Econ 327: Game Theory

Homework #2

University of Oregon

Due: Oct. 10^{th}

Question:	**Q1**	**Q2**	**Q3**	**Q4**	Total
Points:	12	10	8	8	38
Score:					

For homework assignments:

- Complete *all* questions and parts.
- You will be graded on not only the content of your work but on how clearly you present your ideas. Make sure that your handwriting is legible. Please use extra pages if you run out of space but make sure that all parts of a question are in the correct order when you submit.
- You may choose to work with others, but everyone must submit to Canvas individually.

  Please include the names of everyone who you worked with below your own name.

Name ____

Q1. Imagine a sequential moves version of rock-paper-scissors where player 2 gets to pick what they will do after player 1 picks. Please model the game in its extensive form (as a game tree). Assume both player 1 and player 2 only care about the result of the game and have the following preferences over the result of the game: win $\succ$ tie $\succ$ loss. [1]

(a) [2 points] How many decisions does **Player 1** have to make in their *complete strategy*? In other words, how many *decision nodes* will Player 1 have in the game tree?

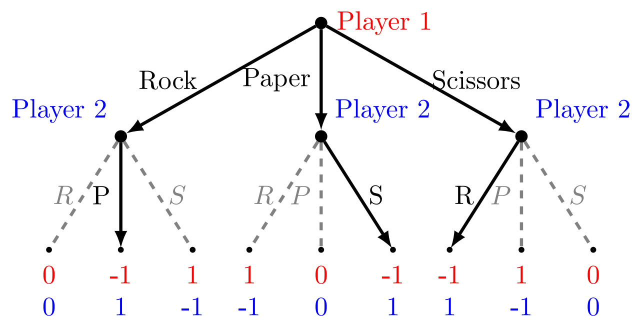
Solution: Player 1 only has **one** decision decision node because they cannot observe Player 2's actions.

(b) [2 points] How many decision nodes does **Player 2** have? Why?

Solution: Player 2 has **three** decision nodes because they have to make a plan of action for each of the three different strategies they could observe Player 1 choose.

(c) [4 points] Draw out the extensive form game. Make sure to clearly label all branches, nodes, and choose any payoff values that reflect the preference ranking above.

Solution:



(d) [4 points] Use **backwards induction** (rollback) to prune branches which are not sequentially rational. Does this game have more than one equilibrium? Why or why not?

Solution: In the figure above, dashed lines represent those which have been pruned.

There are three rollback equilibria because as long as Player 2 chooses whatever action beats Player 1's action, Player 1 is indifferent between any of their three strategies. Backwards induction cannot be used to prune any of Player 1's strategies, so we must stop here.

In this case, there are three equilibria:

- { Rock, (Paper, Scissors, Rock) },
- { Paper, (Paper, Scissors, Rock) },
- { Scissors, (Paper, Scissors, Rock) }

[1] Ethan Holdahl, University of Oregon

Q2. Use the same sequential Rock, Paper, Scissors setup, but now imagine player 1's preferences change because they want to be seen as a 'tough guy'. Given that what they want to play remains the same, they still have the following preferences over the result of the game: win $\succ$ tie $\succ$ loss. However, they now would prefer to lose playing rock than win playing paper or scissors.

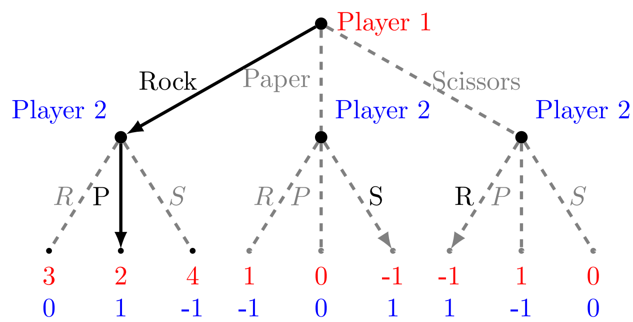
(a) [4 points] Please create a new game tree so the payoffs reflect these new preferences.

Solution:

I represented Player 1's preferences as:

payoff to winning with rock = 4, tie with rock = 3, lose with rock = 2, win with paper or scissors = 1, tie with paper or scissors = 0, lose with paper or scissors = -1.

Because payoffs are *ordinal*, you can choose any values as long as the ranking is the same.



(b) [4 points] Now apply backwards induction to the modified game tree. Clearly show on your game tree which branches survive the pruning process.

Solution: Now because Player 1 prefers to lose with rock to any other loss, we can prune the branches where they choose either paper or scissors. The only branches which survive pruning are Player 1's Rock and Player 2's Paper after Player 1 chooses Rock.

(c) [2 points] What is the rollback equilibrium now?

Solution: { Rock, (Paper, Scissors, Rock) }

or in English; Player 1 plays Rock, Player 2 plays Paper if Player 1 plays Rock, Scissors if Player 1 plays Paper, and Rock if Player 1 plays Scissors.

Q3. At the beginning of the wet season, the herds of gnu which live in the Serengeti migrate South to follow greener grasses. This is also a prime feeding time for the Nile crocodiles which inhabit the Mara river which the gnu must cross on their way.

To simplify the situation, suppose that there are only two crossings which the gnu can initially approach; the shallow *rapids*, or the deeper *channel*. After the gnu arrive at a crossing, suppose the crocodiles can observe where the gnu are gathered and choose which crossing to wait in ambush.

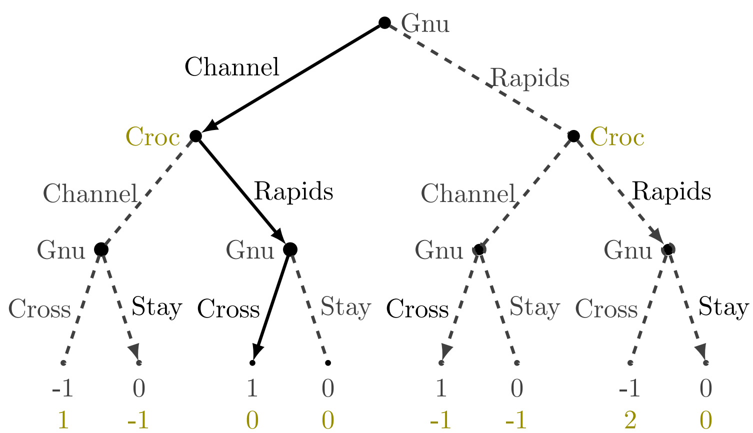
If the gnu cross the river at the same crossing where the crocs are waiting, they are eaten and recieve a payoff of -1. However, if they cross at the opposite crossing from the crocs, they succesfully cross the river and recieve a payoff of 1.

The crocs want to eat gnu, but they also prefer to wait in the rapids where they can sun themselves rather than the channel. If the crocs wait in the rapids and they catch the gnu crossing there, the crocs get a payoff of 2. If the crocs wait in the channel and catch the gnu crossing there, they only get a payoff of 1. If the crocs wait in the rapids but they don't catch any gnu, the crocs earn a payoff of 0. If the crocs wait in the channel but they don't catch any gnu, the crocs get a payoff of -1.

After the gnu have scoped out their choice of crossing, they can choose to either *cross* the river, or *stay* on the banks depending on whether they can see crocs waiting for them. If the gnu decide to stay on the banks, they get a payoff of 0 (and the crocs don't catch them).

(a) [4 points] Draw out the extensive form game. Clearly label all nodes and branches. [2]

Solution:



(b) [4 points] Use backwards induction to solve for the equilibrium.

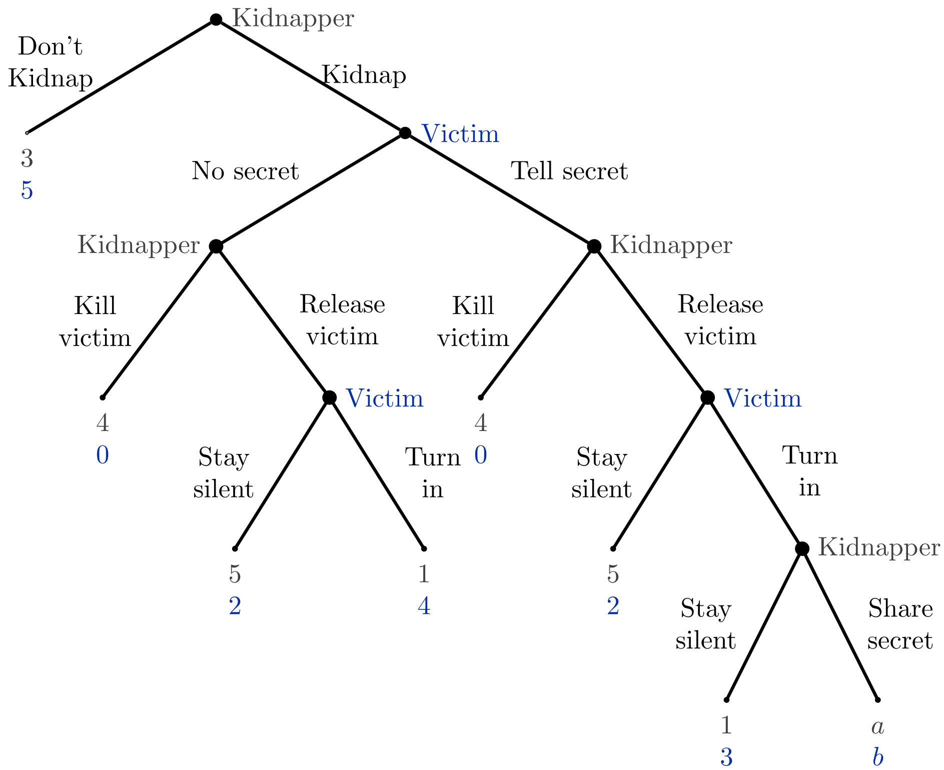
Solution: Dotted lines in the figure above can be pruned.

The only equilibrium is where the Gnu Cross only if the Crocs don't choose the same crossing as them. The Crocs choose the Rapids no matter what because they know the Gnu will respond by Staying if they follow them and they would rather be in the rapids if they don't get to eat. Knowing that the Crocs will stick to the rapids, the Gnu will approach the Channel.

[2]Hint: there should be eight possible outcomes.

Q4. Suppose that a kidnapper first makes a decision whether to kidnap a high-value victim. The problem is that once the ransom has been paid, the kidnapper has an incentive to kill the victim to prevent them from turning them into the police once released.

Consider the solution proposed by Thomas Schelling whereby the victim reveals his darkest secret to the kidnapper which he would hate to be revealed to the world. [3]



(a) [4 points] What values of a and b ensure that the subgame perfect Nash equilibrium results in the victim being released by the kidnapper?

> **Solution:** For the kidnapper to be willing to share the secret, $a > 1$. For the victim to be willing to stay silent when they know the kidnapper will share their secret, $b < 2$.

(b) [4 points] Do you think that the victim revealing an embarassing secret to their kidnapper could be an effective strategy in real life? Why or why not?

> **Solution:** A secret needs to be so costly to the victim that the kidnapper can be sure they would rather stay silent than turn them in. Also, the kidnapper needs proof that it would be suitably embarassing because otherwise, the victim could just invent some story to get free.

[3] Adapted from Harrington, *Games, Strategies, and Decision Making*, 2015, page 297.