Solving Simultaneous Games
EC 327 Game Theory

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iClicker Poll

		Luca	
		Testify	*Keep Quiet*
Guido	*Testify*	$-10, -10$	$0, -20$
	Keep Quiet	$-20, 0$	$-1, -1$

- Suppose that you are Guido in the Prisoner's Dilemma, above. Which strategy would you pick?
    1. Testify
    2. Keep Quiet

iClicker Poll

		Luca	
		Testify	*Keep Quiet*
Guido	*Testify*	$-10, -10$	$0, -20$
	Keep Quiet	$-20, 0$	$-1, -1$
	False Confession	$-20, 0$	$-20, -1$

- What if we change the Prisoner's Dilemma like this? What would you pick if you were Guido?
  1. Testify
  2. Keep Quiet
  3. False Confession

iClicker Poll

		Luca		
		Testify	*Keep Quiet*	*False Confe*
Guido	*Testify*	$-10, -10$	$0, -20$	$0, -20$
	Keep Quiet	$-20, 0$	$-1, -1$	$-1, -20$
	False Confession	$-20, 0$	$-20, -1$	$-10, -1$

- What about this third variation? What would you pick if you were Guido?
  1. Testify
  2. Keep Quiet
  3. False Confession

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- One last question, and this one isn't based on a Prisoner's Dilemma...

iClicker Poll

			Bart	
		R	P	S
3*Lisa	R	0, 0	-1, 1	1, -1
	P	1, -1	0, 0	-1, 1
	S	-1, 1	1, -1	0, 0

- This game models a game of Rock-Paper-Scissors. If you are Lisa, which strategy will you choose?
  1. R(ock)
  2. P(aper)
  3. S(cissors)

Obviously-Wrong Strategies

- The first three of those games contained strategies that were **obviously bad choices**.
- Rock-Paper-Scissors did not.
- One of the simplest things you can do with a strategic-form game is to start by finding and eliminating (ruling out) the strategies which are obviously bad.
- In some games, this can even be enough to identify the Nash Equilibrium!

The Problem of Finding Nash Equilibria

- When we first discussed Nash Equilibria (which I'm going to start abbreviating as NEs), we found them by checking all of the strategy profiles in the game to see which of them were stable.
- This is not a problem in a 2x2 game like we've been working with—but it gets much more time-consuming in games with more players and more strategies per player, not to mention the more complicated games we'll look at later in the term.
- We can make it easier to find NEs with a few useful shortcuts—such as eliminating entire strategies (not just strategy profiles) that can't possibly be part of a NE.

Strict Dominance

- A strategy is said to be strictly dominated if there is some other strategy, in the same player's strategy set, which provides that player a higher payoff, no matter what strategies the other players pick.
  - Another way to phrase it is that a strategy is strictly dominated if some other strategy is a better alternative for the player, no matter what other players do.
- Recall that, in the Prisoner's Dilemma, both Guido and Luca prefer to Testify, no matter whether the other player Testified or kept Quiet: this means that Quiet is strictly dominated, **by Testify**, for both players.
- It is not rational to play a strictly dominated strategy—meaning that in the Prisoner's Dilemma, we can immediately deduce that neither player would play Quiet, and the only remaining strategy profile is (Testify, Testify).

Finding NEs by Elimination

- If all but one of each player's strategies can be eliminated like this (leaving only a single strategy profile), then the remaining strategy profile is a NE.
  - A strictly dominated strategy can never be part of a NE.
- However, it's rare that a player has one strategy which strictly dominates all of their others from the very start, as in the Prisoner's Dilemma. (This is called a strictly dominant strategy.)
- Even if a player doesn't have a strictly dominant strategy, we can still sometimes use elimination to find a NE, by using a process called Iterated Elimination of Strictly Dominated Strategies (IESDS).

Commonly Known Rationality

- Let's assume that, not only is every player rational, they all know that the other players are rational too.
- This means that players can deduce which strategies the **other** players would never play.
- And if a player can eliminate another player's strategy, it may reveal additional strictly dominated strategies that can be eliminated.
- Let's see an example...

Example: IESDS

			P$_2$	
		a	b	c
3*P$_1$	A	1, 1	2, 2	3, 3
	B	2, 0	3, 1	4, 2
	C	3, 1	2, 2	1, 1

- In the game table above, there are no strictly domin**ant** strategies.
    - For Player 1, A is strictly dominated by B, but C is neither dominant nor dominated.
    - And for Player 2, a is strictly dominated by b, but c is also neither dominant nor dominated.
- So, we could eliminate A and a, but we'd still have a 2x2 game left over.

Example: IESDS

		P$_2$		
		a	b	c
3*P$_1$	A	1, 1	2, 2	3, 3
	B	2, 0	3, 1	4, 2
	C	3, 1	2, 2	1, 1

- However, the assumption of Commonly Known Rationality allows Player 1 to **deduce** that Player 2 would never play a.
- Player 1 can eliminate a, just like we did—and once they do, C is strictly dominated by B.
- Player 2 can deduce all of this—and once they eliminate A, a, and C, b is strictly dominated by c.
- This leaves us one strategy per player, and so the NE here is (B, c).

Flaws of IESDS: the Deer Hunt

		Ogg	
		Deer	*Rabbit*
2*Igg	*Deer*	$2, 2$	$0, 1$
	Rabbit	$1, 0$	$1, 1$

- IESDS doesn't always reveal a NE. The strategies involved in a NE don't have to be strictly dominant—they just can't be strictly dominat**ed**.
- As an example, the Deer Hunt game (above) contains no strictly dominated strategies.
- Even if IESDS doesn't reveal a NE, it can still be useful for simplifying a game before applying other methods.

Order Doesn’t Matter

- **In IESDS,** the order in which you eliminate strategies doesn’t matter. You’ll get the same result no matter how you do it—as long as you keep going to the end.
- Consider this even larger game, and suppose we start by looking for Player 1’s strictly dominated strategies:

		P$_2$				
		a	b	c	d	e
5*P$_1$	A	1, 1	2, 2	2, 2	2, 1	4, 1
	B	1, 3	1, 3	2, 2	2, 3	3, 2
	C	1, 2	2, 4	1, 3	2, 3	1, 3
	D	3, 2	2, 3	1, 4	2, 2	1, 2
	E	2, 1	3, 2	3, 2	3, 3	4, 1

Order Doesn't Matter

- And now suppose we start by looking for Player 2's strictly dominated strategies:

				P$_2$		
		a	b	c	d	e
5*P$_1$	A	1, 1	2, 2	2, 2	2, 1	4, 1
	B	1, 3	1, 3	2, 2	2, 3	3, 2
	C	1, 2	2, 4	1, 3	2, 3	1, 3
	D	3, 2	2, 3	1, 4	2, 2	1, 2
	E	2, 1	3, 2	3, 2	3, 3	4, 1

IESDS in a Nutshell

- The process of IESDS can be summed up in three steps:
  1. Search for a strictly dominated strategy belonging to any player. If none exists, stop here: IESDS is completed.
  2. Eliminate (cross out) that strategy. Optionally, re-draw the game table without the eliminated strategy.
  3. Return to step 1.

iClicker Q1

			P$_2$	
		x	y	z
3*P$_1$	X	$1, 3$	$2, 2$	$3, 2$
	Y	$2, 2$	$2, 2$	$4, 3$
	Z	$1, 1$	$0, 2$	$1, 1$

- In the game above, which strategy is strictly dominated?

label=) X
lbbel=) Y
lcbel=) Z
ldbel=) x
lebel=) y

iClicker Q2

P$_2$

		x	y	z
3*P$_1$	X	$1, 3$	$2, 2$	$3, 2$
	Y	$2, 2$	$2, 2$	$4, 3$
	Z	$1, 1$	$0, 2$	$1, 1$

- Strategy Z is strictly dominated. Which strategy strictly dominates it?

label=) X

lbbel=) Y

lcbel=) Both X and Y

iClicker Q3

P$_2$

		x	y	z
3*P$_1$	X	$1, 3$	$2, 2$	$3, 2$
	Y	$2, 2$	$2, 2$	$4, 3$
	Z	$1, 1$	$0, 2$	$1, 1$

- Perform IESDS all the way to completion. What does IESDS tell you about the NE of this game?

label=) The NE is (X, x).
lbbel=) The NE is (X, y).
lcbel=) The NE is (Y, y).
ldbel=) The NE is (Y, z).
lebel=) IESDS by itself does not reveal the NE of this game.

Other Elimination Methods: Weakly Dominated Strategies

- It is also possible to find Nash Equilibria by eliminating weakly dominated strategies (strategies for which there is an alternative that is never worse, and sometimes better).
- We will not spend a lot of time on this method, because it has two serious flaws:
    1. It is possible for a Nash Equilibrium to involve playing a weakly dominated strategy. IEWDS may therefore eliminate actual Nash Equilibria.
    2. Unlike IESDS, the order in which you eliminate weakly dominated strategies matters: you may get different results from different orders of elimination.

Example: Why Weak Dominance is Not Useful

- In the game below, B is weakly dominated for both players.

		Player 2	
		A	B
2*Player 1	A	$2, 2$	$1, 1$
	B	$1, 1$	$1, 1$

- If we eliminate the weakly dominated strategy for both players, then the only remaining strategy profile is (A, A)—and this is a Nash equilibrium.
- However, (B, B) is also a Nash equilibrium: both players get payoff 1, and neither can improve that payoff by changing their own strategy. We failed to find this equilibrium by eliminating weakly dominated strategies.

Another Example: Why Weak Dominance is Not Useful

- In the game below, M and R are weakly dominated.

		Player 2		
		L	M	R
2*Player 1	T	0, 1	1, 0	0, 0
	B	0, 0	0, 0	1, 0

- If we begin by eliminating R, then afterwards, M and B are both weakly dominated, and we would eliminate them, leaving only (T, L).

Another Example: Why Weak Dominance is Not Useful

		Player 2		
		L	M	R
2*Player 1	T	0, 1	1, 0	0, 0
	B	0, 0	0, 0	1, 0

- However, if we begin by eliminating M, then T and R are both weakly dominated, and if we eliminate them, we are left with only (B, L).
- Not only does the outcome of IEWDS depend on what we eliminate first, it still fails to find a **third** Nash equilibrium, which is (B, R).

Other Elimination Methods: Non-Best-Responses

- Generally speaking, we can eliminate any strategy which is not rational to play in a NE.
- It's never rational to play strictly dominated strategies, but it's sometimes rational to play weakly dominated strategies.
- There are other categories of non-rational strategies:
- A strategy is a non-best-response or non-rationalizable strategy if and only if, regardless of what the other players choose, it never provides the best possible payoff.
- I like to describe non-rationalizable strategies as strategies that you'd have to be crazy to think were a good idea, i.e. you can't rationalize playing them.

Strictly Dominated vs. Non-Best-Response

- Non-rationalizability is very similar to strict dominance, but here's the difference:
- Strict dominance is pairwise: a strategy s dominates another, s', if s specifically always gives a better payoff than s'.
- Non-rationalizability is a property of a single strategy: for a strategy to be non-rationalizable, it means that there is always **some** option that gives a better payoff—but the better option doesn't always have to be the same strategy.
- To put it another way: strategy s is not a best response if there is always **some strategy** which is better. To be strictly dominated, there must be **one particular strategy** which is always better.

Example: Non-Best-Responses

- In the game below, there are no strictly dominated strategies, meaning that IESDS will not do anything to simplify it.

		P$_2$		
		a	b	c
3*P$_1$	A	0, 4	1, 2	3, 3
	B	1, 2	0, 3	1, 4
	C	3, 3	1, 2	0, 1

- However, we can see that B is non-rationalizable for Player 1: regardless of whether Player 2 chooses a, b, or c, Player 1 is better off playing either A or C.
- Also, b is non-rationalizable for Player 2.

Another Example: Non-Best-Responses

P$_2$

		a	b	c	d
4*P$_1$	A	1, 0	2, 1	3, 1	4, 2
	B	3, 1	2, 2	2, 0	3, 0
	C	3, 1	4, 0	1, 0	2, 0
	D	4, 2	3, 0	2, 1	1, 1

Iterated Elimination of Non-Best-Responses

- It is not rational to play a non-best-response strategy in a pure-strategy context. (As we'll see much later, it's more complicated in a mixed-strategy context).
- Any strictly dominated strategy is also not a best response, but not all NBR strategies are strictly dominated.
- Because of this, if we eliminate non-best-responses, using the same steps as IESDS, this process will always eliminate the same strategies, and it may eliminate even more!
- This process is, naturally, called Iterative Elimination of Non-Best-Responses (IENBR).

Example: IENBR

- Returning to this same example, we can start by eliminating B.

			P_2	
		a	b	c
3*P_1	A	0, 4	1, 2	3, 3
	B	1, 2	0, 3	1, 4
	C	3, 3	1, 2	0, 1

- Once we do this, we can now see that b and c are non-rationalizable, and eliminate them.
- Finally, we can see that B is non-rationalizable and eliminate it, leaving only the strategy profile (C, a), which is the Nash equilibrium of this game.

Another Example: IENBR

		P$_2$			
		a	b	c	d
4*P$_1$	A	1, 0	2, 1	3, 1	4, 2
	B	3, 1	2, 2	2, 0	3, 0
	C	3, 1	4, 0	1, 0	2, 0
	D	4, 2	3, 0	2, 1	1, 1

iClicker Q4

			P$_2$	
		x	y	z
3*P$_1$	X	$1,3$	$2,2$	$3,2$
	Y	$2,2$	$2,2$	$4,3$
	Z	$1,1$	$0,2$	$1,1$

- After eliminating Z, which is strictly dominated, what strategy is a non-best-response?

label=) X
lbbel=) Y
lcbel=) Z
ldbel=) y
lebel=) z

iClicker Q5

P$_2$

		x	y	z
3*P$_1$	X	$1, 3$	$2, 2$	$3, 2$
	Y	$2, 2$	$2, 2$	$4, 3$
	Z	$1, 1$	$0, 2$	$1, 1$

- If we complete IENBR on this game table, what strategy profile is left at the end?

label=) (X, x)
lbbel=) (X, z)
lcbel=) (Y, x)
ldbel=) (Y, z)
lebel=) There is more than one strategy profile left.

Failures of Elimination Methods

- There's no guarantee that any particular game will contain strategies that are either strictly dominated or non-rationalizable.
- Even when there are strategies we can eliminate, there may not be enough of them to find a NE just by elimination.
- So why bother with this?
- Even if elimination doesn't immediately identify a NE, it can still be helpful to simplify the game before trying other methods.
- Simplifying by elimination is **especially** useful when dealing with mixed strategies—which we'll get to after the midterm.

Best-Responses

- In this discussion of non-best-response strategies, we've sort of skipped over one thing: if a strategy is NOT a non-best-response, then logically, it must sometimes be a best response.
- This is arguably an even more important concept—and one that we'll get to tomorrow.

iClicker Poll

		P_2			
		a	b	c	d
4*P_1	A	1, 0	2, 1	3, 1	4, 2
	B	3, 1	2, 2	2, 0	3, 0
	C	3, 1	4, 0	1, 0	2, 0
	D	4, 2	3, 0	2, 1	1, 1

- Suppose that you KNEW, without a doubt, that Player 2 would play b. If you were Player 1, what strategy would you choose?
    1. A
    2. B
    3. C
    4. D
    5. I would be indifferent between two or more strategies.

iClicker Poll

		P$_2$			
		a	b	c	d
4*P$_1$	A	1, 0	2, 1	3, 1	4, 2
	B	3, 1	2, 2	2, 0	3, 0
	C	3, 1	4, 0	1, 0	2, 0
	D	4, 2	3, 0	2, 1	1, 1

- Suppose that you KNEW, without a doubt, that Player 1 would play C. If you were Player 2, what strategy would you choose?
  1. a
  2. b
  3. c
  4. d
  5. I would be indifferent between two or more strategies.

iClicker Poll

		P_2			
		a	b	c	d
4*P_1	A	1, 0	2, 1	3, 1	4, 2
	B	3, 1	2, 2	2, 0	3, 0
	C	3, 1	4, 0	1, 0	2, 0
	D	4, 2	3, 0	2, 1	1, 1

- Suppose that you KNEW, without a doubt, that Player 2 would play a. If you were Player 1, what strategy would you choose?
  1. A
  2. B
  3. C
  4. D
  5. I would be indifferent between two or more strategies.

iClicker Poll

		P$_2$			
		a	b	c	d
4*P$_1$	A	1, 0	2, 1	3, 1	4, 2
	B	3, 1	2, 2	2, 0	3, 0
	C	3, 1	4, 0	1, 0	2, 0
	D	4, 2	3, 0	2, 1	1, 1

- Suppose that you KNEW, without a doubt, that Player 1 would play D. If you were Player 2, what strategy would you choose?
  1. A
  2. B
  3. C
  4. D
  5. I would be indifferent between two or more strategies.

Best Responses

- If you know what strategy the other player will choose, then you can easily figure out what your best option (or options) are.
- Of course, you don't actually know what the other player will do when you choose your strategy in a game like this—but thinking about the game this way makes it easy to find NEs.
- A strategy s_i is a best response to another player's strategy s_{-i} if and only if it provides the highest payoff possible when the other player chooses s_{-i}.

Some Notes on Best Responses

- The phrase “to another player’s strategy” in the definition of a best response is **important**.
- A strategy can only be a best response **to some strategy of the other player**. There is no such thing as a strategy which is just “a best response.”
- When dealing with strategic-form games with game tables, there is always at least one best response to another player’s strategy—and there may be multiple, if there is more than one strategy which provides the best payoff.

Best Responses in Practice

- It's easy to depict best responses in a game table: we can go through each strategy in the game, and mark each strategy which is a best response to them.
- We do this by marking the payoffs—however, it's important to understand that it's not the payoff itself which is a best response–we're just using them as a convenient way to depict where the best responses are.
    - In other words: don't try to describe the best response using the payoff. The best response is always the strategy.

Example: Best Responses in Game Table

		P$_2$			
		a	b	c	d
4*P$_1$	A	1, 0	2, 1	3, 1	4, 2
	B	3, 1	2, 2	2, 0	3, 0
	C	3, 1	4, 0	1, 0	2, 0
	D	4, 2	3, 0	2, 1	1, 1

- To find Player 1's best responses, we will go through each column of the table (i.e. each of Player 2's strategies) and look for which of Player 1's strategies gives Player 1 the best payoff in that column.

Example: Best Responses in Game Table

		P$_2$			
		a	b	c	d
4*P$_1$	A	1, 0	2, 1	3, 1	4, 2
	B	3, 1	2, 2	2, 0	3, 0
	C	3, 1	4, 0	1, 0	2, 0
	D	4, 2	3, 0	2, 1	1, 1

- Likewise, to find Player 2's best responses, we will go through each **row** of the table (each of Player 1's strategies) and look for which of Player 2's strategies gives **Player 2** the best payoff in that column.
  - The bolded parts are important: you have to look at Player 1's payoffs to find Player 1's best responses, and Player 2's payoffs to find Player 2's best responses.

Another Example: Best Responses in the Deer Hunt

- Recall the Deer Hunt game, in which two cavemen each decide whether to hunt Deer or Rabbit.
- We couldn't find a Nash Equilibrium to this game using elimination methods, because there was nothing to eliminate: no strategies are strictly dominated, and there are no strategies which are non-best-responses.
- Here are Igg's best responses...

		Ogg	
		$Deer$	$Rabbit$
2*Igg	$Deer$	$2, 2$	$0, 1$
	$Rabbit$	$1, 0$	$1, 1$

Another Example: Best Responses in the Deer Hunt

- And here are Ogg's best responses...

		Ogg	
		Deer	*Rabbit*
2*Igg	*Deer*	$2, 2$	$0, 1$
	Rabbit	$1, 0$	$1, 1$

Another Example: Non-Unique Best Responses

- As mentioned earlier, best responses don’t have to be unique.

		Michael	
		$Swerve$	$Straight$
2*Eleanor	$Swerve$	$\underline{1}, 1$	$\underline{1}, 1$
	$Straight$	$\underline{1}, 1$	$0, 0$

- Here, for both players, both Straight and Swerve are best responses to Swerve. When the other player chooses Swerve, both strategies provide payoff of 1.

iClicker Q1

			P_2	
		X	Y	Z
3*P_1	A	3, 3	2, 2	1, 1
	B	4, 2	1, 1	2, 2
	C	1, 1	2, 2	3, 1

- In the game shown above, what is Player 1's best response to X?
  1. A
  2. B
  3. C
  4. A and B are both best responses.
  5. A and C are both best responses.

iClicker Q2

			P$_2$	
		X	Y	Z
3*P$_1$	A	3, 3	2, 2	1, 1
	B	4, 2	1, 1	2, 2
	C	1, 1	2, 2	3, 1

- In the game shown above, what is Player 1's best response to **Y**?
  1. A
  2. B
  3. C
  4. A and B are both best responses.
  5. A and C are both best responses.

iClicker Q3

		P$_2$		
		X	Y	Z
3*P$_1$	A	3, 3	2, 2	1, 1
	B	4, 2	1, 1	2, 2
	C	1, 1	2, 2	3, 1

- In the game shown above, what is **Player 2's** best response to **C**?
  1. X
  2. Y
  3. Z
  4. X and Y are both best responses.
  5. X and Z are both best responses.

Nash Equilibrium from Best Responses

- Recall the various definitions of a Nash Equilibrium:
  - a strategy profile such that no player can obtain a larger payoff by *unilaterally deviating* (changing only their own strategy).
  - A strategy profile such that no single player can make themselves better off by changing only their own strategy.
  - A strategy profile such that, after the game is played, each player is satisfied that they could not have made a better decision.
- Another definition we can use now is "A strategy profile such that each player's strategy is a best responses to the other player's strategy."
- "Playing a best response" is equivalent to "cannot obtain a larger payoff by unilaterally deviating," or any of the other ways to describe this condition.

Nash Equilibrium from Best Responses

- If we find each player's best responses in a game table, and do it using the same table for each player...

P$_2$

		a	b	c	d
4*P$_1$	A	1, 0	2, 1	**3**, 1	**4**, **2**
	B	3, 1	2, **2**	2, 0	3, 0
	C	3, **1**	**4**, 0	1, 0	2, 0
	D	**4**, **2**	3, 0	2, 1	1, 1

- Then any cell of the table in which **all payoffs are marked to indicate a best response** represents a NE.
- Here, the two NEs are (D, a) and (A, d), which we could never have found from elimination.

Another Example: Nash Equilibrium in the Deer Hunt

- We did already find the NEs of the Deer Hunt, but we had to go through and check all four strategy profiles. We can do it much faster by just using best responses:

		Ogg	
		Deer	*Rabbit*
2*Igg	*Deer*	**2,2**	0,1
	Rabbit	1,0	**1,1**

Another Example: When Elimination Does Nothing

- This game has **absolutely no strategies** that can be eliminated: none are strictly dominated or non-rationalizable.
- We can still find the NEs (of which there are quite a few) using best responses:

P$_2$

5*P$_1$		a	b	c	d
	A	1, 1	2, 2	2, 2	2, 1
	B	1, 3	1, 3	2, 2	2, 3
	C	1, 2	2, 4	1, 3	2, 3
	D	3, 2	2, 3	1, 4	2, 2

iClicker Q4

		P$_2$		
		X	Y	Z
3*P$_1$	A	3, 3	2, 2	1, 1
	B	4, 2	1, 1	2, 2
	C	1, 1	2, 2	3, 1

- In the game shown above, which of the following are the Nash Equilibria? (You will need to find best responses for both players.)
  1. (A, X)
  2. (A, Y)
  3. (B, X)
  4. (C, Z)
  5. More than one of the above.

Best Responses and Non-Rationalizability

- Finding best responses first makes it a lot easier to search for non-rationalizable strategies.
- Recall that a non-rationalizable strategy can also be called a non-best response: if none of the payoffs of a strategy are marked to indicate that it is a best-response, it is non-rationalizable. In other words:
  - Any row in which none of Player 1's payoffs are marked to indicate a best response, is non-rationalizable.
  - Any column in which none of Player 2's payoffs are marked to indicate a best response, is non-rationalizable.

Example: Non-Rationalizability from Best Responses

- Finding best responses first makes it a lot easier to search for non-rationalizable strategies.
- Recall that a non-rationalizable strategy can also be called a non-best response: if none of the payoffs of a strategy are marked to indicate that it is a best-response, it is non-rationalizable. In other words:
    - Any row in which none of Player 1's payoffs are marked to indicate a best response, is non-rationalizable.
    - Any column in which none of Player 2's payoffs are marked to indicate a best response, is non-rationalizable.

Example: Non-Rationalizability from Best Responses

		P_2			
		a	b	c	d
4*P_1	A	1, 0	2, 1	**3**, 1	**4**, **2**
	B	3, 1	2, **2**	2, 0	3, 0
	C	3, **1**	**4**, 0	1, 0	2, 0
	D	**4**, **2**	3, 0	2, 1	1, 1

- We found these best responses earlier (as well as the NEs of this game): note that in the row for Player 1's strategy B, none of Player 1's strategies are marked.
- Likewise, none of Player 2's payoffs are marked in column c. B and c are non-rationalizable strategies.

Example: Non-Rationalizability from Best Responses

		P$_2$			
		a	b	c	d
4*P$_1$	A	1, 0	2, 1	**3**, 1	**4**, **2**
	B	3, 1	2, **2**	2, 0	3, 0
	C	3, **1**	**4**, 0	1, 0	2, 0
	D	**4**, **2**	3, 0	2, 1	1, 1

- Performing IENBR reveals additional non-rationalizable strategies:
  - Once we eliminate B, b becomes non-rationalizable.
  - Once we eliminate b, C becomes non-rationalizable.

Classifying NEs

- Nash Equilibria may be either strict or weak.
- A Nash Equilibrium is strict if and only if each player would receive a **smaller** payoff by changing their own strategy.
- If a Nash Equilibrium is not strict—meaning that at least one player could change their own strategy and receive an equal (but not larger) payoff—it is weak.

Intuition on Strict vs. Weak Equilibria

- In any Nash Equilibrium, no player has a reason to change their own strategy—they cannot get a higher payoff this way.
- Strict Nash Equilibria go a little further: not only does no player have a reason to change their own strategy, they also have a reason **not** to, because any other strategy would provide them a worse payoff.
- If a Nash Equilibrium is weak, it means that some player could change their strategy, and get exactly the same payoff they already were. They have no reason to do this, but also no reason **not** to.
- We can also say that a strict Nash Equilibrium is one where each player is playing a strategy which is a **unique** best response to the strategies chosen by other players.

Deer Hunt: Strict Nash Equilibria

		Ogg	
		Deer	*Rabbit*
2*Igg	*Deer*	$\underline{2}, \underline{2}$	$0, 1$
	Rabbit	$1, 0$	$\underline{1}, \underline{1}$

- Here, note that at each Nash Equilibrium, each player has no other strategy providing the same payoff. This is a strict Nash Equilibrium.

Not Quite the Trolley Problem: Weak Nash Equilibria

		Michael	
		$Swerve$	$Straight$
2*Eleanor	$Swerve$	$\underline{1}, \underline{1}$	$\underline{1}, \underline{1}$
	$Straight$	$\underline{1}, \underline{1}$	$0, 0$

- However, in this game, each Nash Equilibrium features at least one player who could still get the same payoff if they change their strategy.

Classifying Games Based on NEs

- Now that we've talked about several ways to find a game's NEs, we can start to talk about classifying games using them.

Prisoners' Dilemmas

		Luca	
		Testify	*Keep Quiet*
2*Guido	*Testify*	$-10, -10$	$0, -20$
	Keep Quiet	$-20, 0$	$-1, -1$

- So far, I've only used the payoffs above to describe the Prisoner's Dilemma.
- But we can change those payoffs—we can also change the story behind the game, and the names of the players and strategies, and it would still count as a Prisoner's Dilemma.
- In general, a Prisoner's Dilemma is any game in which:
  - The players have the same strategies, A and B.
  - A strictly dominates B, making (A, A) the only NE.
  - But (B, B) is better for both players than (A, A).

Other Representations of the Prisoners' Dilemmas

		Luca	
		Testify	*Keep Quiet*
2*Guido	*Testify*	$1, 1$	$3, 0$
	Keep Quiet	$0, 3$	$2, 2$

		P_2	
		A	B
2*P_1	A	$0, 4$	$2, 3$
	B	$-1, 6$	$1, 5$

Coordination Games

- A coordination game is a game in which the players all have the same strategy sets, and the NEs are all of the strategy profiles where the players choose the same strategy.
- The Deer Hunt is a coordination game.

		Ogg	
		Deer	*Rabbit*
2*Igg	*Deer*	**2,2**	0,1
	Rabbit	1,0	**1,1**

Other Coordination Games

Table 1: *

Bach or Stravinsky?

		Stravinsky Fan	
		Bach	Strav.
2*Bach Fan	Bach	**3, 2**	0,0
	Strav.	0,0	**2, 3**

		P_2		
		A	B	C
2*P_1	A	**1, 1**	1, 0	3, 0
	B	0, 1	**2, 2**	2, 1
	B	0, 3	1, 2	**4, 4**

Anti-Coordination Games

- An anti-coordination game is a game in which the players all have the same strategy sets, but the NEs are all of the strategy profiles where the players choose **different** strategies.

Table 2: *

			Buzz	
Chicken			Straight	Swerve
	2*Jim	Straight	-10, -10	**5, -1**
		Swerve	**-1, 5**	0, 0

Symmetric Games

- A symmetric game is a game which is **indifferent to an exchange of players**—in other words, a game where the players are interchangeable.
- Consider the Prisoner's Dilemma: if we swap the players' names, and their positions in the game table, and the order of their payoffs, we get the same game that we started with.
- A two-player game with a game table is symmetric if:
  - The players have the same strategy sets.
  - In the on-diagonal cells of the game table, the players receive equal payoffs.
  - In the "mirrored" off-diagonal cells of the game table, the players' payoffs are reversed.
- Of the games we've just looked at, the Deer Hunt, the 3x3 coordination game, and Chicken are symmetric games. Bach and Stravinsky is not.