# Econ 327: Game Theory

## Homework #1

University of Oregon

Due: Jan.  $26^{th}$ 

Question:	Question 1	Question 2	Question 3	Question 4	Question 5	Total
Points:	20	20	20	20	20	100
Score:						

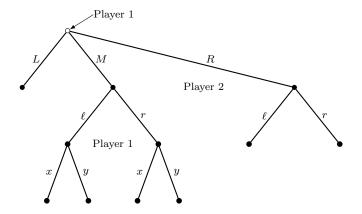
#### For homework assignments:

- Complete *all* questions and parts. I will select one question at random to be graded according to the rubric on Canvas.
- You may choose to work with others, but everyone must submit to Canvas individually. Please include the names of everyone who you worked with below your own name.

Name .			
Name			

#### Question 1. [20 points] Multiple Choice

- (a) Consider three different outcomes, A, B, and C. Outcome A is Pareto efficient, and outcome B is not Pareto efficient. Choose one of the following:
  - A. C cannot be Pareto efficient.
  - B. A can not be Pareto dominated by C.
  - C. C is Pareto dominated by A
  - D. C is Pareto dominated by B
- (b) Consider the game tree below How many strategies does each player have?



- A. Player 1: 9 strategies, Player 2: 4 strategies
- B. Player 1: 12 strategies, Player 2: 4 strategies
- C. Player 1: 7 strategies, Player 2: 2 strategies
- D. Player 1: 7 strategies, Player 2: 4 strategies
- (c) Consider the game tree from the previous question. Which of the following is a complete strategy profile for Player 1?
  - A.  $\{L\}$
  - B.  $\{x \text{ if } \ell\}$
  - C.  $\{L, x \text{ if } \ell, y \text{ if } r\}$
  - D.  $\{L, x, y\}$
- (d) Consider the strategic form game below: In the game above, which strategy is strictly dominated?

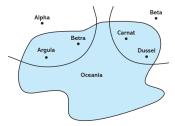
		$P_2$			
		x	y	z	
	a	1,3	2,2	3,2	
$P_1$	b	2,2	2,2	4,3	
	c	1,1	0,2	1,1	

- A. a
- B. *b*
- C. c
- D. x
- (e) Perform Iterative Deletion of Strictly Dominated Strategies for the same game as above all the way to completion. What does IDSDS tell you about the Nash equilibrium of this game?
  - A. The NE is (a, x)

- B. The NE is (a, y)
- C. The NE is (Y, z)
- D. IESDS by itself does not reveal the NE of this game.

Question 2. [20 points] The countries of Oceania and Eurasia are at war. As depicted in the figure, Oceania has four cities — Argula, Betra, Carnat, and Dussel — and it is concerned that one of them is to be bombed by Eurasia. The bombers could come from either base Alpha, which can reach the cities of Argula and Betra; or from base Beta, which can reach either Carnat or Dussel. Eurasia decides which one of these four cities to attack. Oceania doesn't know which one has been selected, but does observe the base from which the bombers are flying. After making that observation, Oceania decides which one (and only one) of its four cities to evacuate.

Assign a payoff of 2 to Oceania if it succeeds in evacuating the city that is to be bombed and a payoff of 1 otherwise. Assign Eurasia a payoff of 1 if the city it bombs was not evacuated and a zero payoff otherwise. Write down the extensive form game. <sup>1</sup>



<sup>&</sup>lt;sup>1</sup>Harrington Games, Strategies, and Decision Making

- Question 3. Imagine a sequential moves version of rock-paper-scissors where player 2 gets to pick what they will do after player 1 picks. Please model the game in its extensive form (as a game tree). Assume both player 1 and player 2 only care about the result of the game and have the following preferences over the result of the game: win  $\succ$  tie  $\succ$  loss. <sup>2</sup>
  - (a) Answer the following questions:
    - i. [2 points] How many nodes are there?
    - ii. [2 points] How many branches are there?
    - iii. [2 points] How many terminal nodes are there?
  - (b) [6 points] Prune the tree as much as possible. How many branches were you able to eliminate? (A complete answer should include your drawing(s) of the game tree)

<sup>&</sup>lt;sup>2</sup>Ethan Holdahl, University of Oregon

Use the same setup, but now imagine player 1's preferences change because they want to be seen as a "tough guy". Given that what they want to play remains the same, they still have the following preferences over the result of the game: win  $\succ$  tie  $\succ$  loss. However, they now would prefer to lose playing rock than win playing paper or scissors. Please create a new game tree so the payoffs reflect these new preferences.

(a) [8 points] Prune the tree as much as possible. How many branches were you able to eliminate? (Include your drawing(s))

**Question 4.** Here's a little ditty, about Jack and Diane, two American kids growing up in the heartland. The game is below.  $^3$ 

		Diane			
		x	y	z	
	a	1,1	2,1	2,0	
Jack	b	2,3	0,2	2,1	
	c	2,1	1,2	3,0	

- (a) [8 points] Find all pure Nash strategy profiles and outcomes if Jack and Diane move simultaneously. Carefully detail and explain your strategy profiles and how they map onto your Nash outcomes.
- (b) [12 points] Find all pure Nash strategy profiles and outcomes if Jack moves first. Carefully detail and explain your strategy profiles and how they map onto your Nash outcomes.

<sup>&</sup>lt;sup>3</sup>Cliff Bekar, Lewis and Clark College

**Question 5**. See the figures below for the data from our in-class activity 2 <sup>4</sup>. where teams took turns taking flags from a starting pool. Note that in Game 1, 6 out of 9 matches were won by the first team to take flags and in Game 2, 3 of 9 matches were won by the starting team.

- (a) [10 points] Using the rollback equilibrium as a predictive model, how many times would you expect the starting team to win when all agents are *perfectly informed*, *rational*, and have *common knowledge of rationality*? Based on the class data, should we *accept* or *reject* this hypothesis?
- (b) [10 points] Based on your observations in class and the results from other groups, which of the assumptions above do you think could be modified to create a more accurate model of this game? What modifications would you make, and what alternative hypotheses could you test?

 $<sup>^4\</sup>mathrm{You}$  can also find the data on Canvas in the Activities folder of the files tab

### Game 1 match histories Match squares indicate rollback solution - 11v19 20 15v16 17v20 1v14 16 2v9 flags left 3v10 4v18 5v8 8 6v7 4 0 round

