Econ 327: Game Theory

Homework #1

University of Oregon

Due: Jan. 26^{th}

Question:	**Question 1**	**Question 2**	**Question 3**	**Question 4**	**Question 5**	Total
Points:	20	20	20	20	20	100
Score:						

For homework assignments:

- Complete *all* questions and parts. I will select one question at random to be graded according to the rubric on Canvas.
- You may choose to work with others, but everyone must submit to Canvas individually. Please include the names of everyone who you worked with below your own name.

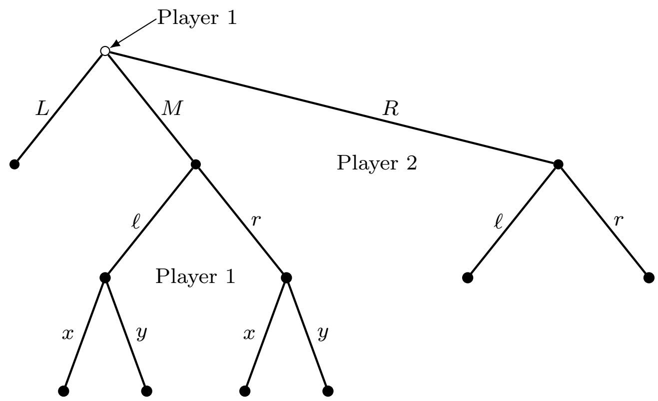
Name ______________________________________

Question 1. [20 points] **Multiple Choice**

(a) Consider three different outcomes, A, B, and C. Outcome A is Pareto efficient, and outcome B is not Pareto efficient. Choose one of the following:

A. C cannot be Pareto efficient.
B. A can not be Pareto dominated by C.
C. C is Pareto dominated by A
D. C is Pareto dominated by B

(b) Consider the game tree below



How many *strategies* does each player have? (recall that a strategy is a complete plan of action for *every* eventuality)

A. Player 1: 9 strategies, Player 2: 4 strategies
B. Player 1: 12 strategies, Player 2: 4 strategies
C. Player 1: 7 strategies, Player 2: 2 strategies
D. Player 1: 7 strategies, Player 2: 4 strategies

(c) Consider the game tree from the previous question. Which of the following is a complete strategy for Player 1?

A. $\{L\}$
B. $\{x \text{ if } \ell\}$
C. $\{L, x \text{ if } \ell,\ y \text{ if } r\}$
D. $\{L,\ x\}$

(d) Consider the strategic form game below: In the game above, which strategy is strictly dominated?

		P_2		
		x	y	z
P_1	a	1,3	2,2	3,2
	b	2,2	2,2	4,3
	c	1,1	0,2	1,1

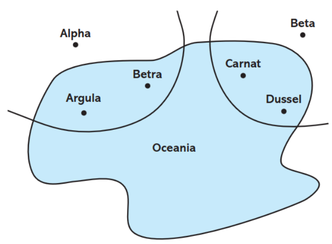
A. a
B. b
C. c
D. x

(e) Perform Iterative Deletion of Strictly Dominated Strategies for the same game as above all the way to completion. What does IDSDS tell you about the Nash equilibrium of this game?

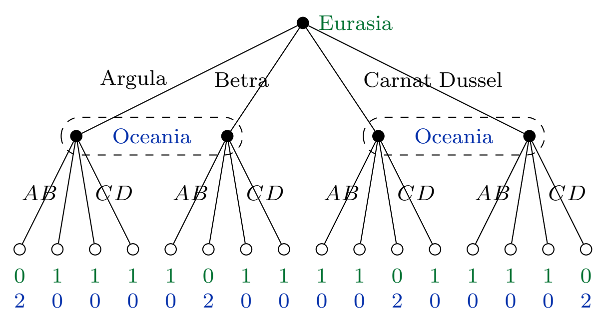
A. The NE is (a, x)
B. The NE is (a, y)
C. The NE is (Y, z)
D. IESDS by itself does not reveal the NE of this game.

Question 2. [20 points] The countries of Oceania and Eurasia are at war. As depicted in the figure, Oceania has four cities — Argula, Betra, Carnat, and Dussel — and it is concerned that one of them is to be bombed by Eurasia. The bombers could come from either base Alpha, which can reach the cities of Argula and Betra; or from base Beta, which can reach either Carnat or Dussel. Eurasia decides which one of these four cities to attack. Oceania doesn't know which one has been selected, but does observe the base from which the bombers are flying. After making that observation, Oceania decides which one (and only one) of its four cities to evacuate.

Assign a payoff of 2 to Oceania if it succeeds in evacuating the city that is to be bombed and a payoff of 1 otherwise. Assign Eurasia a payoff of 1 if the city it bombs was not evacuated and a zero payoff otherwise. Write down the extensive form game. [1]



Solution:



Note that A, B, C, and D in the last row are short for the city names. Eurasia acts first, so the initial node is labelled accordingly. Oceania has only two info sets which are represented with the dashed ovals. The $(0, 2)$ or $(1, 0)$ payoff sets correspond to Oceania choosing the same city that is bombed, or choosing a different city respectively.

[1] Harrington *Games, Strategies, and Decision Making*

Question 3. Imagine a sequential moves version of rock-paper-scissors where player 2 gets to pick what they will do after player 1 picks. Please model the game in its extensive form (as a game tree). Assume both player 1 and player 2 only care about the result of the game and have the following preferences over the result of the game: win $\succ$ tie $\succ$ loss. [2]

(a) Answer the following questions:

i. [2 points] How many nodes are there?

ii. [2 points] How many branches are there?

iii. [2 points] How many terminal nodes are there?

(b) [6 points] Prune the tree as much as possible. How many branches were you able to eliminate? (A complete answer should include your drawing(s) of the game tree)

(c) [8 points] Use the same setup, but now imagine player 1's preferences change because they want to be seen as a "tough guy". Given that what they want to play remains the same, they still have the following preferences over the result of the game: win $\succ$ tie $\succ$ loss. However, they now would prefer to lose playing rock than win playing paper or scissors. Please create a new game tree so the payoffs reflect these new preferences.

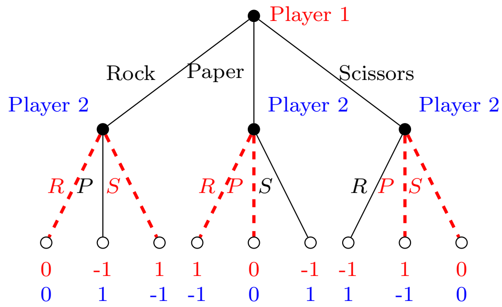
Prune the tree as much as possible.

How many branches were you able to eliminate? (Include your drawing(s))

Solution:

(a) [!h]

[2] Ethan Holdahl, University of Oregon

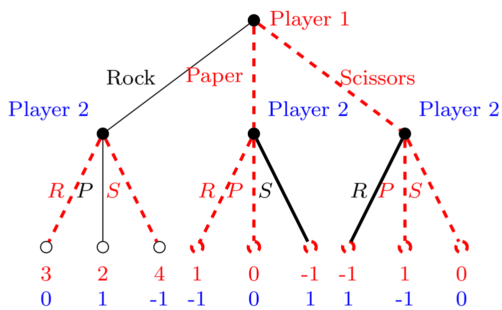

Player 1
Rock
Paper
Scissors
Player 2
Player 2
Player 2
R
P
S
R
P
S
R
P
S
0
0
-1
1
1
-1
1
-1
0
0
-1
1
-1
1
1
-1
0
0

In the figure, dashed lines represent those which have been pruned.

The tree has 4 nodes, 12 branches, and 9 terminal nodes.

(b) In each of Player 2's subgames, we can prune the two branches where they would lose or tie based on what Player 1 chose. We can't prune any more branches because Player 1 is indifferent between all outcomes where they lose.

(c) [!h]



I represented Player 1's preferences as:

payoff to winning with rock = 4, tie with rock = 3, lose with rock = 2, win with paper or scissors = 1, tie with paper or scissors = 0, lose with paper or scissors = -1.

Because payoffs are *ordinal*, you can choose any values as long as the ranking is the same.

Now because Player 1 prefers to lose with rock to any other loss, we can prune the branches where they choose either paper or scissors.

The rollback solution is { Player 1 chooses Rock, (Player 2 chooses paper if rock, scissors if paper, and rock if scissors) }.

Grading:

This question is worth 20 points total, with 10 for correctness, 5 for format, and 5 for explanation.

- For part a, each correct number is worth 1 point.
- For part b, give 1 point if the overall structure is there, 1 point if the payoffs go to the right node, and 1 point if the correct 6 branches are pruned and no others.
- For part a on the second page (sorry), give 1 correctness point for every correctly pruned branch, with -1 point for pruning other branches.

Scores for formatting and explanation are left up to the grader's interpretation of the rubric.

Question 4. Here's a little ditty, about Jack and Diane, two American kids growing up in the heartland. The game is below. [3]

		Diane		
		x	y	z
Jack	a	1,1	2,1	2,0
	b	2,3	0,2	2,1
	c	2,1	1,2	3,0

(a) [8 points] Find all pure Nash strategy profiles and outcomes if Jack and Diane move simultaneously. Carefully detail and explain your strategy profiles and how they map onto your Nash outcomes.

(b) [12 points] Find all pure Nash strategy profiles and outcomes if Jack moves first. Carefully detail and explain your strategy profiles and how they map onto your Nash outcomes.

Solution:

(a) Players: {Jack, Diane}

Strategy sets: $S_{\text{Jack}} = \{a, b, c\}$ $S_{\text{Diane}} = \{x, y, c\}$

For Jack, b and c are best responses to x, a is BR to y, and c is BR to z.

For Diane, x and y are BR to a, x is BR to b, and z is BR to c.

There are two strategy profiles where each player's best responses intersect:

- $N_1 = \{b, x\}$
  Jack's strategy is to choose b, Diane's strategy is to choose x. Neither have regrets about their strategy choice; given that Jack is choosing b, Diane can't get a higher payoff by deviating. Given that Diane is playing x, Jack is indifferent between playing b and c but he can't get a strictly higher payoff by deviating. The resulting outcome is that Jack gets 2, Diane gets 3.
- $N_2 = \{a, y\}$
  Jack's strategy is a, Diane's is y. When Jack plays a, Dianne is indifferent between x and y, but still cannot deviate to a strictly higher payoff. When Diane plays y, Jack's best response is a because $2 > \{0, 1\}$. The outcome is that Jack gets 2 and Diane gets 1.

These are the only two *pure strategy* Nash equilibria because there are no other intersections of pure strategy best responses. Note that even though N_1 Pareto dominates N_2 (Jack is indifferent; $2 = 2$, and Diane is better off; $3 > 1$), there is no *unilateral* deviation that would reach N_1 from N_2.

(b) See the extensive form game tree below.

Now the strategy sets are:

$S_{\text{Jack}} = \{a, b, c\}$ because Jack still has only one decision node.

$S_{\text{Diane}} = \{$ any combination $c_1c_2c_3$ where c can be either x, y, or $z\}$ because Diane now has 3 elements in her information set. She knows whether Jack has chosen a, b, or c before she makes her own choice.

I represented each of Diane's strategies as a triple where the first letter represents her choice at node 1, second letter at node 2, and third letter at node 3. So for example, xyz would be the strategy where she chooses x in node 1, y in node 2, and z in node 3.

[3] Cliff Bekar, Lewis and Clark College

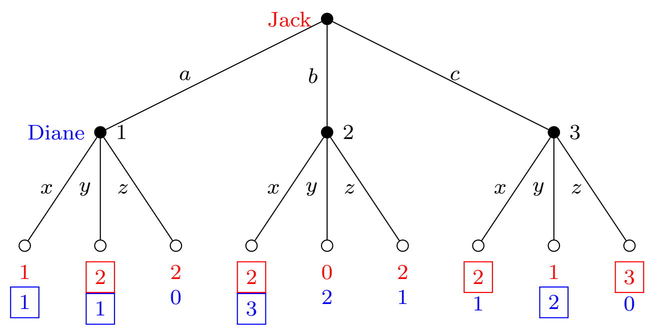
- $\mathbf{N_1} = \{a,\ yc_2c_3 |\text{where } c_2 \text{ is } x,\ y, \text{ or } z;\ c_3 \text{ is } x \text{ or } y\ \}$

  Any set of strategies in which Jack chooses a and Diane chooses y in node 1 but doesn't choose z at node 3 would result in an equilibrium outcome of $(2, 1)$ where Jack cannot deviate to a higher payoff than 2, and Diane also has no regrets with choosing y.

- $\mathbf{N_2} = \{b,\ xc_2c_3 |\text{where } c_2 \text{ could be } x,\ y, \text{ or } z;\ c_3 \text{ could be either } x \text{ or } y\ \}$

  Any set of strategies in which Jack chooses b and Diane doesn't choose z at node 3 would result in Jack choosing b and Diane choosing x given that jack chooses b.

  The equilibrium outcome of any of these Nash strategy profiles would be $(2, 3)$



Question 5. See the figures below for the data from our in-class activity 2 [4]. where teams took turns taking flags from a starting pool. Note that in Game 1, 6 out of 9 matches were won by the first team to take flags and in Game 2, 3 of 9 matches were won by the starting team.

(a) [10 points] Using the rollback equilibrium as a predictive model, how many times would you expect the starting team to win when all agents are *perfectly informed*, *rational*, and have *common knowledge of rationality*? Based on the class data, should we *accept* or *reject* this hypothesis?

(b) [10 points] Based on your observations in class and the results from other groups, which of the assumptions above do you think could be modified to create a more accurate model of this game? What modifications would you make, and what alternative hypotheses could you test?

Solution:

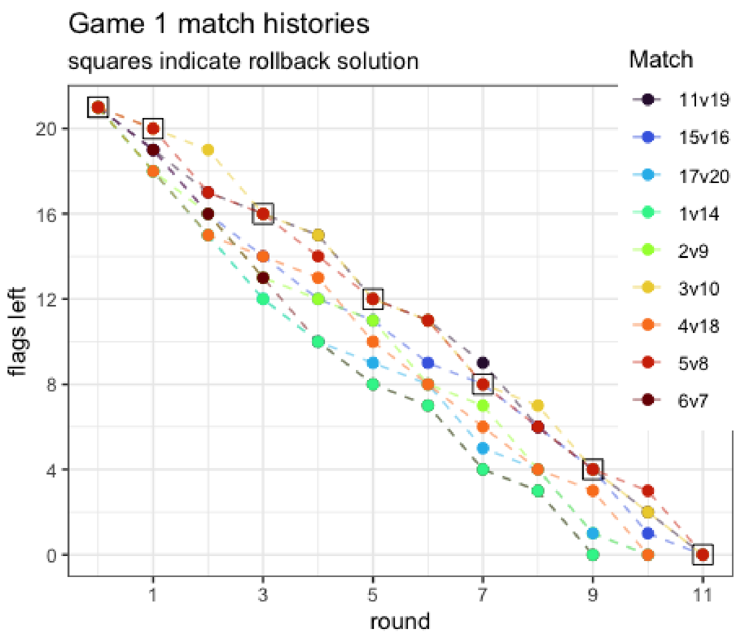
(a) The first team can win 100% of the time by always leaving a multiple of the maximum flags which can be taken plus one. So if all teams know the rules of the game then we would predict that 100% of matches will be won by the starting team.

However, in class only 9 out of 18 matches were won by the starting team. We can pretty easily reject the 100% win hypothesis.

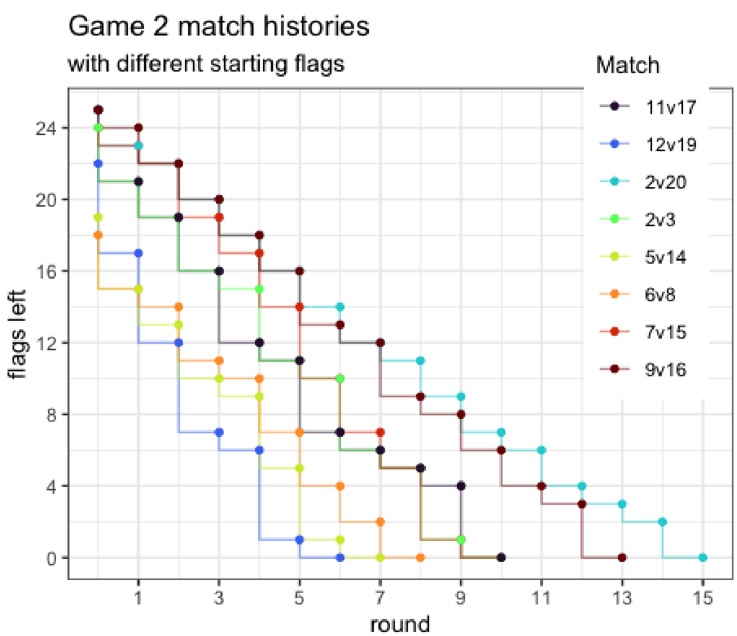
(b) This one is open to interpretation. It might be that not everyone was fully paying attention when the rules were explained and so perfect information doesn't hold.

One simple hypothesis to test could be that teams were choosing flags to take randomly instead of thinking through all of the backwards induction logic. This would give us a prediction of the first team winning 1/2 of the time, which is what we saw in class. However, you might want to replicate this experiment more than 18 times to be more sure that there acutally is no tendency for the starting team to win.

[4] You can also find the data on Canvas in the *Activities* folder of the *files* tab



Game 1 match histories
squares indicate rollback solution
Match
11v19
15v16
17v20
1v14
2v9
3v10
4v18
5v8
6v7
flags left
round



Game 2 match histories
with different starting flags
Match
11v17
12v19
2v20
2v3
5v14
6v8
7v15
9v16
flags left
round