Study of closed dynamical system based on the Kapitsa phenomenon (inverted pendulum)

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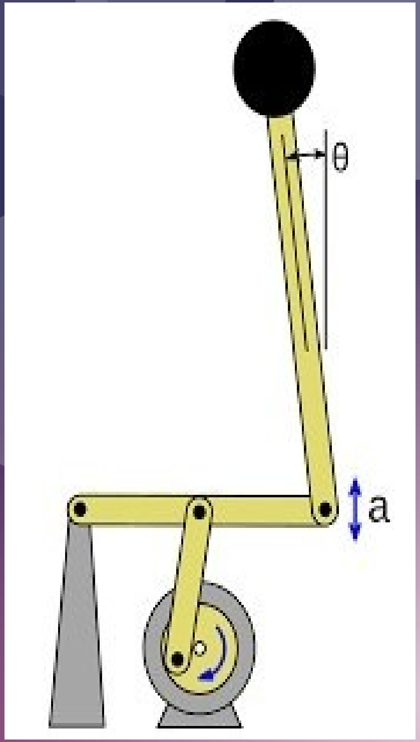
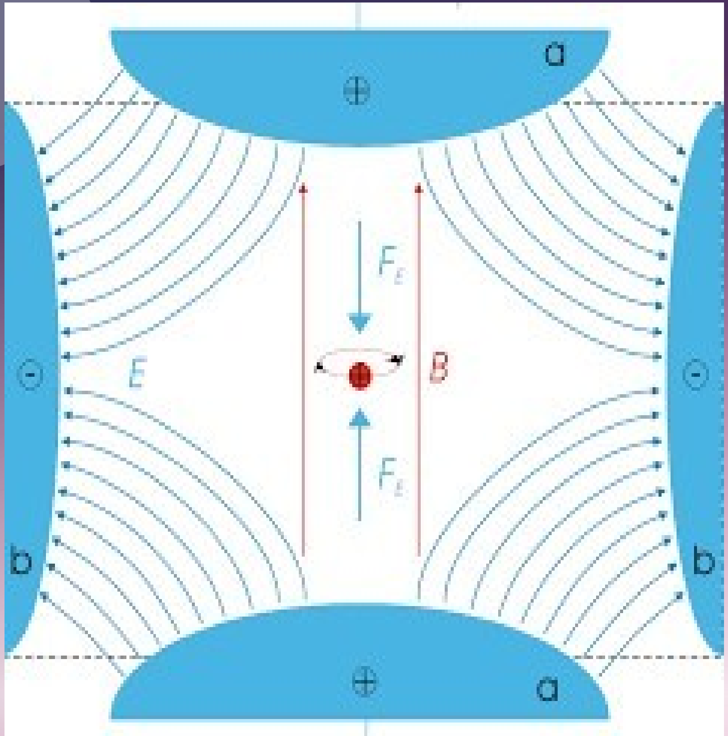
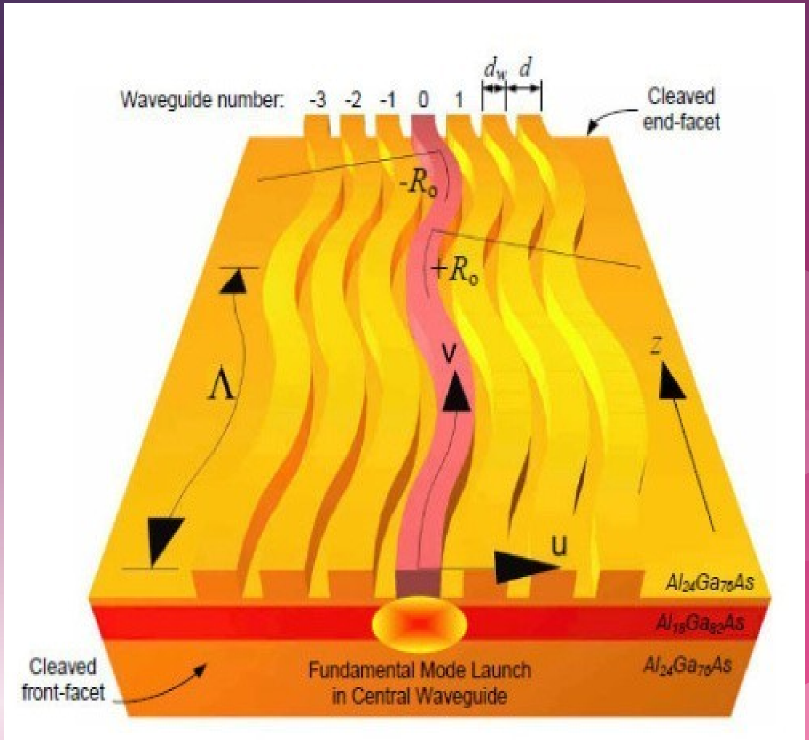
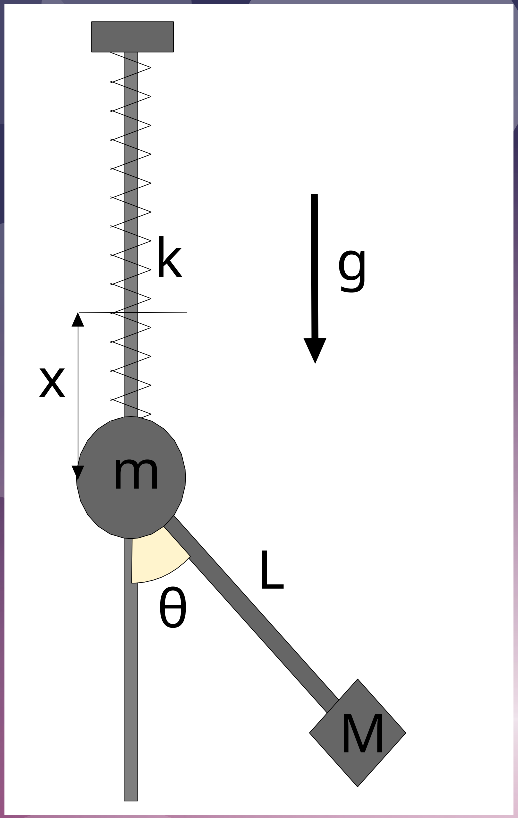
Kapitsa stabilization and analogs







System description



The system – double oscillator.

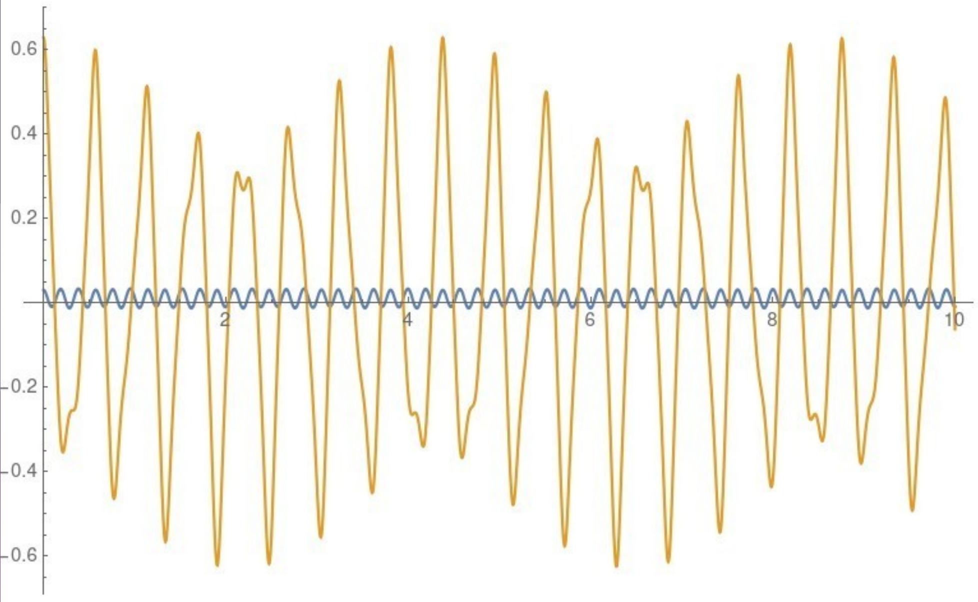
It’s made out of mathematical and spring pendulums connected together.

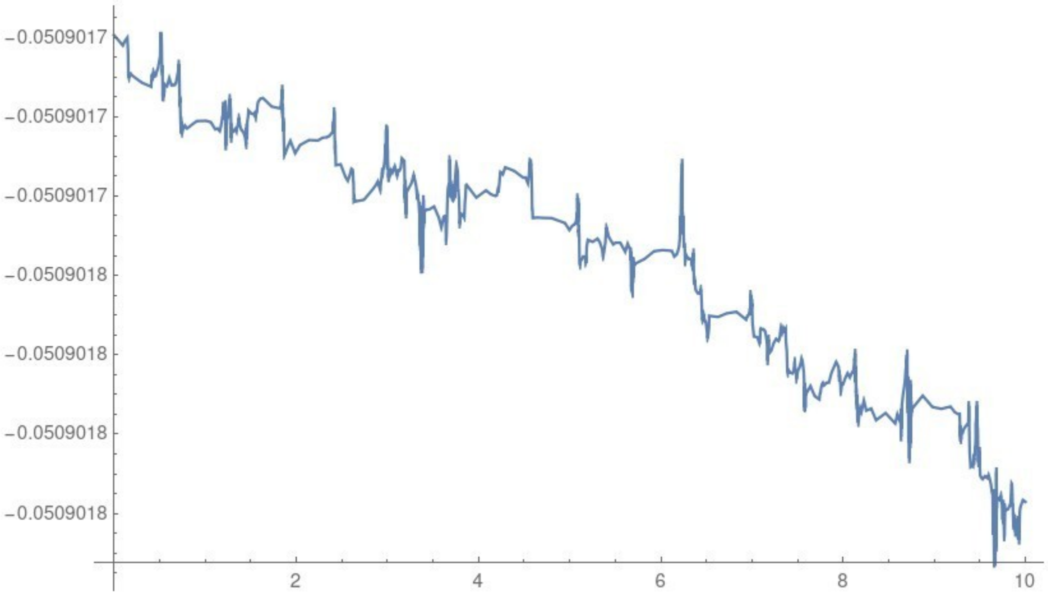
For this system we came up with differential equations describing it’s movement:

$$x''=g+\frac{M\theta'^2L+M(g-x'')\cos[\theta]}{m}\cos[\theta]-\frac{k}{m}x \qquad (1)$$

$$\theta''=-\frac{g-x''[t]}{L}\sin[\theta] \qquad (2)$$

Correction check





Mode discoveries

For the Kapitsa pendulum and double oscillator we’ve initialized simulation with different starting parameters.

We’ve made plots for average mathematical pendulum position and deviation from it.

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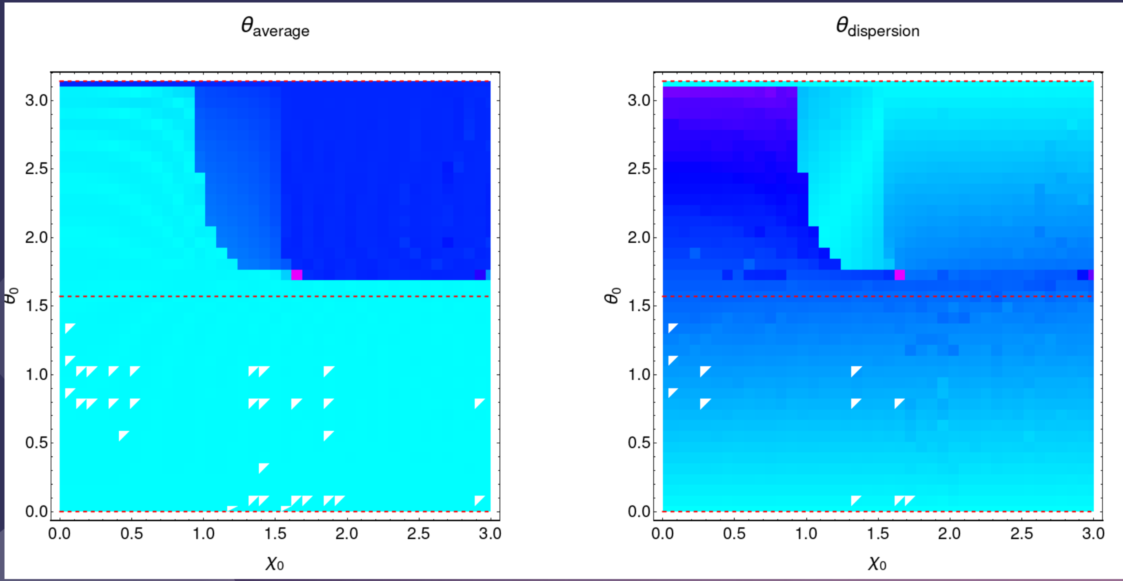
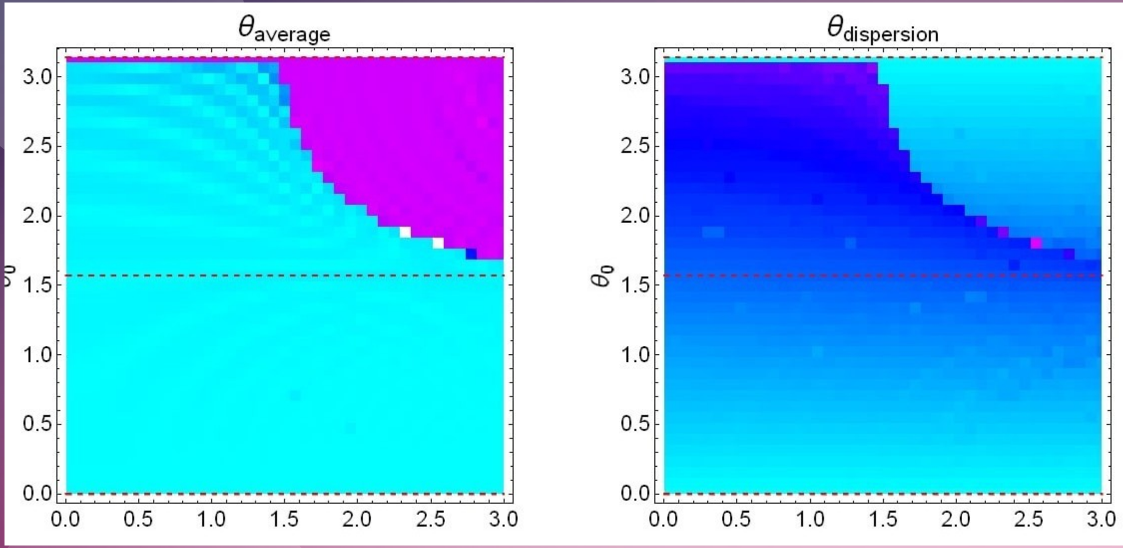
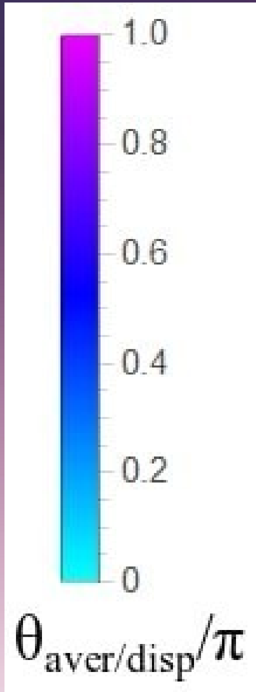
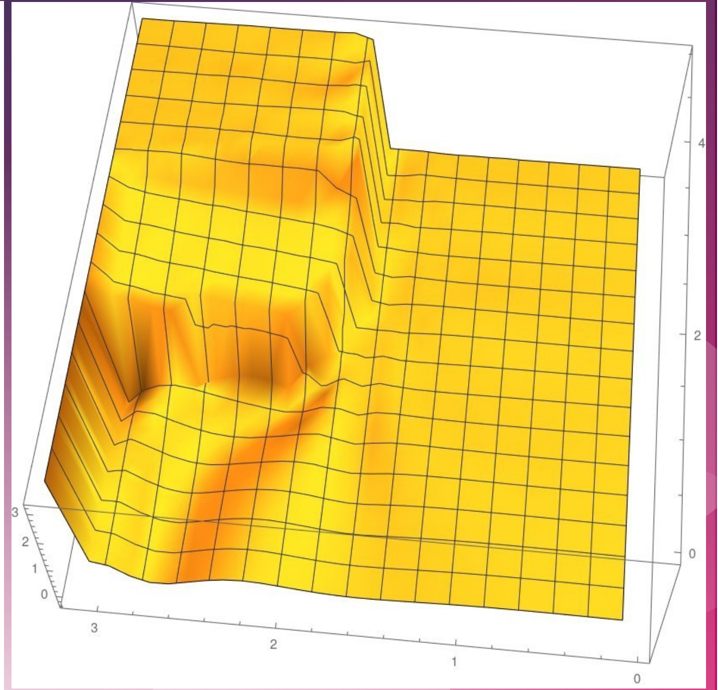
(*Обрахування середнього значення функції*)
mean = NIntegrate[θsol[t], {t, 0, tmax}, PrecisionGoal → 2, MaxRecursion → 3] / (tmax - 0);
(*Обрахування дисперсії у функції*)
dev = NIntegrate[Abs[θsol[t] - mean], {t, 0, tmax}, PrecisionGoal → 2, MaxRecursion → 3] / (tmax - 0);
```

Mode discoveries

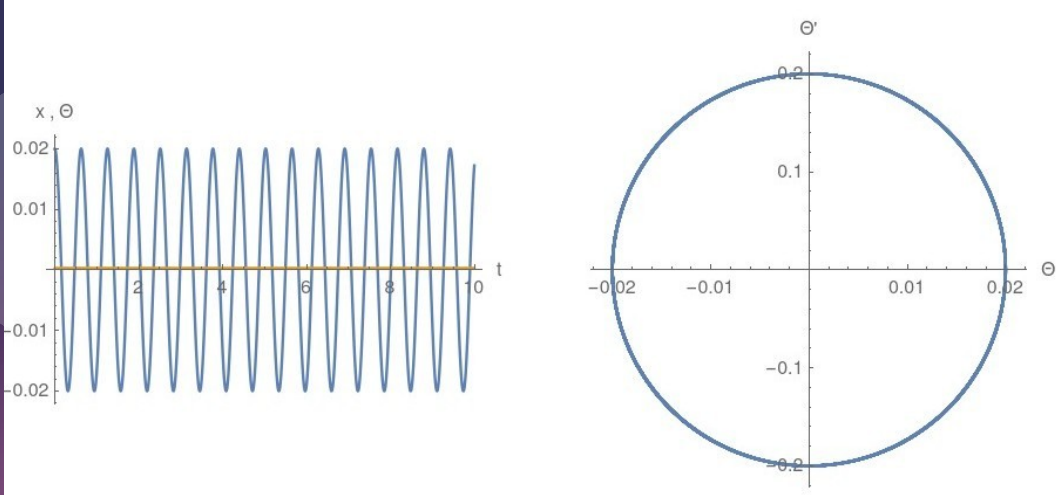




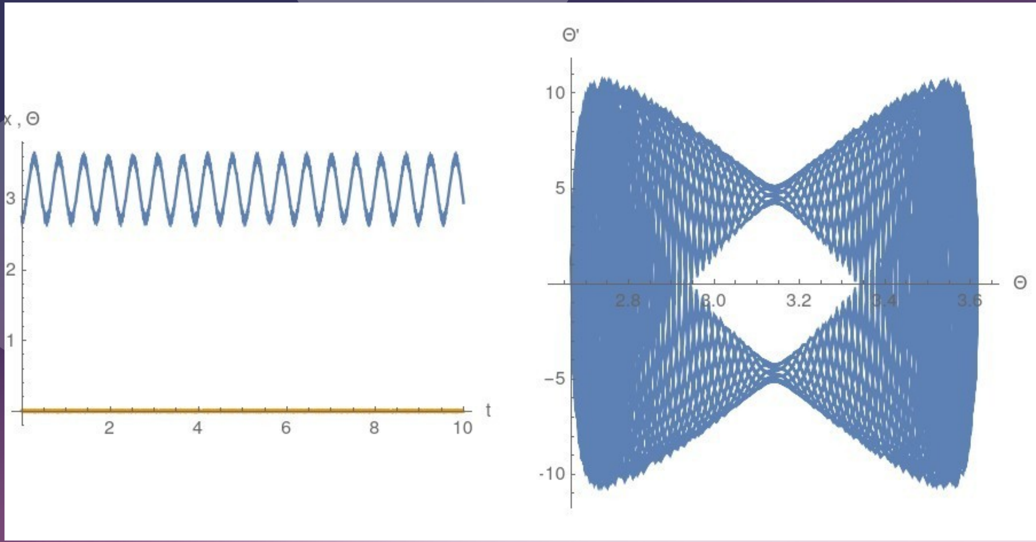




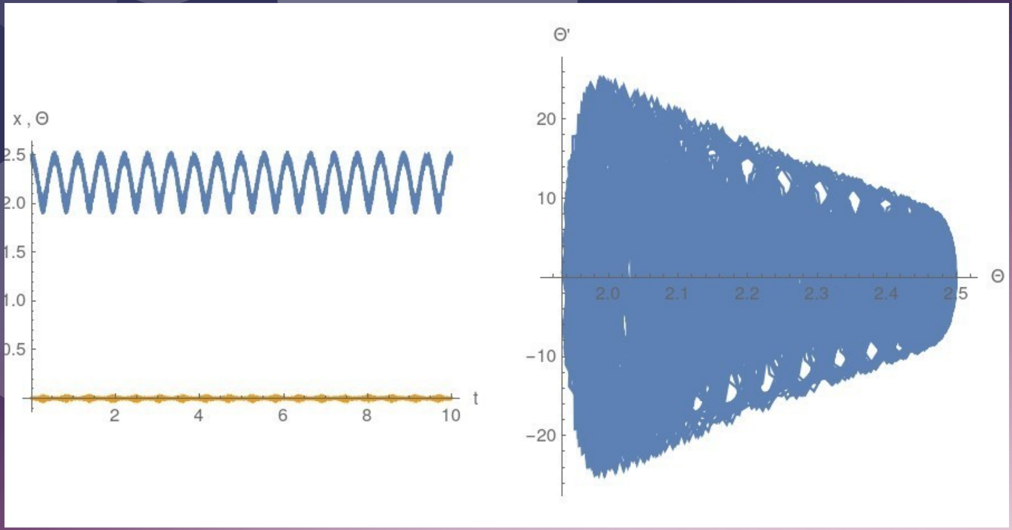
Mode №1



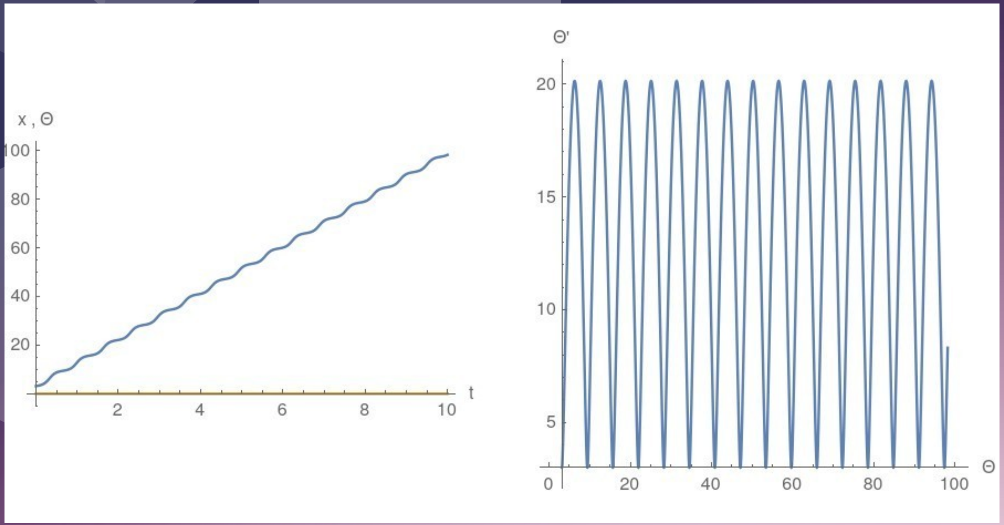
Mode №2



Mode №3



Mode №4



Effective dynamic potential

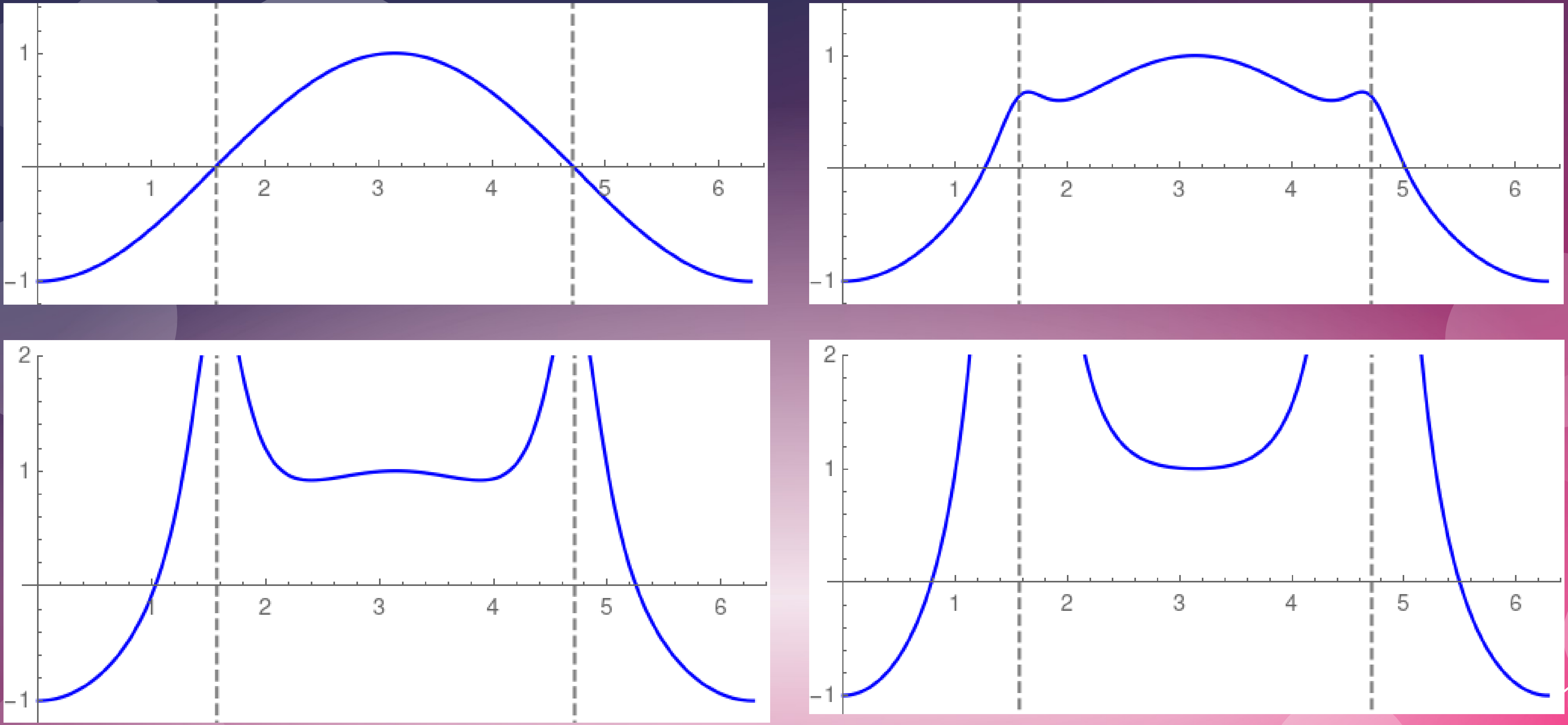
For this system we can’t find static effective potential. It can be approximately described with dynamic potential.

$$U_{еф} = mgL\left(-\cos[\theta] + \frac{a^2\gamma^2}{4\,gL}\sin[\theta]^2\right) \qquad (3)$$

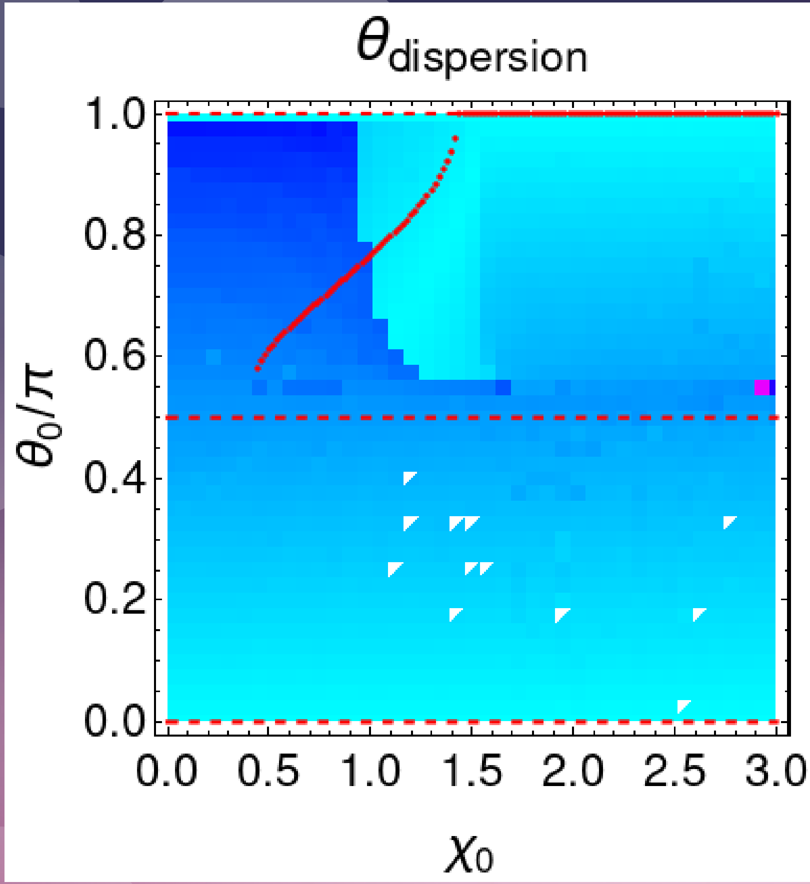
$$\gamma = \sqrt{\frac{k}{m + M\cos[\theta]^2}} \qquad (4)$$

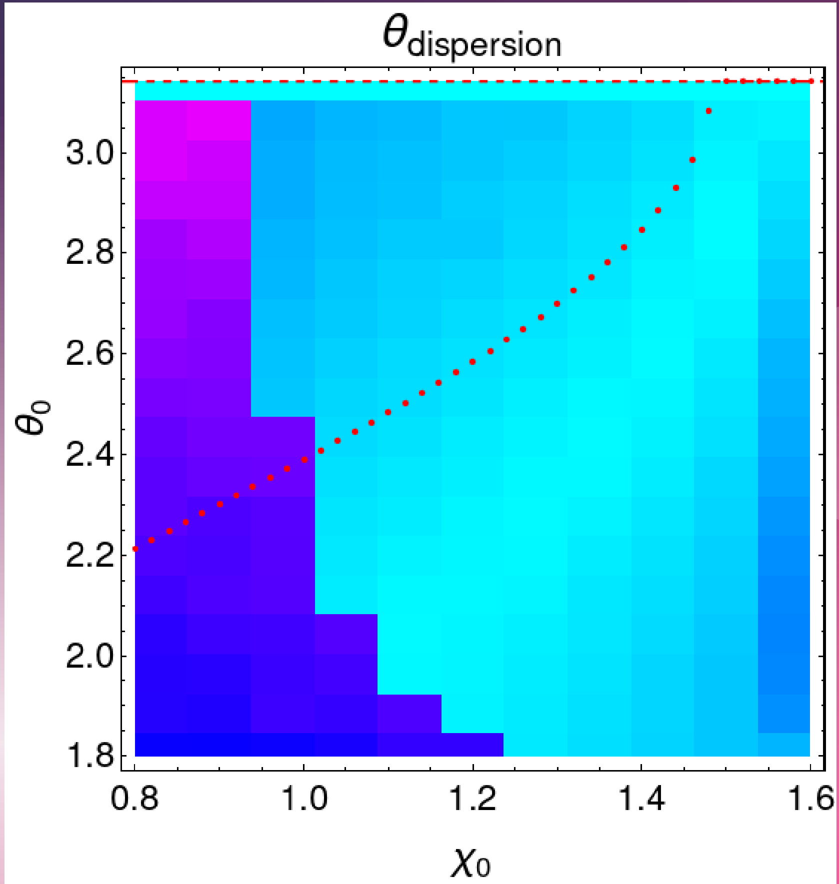
$$a = \frac{(m+M)\,g + \sqrt{(m+M)^2 g^2 + 2\,k\left(MgL\left(\cos[\theta] - \cos[\theta_0]\right) + \frac{k\,x_0^2}{2} - (m+M)\,g\,x_0\right)}}{k} \qquad (5)$$

Dynamic effective potential

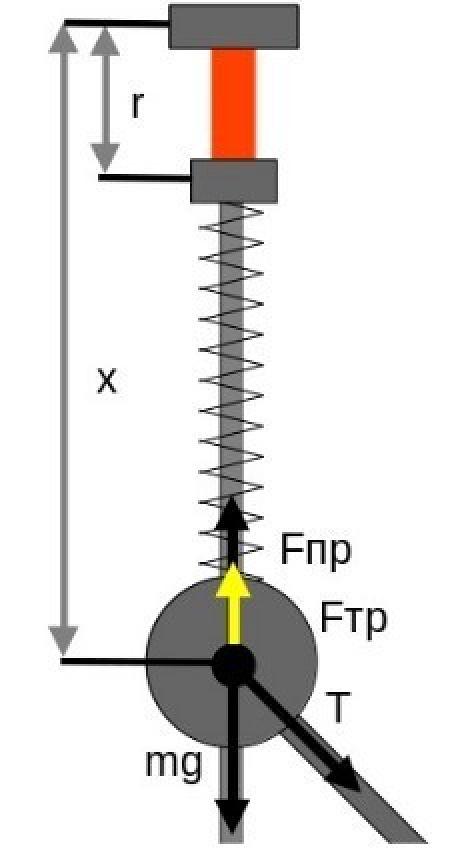


Effective dynamic potential minimum plots





Experimental model



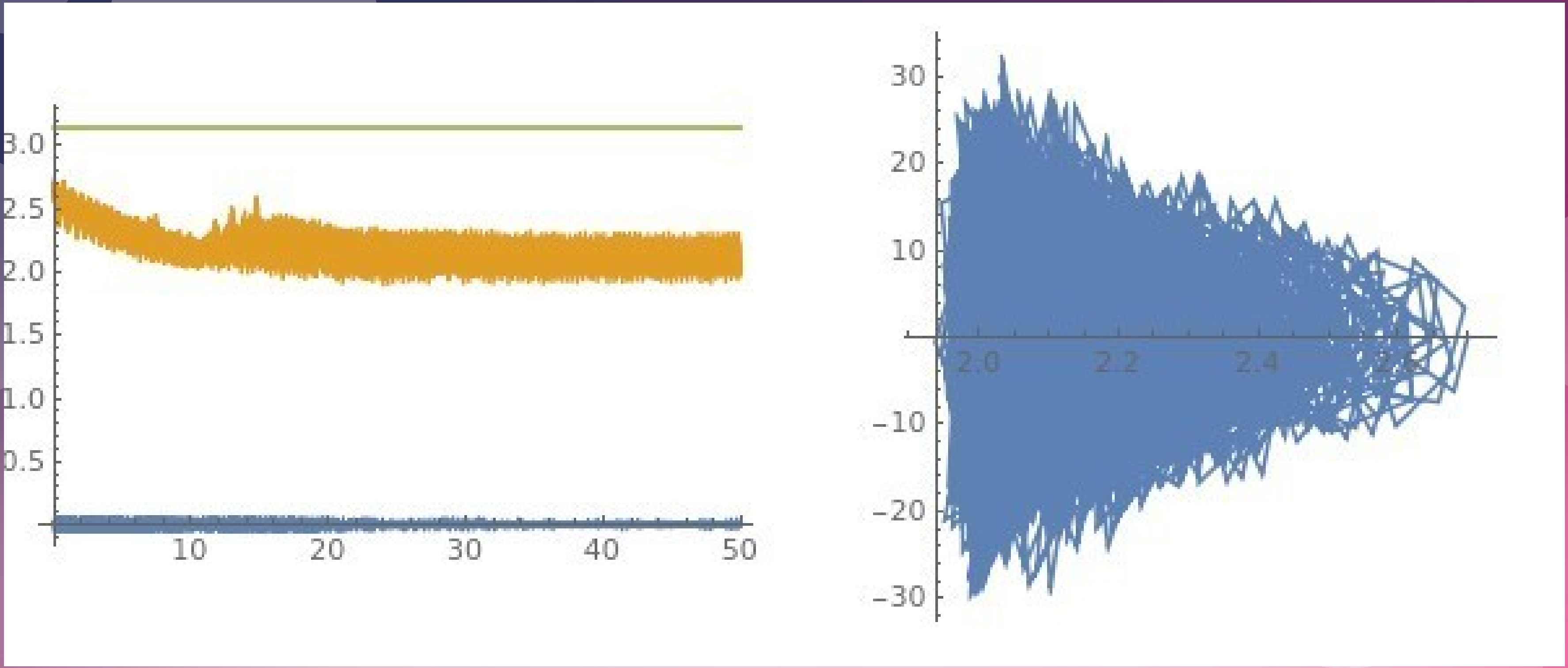
We’re adding friction to the systen and pereodic movement of the suspension point. This will allow system not to lose much energy on friction and will make it possible for stable nontrivial modes to exist.

$$x''=g+\frac{M\theta'^2 L+M(g-x'')\cos[\theta]}{m}\cos[\theta]-\frac{k}{m}x+\frac{f}{m} \quad (3)$$

$$\theta''=-\frac{g-x''[t]}{L}\sin[\theta] \quad (4)$$

$$f[t]=k\,r[t]-F_{mp} \quad (5)$$

Modulation results

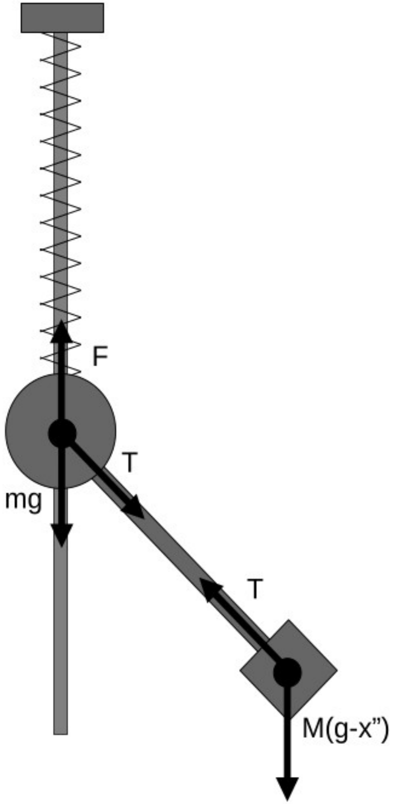


Results

1. New pendulum modes have been found.
2. Effective dynamic potential model is proposed.
3. Experimental model is proposed.

Thanks for your attention!

Additional slide №1



$$m\,x''=mg+T\cos[\alpha]-k\,x \qquad (1)$$

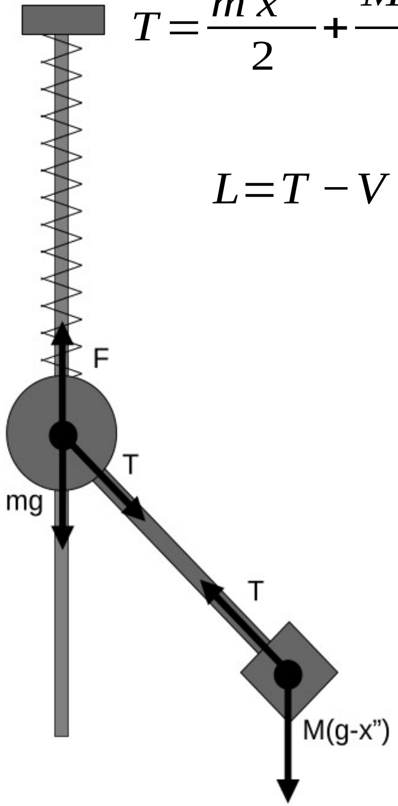
$$\theta''\,L\,M=-M\,(g-x'')\sin[\theta] \qquad (2)$$

$$T=M\,\theta'^2\,L+M\,(g-x'')\cos[\theta] \qquad (3)$$

$$x''=g+\frac{M\theta'^2 L+M(g-x'')\cos[\theta]}{m}\cos[\theta]-\frac{k}{m}x \qquad (4)$$

$$\theta''=-\frac{g-x''[t]}{L}\sin[\theta] \qquad (5)$$

Additional slide №2



$$T=\frac{m\,x'^2}{2}+\frac{M\left(\left(x'+L\theta'\sin[\theta]\right)^2+\left(L\theta'\cos[\theta]\right)^2\right)}{2} \qquad (1)$$

$$V=\frac{k\,x^2}{2}-mgx-Mg\left(x+L\cos[\theta]\right) \qquad (2)$$

$$L=T-V=\frac{(m+M)\,x'^2}{2}+\frac{M\left(L^2\theta'^2+2\,x'\,L\theta'\sin[\theta]\right)}{2}+(m+M)\,gx+MgL\cos[\theta]-\frac{k\,x^2}{2} \qquad (3)$$

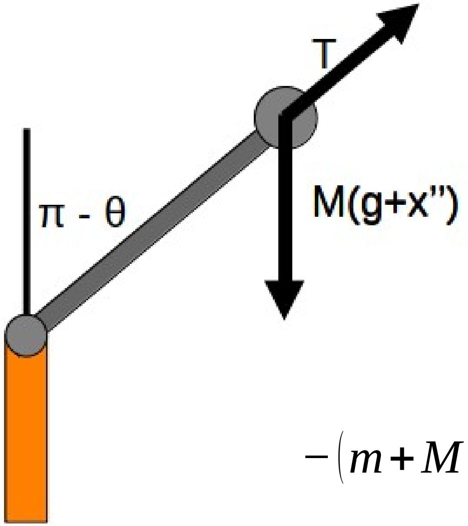
$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{x}}\right)-\frac{\delta L}{\delta x}=0 \qquad (4)$$

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{\theta}}\right)-\frac{\delta L}{\delta \theta}=0 \qquad (5)$$

$$x''=g+\frac{M\theta'^2L+M\left(g-x''\right)\cos[\theta]}{m}\cos[\theta]-\frac{k}{m}x \qquad (6)$$

$$\theta''=-\frac{g-x''[t]}{L}\sin[\theta] \qquad (7)$$

Additional slide №3



$$T = M\left(g + x''\right)\cos\left[\pi - \theta\right] \qquad (1)$$

$$F_{\text{пр эф}} = mg + T\cos\left[\pi - \theta\right] = mg + M\left(g + x''\right)\cos\left[\pi - \theta\right]^2 \qquad (2)$$

$$m_{эф} = \frac{F_{\text{пр эф ср}}}{g} = m + M\cos\left[\theta\right]^2 \, (3) \qquad \gamma = \sqrt{\frac{k}{m + M\cos\left[\theta\right]^2}} \qquad (4)$$

$$-\left(m + M\right) g\, x_0 - MgL\cos\left[\theta_0\right] + \frac{k\, x_0^2}{2} = -\left(m + M\right) g\, a - MgL\cos\left[\theta\right] + \frac{k\, a^2}{2} \qquad (5)$$

$$\frac{k}{2} a^2 - \left(m + M\right) g\, a + MgL\left(\cos\left[\theta_0\right] - \cos\left[\theta\right]\right) + \left(m + M\right) g\, x_0 - \frac{k\, x_0^2}{2} = 0 \qquad (6)$$

$$a = \frac{\left(m + M\right) g + \sqrt{\left(m + M\right)^2 g^2 + 2k\left(MgL\left(\cos\left[\theta\right] - \cos\left[\theta_0\right]\right) + \frac{k\, x_0^2}{2} - \left(m + M\right) g\, x_0\right)}}{k} \qquad (7)$$