Introduction to Game Theory

Sequential Games

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Outline

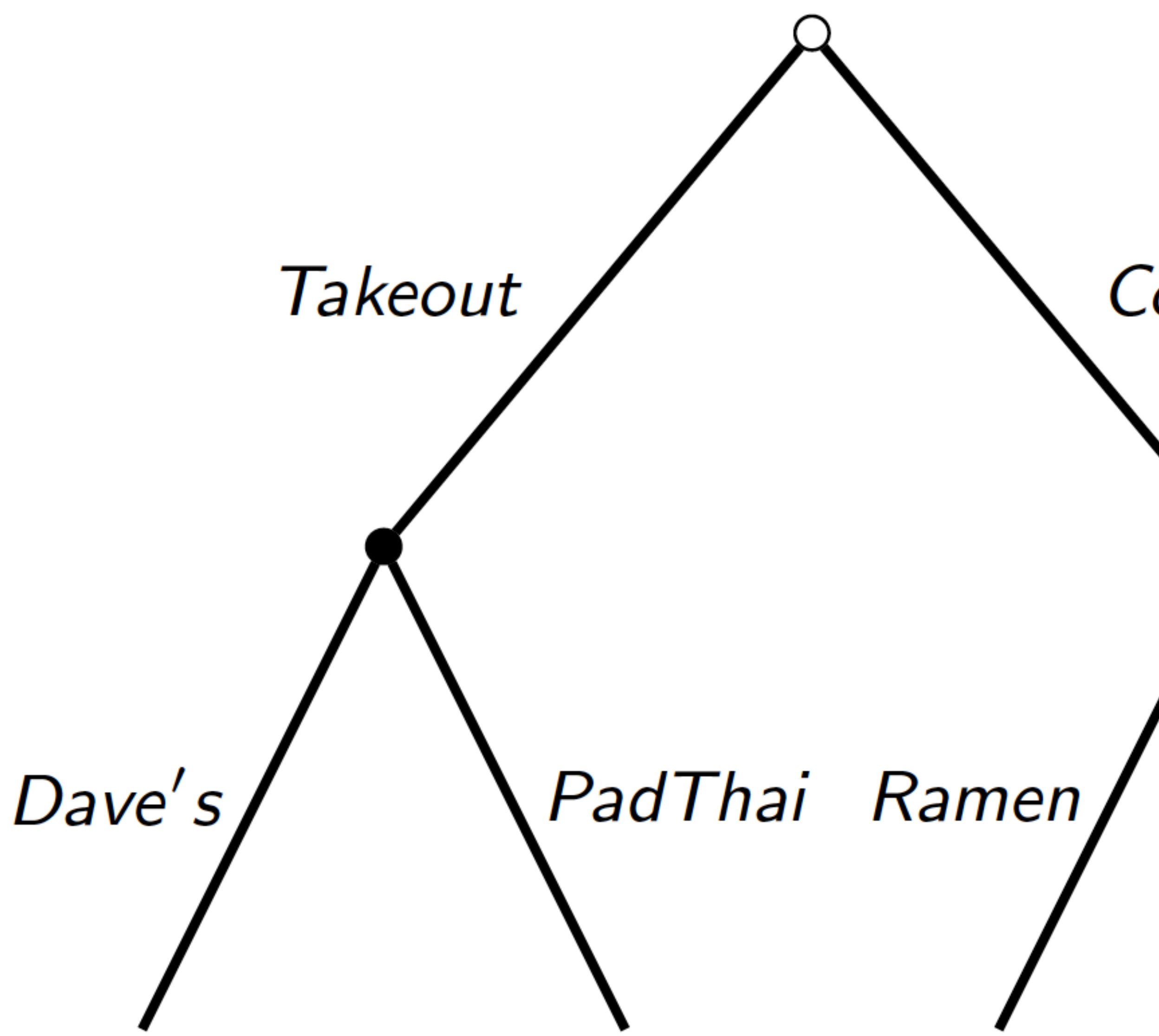
- Game trees
- Backwards Induction
- Efficiency

Extensive Form

Game Trees/Extensive Form as

- Before we learn how to solve a game, it will help
  visualize them
- Because of the ordered nature of sequential gar
  makes sense

A *Decision* Tree



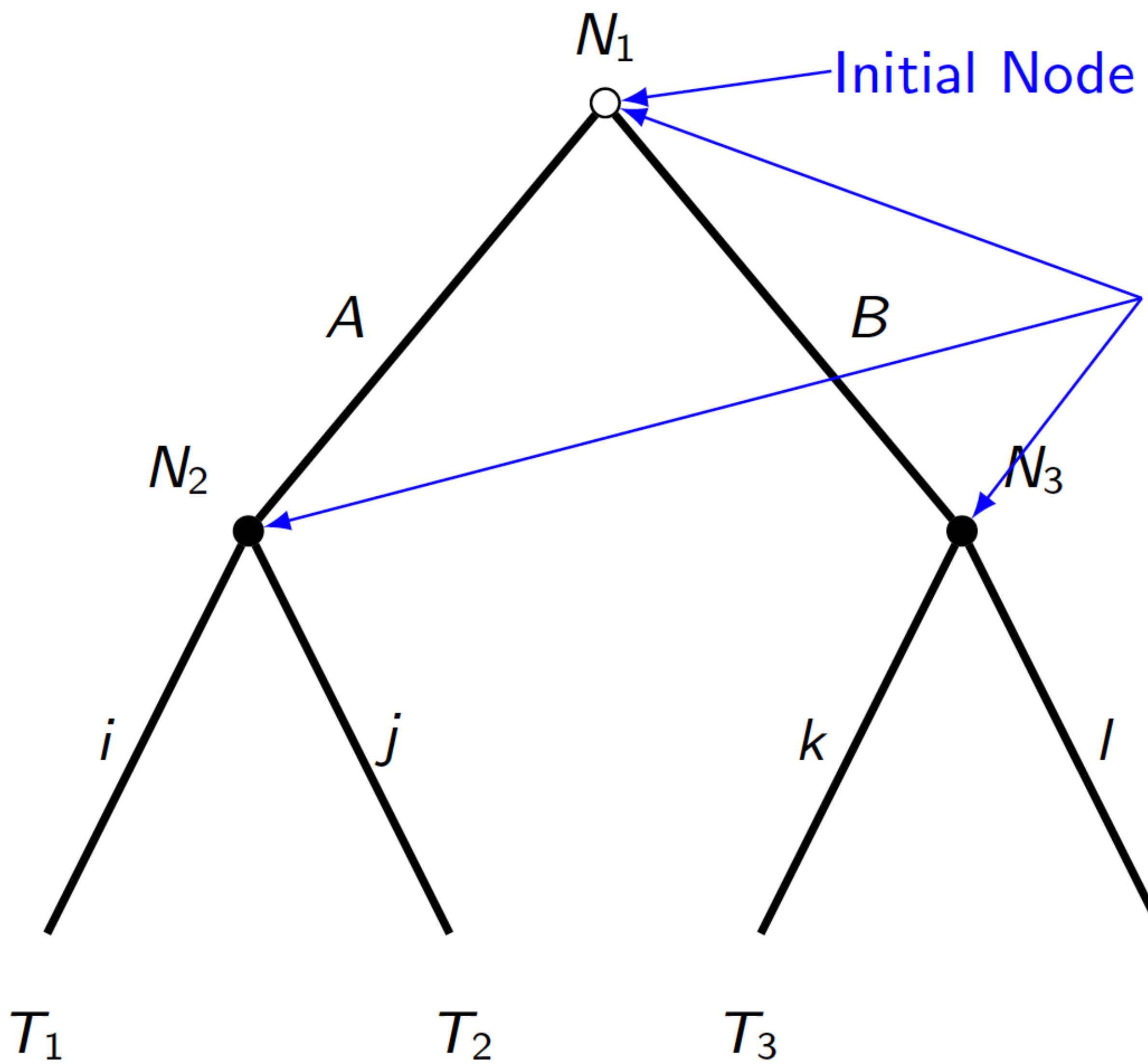
Extensive Form Definition

A **Tree Graph** consists of:

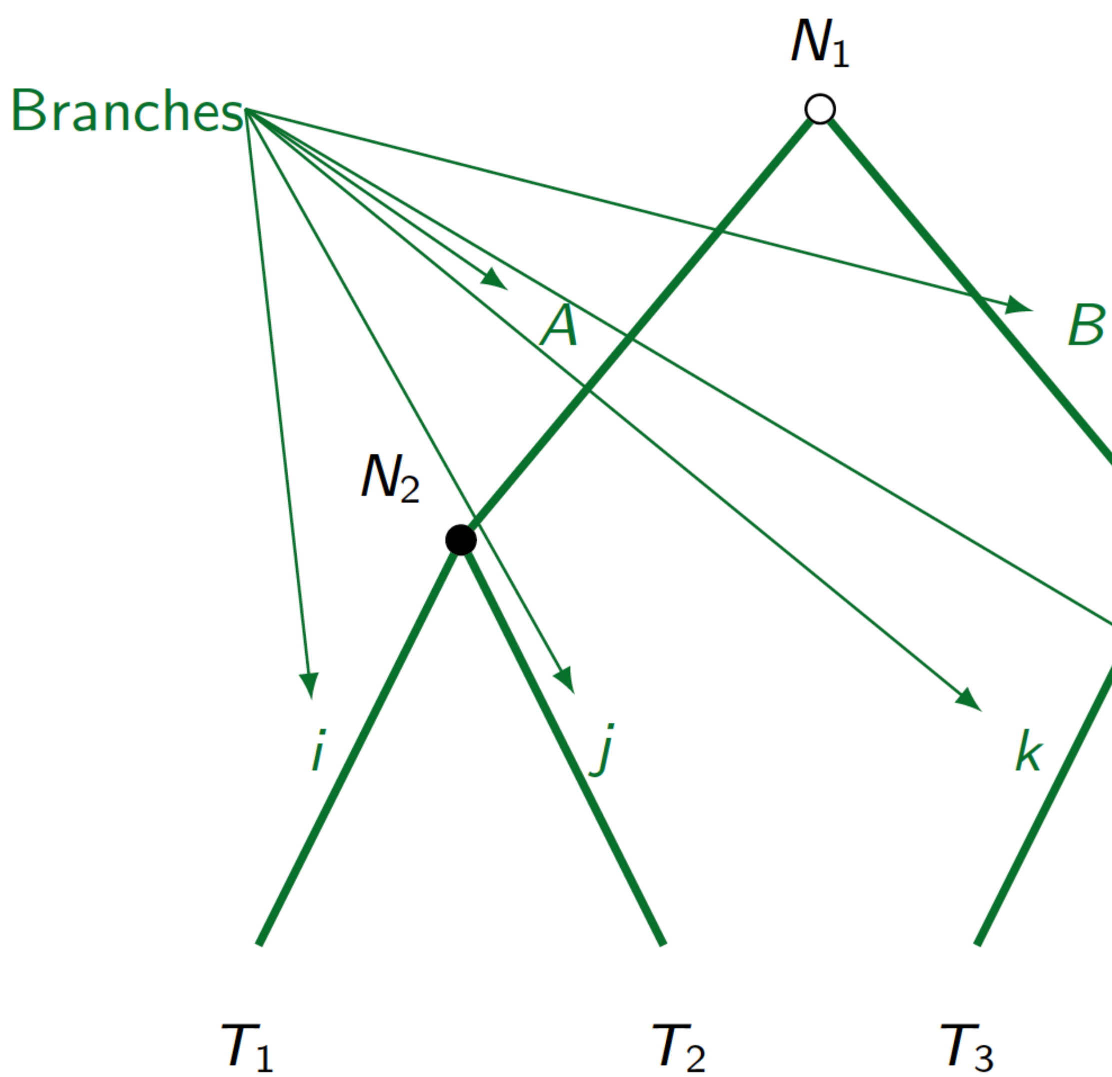
- Multiple **nodes** with an ordered hierarchy startin
  node
- **Branches** coming from each node which connec
- The tree ends in any of the multiple **terminal no**

> **Warning**
>
> Each (non-initial) terminal node may have multiple branches leading f
> branch that *leads to it*.

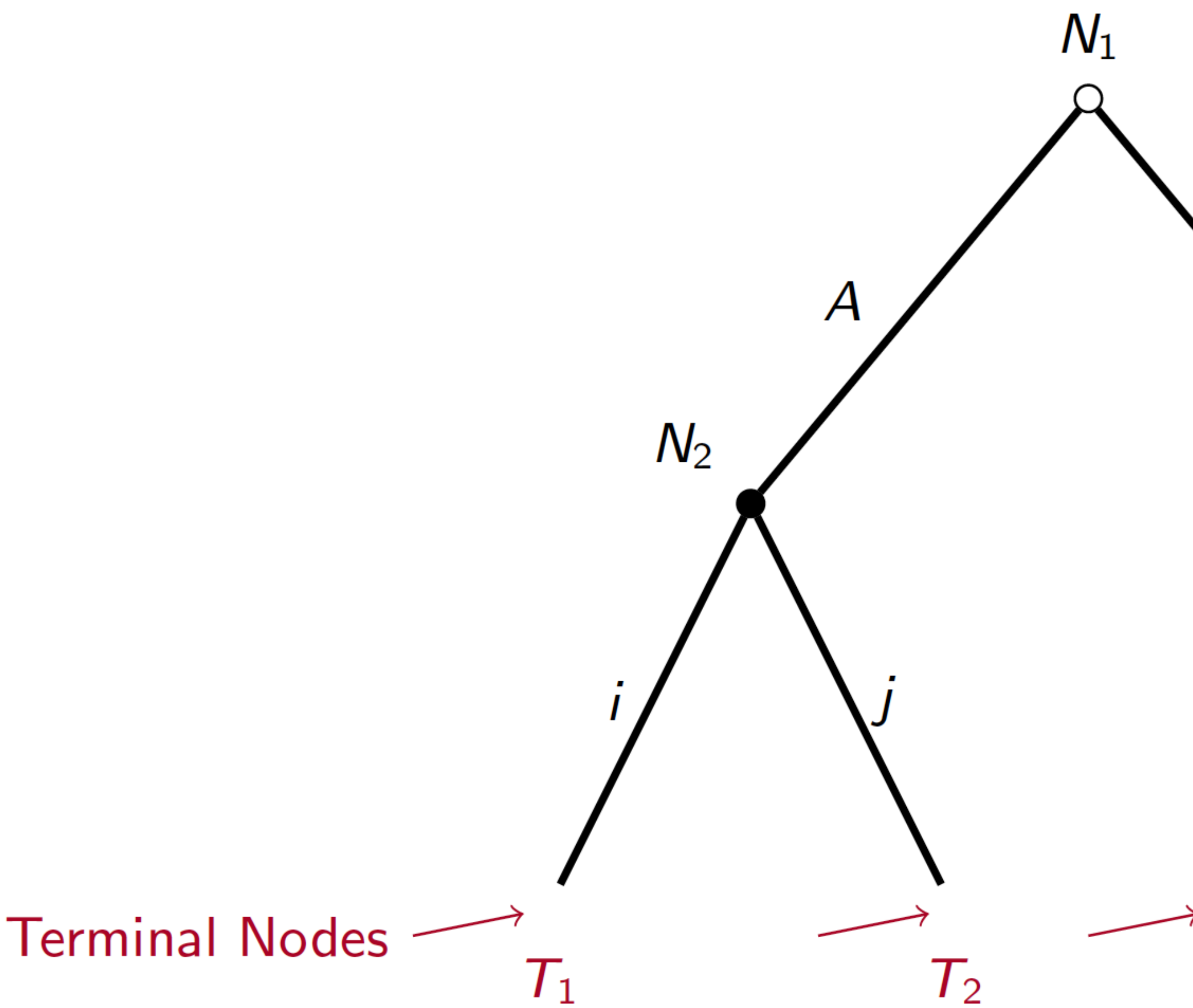
Anatomy of a tree



Anatomy of a tree



Anatomy of a tree



Kidnapping Game [1]

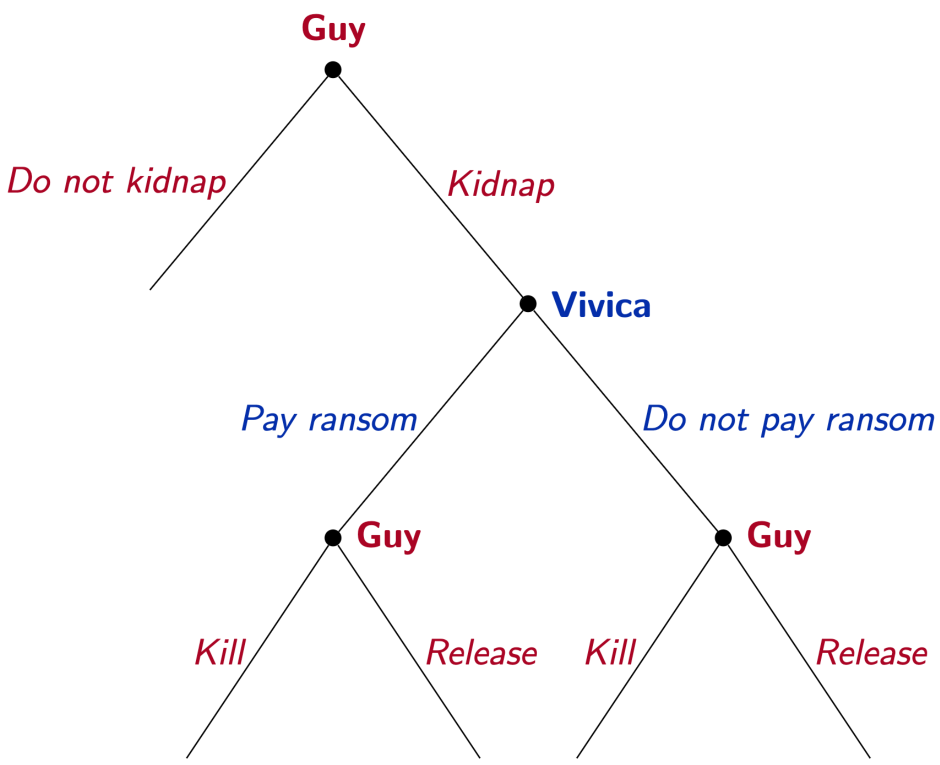
A kidnapper named **Guy** has contacted the victim's
demand a ransom.

To predict what will happen to the victim, **Orlando**
game theoretic model of the situation.

Let's use the language of the tree graph to visualiz

1. Harrington, pg 18

Kidnapping Game



- Who are th
- Where are
- What are t
  they repre
- What do th
  represent?
- Is this a co
  of a game?

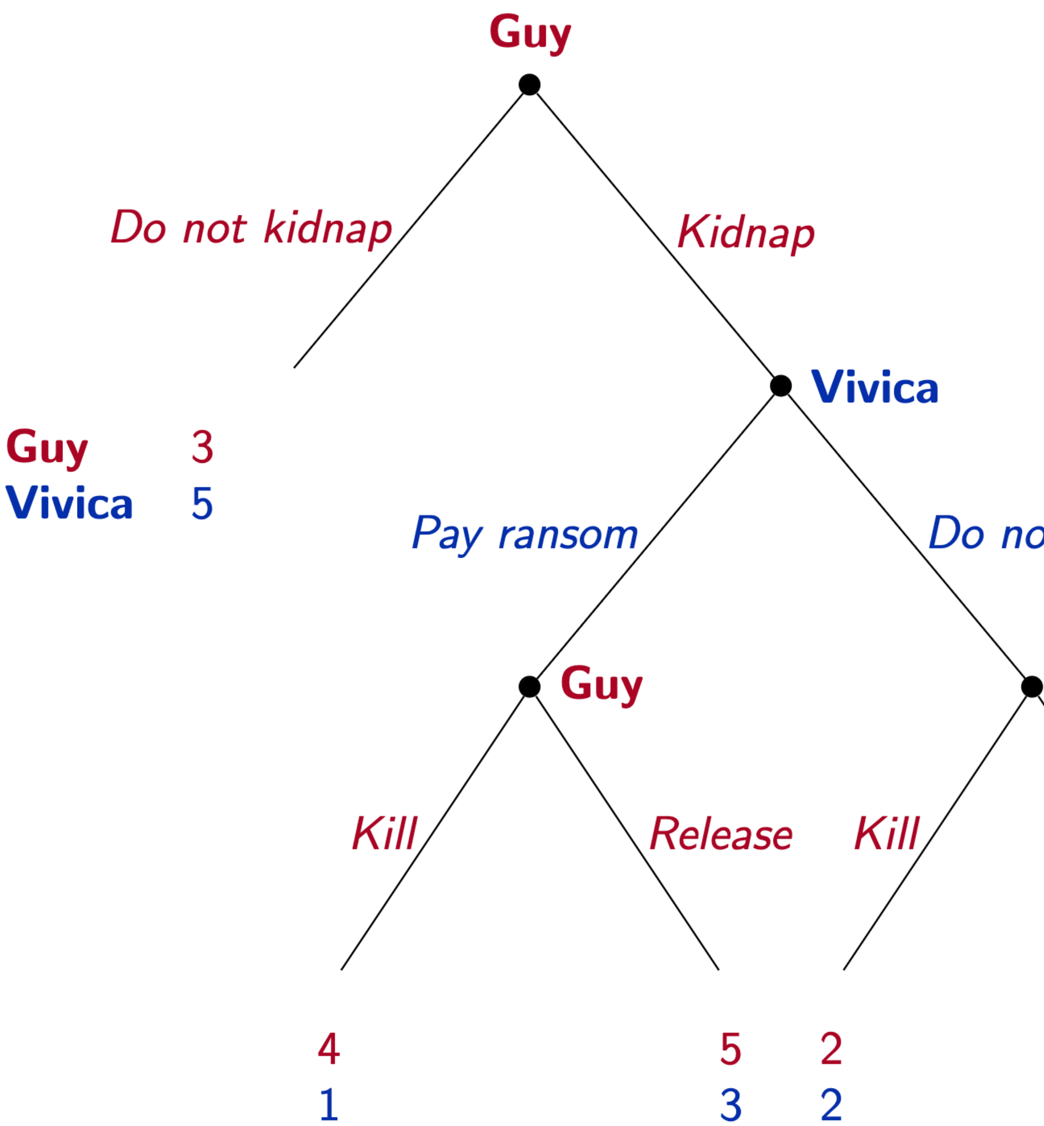
Kidnapping Game payoffs

Outcome
No kidnapping
Kidnapping, ransom paid, Orlando killed
Kidnapping, ransom paid, Orlando relea
Kidnapping, no ransom paid, Orlando ki
Kidnapping, no ransom paid, Orlando re

Kidnapping Game payoffs

Outcome	**Guy**
No kidnapping	3
Kidnapping, ransom paid, Orlando killed	4
Kidnapping, ransom paid, Orlando released	5
Kidnapping, no ransom paid, Orlando killed	2
Kidnapping, no ransom paid, Orlando released	1

Kidnapping game tree with pay



Predictions?

Based on the extensive form game tree with payof

- Do you have any predictions for what strategies
  choose?

a Definition of an Extensive For

- A collection of decision-makers, called **players** c
- A set of **decision nodes**, each represents the inf
  the player of that node
- Strategies for each player which list the **branche**
  represetn the actions a player would take if face
- A **tree diagram** which maps the intersections of
  profiles to the outcomes represented at each **te**

1. for finite, perfect information games

Strategies in Extensive Form Ga

Definition

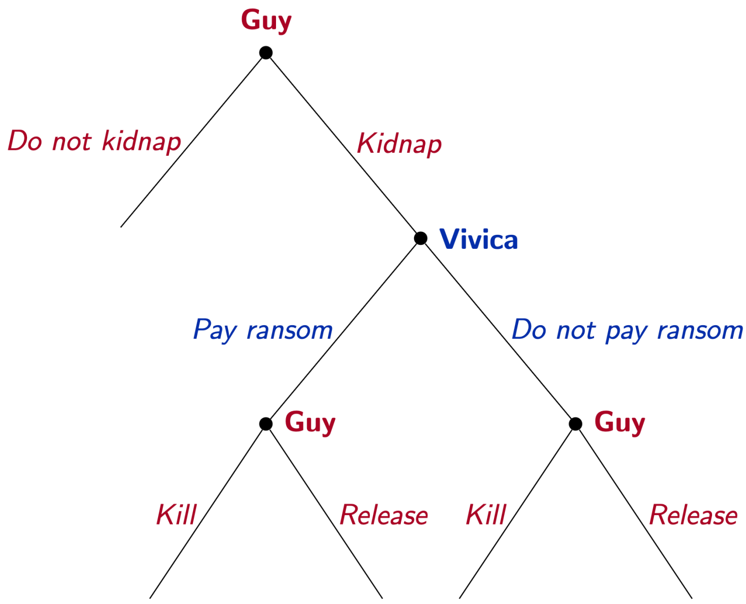
A **strategy** is a **complete plan of action** which assigns an action at *eve*
decision

Warning

Be careful to distinguish between a ***strategy*** and a single *action/choic*

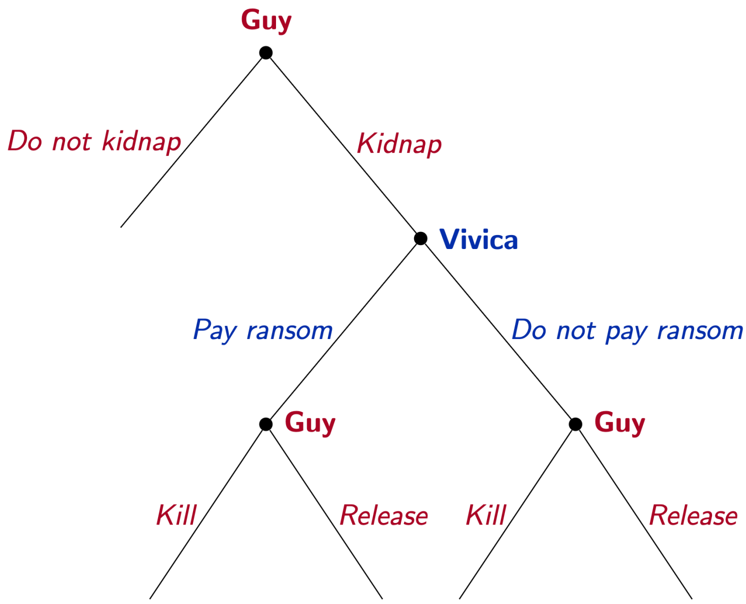
What's the difference?

Apply this definition to the kidr



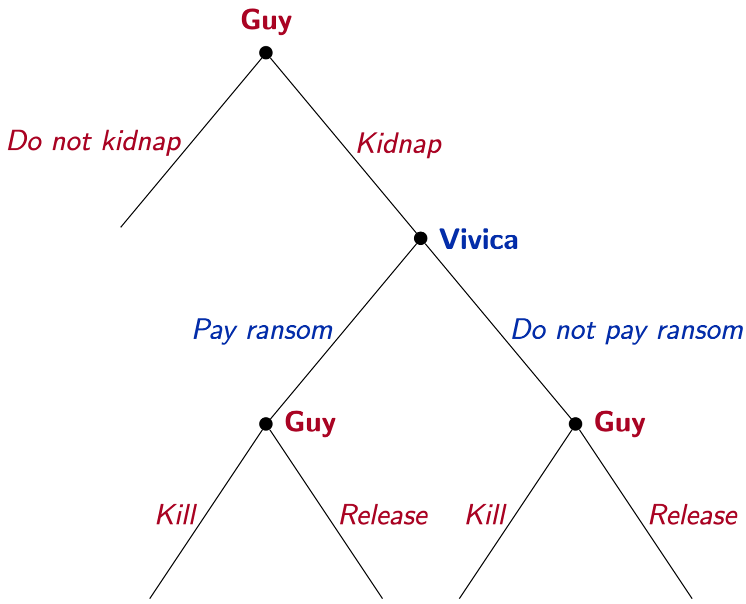
- How many decisio
  - 3
- How many decisio
  - 1

Apply this definition to the kidn



- Write out a compl
  - Only two strate
    - *Pay* the rans
    - or *Don’t* pay

Apply this definition to the kidn



- Write out at comp
  - Let's give some Guy's actions:
    - A ≡ Kidnap
    - I ≡ Don't ki
    - K ≡ Kill Orl
    - L ≡ Let Orla

Apply this definition to the kidr

Guy has 8 total complete strategies:

If Guy Abducts	If Guy Ignore
(A, K, K)	(I, K, K)
(A, L, K)	(I, L, L)
(A, K, L)	(I, K, L)
(A, L, L)	(I, L, L)

Backwards Induction

Solving Sequential Games

Now that we have defined all the parts of what a s
can start to *solve* them.

- A solution in our case will be a prediction of wh
  would do in a sequential game

The smoking decision

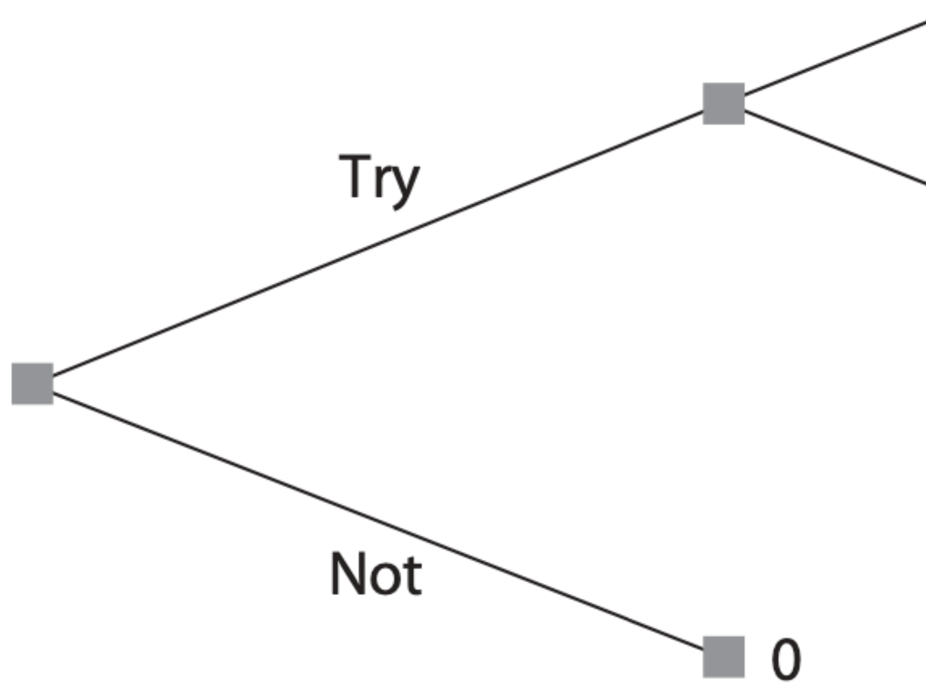


FIGURE 3.2 The Smoking Decision

figures/fig3.2.png

The smoking *game*

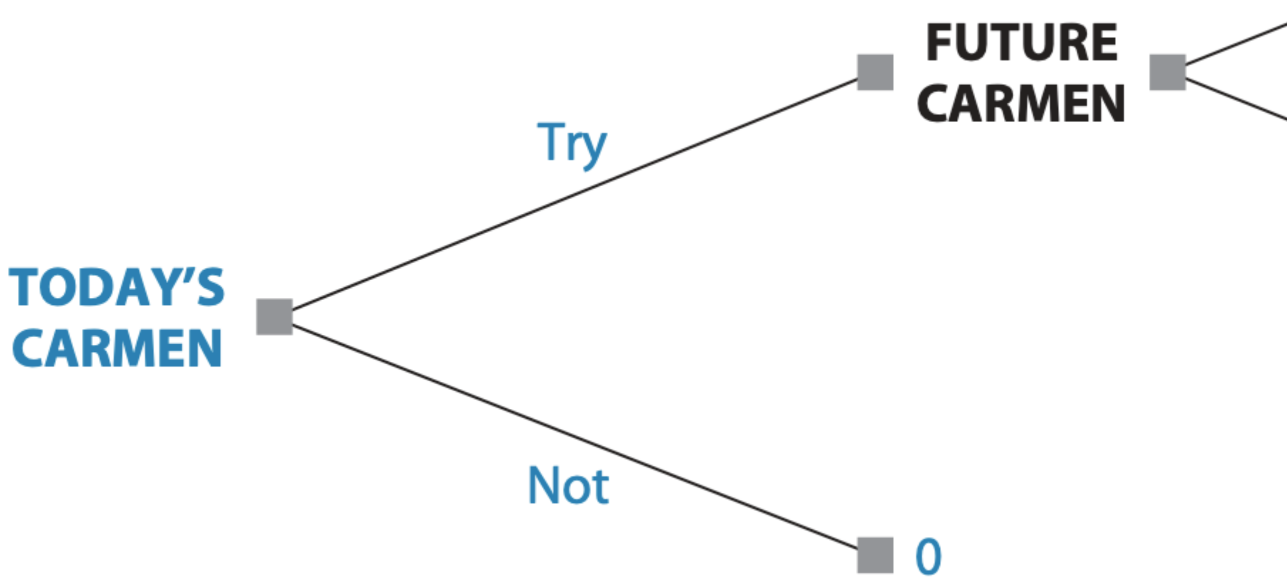


FIGURE 3.3 The Smoking Game

figures/fig3.3.png

‘Pruning’ branches

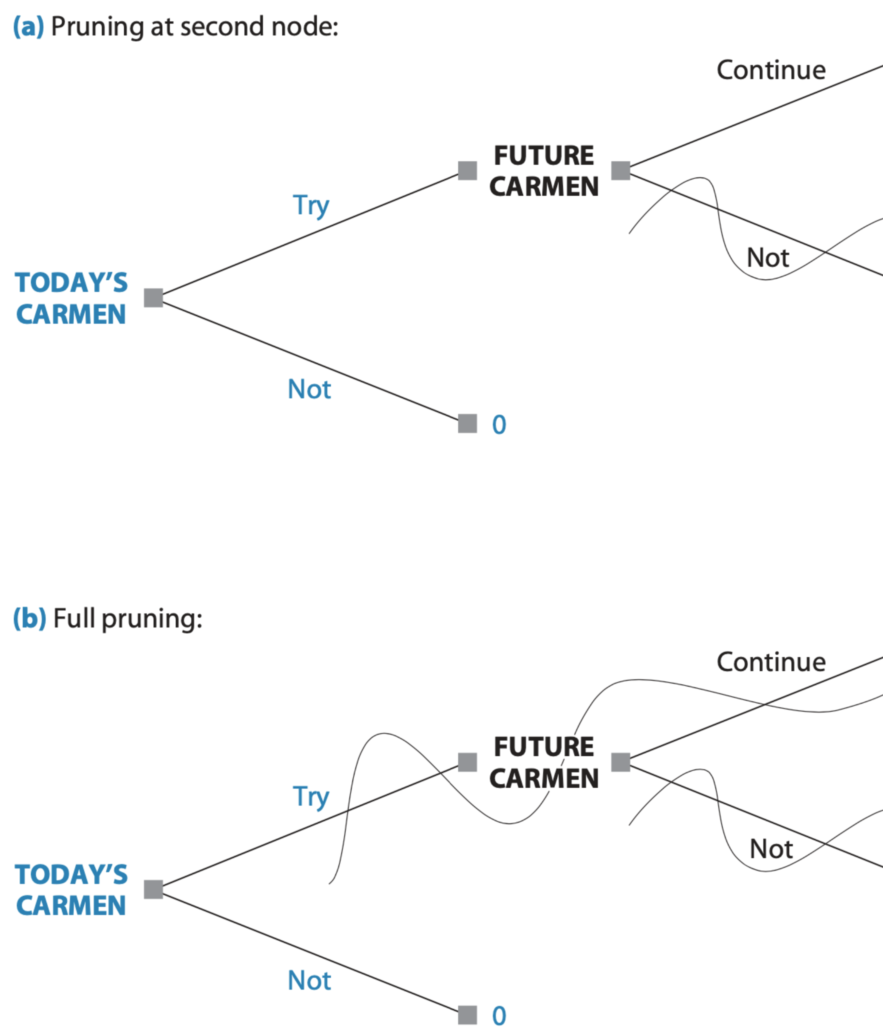


FIGURE 3.4 Pruning the Tree of the Smoking Game

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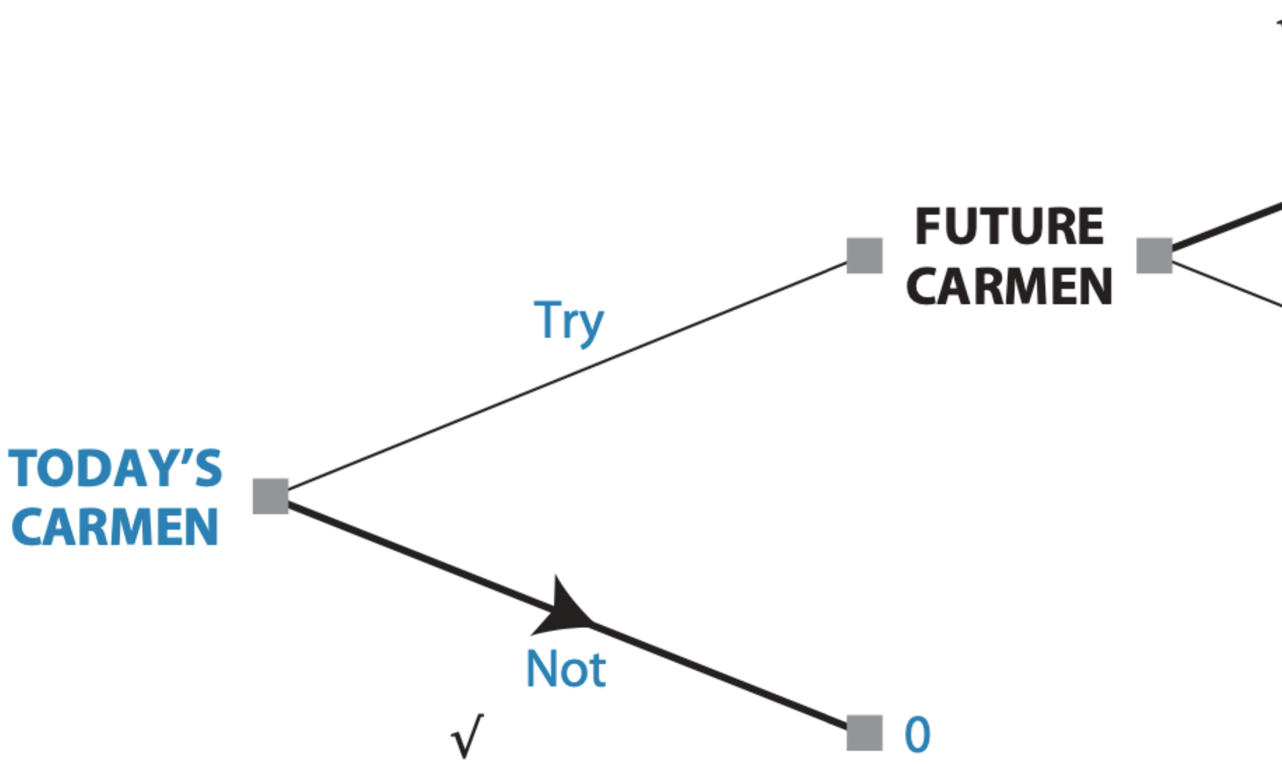

FIGURE 3.5 Showing Branch Selection on the Tree of the Smo

figures/fig3.5.png

Backwards Induction defined

The method of looking at decisions in the future to
now is called **Backwards Induction** or **Rollback**

Definition [1]

When all players do *rollback analysis* to choose their optimal strategie
the ***rollback equilibrium***[2] of the game; the outcome that arises from pl
rollback equilibrium outcome

1. Dixit et al, pg 56
2. aka subgame perfect equilibrium

Group Exercise:

Consider the Flag game but instead of starting with
starts with 5 flags, and instead of being able to pic
can only pick 1 or 2 flags.

1. *Draw the extensive form game tree complete wit*
   teams.
2. *How many total strategies are there for team 1?*
3. *Use pruning to eliminate actions to get to a rollb*
   will win? What is the winning strategy?

Adding more players

We can start to add more complexity with more tha

3-player planting game

- **Emily**, **Nina**, and **Talia** are roommates who want
  communal garden.
- They like to enjoy the benefits of fresh produce
  is costly for them to put the work in.
- **2 or 3 people** working is enough to keep the gar
  0 work, then the garden will die.

Planting Game payoffs

outcome:
I don't contribute, but garden lives
I contribute, and get garden.
I don't contribute, and garden dies
I contribute, but garden dies

Planding Game Tree

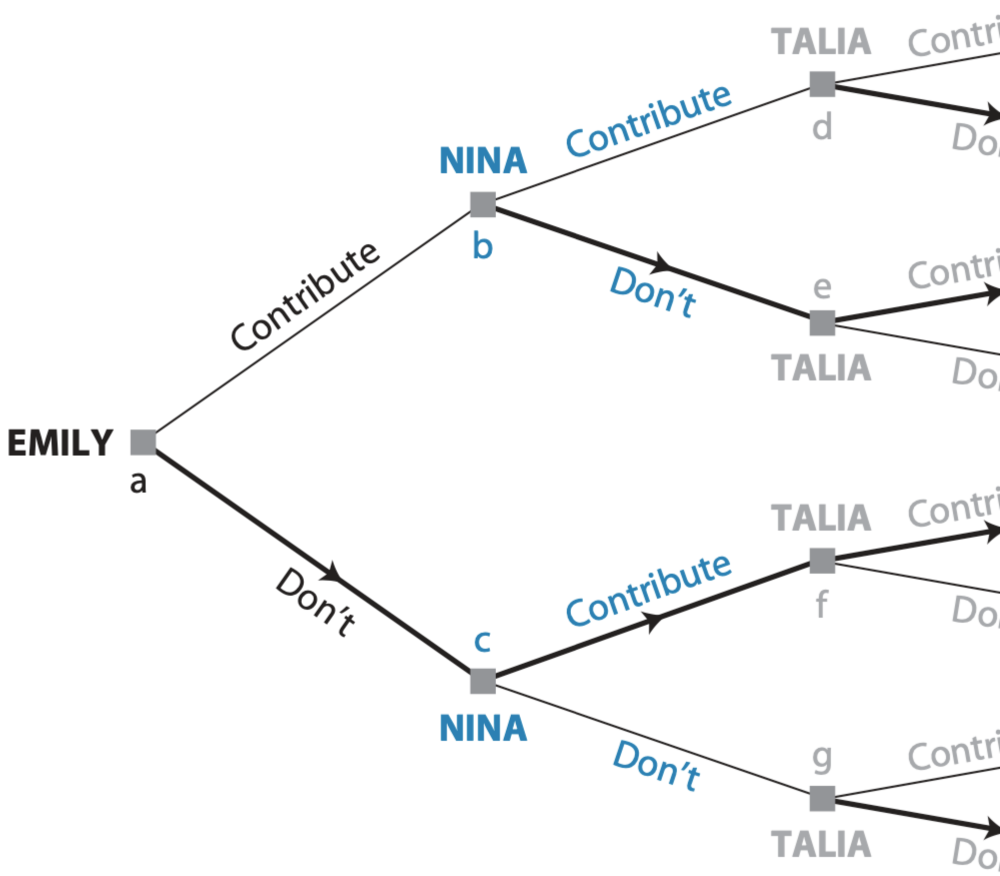

FIGURE 3.6 The Street–Garden Game

figures/fig3.6.png

Equilibrium Path of Play

Note that there is one continuous path we traced f
a final equilibrium outcome.

However, we couldn't have gotten their without the
even though they are never reached in equilibriur

Recall that a **strategy** is a collection of choices at e

Equilibrium Strategies

Even though the players available actions are all c
(Contribute or Don't), this tree provides labels of e
we can say something like:

> "Nina's **strategy** in the rollback equilibrium is {
> ***Contribute*** at **c** }".

- To make it even shorter, let's call this strategy **D**

How many strategies does Talia

- CCCC, CCCD, CCDC, CCDD, CDCC, CDCD, CDDC, CDDD
  DCDD, DDCC, DDCD, DDDC, DDDD
- 16 total strategies

Rollback Equilibrium Strategies

The equilibrium is:

- { **D**^{1}, **DC**^{2}, **DCCD**^{3} }

1. Emily
2. Nina
3. Talia

Adding More Moves

Even a simple game get complicat

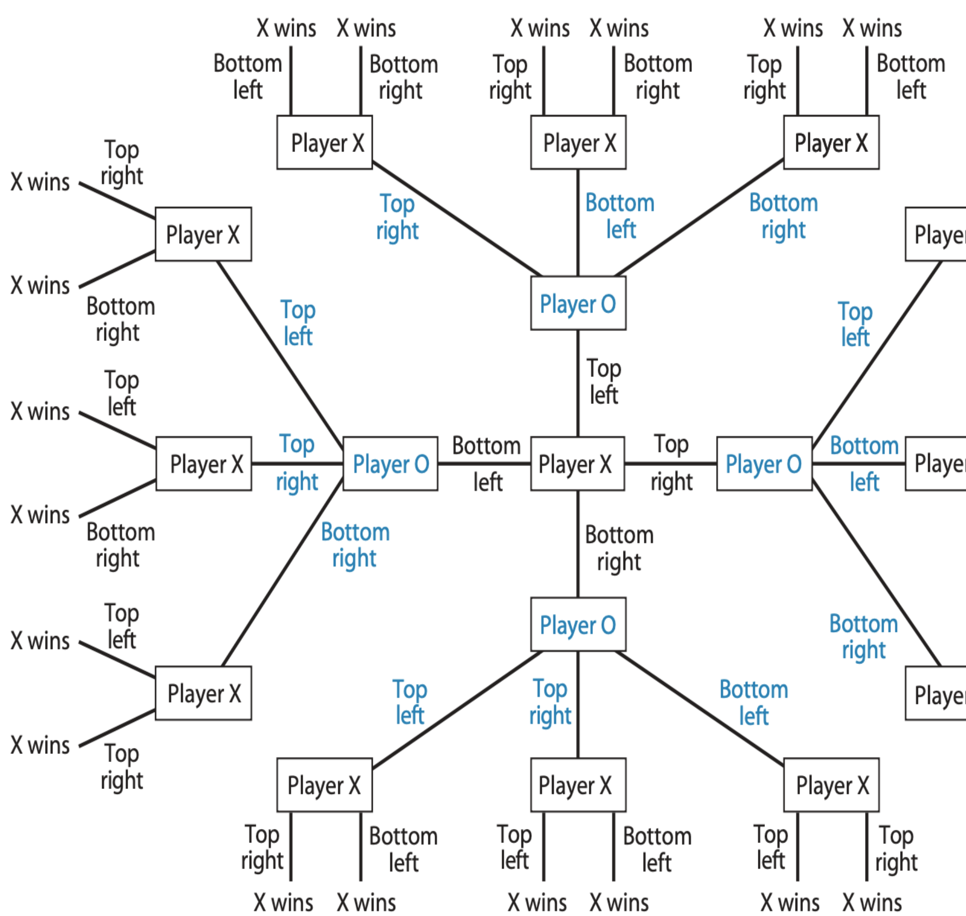

FIGURE 3.7 The Complex Tree for Simple Two-by-Two Tic-Tac-Toe

Tic-Tac-Toe

- Even though it looks complicated, the main bran
  copies of each other
- Most people probably figure out the rollback equ
  it enough
- Insert relevant xkcd here: https://xkcd.com/832/

Chess

- What about more complicated games like chess?
  - technically rollback solvable, but with 10^{120} p
    hasn't been solved by either human or machi
- Players of complicated sequential games often i
  intermediate valuation function to assign payof
  nodes.

Welfare and Efficien

What are the **good** outcomes in the planting game

Can we rank outcomes by collective welfare?

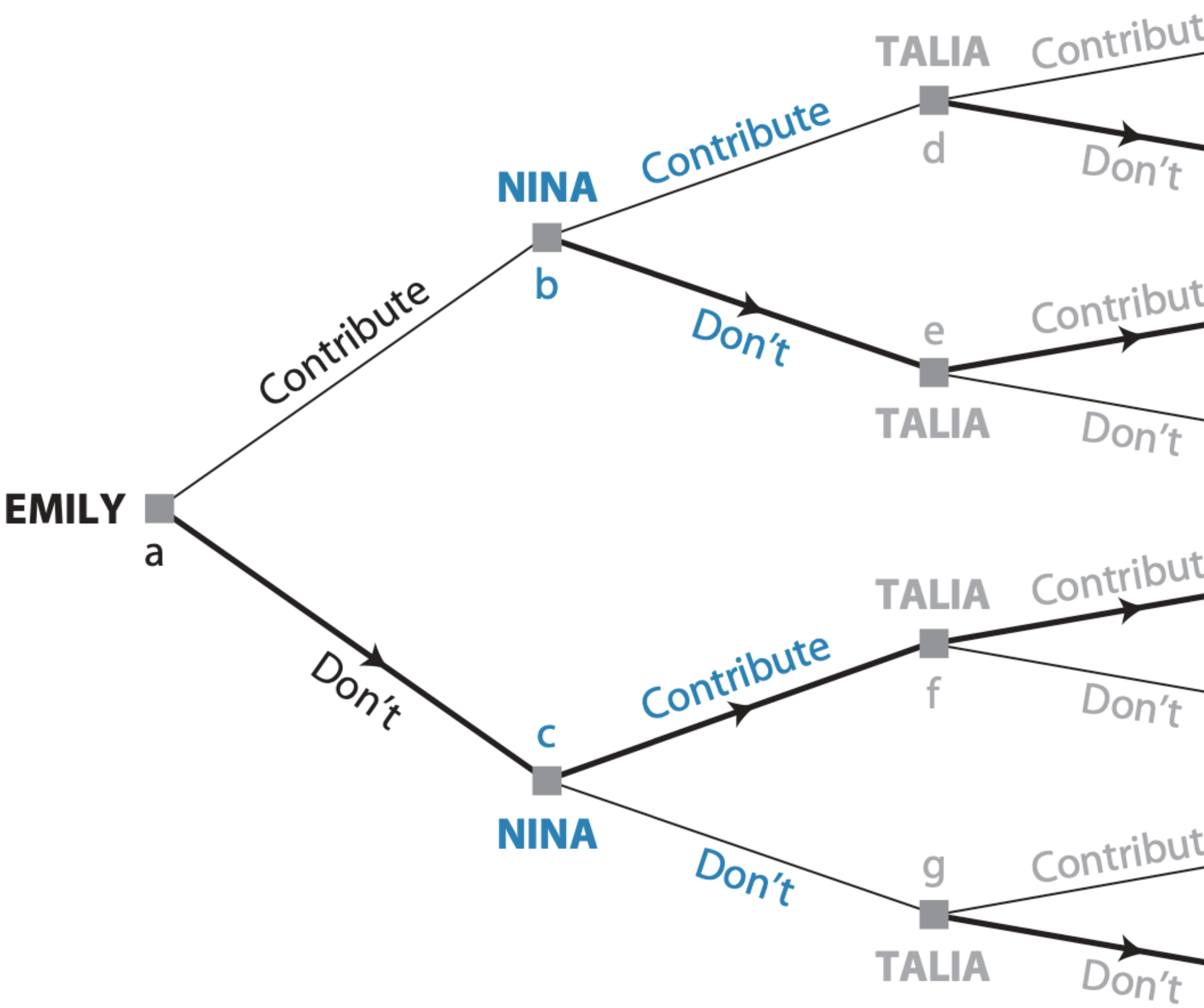

FIGURE 3.6 The Street–Garden Game

Pareto Dominance

Pareto optimality (or efficiency) is econonomists' b
with a ranking of which outcomes are objectively 'b

- For any two outcomes (🎉, 🎊), 🎉 is **Pareto domi**
    1. No one strictly prefers 🎉 to 🎊 - U👤(🎊) $\geq$ U👤
       $\forall$👤 $\in$ { 🙍, 👦, 💆, 🧖, 👦, 😺, . . . }
    2. At least one person strictly🎉 prefers 🎉 to 🎊
       $>$ U👤(🎉)

Pareto Improvement

The move from a policy y to an alternative policy x
improvement if x Pareto dominates y.

- Such a policy change should reasonably be seen
  good
- Another perspective is that *no-one would veto* a

Pareto Efficiency

An outcome is **Pareto Efficient** (Optimal) if no othe
dominates it.

An outcome is **Pareto Inficient** if at least one othe
dominates it.

Ranking the Planting Payoffs

Compare (4,3,3) to (1,2,2)

- Which one is Pareto dominating?

Ranking the Planting Payoffs

Now compare (4,3,3) to (3,4,3) or (3,3,4)

- Which one is Pareto dominating?

Is the rollback equilibrium outcome a *Pareto efficie*

Discussion: Efficiency vs other s comparisons

- How useful is Pareto Efficiency in the real world?
- How else could we group outcomes?
- We might address this later in the class with wha *Cooperative Game Theory*