Econ 327: Game Theory

Practice Final Exam

University of Oregon

December 8th, 2025

ANSWER KEY

Multiple Choice

Question 1. (4 P.)
For the strategic form game below:

		P_2	
		Left	Right
P_1	Up	3,3	9,4
	Down	5,2	6,1

let p be the **probability Player 1 chooses Up** and let q be the probability **Player 2 chooses Left**. Choose the correct Expected Utility expression for Player 1's strategy Up.

a) $3p + 5(1 - q)$
b) $3q + 4(1 - q)$
c) $3p + 9(1 - p)$
d) $3q + 9(1 - q)$ **Correct**

Question 2. (4 P.)
For the strategic form game below:

		P_2	
		Close	Far
P_1	High	9,5	5,1
	Low	6,2	6,8

let p be the **probability Player 1 chooses High** and let q be the probability **Player 2 chooses Close**. Choose the correct Expected Utility expression for **Player 2's strategy Close**.

a) $5q + 1(1 - q)$
b) $2p + 8(1 - q)$
c) $5p + 2(1 - p)$ **Correct**
d) $1p + 8(1 - p)$

		P_2	
		Push	Pull
P_1	Give	6, 9	12, 8
	Take	9, 4	7 ,10

Question 3. (4 P.)
For the strategic form game below:

Which of the following mixed strategy profiles is a Nash equilibrium?

a) $\sigma_1 = (1/2 \text{ Give}, 1/2 \text{ Take})$, $\sigma_2 = (1/2 \text{ Push}, 1/2 \text{ Pull})$
b) $\sigma_1 = (2/5 \text{ Give}, 3/5 \text{ Take})$, $\sigma_2 = (1/2 \text{ Push}, 1/2 \text{ Pull})$
c) $\sigma_1 = (1/3 \text{ Give}, 2/3 \text{ Take})$, $\sigma_2 = (5/6 \text{ Push}, 1/6 \text{ Pull})$
d) $\boldsymbol{\sigma_1 = (6/7 \text{ Give}, 1/7 \text{ Take}), \sigma_2 = (5/8 \text{ Push}, 3/8 \text{ Pull})}$

Question 4. (4 P.)
Consider the strategic form game below:

		P_2	
		X	Y
	A	2,3	6,1
P_1	B	4,2	1,3
	C	3,1	2,4

Suppose Player 1 plays A with probability α, B with probability β, and C with probability γ. When will Player **2** be indifferent between playing X and playing Y?

a) $2\alpha + 4\beta + 3\gamma = 6\alpha + 1\beta + 2\gamma$
b) $3\alpha + 1\alpha = 2\beta + 3\beta = 1\gamma + 4\gamma$
c) $3\alpha + 2\beta + 1\gamma = 1\alpha + 3\beta + 4\gamma$ **Correct**
d) $\alpha = \beta = \gamma$

Question 5. (4 P.)
A player using a **mixed strategy** means that:

a) some parts of their strategy are played simultaneously and other parts are played sequentially
b) they are confused about what action their opponent is taking
c) they will regret not having chosen their a pure strategy instead
d) **they are internally uncertain about which action they will choose because they are acting randomly**

Question 6. (4 P.)
A game featuring **asymmetric information**:

a) **has some players who have access to private information which is not directly observable to others**
b) means that one player has a strategy with no equivalent strategy avaibable to any other player
c) has Nature acting as a player even though she doesn't have any preferences
d) is repeated multiple times by the same players

Question 7. (4 P.)
By **screening**:

a) **a player attempts to learn about some private information held by others by designing an incentive mechanism**
b) a player can reveal their own private information through their actions
c) only players with the 'bad' condition sort into a market
d) only mixed strategies will be played in equilibrium

Question 8. (4 P.)
Consider the strategic form game below:

		P_2		
		Left	Middle	Right
P_1	Up	0,1	9,0	2,3
	Straight	5,9	7,3	1,7
	Down	7,5	10,10	3,5

How many Nash equilibria exist in this simultaneous game, including both **pure** and **mixed** strategies?

a) **One equilibrium**
b) Two equilibria
c) Three equilibria
d) An infinite number of equilibria

Question 9. (4 P.)
An **information set**:

a) is used by game theorists to signal how they want their games to be played
b) tells a player what action to take
c) **contains all decision nodes which a player cannot tell the difference between when they reach that part of the game**
d) holds all pieces of information which are publically observable to all players

Question 10. (4 P.)
Consider the following lottery:

- with probability 1/3 you will receive \$900.
- with probability 2/3 you only receive \$36.

Suppose someone has a risk-averse utility function of $u(\$x) = \sqrt{\$x}$. For what certain amount of dollars, x, will this person be indifferent between taking the certain payment with probability of 1 and taking the lottery defined above?

a) **\$196**
b) \$324
c) \$468
d) \$484

Question 11. (4 P.)
Identify the class concept that most closely describes the situation below:
Conspicuous consumption describes the phenomenon of buying flashy luxury goods with visible branding such as Louis Vuitton, Gucci, Prada, etc. in order to display the buyer's level of wealth to be able to afford such goods.

a) Brinksmanship
b) Mixed Strategy Nash Equilibrium
c) Risk sharing
d) **Signaling**

Question 12. (4 P.)
In the **Prisoner's Dilemma**, mutual cooperation:

a) is a dominant strategy equilibrium
b) **Pareto dominates the outcome of mutual defection**
c) is stable
d) is a credible threat

Question 13. (4 P.)
The Folk Theorem states that:

a) **Any individually rational and feasible outcome can be reached in a repeated game for some sufficiently high enough discount factor.**
b) All Pareto optimal outcomes can always be reached in a Nash equilibrium.
c) No matter how hard you try, some folks will just never cooperate
d) The Prisoners' Dilemma is the only game with a unique Nash equilibrium.

Question 14. (4 P.)
Consider the Prisoners' Dilemma game with payoffs as shown in the strategic form table below:

		P_2	
		Cooperate	Cheat
P_1	Cooperate	16, 16	8, 36
	Cheat	36, 8	12, 12

Suppose Player 2 is utilizing a **Tit-for-Tat** strategy in which they will start off cooperating, and after that they will play whatever strategy their oponent used in the previous round.
Which of the following represents Player 1's present value of cheating in the first period and then going cooperating in all following periods?

a) $36 + 0\delta + 0\delta^2 + 0\delta^3 + ... = 36$
b) $36 + 8\delta + 16\delta^2 + 16\delta^3 + ... = 36 + 8\delta + 16\frac{\delta^2}{1-\delta}$ **Correct**
c) $16 + 16\delta + 16\delta^2 + 16\delta^3 + ... = \frac{16}{1-\delta}$
d) $36 + 12\delta + 12\delta^2 + 12\delta^3 + ... = 36 + \frac{12\delta}{1-\delta}$

		P_2	
		Cooperate	Cheat
P_1	Cooperate	3, 3	1, 4
	Cheat	4, 1	2, 2

Question 15. (4 P.)
Consider the Prisoners' Dilemma game with payoffs as shown in the strategic form table below:

Suppose Player 2 is utilizing a **Grim Trigger** strategy in which they will start off cooperating, and continue to cooperate unless their opponent has ever played Cheat, in which case they will play Cheat in all periods following.
Which of the following represents Player 1's present value of cheating in the first period (and in all following periods)? δ is the per-period discount rate.

a) $4 + 2\delta + 2\delta^2 + 2\delta^3 + ... = 4 + 2\frac{\delta}{1-\delta}$ **Correct**
b) $3 + 3\delta + 3\delta^2 + 3\delta^3 + ... = \frac{3}{1-\delta}$
c) $4 + 3\delta + 3\delta^2 + 3\delta^3 + ... = 4 + 3\frac{\delta}{1-\delta}$
d) $4 + 1\delta + 2\delta^2 + 2\delta^3 + ... = 4 + 1\delta + 2\frac{\delta^2}{1-\delta}$

Long Answer

Question 16. (12 P.)
Mixed Strategies: Consider the strategic form game below:

		P_2			
		Hall	**Office**	**Library**	**Bathroom**
P_1	**Roof**	0 , 2	1 , 1	0 , 2	5, 0
	Mezzanine	1 , 1	0 , 2	0 , 2	4, 0
	Ground	0 , 2	0 , 2	1 , 0	3, -1

a) (4 P.) Find any **pure strategy** Nash equilibria

$$BR_1(Hall) = Mezzanine$$
$$BR_1(Office) = Roof$$
$$BR_1(Library) = Ground$$
$$BR_1(Bathroom) = Roof$$

$$BR_2(Roof) = \{Hall, Library\}$$
$$BR_2(Mezzanine) = \{Office, Library\}$$
$$BR_2(Ground) = \{Hall, Office\}$$

No intersection of best responses, so **No PSNE**

b) (4 P.) Consider the following mixed strategy profile:

- Player 1 plays 1/3 **Roof**, 0 **Mezzanine**, and 2/3 **Ground**
- Player 2 plays 0 **Hall**, 1/2 **Office**, 1/2 **Library**, and 0 **Bathroom**

Check whether this is a **mixed strategy Nash equilibrium** and explain why or why not.

What is P_1's best response to $(1/2Office,\ 1/2Library)$?

$$EU_1(Roof) = 0(0) + 1/2(1) + 1/2(0) + 0(5) = \underline{1/2}$$
$$EU_1(Mezz) = 0(1) + 1/2(0) + 1/2(0) + 0(4) = 0$$
$$EU_1(Grnd) = 0(0) + 1/2(0) + 1/2(1) + 0(3) = \underline{1/2}$$

So the best response to $(1/2Office,\ 1/2Library)$ is any strategy which mixes between Roof and Ground with positive probability (and never Mezzanine).
What is P_2's best response to $(1/3Roof,\ 2/3Ground)$?

$$EU_1(Hall) = 1/3(2) + 0(1) + 2/3(2) = \underline{2}$$
$$EU_1(Offc) = 1/3(1) + 0(2) + 2/3(2) = 5/3$$
$$EU_1(Libr) = 1/3(2) + 0(2) + 2/3(0) = 2/3$$
$$EU_1(Bath) = 1/3(0) + 0(0) + 2/3(1) = 1/2$$

So the best response to $(1/3Roof,\ 2/3Ground)$ is to play $Hall$ with probability of one.
***Not* a Nash equilibrium.** Player 2 would prefer to play $(Hall)$ over $(1/2Office,\ 1/2Library)$.

c) (4 P.) Now consider the strategy profile:

- Player 1 plays 2/5 **Roof**, 2/5 **Mezzanine**, and 1/5 **Ground**
- Player 2 plays 1/3 **Hall**, 1/3 **Office**, 1/3 **Library**, and 0 **Bathroom**

Check whether this is a **mixed strategy Nash equilibrium** and explain why or why not.

What is P_1's best response to $(1/3Hall,\ 1/3Office,\ 1/3Library)$?

$$EU_1(Roof) = 1/3(0) + 1/3(1) + 1/3(0) + 0(5) = \underline{1/3}$$
$$EU_1(Mezz) = 1/3(1) + 1/3(0) + 1/3(0) + 0(4) = \underline{1/3}$$
$$EU_1(Grnd) = 1/3(0) + 1/3(0) + 1/3(1) + 0(3) = \underline{1/3}$$

So the best response is any strategy which mixes between Roof, Mezz., and Ground with positive probability

What is P_2's best response to $(2/5Roof,\ 2/5Mezzanine,\ 2/3Ground)$?

$$EU_1(Hall) = 2/5(2) + 2/5(1) + 1/5(2) = \underline{8/5}$$
$$EU_1(Offc) = 2/5(1) + 2/5(2) + 1/5(2) = \underline{8/5}$$
$$EU_1(Libr) = 2/5(2) + 2/5(2) + 1/5(0) = \underline{8/5}$$
$$EU_1(Bath) = 2/5(0) + 2/5(0) + 1/5(1) = 1/5$$

So the best response is any strategy which mixes between Hall, Office, and Library with positive probability (and never Bathroom).

This *is* a Nash equilibrium. Both players are best responding to the other's strategy.

Question 17. (8 P.)

Screening: You are the Dean of the Faculty at St. Anford University. You hire Assistant Professors for a probationary period of 7 years, after which they come up for tenure and are either promoted and gain a job for life or turned down, in which case they must find another job elsewhere. Your Assistant Professors come in two types, Good and Brilliant. Any types worse than Good have already been weeded out in the hiring process, but you cannot directly distinguish between Good and Brilliant types. Each individual Assistant Professor knows whether he or she is Brilliant or merely Good. You would like to tenure only the Brilliant types. The payoff from a tenured career at St. Anford is \$6 million; think of this as the expected discounted present value of salaries, consulting fees, and book royalties, plus the monetary equivalent of the pride and joy that the faculty member and his or her family would get from being tenured at St. Anford. Anyone denied tenure at St. Anford will get a fac- ulty position at Boondocks College, and the present value of that career is \$1 million. Your faculty can do research and publish the findings. But each publication requires effort and time and causes strain on the family; all these are costly to the faculty member. The monetary equivalent of this cost is \$25,000 per publication for a Brilliant Assistant Professor and \$50,000 per publication for a Good one. You can set a minimum number, N, of publications that an Assistant Professor must produce in order to achieve tenure.

a) (4 P.) What is the minimum number N you could require so that only *brilliant* professors apply and *good* professors don't apply?

Payoffs in thousands:

$$\begin{aligned} U_{\text{Good}}(\text{Apply to St. Anford}) &= 6000 - 50N \\ U_{\text{Good}}(\text{Boondocks}) &= 1000 \\ 6000 - 50N &< 1000 \\ &\Rightarrow N > 100 \end{aligned}$$

Good APs will choose Boondocks over St. Anford as long as $N > 100$

b) (4 P.) What is the maximum number N that you could require so that *brilliant* professors still want to apply?

Payoffs in thousands:

$$\begin{aligned} U_{\text{Brilliant}}(\text{Apply to St. Anford}) &= 6000 - 25N \\ U_{\text{Brilliant}}(\text{Boondocks}) &= 1000 \\ 6000 - 25N &> 1000 \\ &\Rightarrow N < 200 \end{aligned}$$

Brilliant APs will choose St. Anford over Boondocks as long as $N < 200$

Question 18. (20 P.)

Baysian Games: Consider a Wild West shootout between Earp and the Stranger.
With probability .75, the Stranger is a Gunslinger type and the table shows Earp's and the Stranger's payoffs

		Gunslinger Stranger	
		Draw	Wait
Earp	Draw	2, 3	3, 1
	Wait	1, 4	8, 2

But with probability .25, the Stranger is a Cowpoke type and the table shows Earp's and the Stranger's payoffs

		Cowpoke Stranger	
		Draw	Wait
Earp	Draw	5, 2	4, 1
	Wait	6, 3	8, 4

a) (4 P.) What is the Nash equilibrium **when the Stranger is always a Gunslinger**?

NE: {Earp `Draws`, Gunslinger `Draws`}

b) (4 P.) What is the Nash equilibrium **when the Stranger is always a Cowpoke**?

NE: {Earp `Waits`, Gunslinger `Waits`}

c) (4 P.) What is the Nash equilibrium when Earp believes the Stranger is a **Gunslinger with probability 0.75**?

$$EU_{\text{Earp}}(\text{Draw}) = 3/4(2) + 1/4(4) = 2.5$$
$$EU_{\text{Earp}}(\text{Wait}) = 3/4(1) + 1/4(8) = 2.75$$

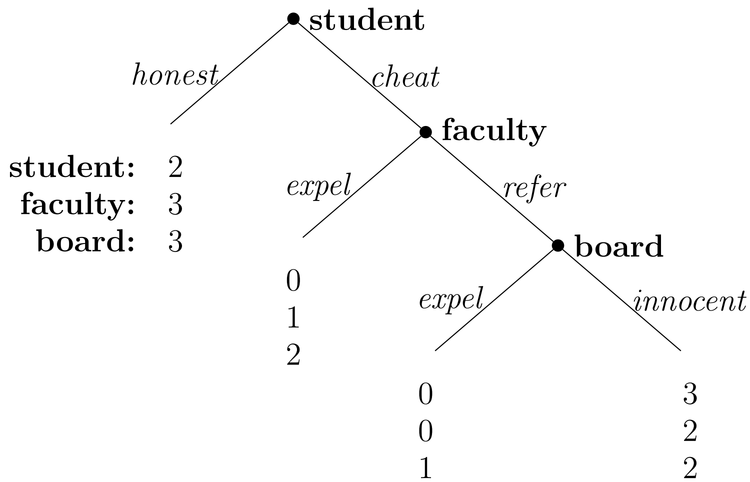
So Earp should wait if he believes it is likely enough he is against a Gunslinger.
BNE: {(Earp `Waits`, believing that prob Stranger is Gunslinger is .75), (Cowpoke `Waits`, Gunslinger `Draws`) }

d) (4 P.) Consider a strategic move variation where the Gunslinger can commit to only playing Wait before Nature has assigned them a type.
Is this type of commitment *credible*? Why or why not?

Not credible for the Gunslinger to commit to `Wait` because `Draw` strictly dominates `Wait`.

Question 19. (12 P.)

Bayesian Game: Consider a situation in which a student can decide to cheat or be honest on an exam. If the faculty thinks the student has cheated, the faculty member has to decide whether to expel them from the college or refer them to the Honor Board. The Honor Board has to decide whether to expel the student or find them innocent. The payoffs are ordered, student, faculty, and college. Assume the board shares the college's payoffs.



a) (4 P.) Find the Subgame Perfect Nash Equilibrium.

Backwards Induction:

- **board** will find *innocent*
- **faculty** will *refer* knowing board will find *innocent*
- **student** will *cheat* knowing faculty will refer, board will find innocent.

SPNE: $\{cheat, refer, innocent\}$

b) (4 P.) Assume now that the board acts like *Nature*, making no deliberate choice but instead expels a guilty student q% of the time. Solve for the range of q such that the student chooses *honest* in the SPNE.

If board acts randomly, **faculty** will *expel* when:

$$\begin{aligned} EU_f(expel) &\geq EU_f(refer) \\ 1 &\geq 0q + 2(1-q) \\ 1 &\geq 2 - 2q \\ q &\geq 1/2 \end{aligned}$$

SPNE$_1$:

- Student chooses *honest*
- Faculty *expel*s
- Board *expel*s more than 50% of the cases refered to them

If the faculty chooses to *refer*, there is still a possibility the threat of expulsion will incentivize the student to stay *honest*. Suppose the $q < 1/2$ so the faculty always *refer*s:

$$\begin{aligned} EU_s(honest) &> EU_s(cheat) \\ \Rightarrow 2 &> 0q + 3(1-q) \\ 2 &> 3 - 3q \\ q &> 1/3 \end{aligned}$$

SPNE$_2$:

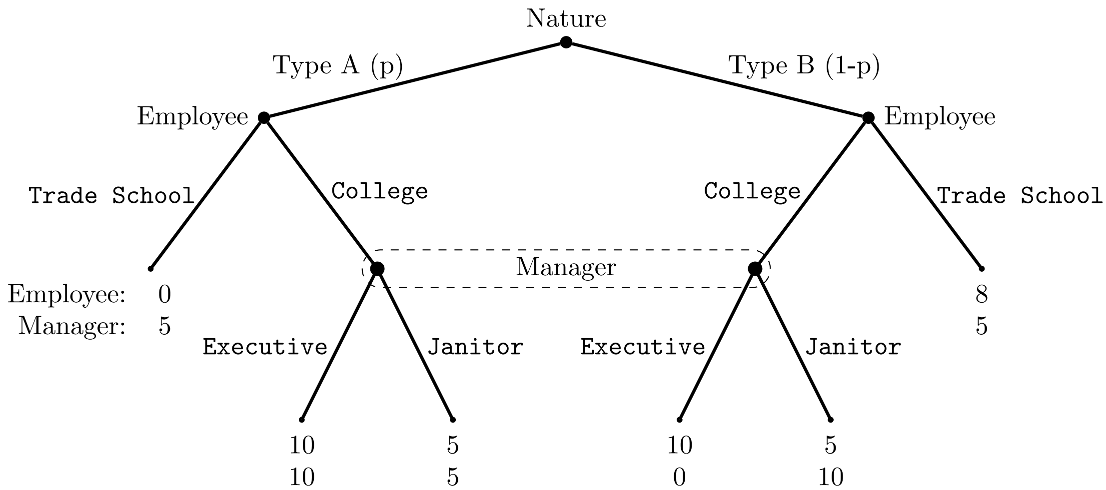
- Student chooses *honest*
- Faculty *refer*s
- Board *expel*s $\frac{1}{3} < q < \frac{1}{2}$ percent of the cases.

c) (4 P.) Relative to the pure threat of explusion alone, who gains and who loses from the existence of an honor board that expels probabalistically?

Student loses, because they get their 2nd favorite outcome instead of favorite. Faculty and board both gain because their favorite outcome is achieved in equilibrium.

Question 20. (16 P.)

Signaling: Consider a Bayesian game where Nature determines whether an employee is an A type and more suited for executive roles or a B type who are more suited for janitorial work. The Manager cannot observe the hidden type of an employee, but employees may choose to go to college or not.
The extensive form game is shown below:



a) (4 P.) Suppose that $p = 3/4$. Suppose that the Manager's pure strategy is to always hire College grads as *Executives*. Solve for the Subgame-perfect Bayes-Nash Equilibrium (SPBNE).
Is this a *separating* or a *pooling* equilibrium?

Case 1: If Managers always hire College grads as *Executives*:

$$EU_{\text{A}}(\texttt{College}) = \underline{10}$$
$$EU_{\text{A}}(\texttt{Trade School}) = 0$$
$$EU_{\text{B}}(\texttt{College}) = \underline{10}$$
$$EU_{\text{B}}(\texttt{Trade School}) = 8$$

So Manager should believe that all employees will go to college, $prob(\text{College}|\text{A}) = prob(\text{College}|\text{B}) = 1$.

$$EU_{\text{Manager}}(\texttt{Executive}|\text{College}) = 10\left(\frac{1\times p}{p+(1-p)}\right) + 0\left(\frac{1\times(1-p)}{p+(1-p)}\right) = 10p = 10(3/4) = \underline{7.5}$$
$$EU_{\text{Manager}}(\texttt{Janitor}|\text{College}) = 5\left(\frac{1\times p}{p+(1-p)}\right) + 10\left(\frac{1\times(1-p)}{p+(1-p)}\right) = 10 - 5(3/4) = 6.25$$

Pooling **SPBNE:**

- Manager: hire `Executive` with the belief that prob(A|College) = 3/4.
- Employee: `College` if Type A, `College` if Type B with the belief that prob(Executive|College) = 1.

b) (4 P.) Suppose that $p = 3/4$. Is there a *separating equilibrium* in pure strategies where all A types go to college, and all B types go to trade schools?

If Manager believes that all Type As go to college, $prob(\text{College}|\text{A}) = 1$, and all Type Bs go to trade schools; $prob(\text{College}|\text{B}) = 0$, then they should only hire `Executive`s.

$$EU_{\text{Manager}}(\texttt{Executive}|\text{College}) = 10\left(\frac{1 \times p}{p+0}\right) + 0\left(\frac{1 \times 0}{p+0}\right) = \underline{10}$$
$$EU_{\text{Manager}}(\texttt{Janitor}|\text{College}) = 5\left(\frac{1 \times p}{p+0}\right) + 5\left(\frac{1 \times 0}{p+0}\right) = 5$$

However, if Employees believe that the chances of being hired as executives conditional on choosing `College` is equal to 1, then they will all pool in the `College` strategy as in part (a).
No separating equilibrium in pure strategies

c) (4 P.) Suppose that now $p = 1/2$.
Define mixed strategies for both players and use them to solve for a *semi-separating* equilibrium.

Let q be the probability a Manager hires on an `Executive`.

$$EU_{\text{A}}(\texttt{College}) = \underline{10q + 5(1-q)}$$
$$EU_{\text{A}}(\texttt{Trade School}) = 0$$
$$EU_{\text{B}}(\texttt{College}) = \underline{10q + 5(1-q)}$$
$$EU_{\text{B}}(\texttt{Trade School}) = \underline{8}$$

Type As will always go to college, but Type Bs will only go to college if $10q + 5(1-q) > 8$, and will mix if their expected utilities from college and trade school are equal.

$$10q + 5(1-q) = 8$$
$$\Rightarrow q = 3/5$$

Now let r be the probability that a Type B Employee goes to College.

$$EU_{\text{Manager}}(\texttt{Executive}|\text{College}) = 10\left(\frac{1 \times 1/2}{1/2 + 1/2(r)}\right) + 0\left(\frac{r \times 1/2}{1/2 + 1/2(r)}\right) = 10\frac{1}{1+r}$$
$$EU_{\text{Manager}}(\texttt{Janitor}|\text{College}) = 5\left(\frac{1 \times 1/2}{1/2 + 1/2(r)}\right) + 10\left(\frac{r \times 1/2}{1/2 + 1/2(r)}\right) = 5\frac{1}{1+r} + 10\frac{r}{1+r}$$

So a Manager will be willing to mix their hiring between executives and janitors when:

$$10 = 5 + 10r \Rightarrow r = 1/2$$

***Semi-separating* SPBNE:**

- Manager's mixed strategy: hire `Executive` with probability 3/5, `Janitor` with 2/5 probability,
  - beliefs: $\text{prob}(\text{A}|\text{College}) = \frac{(1)(1/2)}{(1)(1/2)+(1/2)(1/2)} = \frac{1}{3/2} = 2/3$
- Employee's strategy:
  - if Type A $\rightarrow$ `College`,
  - if Type B $\rightarrow$ (`College` with prob 1/2, `Trade School` with prob 1/2)
  - beliefs: probability of being hired as `Executive` after college = 3/5

d) (4 P.) What is the *signalling* value of an employee choosing `College`? Use Bayes rule to compare the ex-ante probability $p = 1/2$ of a Type A to the updated belief of a Manager as to the Employee being Type A conditional on observing college in the semi-separating equilibrium in part (c).

In the semi-separating equilibrium above, the `College` strategy has signaling value.
The signal of college gives the Manager a better idea of the Employee's true type than their naive estimate of $p = 1/2$.

$$prob(\mathrm{A}|\texttt{College}) = \frac{prob(\texttt{College}|\mathrm{A}) \times prob(\mathrm{A})}{prob(\texttt{College})} = \frac{1 \times p}{1p + r(1-p)} = 2/3 > p = 1/2$$

This is because all Type A's choose college in equilibrium, but only a fraction, $r = 1/2$, of Type B's choose college.

Question 21. (16 P.)
Repeated Games: Consider the strategic form game below:

		Column	
		Cooperate	Defect
Row	Cooperate	8 , 8	0, 10
	Defect	10 , 0	3 , 3

a) (4 P.) What will happen when this game is a *one-shot* game and neither player can make any strategic moves?

Prisoner's Dilemma where (Defect, Defect) is the only NE

b) (2 P.) Will this outcome be *Pareto optimal*?

No, the Nash outcome of (3,3) is Pareto dominated by the (Coop, Coop) outcome of (8,8) **not Pareto optimal**

c) (4 P.) What could you change about the structure of this game to ensure that a socially optimal outcome will be reached in equilibrium?

Examples:

- reputation
- strategic moves: committing to Cooperation, etc.
- repeated play with indefinite or infinite horizon w/ common knowledge of history, etc.

d) (6 P.) Suppose that both players have a *discount factor* of $\delta = 3/4$. Can a strategy profile of both players using *grim trigger* strategies be sustained in the game where the strategic form game above is repeated infinitely?
Show all calculations and explain your answer.

$$PV(\text{Coop}) = 8 + 8\delta + 8\delta^2 + ... \to = 8 + \frac{8\delta}{1-\delta}$$
$$PV(\text{Defect}) = 10 + 3\delta + 3\delta^2 + ... \to = 10 + \frac{3\delta}{1-\delta}$$

Either player will continue to Cooperate forever as long as the payoff from doing so if greater than the payoff of *Defecting once*:

$$8 + \frac{8\delta}{1-\delta} \geq 10 + \frac{3\delta}{1-\delta}$$
$$\Rightarrow \delta^* \geq \frac{2}{5}$$

So as long as both players' discount factor is higher than 2/5 (3/4 > 2/5 so this is high enough), they are patient enough to sustain Cooperation.