Econ 327: Game Theory

Homework #4

University of Oregon

Due: Oct. 24^{th}

Question:	Q1	Q2	Q3	Q4	Total
Points:	12	16	8	4	40
Score:					

For homework assignments:

- Complete *all* questions and parts.
- You will be graded on not only the content of your work but on how clearly you present your ideas. Make sure that your handwriting is legible. Please use extra pages if you run out of space but make sure that all parts of a question are in the correct order when you submit.
- You may choose to work with others, but everyone must submit to Canvas individually.

  Please include the names of everyone who you worked with below your own name.

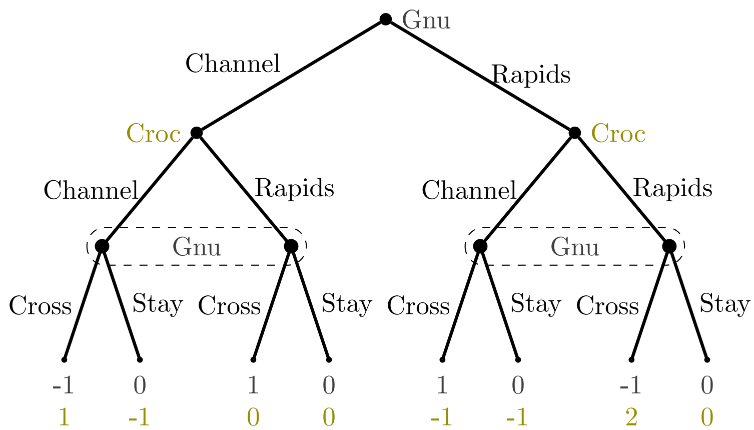
Name ______________________________

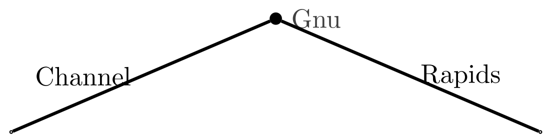
Q1. Recall the gnu and croc river crossing game from Homework 2, Question 3.

Now, consider what happens if the gnu have bad eyesight and can't see where the crocs are choosing to lie in wait.

(a) [4 points] Draw out the new extensive form game.

Solution: Either of the following extensive forms is acceptable:





		Croc	
		Channel	Rapids
Gnu	Cross	-1,1	1,0
	Stay	0,-1	0,0

		Croc	
		Channel	Rapids
Gnu	Cross	1,-1	-1,2
	Stay	0,-1	0,0

(b) [4 points] Is there a Nash equilibrium in which the Gnu always cross at either the channel or rapids? Why or why not?

Solution: In the subgame after the gnu initially choose rapids, the only stable outcome is Gnu Stay, crocs wait in Rapids. In the subgame after Gnu initially choose channel, Gnu crossing is not stable because crocs would wait in the Channel. So no pure strategy SPNE where gnu cross.

(c) [4 points] How does your prediction compare to the perfect information version from homework 2? Explain intuitively.

Solution: If the Gnu can't see the Crocs, they can't use a strategy where they only selectively cross the Channel if the Crocs are there. If they choose to Cross the Channel in this case, then the Crocs will decide it's worth it to go to the Channel. So the equilibrium from earlier doesn't apply because the information sets of the extensive form game have changed.

Q2. Consider a modified deer hunt game with continuous levels of effort.

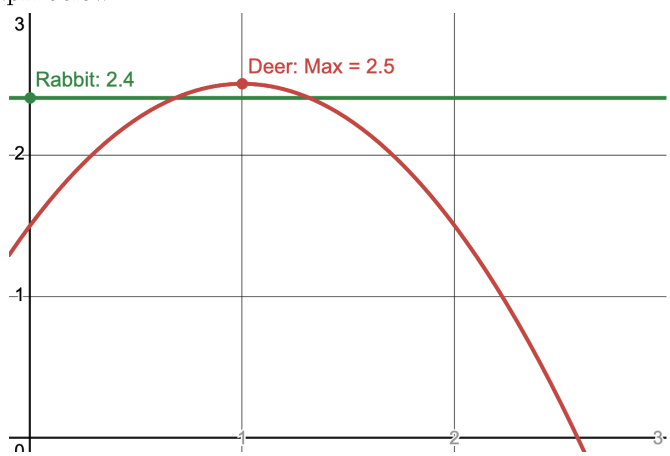
Hunters can either hunt deer together, or rabbit separately. Rabbits take zero effort to hunt successfully and give a single hunter a payoff of 3. If either hunter decides to hunt deer, they also have to choose a level of effort, e_i.

The number of deer that each hunter catches depends on the effort of the other hunter. There is some potential for mutual benefit in higher levels of joint effort, but if too much effort is exerted on deer, they will get overhunted.

Assume that the payoffs to hunting deer or rabbit are:

$$\pi_1(Deer_1, e_1, e_2) = 3e_1 + 1.5e_2 - e_1e_2 - e_1^2$$
$$\pi_2(Deer_2, e_1, e_2) = 3e_2 + 1.5e_1 - e_1e_2 - e_2^2$$
$$\pi_{1,2}(Rabbit) = 2.4$$

(a) [4 points] Suppose that hunter 2 chooses to hunt deer with effort level $e_2 = 1$. Draw out hunter 1's payoffs for both of his *Deer* and *Rabbit* strategies as a function of his own level of effort.

> **Solution:** See graph below:
>
> 
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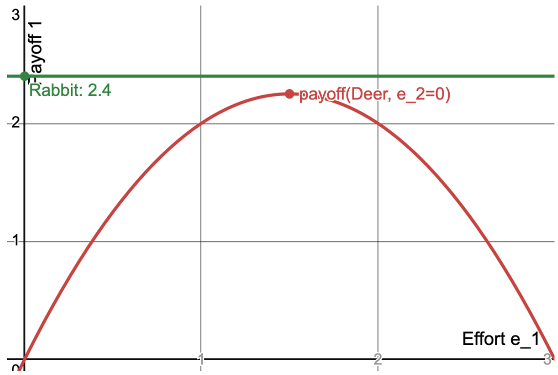
>
> Payoff to rabbit is constant at 2.4. Payoff to deer is maximised when level of effort is $e_1 = 1$, resulting in a payoff of 2.5.

(b) [2 points] What is hunter 1's best response to hunter 2 choosing $e_2 = 1$? If deer, how much effort should he put in?

> **Solution:** Hunt Deer with effort $e_1 = 1$.

(c) [2 points] What is hunter 1's best response to hunter 2 choosing $e_2 = 0$? If deer, how much effort should he put in?

> **Solution:** Hunt Rabbit. ($e_1 = 0$)



If the other hunter puts in zero effort, then hunting rabbit is better than hunting deer alone at any level of effort. The maximum payoff to hunting deer of 2.25 < 2.4 from hunting rabbit.

(d) [4 points] Is there a Nash equilibrium where both hunt **Deer**? If yes, what levels of effort will they put in? If not, explain why.

Solution: Both hunting deer is a Nash as long as the maximum payoff is greater than 2.4. For example, $\{(Deer_1, e_1 = 1)(Deer_2, e_2 = 2)\}$ is a Nash because each earn a payoff of 2.5.

(e) [4 points] Is there a Nash equilibrium where both hunt **Rabbit**? Explain why or why not.

Solution: Yes, there is a Nash where As long as both are hunting rabbit and putting in zero effort to hunt deer, there is no incentive to switch from rabbit.

Q3. A game theorist is walking down the street in his neighborhood and finds \$20. Just as he picks it up, two neighborhood kids, Jane and Tim, run up to him, asking if they can have it. Because game theorists are generous by nature, he says he's willing to let them have the \$20, but only according to the following procedure: Jane and Tim are each to submit a written request as to their share of the \$20. Let t denote the amount that Tim requests for himself and j be the amount that Jane requests for herself. Tim and Jane must choose j and t from the interval $[0, 20]$. If $j + t \leq 20$, then the two receive what they requested, and the remainder, $20 - j - t$, is split equally between them. If, however, $j + t > 20$, then they get nothing, and the game theorist keeps the \$20. Tim and Jane are the players in this game. Assume that each of them has a payoff equal to the amount of money that he or she receives. [1]

(a) [4 points] Describe Tim's and Jane's best response rules.

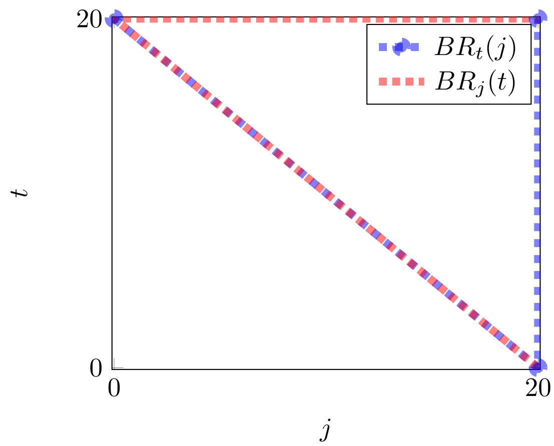
Solution:
Tim's best response rule:

$$BR_t(j) = \begin{cases} 20 - j & \text{if } j < 20 \\ [0, 20] & \text{if } j = 20 \end{cases}$$

Jane's best response rule:

$$BR_j(t) = \begin{cases} 20 - t & \text{if } t < 20 \\ [0, 20] & \text{if } t = 20 \end{cases}$$

(b) [4 points] Find all Nash equilibria. Be careful about corner solutions.

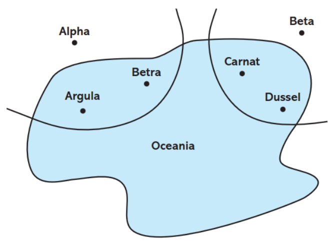


Solution:
The NE are any points on the graph where the $BR_j(t) = t$ and $BR_t(j) = t$. This includes the diagonal line which is the set of all pairs, j and t such that $j + t = 20$ where $j, t < 20$. When $j = 20$, there is one NE when $t = 0$ and another when $t = 20$. When $t = 20$, there is one NE when $j = 0$ (and $j = 20$ is still NE).

[1] Harrington *Games, Strategies, and Decision Making*

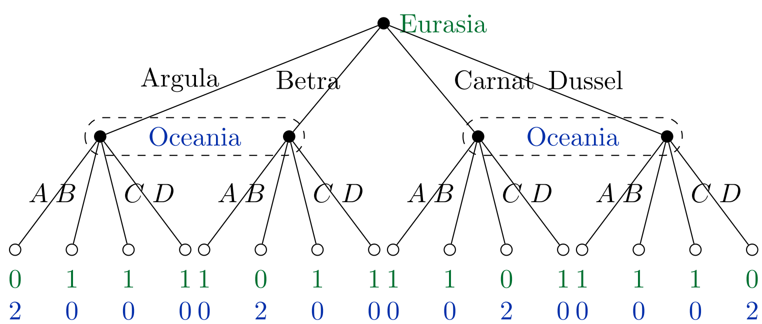
Q4. The countries of Oceania and Eurasia are at war. As depicted in the figure, Oceania has four cities — Argula, Betra, Carnat, and Dussel — and it is concerned that one of them is to be bombed by Eurasia. The bombers could come from either base Alpha, which can reach the cities of Argula and Betra; or from base Beta, which can reach either Carnat or Dussel. Eurasia decides which one of these four cities to attack. Oceania doesn't know which one has been selected, but does observe the base from which the bombers are flying. After making that observation, Oceania decides which one (and only one) of its four cities to evacuate.

Assign a payoff of 2 to Oceania if it succeeds in evacuating the city that is to be bombed and a payoff of 1 otherwise. Assign Eurasia a payoff of 1 if the city it bombs was not evacuated and a zero payoff otherwise. [2]



(a) [4 points] Write down the extensive form game.

Solution:



Note that A, B, C, and D in the last row are short for the city names. Eurasia acts first, so the initial node is labelled accordingly. Oceania has only two info sets which are represented with the dashed ovals. The $(0, 2)$ or $(1, 0)$ payoff sets correspond to Oceania choosing the same city that is bombed, or choosing a different city respectively.

[2] Harrington *Games, Strategies, and Decision Making*