Repeated Games

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EC327 Game Theory

- So far, we have only seen games as either **one-shot** simultaneous or **finitely sequential**
- However, these representations can only do so much to represent the many complicated social interactions in which **repeated interactions** between the same players matter
- Specifically, in the Strategic Moves section, we discussed how reputation could play a role, but only allowed it to show up in the single-shot game through changing the payoffs

- In games with **finite** numbers of player actions, we can always use backwards induction to find equilibria
- But often players *do not know* when certain social interactions will end, and so it won't be reasonable to assume that they can backwards induct

- When games are repeated over time, we will use *discount rates* to represent how patient players are
- We can combine this with probability that a game will end at each stage in what we will call an effective rate of return

Trench Warfare as Repeated Prisonners' Dilemma

Trench Warfare in WWI

- On the Western Front, early advances ground to a halt and stagnated into trench warfare
- Technologies like artillery and machine guns made the war one of the bloodiest in human history

Trench Warfare in WWI

FIGURE 13.1 Trench Warfare Game

		German soldiers	
		Kill	*Miss*
Allied soldiers	*Kill*	2,2	6,0
	Miss	0,6	4,4

What's the **NE**?

Unexpected Truces Emerge

Christmas Day Truce, 1914:

Image Credit: Stephanie Lecocq/European Pressphoto Agency

Unexpected Truces Emerge

So regular were [the Germans] in their choice of targets, times of shooting, and number of rounds fired, that, after being in the line one or two days, Colonel Jones had discovered their system, and knew to a minute where the next shell would fall. His calculations were very accurate, and he was able to take what seemed to uninitiated Staff Officers big risks, knowing that the shelling would stop before he reached the place being shelled.

I was having tea with A Company when we heard a lot of shouting and went out to investigate. We found our men and the Germans standing on their respective parapets. Suddenly a salvo arrived but did no damage. Naturally both sides got down and our men started swearing at the Germans, when all at once a brave German got on to his parapet and shouted out "We are very sorry about that; we hope no one was hurt. It is not our fault, it is that damned Prussian artillery."

The puzzle of trench truces

- How did cooperation between enemy armies achieved and sustained?
- One answer might be that in these parts of the front, interactions were **repeated** between the same units

Constructing a Repeated Game

- Suppose that Allied and German forces anticipate that they will play this game T times

FIGURE 13.1 **Trench Warfare Game**

German soldiers

Allied soldiers	Kill	Miss
Kill	2,2	6,0
Miss	0,6	4,4

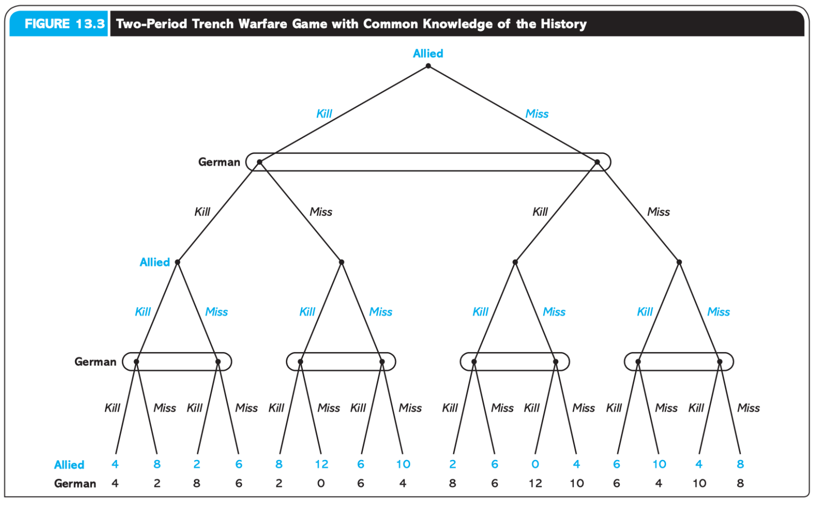
- A *strategy* will be made up of T *actions*; one for each time this stage game is played

Constructing a Repeated Game

To represent this as an extensive form tree, lets suppose that $T = 2$: and suppose that the history of all past plays are **common knowledge**

- each player will have five info sets; one for day 1, and four in day 2
- What does the extensive form game look like?

Constructing a Repeated Game



Constructing a Repeated Game

Let's generalize what a strategy in *any* finitely repeated game with common knowledge will look like:

- If a game has T periods, and each player has m actions at each stage,
- there is one initial info set, m^2 info sets in period 2, m^4 info sets in period 3, ..., $m^{2(T-1)}$ in the last period
- A complete strategy is made up of $1 + m^2 + m^4 + ... + m^{2(T-1)}$ actions

In an infinitely repeated game, there will be an infinite number of actions in each strategy

Constructing a Repeated Game

How to model streams payoffs over time?

- We could just add up all of the per-stage payoffs across an entire history
- But for infinitely-long histories, this sum would blow up and not make much sense
- Instead, we will use present value calculations

Present Values

Suppose that I have an income stream where I earn w_t dollars in every year t

- Suppose that there is a single discount factor δ which captures how much I value income tomorrow compared to income today
- My present value over my whole income stream is

$$w_1 + \delta w_2 + \delta^2 w_3 + \delta^3 w_4 + ... + \delta^{T-1} w_T$$

- It makes sense to assume that $0 < \delta < 1$ because I should probably care about tomorrow to some extent, but not as much as today

Present Values

What about calculating a present value of an infinte stream of payoffs?

- It turns out:

$$x + \delta x + \delta^2 x + \delta^3 x + ... + \delta^\infty x$$

- actually converges to $\frac{x}{1-\delta}$ as long as $\delta < 1$

Check Your Understanding

Suppose you are deciding between three different payoff streams:

Period	Stream A	Stream B	Stream C
1	15	25	5
2	15	15	10
3	15	10	20
4	15	5	30

Which has the **highest present value** when $\delta = 0.8$?

Check Your Understanding

Suppose you are deciding between three different payoff streams:

Period	Stream A	Stream B	Stream C
1	15	25	5
2	15	15	10
3	15	10	20
4	15	5	30

Which has the **highest present value** when $\delta = 0.8$?

Stream A: 44.28, **Stream B: 45.9**, Stream C: 41.16

Going back to the trenches

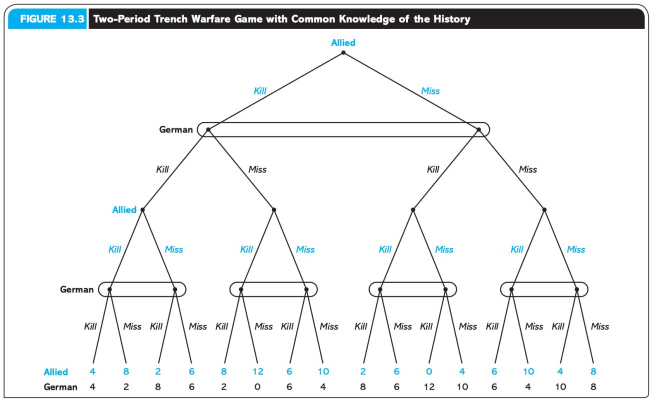


FIGURE 13.3 Two-Period Trench Warfare Game with Common Knowledge of the History

This was our extensive form game for only 2 periods

Going back to the trenches

Now suppose that we have a potentially very large T

How can we find a SPNE?

Going back to the trenches

Suppose that we are already at the last period T of the T-period trench warfare game

Suppose that the Allies total payoff stream value so far is A^{T-1} and the Germans is G^{T-1}

FIGURE 13.7 **Period *T* Subgame of the *T*-Period Trench Warfare Game**

German soldiers

Allied soldiers		*Kill*	*Miss*
	Kill	$A^{T-1}+2, G^{T-1}+2$	$A^{T-1}+6, G^{T-1}$
	Miss	$A^{T-1}, G^{T-1}+6$	$A^{T-1}+4, G^{T-1}+4$

What will happen?

Suppose that we are already at the last period T of the T-period trench warfare game

Suppose that the Allies total payoff stream value so far is A^{T-1} and the Germans is G^{T-1}

FIGURE 13.7 **Period *T* Subgame of the *T*-Period Trench Warfare Game**

		German soldiers	
		Kill	***Miss***
Allied soldiers	***Kill***	$A^{T-1}+2, G^{T-1}+2$	$A^{T-1}+6, G^{T-1}$
	Miss	$A^{T-1}, G^{T-1}+6$	$A^{T-1}+4, G^{T-1}+4$

What will happen?

- Allies will Shoot to **Kill**, Germans will Shoot to **Kill**

Going back to the trenches

Now that we know the T stage will end in $(Kill,\ Kill)$, we can look one period back to what will happen in $T-1$:

FIGURE 13.8 Period $T-1$ Subgame of the T-Period Trench Warfare Game

Allied soldiers \ German soldiers	*Kill*	*Miss*
Kill	$A^{T-2}+4, G^{T-2}+4$	$A^{T-2}+8, G^{T-2}+2$
Miss	$A^{T-2}+2, G^{T-2}+8$	$A^{T-2}+6, G^{T-2}+6$

What will happen?

Going back to the trenches

Now that we know the T stage will end in $(Kill,\ Kill)$, we can look one period back to what will happen in $T-1$:

FIGURE 13.8 **Period $T-1$ Subgame of the T-Period Trench Warfare Game**

		German soldiers	
		Kill	*Miss*
Allied soldiers	*Kill*	$A^{T-2}+4, G^{T-2}+4$	$A^{T-2}+8, G^{T-2}+2$
	Miss	$A^{T-2}+2, G^{T-2}+8$	$A^{T-2}+6, G^{T-2}+6$

What will happen?

- Both will shoot to **Kill** in $T-1$, knowing they will both shoot to kill in T

By now, you should get the idea:

Insight

If the stage game has a unique NE, then any finitely repeated version will have a unique SPNE which is just the repetition of the single-stage NE. No cooperation is sustainable

So what was going on with those spontaneous truces?

Infinitely Repeated Trench Game

- The problem with that last equilibrium we found was that you have to know *exactly when the game will end* to use backwards induction
- But for World War I infantrymen, they didn't know how long it would be until the fronts shifted or their division was rotated out
- We will have to extend our models to allow for indefinite horizons

Repeated Prisoners' Dilemma with Uncertain Second Stage

Suppose that the first stage of the game is the Trenches Game:

		German soldiers	
		Kill	Miss
Allied Soldiers	Kill	4, 4	8, 2
	Miss	2, 8	6, 6

But with probability p, the game repeats in the second round and with probability $1 - p$, it ends after the first round

Repeated Prisoners' Dilemma with Uncertain Second Stage

Consider the following strategy:

$$\begin{cases} \text{In stage } 1 & : \textit{Miss} \\ \text{In stage } 2 & : \begin{cases} \textit{Miss} \text{ if the other player Missed in stage 1} \\ \textit{Kill} \text{ if the other player Killed in stage 1} \end{cases} \end{cases}$$

Let's call this strategy *Punisher* because it starts off friendly, but will try to punish someone who defects in the first round by defecting in the second round.

Repeated Prisoners' Dilemma with Uncertain Second Stage

Suppose you are playing against a *Punisher* in this game.

		German soldiers	
		Kill	Miss
Allied Soldiers	Kill	4, 4	8, 2
	Miss	2, 8	6, 6

- What is your **expected utility** of playing *Kill*, *Kill*?

Repeated Prisoners' Dilemma with Uncertain Second Stage

Suppose you are playing against a *Punisher* in this game.

		German soldiers	
		Kill	Miss
Allied Soldiers	Kill	4, 4	8, 2
	Miss	2, 8	6, 6

- What is your **expected utility** of playing *Kill*, *Kill*?
- 8 in the first stage, 4 in the second stage,
- so $EU(Kill,\ Kill) = 8 + 4p$

Repeated Prisoners' Dilemma with Uncertain Second Stage

Suppose you are playing against a *Punisher* in this game.

		German soldiers	
		Kill	Miss
Allied Soldiers	Kill	4, 4	8, 2
	Miss	2, 8	6, 6

What is your **expected utility** of playing *Miss, Kill*?

Repeated Prisoners' Dilemma with Uncertain Second Stage

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		German soldiers	
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	Miss	2, 8	6, 6

What is your **expected utility** of playing *Miss, Kill*?

- $4 + 8p$

What about from playing *Kill, Miss?*

Repeated Prisoners' Dilemma with Uncertain Second Stage

Suppose you are playing against a *Punisher* in this game.

		German soldiers	
		Kill	Miss
Allied Soldiers	Kill	4, 4	8, 2
	Miss	2, 8	6, 6

What is your **expected utility** of playing *Miss, Kill*?

- $4 + 8p$

What about from playing *Kill, Miss?*

- $8 + 2p$

Would you rather defect earlier or later?

Repeated Prisoners' Dilemma with Uncertain Second Stage

Suppose you are playing against a *Punisher* in this game.

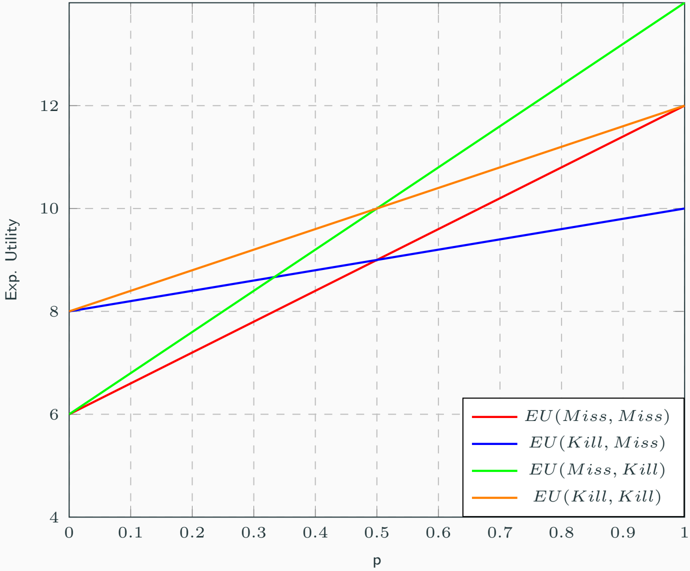
		German soldiers	
		Kill	Miss
Allied Soldiers	Kill	4, 4	8, 2
	Miss	2, 8	6, 6

What is your **expected utility** of playing *Miss, Miss*?

- $6 + 6p$

Repeated Prisoners’ Dilemma with Uncertain Second Stage

Let’s graph your expected utilties of each strategy:



Repeated Prisoners' Dilemma with Uncertain Second Stage

Now you should be getting some of the intuition for how cooperative equilibria might be achieved.

- We need the payoffs of the last period to be uncertain (or never reached)
- If trying to cheat a *Punisher* or *Grim Trigger* strategy, it is better to start cheating them sooner rather than later
- In order for the equilibrium to have both players always cooperating, defecting in at least one period must not be a dominant strategy

Infinitely Repeated Games

Suppose the probability that at each stage, with probability p, the game continues and with $(1-p)$, the game ends and you get $u = 0$

The **expected present value** of a stream of payoffs $u_1, u_2, ...$ is then:

$$V = u_1 + pdu_2 + p^2d^2u_3 + ... = \sum_{t=1}^{\infty}(pd)^{t-1}u_t$$

Infinitely Repeated Games

Now if we let $\delta = pd$ represent the discount factor from both time preferences and the likelihood of the game terminating:

$$V = \sum_{t=1}^{\infty} (pd)^{t-1} u_t = \sum_{t=1}^{\infty} \delta^{t-1} u_t$$

Which is exactly the same as the expected present value of a stream of **infinite** payments

SPNE in Repeated Games

A strategy profile is SPNE if and only if in each period and for each history, the prescribed action is optimal given:

- the other players act according to their strategies in the current period
- all players act according to their strategies in all future periods

Grim Trigger in the Trench Game

Consider the following strategy:

- In period 1, choose miss
- In period $t > 1$, choose miss if both chose miss in all past periods, else choose kill

This type of strategy is known as Grim Trigger because this type of player starts out cooperative, but if wronged once, they will always shoot to kill

Cooperative Equilibrium in the Trenches

Revisiting the Christmas Truce:

- Suppose that the Allies play the Grim Trigger Strategy
- When will the Germans want to Shoot to Miss?

Cooperative Equilibrium in the Trenches

Revisiting the Christmas Truce:

- Suppose that the Allies play the Grim Trigger Strategy
- When will the Germans want to Shoot to Miss?
  - when $pv(Cheat) < pv(Coop)$
  - $pv(Cheat) = 8 + 4\delta + 4\delta^2 + ...$
  - $pv(Coop) = 6 + 6\delta + 6\delta^2 + ...$
  - So Cheat when $8 + 4\sum_{t=2}^{\infty} \delta^t < 6 + 6\sum_{t=2}^{\infty}$
  - or when $\delta > \frac{1}{2}$

Cooperative Equilibrium in the Trenches

Revisiting the Christmas Truce:

- So there is a Nash equilibrium with both players choosing to miss
- as long as one of them is playing the Grim Trigger strategy and the other has a discount rate $\delta > \frac{1}{2}$

General Repeated Prisoners' Dilemma

General Prisoners' Dilemma

		Column	
		Defect	Cooperate
Row	Defect	D,D	H,L
	Cooperate	L,H	C,C

What ordering of payoffs D, H, L, and C make this a **Prisoners' Dilemma**?

a) $C > D > H > L$

b) $H > D > C > L$

c) $H > C > D > L$

d) $C > H > L > D$

General Prisoners' Dilemma

		Column	
		Defect	Cooperate
Row	Defect	D,D	H,L
	Cooperate	L,H	C,C

What ordering of payoffs D, H, L, and C make this a **Prisoners' Dilemma**?

a) $C > D > H > L$

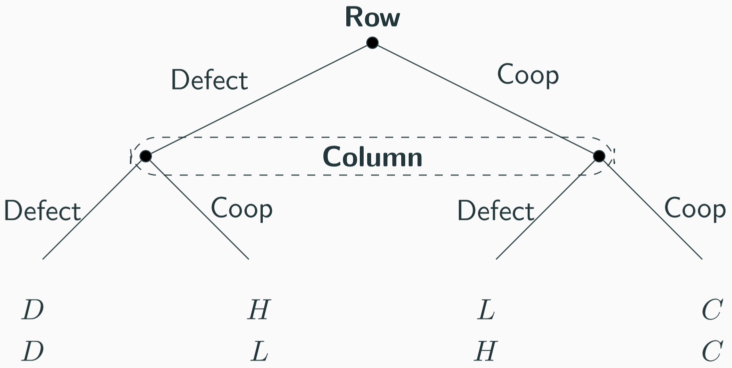
b) $H > D > C > L$

c) $H > C > D > L$

d) $C > H > L > D$

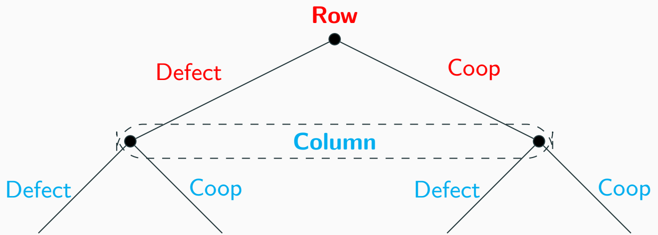
Answer: **(c)**!

A single-stage prisoners' dilemma in extensive form:



General Prisoners’ Dilemma

A **two-stage** prisoners’ dilemma in mixed extensive form:



	Def.	Coop.
Def.	$D+\delta D$, $D+\delta D$	$D+\delta H$, $D+\delta L$
Coop.	$D+\delta L$, $D+\delta H$	$D+\delta C$, $D+\delta C$

	Def.	Coop.
Def.	$H+\delta D$, $L+\delta D$	$H+\delta H$, $L+\delta L$
Coop.	$H+\delta L$, $L+\delta H$	$H+\delta C$, $L+\delta C$

	Def.	Coop.
Def.	$L+\delta D$, $H+\delta D$	$L+\delta H$, $H+\delta L$
Coop.	$L+\delta L$, $H+\delta H$	$L+\delta C$, $H+\delta C$

	Def.	Coop.
Def.	$C+\delta D$, $C+\delta D$	$C+\delta H$, $C+\delta L$
Coop.	$C+\delta L$, $C+\delta H$	$C+\delta C$, $C+\delta C$

Recall that δ is the subjective discount rate from stage to stage

$\mathbb{T}$-stage repeated Prisoners' Dilemma

A complete strategy in a $\mathbb{T}$-stage repeated game will look like:

$$S_{t=1}^{\mathbb{T}} = \begin{cases} \text{In stage } t = 1 & \text{take action } A_0 \\ \text{In stage } t > 1 & \begin{cases} \text{If history so far was } h_t, \text{take action } A_t(h_t) \\ \text{Else if history was } h_t', \text{take action } A_t(h_t') \\ \ldots \end{cases} \end{cases}$$

We can see that the number of possible strategies increases exponentially as $\mathbb{T}$ gets larger

$\mathbb{T}$-stage repeated Prisoners' Dilemma

Suppose that $\mathbb{T}$ is a very large number, but we have played to the very last stage of a repeated Prisoners' Dilemma with that many stages:

		Column	
		Defect	Cooperate
Row	Defect	Tot.$^R + D$, Tot.$^C + D$	Tot.$^R + H$, Tot.$^C + L$
	Cooperate	Tot.$^R + L$, Tot.$^C + H$	Tot.$^R + C$, Tot.$^C + C$

Let Tot.^{R} and Tot.^{C} represent the total payoffs that both players have earned over stages 0 to $\mathbb{T} - 1$

$\mathbb{T}$-stage repeated Prisoners' Dilemma

		Column	
		Defect	Cooperate
Row	Defect	Tot.$^R + D$, Tot.$^C + D$	Tot.$^R + H$, Tot.$^C + L$
	Cooperate	Tot.$^R + L$, Tot.$^C + H$	Tot.$^R + C$, Tot.$^C + C$

Notice that the equilibrium of this subgame is still *Defect*, *Defect* because Tot.^{R} and Tot.^{C} are already decided by prior actions.

Back to the General Form Prisoners' Dilemma

		Column	
		Defect	Cooperate
Row	Defect	D,D	H,L
	Cooperate	L,H	C,C

Now let's suppose that this game is repeated for an **infinite number of stages**.

Extending Plays to Infinity

Suppose the game is in the 'good' equilibrium where all players always play *Cooperate*.

- What is **present value** from this equilibrium?

$$\begin{aligned} pv\left(\{C\}_{t=1}^{\infty}\right) &= C + \delta C + \delta^2 C + \delta^3 C + \ldots \\ &= C \sum_{t=1}^{\infty} \delta^{t-1} \\ &= C \frac{1}{1-\delta} \end{aligned}$$

Extending Plays to Infinity

Let's extend the *Punisher* strategy we had from the two-stage game into the *Grim Trigger strategy* of the general infinite horizon game:

$$\begin{cases} \text{In stage } 1 & : \textit{Cooperate} \\ \text{In stage } t \geq 2 & : \begin{cases} \textit{Cooperate} \text{ if only cooperation has happened so far} \\ \textit{Defect} \text{ if anyone has } \textit{ever} \text{ Defected in the past} \end{cases} \end{cases}$$

Grim Trigger SPNE in Repeated PD

Is both players playing *Grim Trigger* stable?

- Does a player have an incentive to *Defect* against *Grim Trigger*:

$$pv(Always\ Coop) \geq pv(Defect\ once)$$
$$C + \delta C + \delta^2 C + ... \geq H + \delta D + \delta^2 D + ...$$
$$C + C \sum_{t=2}^{\infty} \delta^t \geq H + D \sum_{t=2}^{\infty} \delta^t$$
$$C + C\delta \sum_{t=2}^{\infty} \delta^{t-1} \geq H + D\delta \sum_{t=2}^{\infty} \delta^{t-1}$$
$$C + \frac{\delta C}{1-\delta} \geq H + \frac{\delta D}{1-\delta}$$
$$\frac{\delta}{1-\delta} \geq \frac{H-C}{C-D}$$

Grim Trigger SPNE in Repeated PD

How do we interpret this statement:

$$\text{Cooperation is stable when } R \geq \frac{C-D}{H-C}$$

- Recall that the definition of the Prisoners' Dilemma was that $H > C > D > L$
- So this means $\frac{H-C}{C-D}$ is positive and less than 1
- As the $H - C$, the relative benefit of defecting increases, it gets harder to sustain cooperation
- It also gets harder to sustain cooperation as the relative penalty of defecting, $H - D$, shrinks

Other Strategies in Repeated Games

So far we've only looked at one example of a type of strategy in repeated game, *Grim Trigger*.

- Can you think of some others?
  - Recall that a complete strategy for a repeated game needs:
    - An initial move at $t = 1$
    - A plan of action for *every* possible history in *every* later stage $t \geq 2$
  - Ideally you would be able to tell a computer how to implement your strategy

Other Strategies in Repeated Games

Telling a computer how to implement strategies is exactly what Robert Axelrod did in a famous tournament in 1980.

- He invited people to submit their programs which would play 200 rounds of the prisoners' dilemma against each other
- The winning program was the one which had the highest total score after playing 200 rounds against all other programs
- What types of strategies do you think would succeed?

An Unexpected Winner

The winning program was named `TIT FOR TAT`

Surprisingly, it was fairly simple:

$$\begin{cases} \text{In stage } 1 & : \textit{Cooperate} \\ \text{In stage } t \geq 2 & : \begin{cases} \text{repeat what the other player did in } t-1 \end{cases} \end{cases}$$

Tit-for-Tat

Like *Grim Trigger*, *Tit-for-Tat* can punish other players for defecting.

- If a player plays *Defect*, it will copy them with *Defect* next round

But unlike *Grim Trigger* it has a short memory; or is very forgiving

- If the player who defected goes back to playing cooperatively, *Tit-for-Tat* will go back to cooperating too

Axelrod's Tournament

If you want to learn more:

- Read the original paper:
  Axelrod, Robert; Hamilton, William D. (27 March 1981), "The Evolution of Cooperation" (PDF), *Science*, 211 (4489): 1390–96
- The 1984 Book *The Evolution of Cooperation*, Basic Books
- Run the tournament yourself in python!
  https://github.com/Axelrod-Python/Axelrod
- Play this fun and short web game!
  https://ncase.me/trust/

Other Repeated Games

A More Complicated Game

		Player 2		
		x	y	z
Player 1	x	5, 5	2, 7	1, 3
	y	7, 2	3, 3	0, 1
	z	3, 1	1, 0	2, 2

What are the **pure strategy Nash equilibria** of the *one-shot* game?

A More Complicated Game

		Player 2		
		x	y	z
	x	5, 5	2, 7	1, 3
Player 1	y	7, 2	3, 3	0, 1
	z	3, 1	1, 0	2, 2

What are the **pure strategy Nash equilibria** of the *one-shot* game?

- (y,y) and (z,z)

Repeated Game with 3 strategies per period

Now suppose that this game is played repeatedly an infinite number of times.

- Can we do better than the single period equilibrium?

Grim Trigger

Player 1

$$\begin{cases} t = 0 & \text{Play } x \\ t > 0 & \begin{cases} \text{Play } x \text{ if only } x \text{ has been played} \\ \text{Play } y \text{ if anything other than } x \text{has been played} \end{cases} \end{cases}$$

Player 2

- $EV_{Coop} = \frac{5}{1-\delta}$
- $EV_{Cheat} = 7 + \frac{3\delta}{1-\delta}$

Solve for the value of δ for which this is a **SPNE**

Grim Trigger

Solve for the value of δ for which this is a **SPNE**

$$\begin{aligned}\frac{5}{1-\delta} &\geq 7 + \frac{3\delta}{1-\delta} \\ 5 + \frac{5\delta}{1-\delta} &\geq 7 + \frac{3\delta}{1-\delta} \\ \frac{(5-3)\delta}{1-\delta} &\geq 7 - 5 \\ \delta &\geq \frac{1}{2}\end{aligned}$$

Tit-for-Tat

Player 1

$$\begin{cases} t = 0 & \text{Play } x \\ t > 0 & \text{Play Player 2's strategy from } t-1 \end{cases}$$

Player 2

- $EV_{Coop} = \frac{5}{1-\delta}$
- $EV_{Cheat} = 7 + 2\delta + \frac{5\delta^2}{1-\delta}$

Tit-for-Tat

Solve for the value of δ for which this is a **SPNE**

Tit-for-Tat

Solve for the value of δ for which this is a **SPNE**

$$
\begin{aligned}
5 + 5\delta + \frac{5\delta^2}{1-\delta} &\geq 7 + 2\delta + \frac{5\delta^2}{1-\delta} \\
5 + 5\delta &\geq 7 + 2\delta \\
\delta &\geq \frac{2}{3}
\end{aligned}
$$

A reciprocating cooperation strategy

$$\underline{\text{Player 1}} \begin{cases} t = 0 & \text{Play } y \\ t > 0 & \begin{cases} \text{Play } y \text{ if } t \text{ is even} \\ \text{Play } x \text{ if } t \text{ is odd} \\ \text{Play } z \text{ forever} \\ \quad \text{if P2 played } y \\ \quad \text{when } t \text{ is even} \end{cases} \end{cases}$$

$$\underline{\text{Player 2}} \begin{cases} t = 0 & \text{Play } x \\ t > 0 & \begin{cases} \text{Play } y \text{ if } t \text{ is odd} \\ \text{Play } x \text{ if } t \text{ is even} \\ \text{Play } z \text{ forever} \\ \quad \text{if P2 played } y \\ \quad \text{when } t \text{ is odd} \end{cases} \end{cases}$$

When can cooperation be achieved?

With all of these different ways of achieving repeated cooperation, you might be wondering if there is a way to tell what strategies can actually work

Folk Theorem

Any strategy is a potential SPNE for a **repeated** stage game if:

- Both agents are sufficiently patient and far-sighted (high enough δ)
- The payoffs from the cooperative strategy profile satisfy the two properties:
  - **Individually Rational:** the payoffs to each agent (weakly) exceed their minimax payoffs in the stage game
  - **Feasibility:** the payoffs are weighted averages of the payoffs found in the stage game

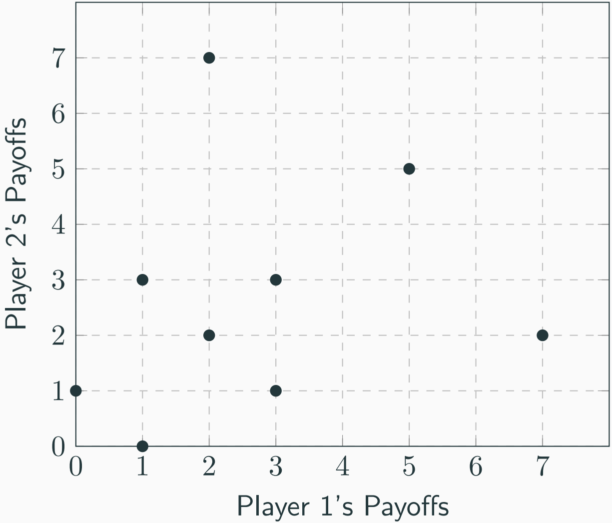
Folk Theorem with 3x3 Repeated Game Example

		Player 2		
		x	y	z
Player 1	x	5, 5	2, 7	1, 3
	y	7, 2	3, 3	0, 1
	z	3, 1	1, 0	2, 2

The Minimax equilibrium is (z,z)

- it *minimizes* the *maximum* payoff that your opponent could get
- The Minimax payoffs in this stage game are (2, 2)
- Intuitively, this is the *safe* option: you can always fall back on it if cooperation fails

Folk Theorem with 3x3 Repeated Game Example payoffs



Folk Theorem with 3x3 Repeated Game Example

- The shaded region of the graph shows us all of the strategy profiles which could be sustained by the **Folk Theorem**
- This shows us why that strategy profile of alternating between (x, y) and (y, x) worked:
  - even though getting 2 on even or odd periods was no better than the Minimax payoffs, because you could alternate with the higher payoff of 7 you could do better as long as you are patient enough
  - this mix between (2, 7) and (7, 2) is *within the convex hull* of sustainable payoffs

Cooperation in Repeated Games

- As you can probably tell, there are an infinite number of strategy profiles which can achieve cooperation
    - We could allow for mixed strategies, which would work similar to the alternating example we saw
    - The Folk Theorem tells us that all we need is for all players to be patient enough
    - and also that the past plays are common knowledge

Importance of the Folk Theorem

Why does this matter for real life?

- Most strategic interactions in your life are repeated
  - Sharing chores with your roommates
  - Interacting in class with me every week
  - Being nice to the barista at your regular cafe

Importance of the Folk Theorem

- Even when you don't repeatedly interact with the same exact people, you still see cooperative outcomes
- **Institutions, Reputations, and Social Structures** all serve to allow for past interactions to be common knowledge
- The history of humanity is built on how we arrange our strategic interactions in ways so that people are incentivized to play nice with others

Importance of the Folk Theorem

Some caveats:

- People can't know exactly when the game will end; if they don't have any incentive from future cooperative gains, they will always defect
  - Institutions have to *seem* like they are infinitely lived (compared to finitely lived humans)
- Cooperative equilibria must be better than peoples' outside options
  - If you make your institution too costly for people to engage with, they will opt out
- People need to be patient enough to make cooperation worth it