EU_d(\text{Don't rec. when beneficial}, (T, NT))$

2. and when $EU_d(\text{Don't rec. when useless}, (T, NT)) \geq EU_d(\text{Rec. when useless}, (T, NT))$

Solve for the range of a where this is a NE.

Defensive Medicine - Conclusions

Interpreting our findings:

- When $a = 0$, the doctor's interests are *perfectly* aligned with the patient's.
- When $a \leq 5$, the doctor's interests are *partially* aligned with the patient's interests, and there is an equilibrium where the doctor gives truthful recommendations.
- When $a > 5$, there is only a babbling equilibrium because the doctor's incentives are to not be truthful. Even if they did give a truthful recommendation, the patient would have no reason to believe it would be *credible*.

Defensive Medicine - Conclusions

Connecting with our real-world observations:

- We don't know what doctors' subjective costs of malpractice threats are (a).
- But we can observe their *behaviors*.
- If we see that doctors recommend more tests than are beneficial, it might reveal that a is quite large.

Revealed Preference

The idea that people reveal their true preferences by the choices they make.

Adverse Selection

Adverse Selection - Definition

Adverse Selection

When one player knows something about the outcomes that others don't and direct communication will not *credibly* signal their information, there can be separating equilibria in which only those players with the 'undesirable' states of information will self-*select* into engaging in the market.

Insurance Markets

- Potential buyers of insurance have different risk levels that they know about themselves but are not easily (or legally) observable to insurance plan providers.
  - Underlying health conditions, riskier driving habits, etc.
- Insurance providers have to pay out more often on these riskier customers.
- However, riskier customers are the exact types who will find insurance plans more attractive.

Insurance Markets

- What strategies could an insurance provider take to *screen* out risky customers from safe customers?
- Any ideas?

Insurance Markets

- Suppose there are only two categories of risk level.
- An insurance provider could offer two different plans:
  - Plan 1: has a lower premium but covers a lower percent of the customer's loss.
  - Plan 2: has a higher premium but covers a higher percent of the loss.

Insurance Markets

- How should the insurance provider structure the two plans so that a *separating equilibrium* is achieved in which all risky types choose a different plan than the safe types?
  - Make Plan 2 only attractive to safe types by setting the price higher than the risky types' willingness to pay for the extra security.
  - and Plan 2 a better option for safe types than Plan 1.
    - We call this incentive compatible.
  - Make Plan 1 a better option for risky types than going without insturance altogether
    - We call this individually rational.

Adverse Selection - Market for Lemons

In economics, one of the most famous examples of adverse selection comes from George Akerlof's 1970 paper, "The Market for Lemons: Qualitative Uncertainty and the Market Mechanism".

- He analyses the used car market, in which crappy cars are called 'lemons'

Adverse Selection - Market for Lemons

- Suppose there are only two types of cars:
    - good quality cars are valued at $12,500 to the seller
    - and lemons are worth $3,000.
- Suppose a potential buyer would be willing to pay more than these values.
    - He would be willing to pay $16,000 for a car he knows is good
    - and $6,000 for a car he knows is a lemon.

Adverse Selection - Market for Lemons

In a perfectly competitive market with perfect info and a large number of buyers:

- buyer competition will drive up prices to \$16,000 for good cars and \$6,000 for lemons.

Adverse Selection - Market for Lemons

However, buyers don't know the true value of any car by looking at them, but the seller knows whether their car is a lemon or not.

- Now we have a market with aymmetric info
- When the type of car is unobservable, there can only be *one price* in the market

Adverse Selection - Market for Lemons

Suppose that a fraction f of cars are lemons
Draw the extensive form game tree

Market for Lemons - Buyer's Perspective

For the buyer:
write out the expected utility of buying a car at price p:

Market for Lemons - Seller's Perspective

For the seller:

- the expected utility of selling a lemon is:
- the expected utility of selling a good car is:

Market for Lemons - Market Clearing

When will all buyers and sellers want to trade?

Market for Lemons - Market Unraveling

When will there be a *pooling equilibrium* where only one type of car is sold?

What type of car is sold in this equilibrium?

Market for Lemons - Conclusions

"Verbal declarations are costless and therefore useless. Anyone can lie about why he is selling the car. One can offer to let the buyer have the car checked. The lemon owner can make the same offer. It's a bluff. If called, nothing is lost. Besides, such checks are costly.
Reliability reports from the owner's mechanic are untrustworthy. The clever nonlemon owner might pay for the checkup but let the purchaser choose the inspector. The problem for the owner, then, is to keep the inspection cost down. Guarantees do not work. The seller may move to Cleveland, leaving no forwarding address."

- A. Michael Spence, *Market Signaling: Information Transfer in Hiring and Related Screening Processes*

Market for Lemons - Discussion

- How well do you think that the used car market in the real world reflects the Market for Lemons model?
- What other factors could there be that allow used car sellers to *credibly signal* the quality of their cars?

Risk Preferences

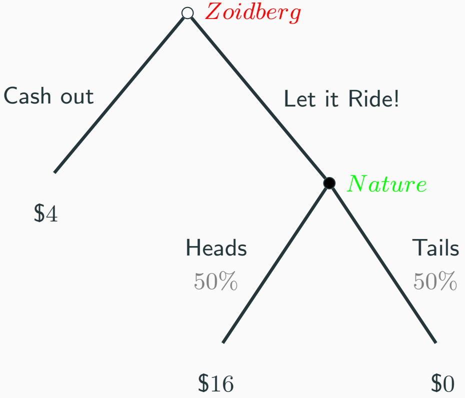
Imperfect Info: Dealing with Risk

Some of you have already started asking about whether different preferences for risk might change the solutions to some of the games we have looked at.

- So far, I have abstracted away from these questions by saying that we should take the payoffs given as the agents' *true subjective utilities*.
- But we can also use the tools of **utility functions** we introduced in the beginning of the class to more explicitly model risk preferences.

Playing with Chance

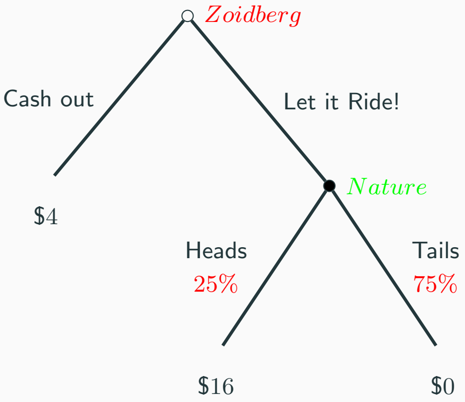
To review, suppose that Nature plays probabilistically with 50:50 odds:



What is Zoidberg's **expected value** of *Let it Ride*?

Playing with Chance

What if the coin is *unfair*?



What is the **expected value** now?

Should Zoidberg take the gamble?

Risk Preferences

Is **expected value** always the same as expected utility?

- Suppose that I offered you a different gamble:
  - **Option 1:** You get \$1,000,000 with 50% chance, \$0 with 50% chance.
  - **Option 2:** You get \$400,000 for certain.
- Which would you choose?

Risk Preferences

Why would I prefer **Option 2** when it gives a lower *expected payout*?

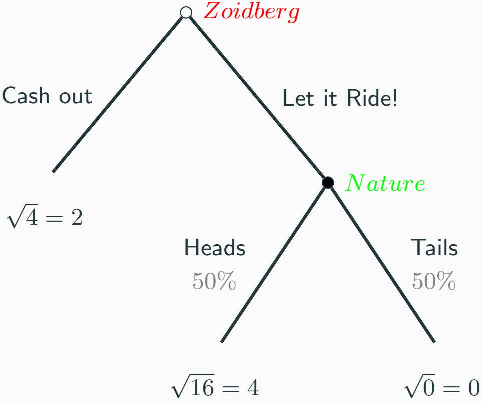
- $400,000 is still a life changing amount of money,
- but my marginal benefit from going from $400,000 to $1,000,000 is less than my marginal benefit from going from $0 to $400,000.
- The **risk** of going home empty handed isn't worth the payout that is a marginally larger life-changing amount of money.

Risk Preference: Risk Aversion

What would a utility function with diminishing marginal benefit look like?

Playing with Chance: Risk Averse Utility

Now suppose that Zoidberg's utility is $U_Z(\$x) = \sqrt{x}$



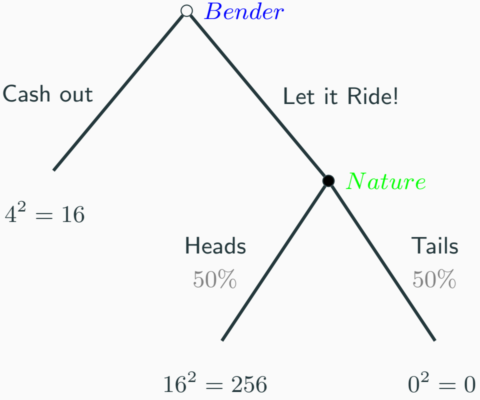
Would Zoidberg be willing to cash out?

Risk Preference: Risk Seeking

What would a utility function with increasing marginal benefit look like?

Playing with Chance: Risk Seeking Utility

Now suppose that Bender's utility is $U_B(\$x) = x^2$



Would Bender be willing to cash out?

Measuring Risk Preferences

Both utility functions are **rational**, but lead to much different behavior.

How can we tell whether someone is risk averse or risk seeking?

Measuring Risk Preferences

Both utility functions are **rational**, but lead to much different behavior.

How can we tell whether someone is risk averse or risk seeking?

- look at their revealed preferences through the choices they make.

Measuring Risk Preferences - Certainty Equivalent

- Suppose that you don't know my risk preference
- but you can offer me a choice between:
  - A **lottery** L between \$$A$ and \$$B$ with probability p
  - or a **certain** amount \$$x$ of your choosing.
- The certain amount x that would make me *indifferent* between the lottery and taking the sure payment is called the certainty equivalent of L.

Measuring Risk Preferences - Certainty Equivalent

- If my **certainty equivalent** is less than the expected value of the lottery
  - you know that I am risk averse
- If the **certainty equivalent** $> \mathbb{E}[L]$
  - I am risk seeking
- If the **certainty equivalent** $= \mathbb{E}[L]$
  - I am risk neutral

Semiseparating Equilibria

Equilibria in 2-Player Signaling Games

- So far we have covered the general concepts of incomplete information.
- We saw how adverse selection can arise in games with many players.
- But now we will solve for equilibria in the case of a simpler 2-player game.

Semiseparating Equilibria

- We saw Pooling Equilibria in which all types take the same action
  - aka *'babbling equilibria'*
- And we also saw Separating Equilibria in which different types take *completely different* actions
  - sometimes called *'cheap talk equilibria'*

Market Entry Game

- **Players:** competing auto manufacturers: Tudor and Fordor
- Tudor is a current monopolist in the auto industry
- Fordor is a potential entrant in the market
- Tudor has **private information** on how tough they will be able to compete against a Fordor entrant.

Market Entry Game

Sequential Game

- **Stage 1:** Tudor sets price $\in \{low,\ high\}$
- **Stage 2:** Fordor makes entry decision $\in \{in,\ out\}$

Payouts:

- Profits for each firm are market price - production costs
  - **Market Demand:** $P = 25 - Q$

Market Entry Game

Costs:

- Fordor's upfront cost of entry: 40
- Fordor's per-unit cost: 10
- Tudor's costs:
  - If **high-cost**: 15
  - If **low-cost**: 5

Market Entry Game

Payouts:

- If Tudor is high-cost:
  - and Fordor stays out: $\Pi_{T1} = 5 * (20 - 15) = 25$ and $\Pi_{T2} = 25$, $\Pi_F = 0$
  - and Fordor enters: $\Pi_T = 25 + 3$, $\Pi_F = 45$ - startup cost of 40
- If Tudor is low-cost:
  - and Fordor stays out: $\Pi_{T1} = 100$ and $\Pi_{T2} = 100$, $\Pi_F = 0$
  - Fordor enters: $\Pi_T = 100 + 69$, $\Pi_F = 11$ - startup cost of 40

Market Entry - Extensive Form Game

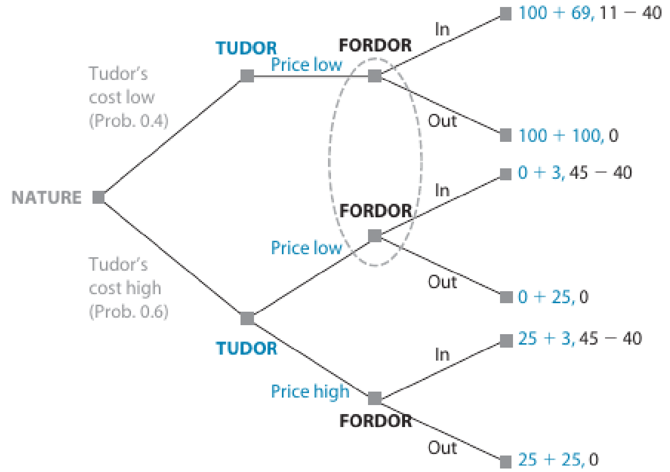


FIGURE 8.7 Extensive Form of Entry Game: Tudor's Low Cost Is 5

Signaling Strategies

- Tudor might use its price as a signal of its cost.
- A *low-cost* firm would charge a lower price, so Tudor might hope to keep its price low to show Fordor that they are a low-cost firm and therefore more difficult to fight.
- However, Tudor might also try to **bluff** Fordor into staying out.

Market Entry Game - Separating Equilibrium

Checking for Separating Equilibrium:

1. **Step 1:** Prune strategies using rollback:
   - What should Fordor do if they see a **high price**?

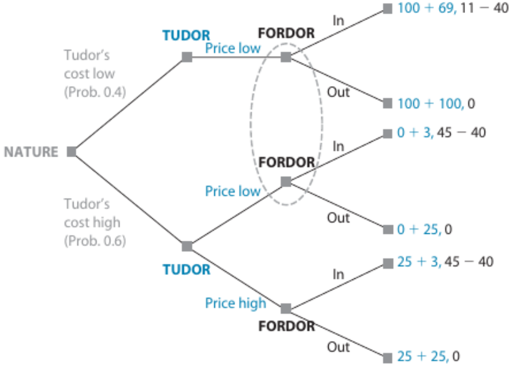


FIGURE 8.7 Extensive Form of Entry Game: Tudor's Low Cost Is 5

Market Entry Game - Separating Equilibrium

Checking for Separating Equilibrium:

- How many Strategies does each player have?
  - (After pruning *Out if Price High* for Fordor)

Checking for Separating Equilibrium:

Step 2: Represent game in *Strategic Form:*

		FORDOR	
		Regardless (II)	Conditional (OI)
TUDOR	Bluff (LL)	$169 \times 0.4 + 3 \times 0.6 = 69.4$, $-29 \times 0.4 + 5 \times 0.6 = -8.6$	$200 \times 0.4 + 25 \times 0.6 = 95$, 0
	Honest (LH)	$169 \times 0.4 + 28 \times 0.6 = 84.4$, $-29 \times 0.4 + 5 \times 0.6 = -8.6$	$200 \times 0.4 + 28 \times 0.6 = 96.8$, $5 \times 0.6 = 3$

FIGURE 8.8 Strategic Form of Entry Game: Tudor's Low Cost Is 5

Market Entry Game - Separating Equilibrium

Checking for Separating Equilibrium:

Step 3: Look for NE in the *Strategic Form*

		FORDOR	
		Regardless (II)	Conditional (OI)
TUDOR	Bluff (LL)	$169 \times 0.4 + 3 \times 0.6 = 69.4$, $-29 \times 0.4 + 5 \times 0.6 = -8.6$	$200 \times 0.4 + 25 \times 0.6 = 95$, 0
	Honest (LH)	$169 \times 0.4 + 28 \times 0.6 = 84.4$, $-29 \times 0.4 + 5 \times 0.6 = -8.6$	$200 \times 0.4 + 28 \times 0.6 = 96.8$, $5 \times 0.6 = 3$

FIGURE 8.8 Strategic Form of Entry Game: Tudor's Low Cost Is 5

Market Entry Game - Separating Equilibrium

Checking for Separating Equilibrium:

- So when Tudor's Low Cost is 5, the Nash Equilibrium is (Honest, Conditional)
- This is a *Separating* equilibrium, because the Tudor's action of *Price High* or *Price Low* completely reveals their type to Fordor.

Market Entry Game

Is it guaranteed that this game will *always* result in complete separation of types?

- What if we change the Tudor's Low Cost to 10 instead of 5?

Market Entry Game - Pooling Equilibrium

Can you prune any strategies?

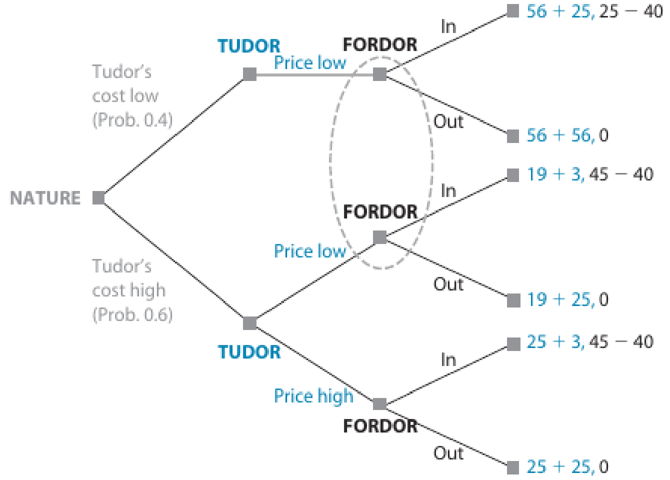


FIGURE 8.9 Extensive Form of Entry Game: Tudor's Low Cost Is 10

Market Entry Game - Pooling Equilibrium

Now what is the Nash Equilibrium of this game?

		FORDOR	
		Regardless (II)	Conditional (OI)
TUDOR	Bluff (LL)	$81 \times 0.4 + 22 \times 0.6 = 45.6$, $-15 \times 0.4 + 5 \times 0.6 = -3$	$112 \times 0.4 + 44 \times 0.6 = 71.2$, 0
	Honest (LH)	$81 \times 0.4 + 28 \times 0.6 = 49.2$, $-15 \times 0.4 + 5 \times 0.6 = -3$	$112 \times 0.4 + 28 \times 0.6 = 61.6$, $5 \times 0.6 = 3$

FIGURE 8.10 Strategic Form of Entry Game: Tudor's Low Cost Is 10

Market Entry Game - Pooling Equilibrium

- So when Tudor's Low Cost is 10, the Nash Equilibrium is (Bluff, Conditional)
- This is a *Pooling* equilibrium, because Tudor always takes the same action of *Price Low*.
  This gives Fordor no signal of their type, but Fordor still doesn't have any incentive to change their strategy.

Market Entry Game

- So far, we found that depending on the relative difference between a *low-cost* Tudor and a *high-cost* Tudor, there may either be a **Pooling** or **Separating** equilibrium.
- But there might also be an equilibrium somewhere in between: where there is *partial* sorting of types
- We call this type of equilibrium **Semiseparating**

Market Entry Game - Semiseparating

Now let's change the original probability that a Tudor is low cost from .4 to .1

- (But keep all of the payoffs the same as in the last case)

Market Entry Game - Semiseparating

Can you find a Nash Equilibrium with the new expected utilities?

		FORDOR	
		Regardless (II)	Conditional (OI)
TUDOR	Bluff (LL)	$81 \times 0.1 + 22 \times 0.9 = 27.9$, $-15 \times 0.1 + 5 \times 0.9 = 3$	$112 \times 0.1 + 44 \times 0.9 = 50.8$, 0
	Honest (LH)	$81 \times 0.1 + 28 \times 0.9 = 33.3$, $-15 \times 0.1 + 5 \times 0.9 = 3$	$112 \times 0.1 + 28 \times 0.9 = 36.4$, $5 \times 0.9 = 4.5$

FIGURE 8.11 Strategic Form of Entry Game: Tudor's Low Cost Is 10 with Probability 0.1

Looking for Mixed Strategy Nash Equilibrium

- Suppose Tudor plays Bluff with probability p, Honest with $1 - p$
- When will Fordor play a mixed strategy?

Looking for Mixed Strategy Nash Equilibrium

- Suppose Fordor plays Regardless with probability q, Conditional with $1 - q$
- When will Tudor play a mixed strategy?

Market Entry Game - Semiseparating

- So this version of the game has the MSNE:
  { (1/3 Bluff, 2/3 Honest), (16/22 Regardless, 6/22 Conditional) }
  - In this equilibrium, instead of *complete separation* or *complete pooling*, we have *semiseparating*
  - A high price conveys full information to Fordor, but a low price could mean that the Tudor is *either* a **low-price** *or* a **high-price** type.

Bayes' Rule

		TUDOR'S PRICE		Sum of row
		Low	High	
TUDOR'S COST	Low	0.1	0	0.1
	High	$0.9 \times 1/3 = 0.3$	$0.9 \times 2/3 = 0.6$	0.9
Sum of column		0.4	0.6	

FIGURE 8.12 Applying Bayes' Theorem to the Entry Game

The CEO's new clothes

clip

Dinesh has let the power of a CEO position go to his head. His new confidence/vanity has led him to trying out a new hairstyle, but he starts to suspect that there is a non-zero probability that he looks **ridiculous** to other people.

The CEO's new clothes

Suppose that looking **ridiculous** is not something that Dinesh can subjectively observe about himself, but is only observable by the people around him.

Gilfoyle can observe whether or not Dinesh looks **ridiculous** and would like it if Dinesh embarrassed himself by looking **ridiculous** in public.

But he knows that if Dinesh thinks he looks **ridiculous** , he will want to change his look back to the more boring (but less risky) style he had as a nerdy programmer.

The CEO's new clothes

Suppose that Dinesh's preferences (from best to worst) are as follows:

- He wears his *new style* proudly and people think he is **cool**
- He wears his *old style* and people think he looks average
- He wears his *new style* but people think it is **ridiculous**

Suppose that Gilfoyle's preferences are:

- Dinesh looks **ridiculous** with the *new style*, continues to wear it and gets embarrassed in public
- Dinesh goes back to his *old style*
- Dinesh looks **cool** with the *new style*, continues to wear it and people think he's cool.

The CEO's new clothes

Model this as an asymmetric information game where Gilfoyle has the private information of whether Dinesh looks **ridiculous**.