Econ 327: Game Theory

Homework #5

University of Oregon

Due: Nov. 7^{th}

Question:	**Q1**	**Q2**	**Q3**	Total
Points:	16	12	16	44
Score:				

For homework assignments:

- Complete *all* questions and parts.
- You will be graded on not only the content of your work but on how clearly you present your ideas. Make sure that your handwriting is legible. Please use extra pages if you run out of space but make sure that all parts of a question are in the correct order when you submit.
- You may choose to work with others, but everyone must submit to Canvas individually.

  Please include the names of everyone who you worked with below your own name.

Name ____

Q1. Follow the steps for the game below:

		P_2	
		B	D
P_1	J	6,6	4, 0
	K	0,3	10,15

(a) [4 points] Define mixed strategies for each player. Make sure to define all variables you introduce.

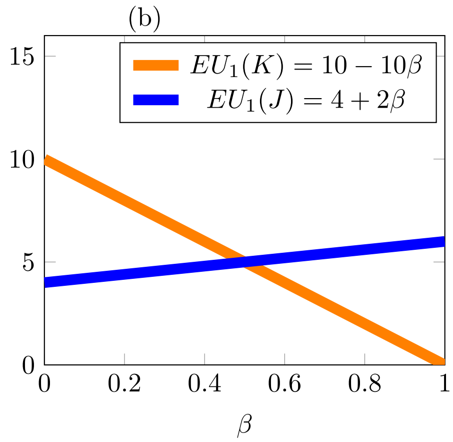
Solution:
Let α be the probability that P_1 plays J.
Player 1's mixed strategy will be $(\alpha\mathbf{J}, (\mathbf{1}-\alpha)\mathbf{K})$ (and zero weight on H and L).
Let β be the probability that P_2 plays B.
Player 2's mixed strategy will be $(\beta\mathbf{B}, (\mathbf{1}-\beta)\mathbf{D})$ (and zero weight on A and C).

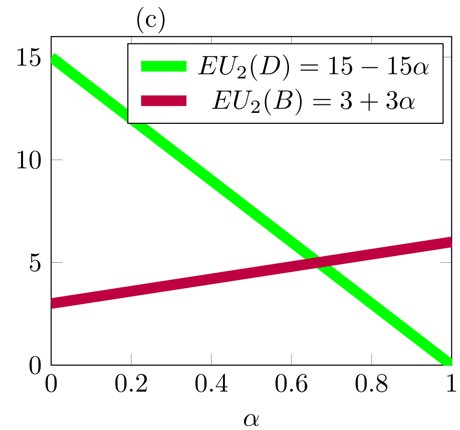
The notation doesn't have to match, but a correct answer will have one probability associated with each player and the remaining probability on the proper player's other strategy played in equilibrium.

(b) [4 points] Graph Player 1's expected utilities as functions of Player 2's mixed strategy you defined in part (a).

(c) [4 points] Graph Player 2's expected utilities as functions of Player 1's mixed strategy you defined in part (a).

Solution:



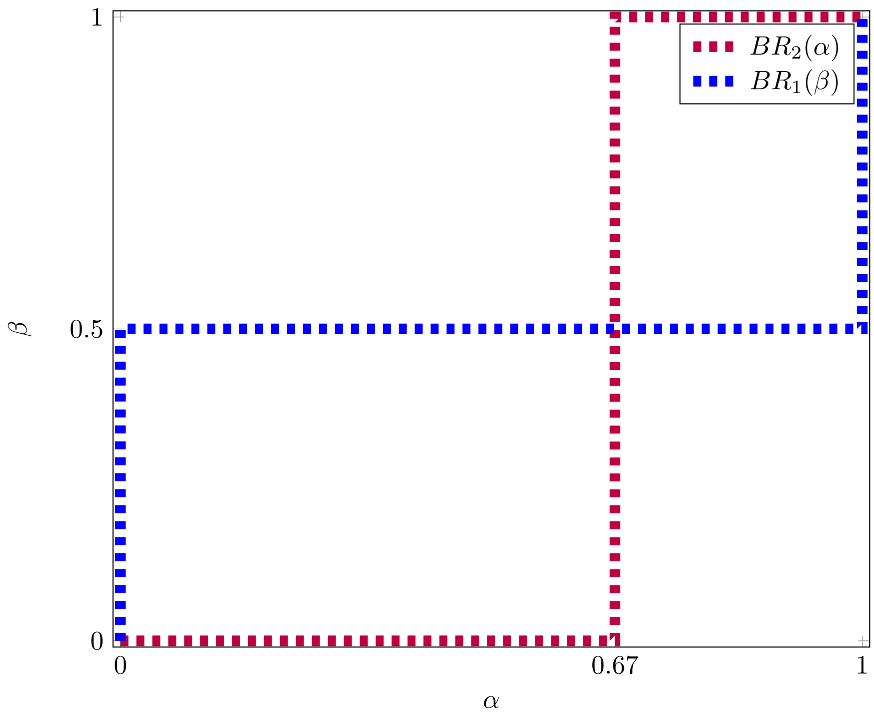


(d) [4 points] Solve for all Nash equilibria in this game (mixed and pure strategies). A complete answer will include all calculations used and a graph of best response functions.

Solution:

When will Player 1 be indifferent between J and K:
$$\begin{aligned} EU_1(J) &= EU_1(K) \\ 6\beta + 4(1-\beta) &= 0\beta + 10(1-\beta) \\ 4 + 2\beta &= 10 - 10\beta \\ \beta &= 1/2 \end{aligned}$$

When will Player 2 be indifferent between B and D:
$$\begin{aligned} EU_2(B) &= EU_2(D) \\ 6\alpha + 3(1-\alpha) &= 0\alpha + 15(1-\alpha) \\ 3 + 3\alpha &= 15 - 15\alpha \\ \alpha &= 2/3 \end{aligned}$$



- **MSNE:** $\{\sigma_1 = (2/3J, 1/3K), \sigma_2 = (1/2B, 1/2D)\}$
- **PSNE:**
  - $\{\sigma_1 = K, \sigma_2 = D\}$
  - $\{\sigma_1 = J, \sigma_2 = B\}$

Q2. Consider the strategic form game below:

		P_2		
		Left	**Center**	**Right**
	Top	3 , 1	3 , 2	0 , 2
P_1	**Middle**	1 , 2	2 , 1	1 , 2
	Bottom	0 , 2	3 , 2	3 , 1

(a) [4 points] Find any **pure strategy** Nash equilibria

> **Solution:** (Bottom, Right) is the only pure-strategy Nash

(b) [4 points] Consider the following mixed strategy profile:

- Player 1 plays 1/3 **Top**, 1/3 **Middle**, and 1/3 **Bottom**
- Player 2 plays 1/3 **Left**, 1/3 **Center**, 1/3 **Right**

Check whether this is a **mixed strategy Nash equilibrium** and explain why or why not.

> **Solution:**
>
> $$\begin{aligned} EU_1(\text{Top}) &= 3(1/3) + 3(1/3) + 0(1/3) = 2 \\ EU_1(\text{Middle}) &= 1(1/3) + 2(1/3) + 1(1/3) = \frac{4}{3} \\ EU_1(\text{Bottom}) &= 0(1/3) + 3(1/3) + 3(1/3) = 2 \end{aligned}$$
>
> So Player 1 would unilaterally deviate to playing less of Middle which has a strictly lower payoff than Top or Bottom, conditional on Player 2 playing (1/3, 1/3, 1/3). ✗
> **Not an MSNE**

(c) [4 points] Now consider the strategy profile:

- Player 1 plays 1/3 **Top**, 1/3 **Middle**, and 1/3 **Bottom**
- Player 2 plays 1/2 **Left**, 0 **Center**, 1/2 **Right**

Check whether this is a **mixed strategy Nash equilibrium** and explain why or why not.

> **Solution:**
>
> $$\begin{aligned} EU_1(\text{Top}) &= 3(1/2) + 3(0) + 0(1/2) = \frac{3}{2} \\ EU_1(\text{Middle}) &= 1(1/2) + 2(0) + 1(1/2) = 1 \\ EU_1(\text{Bottom}) &= 0(1/2) + 3(0) + 3(1/2) = \frac{3}{2} \end{aligned}$$
>
> So Player 1 would unilaterally deviate to playing less of Middle which has a strictly lower payoff than Top or Bottom, conditional on Player 2 playing (1/2, 0, 1/2). ✗
> **Not an MSNE**

(d) Now consider the strategy profile:

- Player 1 plays 1/4 **Top**, 0 **Middle**, and 3/4 **Bottom**
- Player 2 plays 0 **Left**, 1 **Center**, 0 **Right**

Check whether this is a **mixed strategy Nash equilibrium** and explain why or why not.

Solution:

$$EU_1(\text{Top}) = 3(0) + 3(1) + 0(0) = 3$$
$$EU_1(\text{Middle}) = 1(0) + 2(1) + 1(0) = 2$$
$$EU_1(\text{Bottom}) = 0(0) + 3(1) + 3(0) = 3$$

So Player 1 is indifferent between Top and Bottom, and will never play Middle. This is consistent w/ them playing (1/4, 0, 3/4) mixed-strategy ✓.

$$EU_2(\text{Left}) = 1(1/4) + 2(0) + 2(3/4) = \frac{7}{4}$$
$$EU_2(\text{Center}) = 2(1/4) + 1(0) + 2(3/4) = 2$$
$$EU_2(\text{Right}) = 2(1/4) + 2(0) + 1(3/4) = \frac{5}{4}$$

So Player 2's strictly dominat strategy is only Center ✓.
{(1/4, 0, 3/4), Center} is an MSNE

Q3. Consider the following game:[1]

		Colin	
		Yes	No
Rowena	Yes	x,x	0,1
	No	1,0	1,1

(a) [4 points] For what values of x does this game have a unique Nash equilibrium? What is that equilibrium?

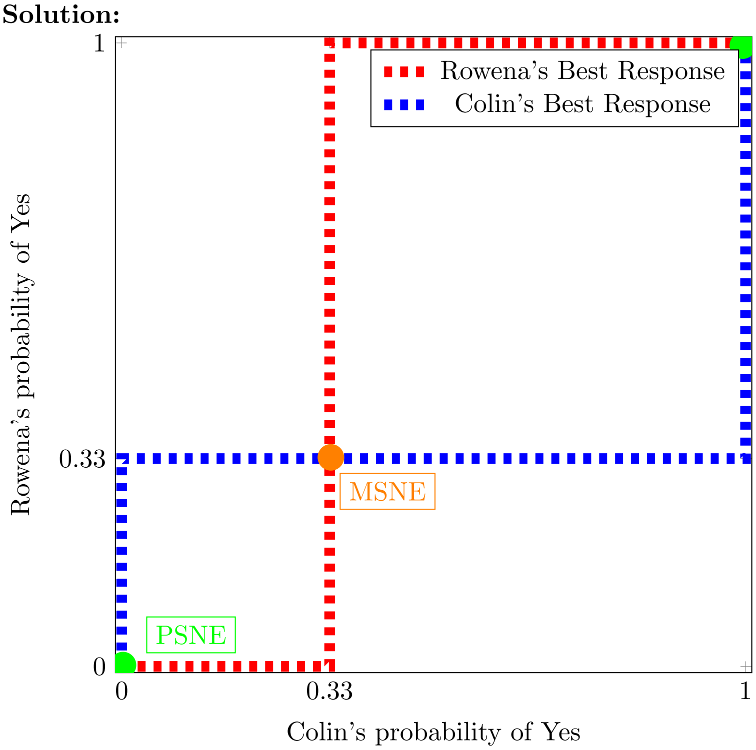
> **Solution:** If $x < 1$, then `No` is a dominant strategy and (`No, No)` is the unique Nash.

(b) [4 points] For what values of x does this game have a mixed strategy equilibrium? With what probability, expressed in terms of x does each player play Yes in this mixed-strategy equilibrium?

> **Solution:** For there to be an MSNE, a player must be indifferent between pure strategies. This is a symmetric game. Either player is indifferent between `Yes` and `No` if:
>
> $$px + 0(1-p) = 1$$
>
> where p is the probability the *other player* chooses `Yes`.
> For p to be a meaningful probability, $0 \leq \frac{1}{x} \leq 1$, or $\underline{x \geq 1}$.

(c) [4 points] Let $x = 3$. Graph the best-response curves of Rowena and Colin against each other's mixed strategy probability on the same graph. Label all Nash equilibria in pure and mixed strategies.

Solution:



[1] Dixit, Skeath, & McAdams, *Games of Strategy*, 4th Edition

(d) [4 points] Let $x = 1$. Graph the best-response curves of Rowena and Colin against each other's mixed strategy probability on the same graph. Label all the Nash equilibria in pure and mixed strategies.

Solution: