Ordered Choices and Heterogeneity in Attribute Processing

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Abstract

A growing number of empirical studies involve the assessment of influences on a choice amongst ordered discrete alternatives. Ordered logit and probit models are well known, including extensions to accommodate random parameters and heteroscedasticity in unobserved variance. This paper extends the ordered choice random parameter model to permit random parameterisation of thresholds and decomposition to establish observed sources of systematic variation in the threshold parameter distribution. We illustrate the empirical gains of this model, over the traditional ordered choice model, by identifying candidate influences on the role that specific attributes play, in the sense of being ignored or not (including being aggregated where they are in common-metric units), in an individual's choice amongst unlabelled attribute packages of alternative tolled and nontolled routes for the commuting trip. The empirical ordering represents the number of attributes attended to from the full fixed set. The evidence suggests that there is significant heterogeneity associated with the thresholds, that can be connected to systematic sources associated with the respondent (that is, gender) and the choice experiment, and hence the generalised extension of the ordered choice model is an improvement, behaviourally, over the simpler model.

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1.0 Introduction

A growing number of empirical studies involve the assessment of influences on a choice amongst ordered discrete alternatives. Ordered logit and probit models are well known, including extensions to accommodate random parameters and heteroscedasticity in unobserved variance (see, for example, Bhat and Pulugurta, 1998; Greene, 2007). The ordered choice model allows for non-linear effects of any variable on the probabilities associated with each ordered level (see, for example, Eluru *et al.*, 2008). However, the traditional ordered choice model is potentially limited, behaviourally, in that it holds the threshold values to be fixed. This can lead to inconsistent (that is, incorrect) estimates of the effects of variables. Extending the ordered choice random parameter model to account for threshold random heterogeneity, as well as underlying systematic sources of explanation for unobserved heterogeneity, is a logical extension in line with the growing interest in choice analysis in establishing additional candidate sources of observed and unobserved taste heterogeneity. ¹

A substantive application herein is used to illustrate the behavioural gains from generalising the ordered choice model to accommodate random thresholds in the presence of random parameters. It is focused on the influences on the role that a specific attribute processing strategy, of preserving each attribute or ignoring it, plays when choosing amongst unlabelled attribute packages of alternative tolled and non-tolled routes for the commuting trip in a stated choice experiment (see Hensher et al., 2005; Hensher, 2006b, 2008). The ordering represents the number of attributes attended to from the full set. Despite a growing number of studies focusing on these issues (see, for example, Swait, 2001; Cantillo et al., 2006; Hensher, 2006; Campbell et al., 2008), the entire domain of every attribute is treated as relevant to some degree, and included in the utility expressions for every individual. While acknowledging the extensive study of non-linearity in attribute specification, which permits varying marginal (dis)utility over an attribute's range, including account for asymmetric preferences under conditions of gain and loss (see Hess et al., 2008), this is not the same as establishing ex-ante the extent to which a specific attribute might be totally excluded from consideration for all manner of reasons, including the influence of the design of a choice experiment when stated choice data is being used.

¹A number of authors have introduced random thresholds (for example, Cameron and Heckman, 1998; Cunha *et al.*, 2007; Eluru *et al.*, 2008) but have not integrated this into a generalised model with random parameters and/or decomposition of random thresholds by systematic sources.

The paper is organised as follows. The next section sets out the econometric specification of the generalised ordered choice model, focusing on the derivation of the random threshold structure and its behavioural appeal. We then introduce the empirical context used to test this new model, focusing on the design of the stated choice experiment and associated questions used to define the choice setting and the process used by each respondent in establishing relevance of each attribute. The empirical analysis that follows presents the estimated models — a traditional model and the extended ordered choice model, together with the associated marginal effects that are the basis of behavioural assessment. The paper concludes with some observations on the merits of the extended model form.

2.0 Generalisations of the Ordered Choice Model to Accommodate Preference Heterogeneity

2.1 The traditional ordered probit model

The ordered probit model was proposed by Zavoina and McElvey (1975) for the analysis of categorical, non-quantitative choices, outcomes, and responses. Familiar applications now include bond ratings, discrete opinion surveys such as those on political questions, obesity measures (Greene *et al.*, 2008), preferences in consumption, and satisfaction and health status surveys such as those analysed by Boes and Winkelmann (2004, 2007).

The model foundation is an underlying random utility or latent regression model,

$$y_i^* = \mathbf{\beta}' \mathbf{x}_i + \mathbf{\varepsilon}_i, \tag{1}$$

in which the continuous latent utility, y_i^* is observed in discrete form through a censoring mechanism (equation (2)):

$$y_{i} = 0 \quad \text{if} \quad \mu_{-1} < y_{i}^{*} \leq \mu_{0},$$

$$= 1 \quad \text{if} \quad \mu_{0} < y_{i}^{*} \leq \mu_{1},$$

$$= 2 \quad \text{if} \quad \mu_{1} < y_{i}^{*} \leq \mu_{2},$$

$$= \dots$$

$$= J \quad \text{if} \quad \mu_{j-1} < y_{i}^{*} \leq \mu_{J}.$$
(2)

The model contains the unknown marginal utilities, β , as well as J+2 unknown threshold parameters, μ_j , all to be estimated using a sample of n observations, indexed by $i=1,\ldots,n$. The data consist of the covariates, \mathbf{x}_i and the observed discrete outcome, $y_i=0,1,\ldots,J$. The assumption of

the properties of the 'disturbance', ε_i , completes the model specification. The conventional assumptions are that ε_i is a continuous disturbance with conventional cumulative distribution function (cdf), $F(\varepsilon_i|\mathbf{x}_i) = F(\varepsilon_i)$ with support equal to the real line, and with density $f(\varepsilon_i) = F'(\varepsilon_i)$. The assumption of the distribution of ε_i includes independence from (or exogeneity of) \mathbf{x}_i . The probabilities associated with the observed outcomes are given as equation (3):

$$Prob[y_i = j | \mathbf{x}_i]$$

$$= Prob[\varepsilon_i \leq \mu_i - \beta' \mathbf{x}_i] - Prob[\mu_{i-1} - \beta' \mathbf{x}_i], \qquad j = 0, 1, \dots, J. \quad (3)$$

Several normalisations are needed to identify the model parameters. First, given the continuity assumption, in order to preserve the positive signs of the probabilities, we require $\mu_j > \mu_{j-1}$. Second, if the support is to be the entire real line, then $\mu_{-1} = -\infty$ and $\mu_J = +\infty$. Finally, assuming (as we will) that \mathbf{x}_i contains a constant term, we will require that $\mu_0 = 0$. With a constant term present, if this normalisation is not imposed, then adding any nonzero constant to μ_0 and the same constant to the intercept term in $\boldsymbol{\beta}$ will leave the probability unchanged. Given the assumption of an overall constant, only J-1 threshold parameters are needed to partition the real line into the J+1 distinct intervals.

Given that data such as ranking data defining the observed ordered choice contain no unconditional information on scaling of the underlying unobserved variable, if y_i^* is scaled by any positive value, then scaling the unknown μ_j and β by the same value preserves the observed outcomes; and hence a free unconditional variance parameter, $\operatorname{Var}[\varepsilon_i] = \sigma_\varepsilon^2$, is not identified without further restriction. We thus impose the identifying restriction $\sigma_\varepsilon = a$ known constant, $\bar{\sigma}$. The usual approach to this normalisation, assuming that ε is independent of \mathbf{x} , is to assume that $\operatorname{Var}[\varepsilon_i|\mathbf{x}_i] = 1$ in the probit model and $\pi^2/3$ in the logit model — in both cases to eliminate the free structural scaling parameter. The standard treatments in the received literature complete the ordered choice model by assuming either a standard normal distribution for ε_i , producing the ordered probit model or a standardised logistic distribution (mean zero, variance $\pi^2/3$), which produces the ordered logit model. Applications appear to be well divided between the two. A compelling case for a particular distribution remains to be put forth.

With the full set of normalisations in place, the likelihood function for estimation of the model parameters is based on the implied probabilities given in equation (4):

Prob[
$$y_i = j | \mathbf{x}_i$$
]
= $F(\mu_i - \beta' \mathbf{x}_i) - F(\mu_{i-1} - \beta' \mathbf{x}_i) > 0, \quad j = 0, 1, ..., J.$ (4)

Estimation of the parameters is a straightforward problem in maximum likelihood estimation (see, for example, Pratt, 1981; Greene, 2008). Interpretation of the model parameters is, however, much less so (see, for example, Daykin and Moffitt, 2002). There is no natural conditional mean function, so in order to attach behavioural meaning to the parameters, one typically refers to the probabilities themselves. The partial effects in the ordered choice model are:

$$\frac{\partial \operatorname{Prob}[\mathbf{y}_{i} = j | \mathbf{x}_{i}]}{\partial \mathbf{x}_{i}} = \left[f(\mathbf{\mu}_{j-1} - \boldsymbol{\beta}' \mathbf{x}_{i}) - f(\mathbf{\mu}_{j} - \boldsymbol{\beta}' \mathbf{x}_{i}) \right] \boldsymbol{\beta}. \tag{5}$$

The result shows that neither the sign nor the magnitude of a coefficient is informative about the corresponding behavioural characteristic in the model, so the direct interpretation of the coefficients (or their 'significance') is fundamentally ambiguous. A counterpart result for a dummy variable in the model would be obtained by using a difference of probabilities, rather than a derivative (Boes and Winkelmann, 2007; Greene, 2008, Ch. E22). One might also be interested in cumulative values of the partial effects, such as shown in equation (6) (see, for example, Brewer *et al.*, 2006). The last term in this set is zero by construction.

$$\frac{\partial \operatorname{Prob}[y_i \leq j | \mathbf{x}_i]}{\partial \mathbf{x}_i} = \left(\sum_{m=0}^{j} \left[f(\mu_{m-1} - \boldsymbol{\beta}' \mathbf{x}_i) - f(\mu_m - \boldsymbol{\beta}' \mathbf{x}_i) \right] \right) \boldsymbol{\beta}. \tag{6}$$

2.2 A generalised ordered choice model

A number of authors, beginning with Terza (1985), have questioned some of the less flexible aspects of the model specification. The partial effects shown above vary with the data and the parameters. It can be shown that for the probit and logit models, this set of partial derivatives will change sign exactly once in the sequence from 0 to J, a property that Boes and Winkelmann (2007) label the 'single crossing' characteristic. Boes and Winkelmann (2007) also note that for any two continuous covariates, x_{ik} and x_{il}

$$\frac{\partial \operatorname{Prob}[y_i = j | \mathbf{x}_i] / \partial x_{i,k}}{\partial \operatorname{Prob}[y_i = j | \mathbf{x}_i] / \partial x_{i,l}} = \frac{\beta_k}{\beta_l}.$$
 (7)

This result in (7) is independent of the outcomes. The ordered choice models above have the property in equation (8); that is, the partial effects are each a multiple of the same β :

$$\partial \operatorname{Prob}[y_i \geqslant j | \mathbf{x}_i] / \partial \mathbf{x}_i = K_j \boldsymbol{\beta},$$
 (8)

where K_j depends on X_j . This is a feature of the model that has been labelled the 'parallel regressions' assumption. Another way to view this feature of

the ordered choice model is through the J implied binary choices implied by (8). Let z_{ij} denote the binary variable defined by

$$z_{ij} = 1$$
 if $y > j$, $j = 0, 1, ..., J - 1$.

The choice model implies

$$Prob[z_{ij} = 1 | x_i] = F(\beta' x_i - \mu_j).$$

The threshold parameter can be absorbed into the constant term. In principle, one can fit these J-1 binary choice models separately. That the same β appears in all of the models is implied by the ordered choice model. However, one need not impose this restriction; the binary choice models can be fit separately and independently. Thus, the null hypothesis of the ordered choice model is that the \(\beta \) in the binary choice equations are all the same (apart from the constant terms). A standard test of this null hypothesis, due to Brant (1990), is used to detect the condition that the β_i vectors are different. The Brant test frequently rejects the null hypothesis of a common slope vector in the ordered choice model. It is unclear what the alternative hypothesis should be in this context. The generalised ordered choice model that might seem to be the natural alternative is, in fact, internally inconsistent — it does not constrain the probabilities of the outcomes to be positive. It would seem that the Brant test is more about functional form or, perhaps, some other specification error (see Greene and Hensher, 2009, Ch. 6).

Recent analyses, for example, Long (1993), Long and Frees (2005), and Williams (2006), have proposed a 'generalised ordered choice model'. An extended form of the ordered choice model that has attracted much (perhaps most) of the recent attention, is the 'Generalised Ordered Logit' (or Probit) model for example, by Williams (2006). This model is defined in equation (9):

 $Prob[y_i = j | \mathbf{x}_i]$

=
$$\operatorname{Prob}[\varepsilon_i \leqslant \mu_j - \beta_j' \mathbf{x}_i] - \operatorname{Prob}[\mu_{j-1} - \beta_{j-1}' \mathbf{x}_i], \quad j = 0, 1, \dots, J,$$
 (9)

where $\beta_{-1} = 0$ (see, for example, Long, 1997; Long and Frees, 2006; Williams, 2006). The extension provides for a separate vector of marginal utilities for each *j*th outcome. Bhat and Zhao (2002) introduce heteroscedasticity across observational units, in a spatial ordered response analysis context, along the lines of the generalised ordered logit form.

The generalisation of the model suggested above deals with both problems (single crossing and parallel regressions), but it creates new ones. The heterogeneity in the parameter vector is an artefact of the coding of the dependent variable, not a manifestation of underlying heterogeneity in the dependent variable induced by behavioural differences. It is unclear what it means for the marginal utility parameters to be structured in this way. Consider, for example, that there is no underlying structure that could be written down in such a way as to provide a means of simulating the data-generating mechanism. By implication, $y_i^* = \beta_j' \mathbf{x}_i + \epsilon_i$ if $y_i = j$; that is, the model structure is endogenous — one could not simulate a value of y_i from the data-generating mechanism without knowing in advance the value being simulated. There is no reduced form. The more difficult problem of this generalisation is that the probabilities in this model need not be positive, and there is no parametric restriction (other than the restrictive model version we started with) that could achieve this. The probability model is internally inconsistent. The restrictions would have to be functions of the data. The problem is noted by Williams (2006), but dismissed as a minor issue. Boes and Winkelmann (2007) suggest that the problem could be handled through a 'nonlinear specification'. Essentially, this generalised choice model does not treat the outcome as a single choice, even though that is what it is.

To put a more positive view, we might interpret this as a semiparametric approach to modelling what is underlying heterogeneity. However, it is not clear why this heterogeneity should be manifest in parameter variation across the outcomes instead of across the individuals in the sample. One would assume that the failure of the Brant test to support the model with parameter homogeneity is, indeed, signalling some failure of the model. A shortcoming of the functional form as listed above (compared to a different internally consistent specification) is certainly a possibility. We hypothesise that it might also be picking up unobserved heterogeneity across individuals. The model we develop here accounts for individual heterogeneity in several possible forms.

2.3 Modelling observed and unobserved heterogeneity

Since Terza (1985), with the exception of Pudney and Shields (2000), most of the 'generalisations' suggested for the ordered choice models have been about functional form — the single crossing feature and the parallel regressions (see also, Greene, 2008). Our interest in this paper is, rather, in a specification that accommodates both observed and unobserved heterogeneity across individuals. We suggest that the basic model structure, when fully specified, provides for sufficient non-linearity to capture the important features of choice behaviour. The generalisation that interests us herein will incorporate both observed and unobserved heterogeneity in the model itself.

The basic model assumes that the thresholds μ_j are the same for every individual in the sample. Terza (1985), Bhat and Pulugurta (1998),

Pudney and Shields (2000), Boes and Winkelmann (2007), and Greene *et al.* (2008), all present cases that suggest that individual variation in the set of thresholds is a degree of heterogeneity that is likely to be present in the data, but is not accommodated in the model. Pudney and Shields discuss a clear example in the context of job promotion, in which the steps on the promotion ladder for nurses are somewhat individual-specific.

Greene (2002, 2008) argues that the fixed parameter version of the ordered choice model, and more generally, many microeconometric specifications, do not adequately account for the underlying, unobserved heterogeneity likely to be present in observed data. Further extensions of the ordered choice model presented in Greene (2008) include full random parameters treatments and discrete approximations under the form of latent class, or finite mixture models. These two specific extensions are also listed by Boes and Winkelmann (2004, 2007), who also describe a common effects model for panel data, and Bhat and Pulugurta (1998) as candidates for extending the model.

The model that assumes homogeneity of the preference parameters, β , across individuals, also assumes homogeneity in the scaling of the random term, ε_i ; that is, the homoscedasticity assumption, $\operatorname{Var}[\varepsilon_i | \mathbf{x}_i] = 1$ is restrictive in the same way that the homogeneity assumption is. Heteroscedasticity in terms of observables in the ordered choice model is proposed in Greene (1997) and reappears as a theme in Williams (2006).

The model proposed here generalises the ordered choice model in the directions of accommodating heterogeneity, rather than in the direction of adding nonlinearities to the underlying functional form. The earliest extensions of the ordered choice model focused on the threshold parameters. Terza's (1985) extension suggested

$$\mu_{ii} = \mu_i + \delta' \mathbf{z}_i, \tag{10}$$

where \mathbf{z}_i are individual-specific exogenous variables that represent sources of systematic variation around the mean estimate of a threshold parameter. The analysis of this model continued with Pudney and Shields's (2000) 'Generalised Ordered Probit Model', whose motivation, like Terza's, was to accommodate *observable* individual heterogeneity in the threshold parameters as well as in the mean of the regression. We (and Pudney and Shields) note an obvious problem of identification in this specification. Consider the generic probability with this extension,

Prob
$$[y_i \leq j | \mathbf{x}_i, \mathbf{z}_i] = F(\mu_j + \delta' \mathbf{z}_i - \beta' \mathbf{x}_i)$$

= $F[\mu_i - (\delta^* \mathbf{z}_i + -\beta' \mathbf{x}_i)], \qquad \delta^* = -\delta.$ (11)

It is less than obvious whether the variables \mathbf{z}_i are actually in the threshold or in the mean of the regression. Either interpretation is consistent with the

model. Pudney and Shields argue that the distinction is of no substantive consequence for their analysis.

Formal modelling of heterogeneity in the parameters as representing a feature of the underlying data, also appears in Greene (2002) (version 8.0) and Boes and Winkelmann (2004), both of whom suggest a random parameters (RP) approach to the model. In Boes and Winkelmann, it is noted that the nature of an RP specification induces heteroscedasticity, and could be modelled as such. The model would appear as follows:

$$\mathbf{\beta}_i = \mathbf{\beta} + \mathbf{u}_i,\tag{12}$$

where $\mathbf{u}_i \sim \mathbb{N}[\mathbf{0}, \mathbf{\Omega}]$. Inserting this in the base case model and simplifying, we obtain equation (13):

$$\operatorname{Prob}[y_{i} \leq j | \mathbf{x}_{i}] = \operatorname{Prob}[\varepsilon_{i} + \mathbf{u}_{i}' \mathbf{x}_{i} \leq \mu_{j} - \boldsymbol{\beta}' \mathbf{x}_{i}]$$
$$= F\left(\frac{\mu_{j} - \boldsymbol{\beta}' \mathbf{x}_{i}}{\sqrt{1 + \mathbf{x}_{i}' \boldsymbol{\Omega} \mathbf{x}_{i}}}\right). \tag{13}$$

Equation (13) could be estimated by ordinary means, albeit with a new source of nonlinearity — the elements of Ω must now be estimated as well.² Boes and Winkelmann (2004, 2007) did not pursue this approach. Greene (2002) analyses essentially the same model, but proposes to estimate the parameters by maximum simulated likelihood.

Curiously, none of the studies listed above focus on the issue of scaling, although Williams (2006), citing Allison (1999) does mention it. A heteroscedastic ordered probit model with the functional form in (14) appears at length in Greene (1997), and is discussed in some detail in Williams (2006):

$$Var[\varepsilon_i | \mathbf{h}_i] = \exp(\gamma' \mathbf{h}_i)^2. \tag{14}$$

In microeconomic data, scaling of the underlying preferences is as important a source of heterogeneity as displacement of the mean, perhaps even more so, but it has received considerably less attention than heterogeneity in location.

In what follows, we will propose a formulation of the ordered choice model that treats heterogeneity in a unified, internally consistent fashion. The model contains three points at which individual heterogeneity can substantively appear: in the random utility model (the marginal utilities), in the threshold parameters, and in the scaling (variance) of the random

²The authors' suggestion that this could be handled semiparametrically without specifying a distribution for \mathbf{u}_i is incorrect, because the resulting heteroscedastic probability written above only preserves the standard normal form assumed if \mathbf{u}_i is normally distributed as well as ε_i .

components. As argued above, this form of treatment seems more likely to capture the salient features of the data-generating mechanism than the received 'generalised ordered logit model', which is more narrowly focused on functional form.

2.4 Random thresholds and heterogeneity in the ordered choice model We depart from the base case of the usual ordered choice model,

Prob[
$$y_i = j | \mathbf{x}_i$$
]
= $F(\mu_j - \beta' \mathbf{x}_i) - F(\mu_{j-1} - \beta' \mathbf{x}_i) > 0, \quad j = 0, 1, ..., J.$ (15)

In order to model heterogeneity in the utility functions across individuals, we construct a hierarchical model in which the coefficients vary with observable variables, z_i (typically demographics such as age and gender), and randomly due to individual specific unobservables, v_i . The coefficients appear as:

$$\boldsymbol{\beta}_i = \boldsymbol{\beta} + \Delta \mathbf{z}_i + \mathbf{\Gamma} \mathbf{v}_i, \tag{16}$$

where Γ is a lower triangular matrix and $\mathbf{v}_i \sim \mathbb{N}[\mathbf{0}, \mathbf{I}]$. The coefficient vector in the utility function, $\boldsymbol{\beta}_i$ is normally distributed across individuals with conditional mean

$$E[\mathbf{\beta}_i | \mathbf{x}_i, \mathbf{z}_i] = \mathbf{\beta} + \mathbf{\Delta} \mathbf{z}_i, \tag{17}$$

and conditional variance

$$Var[\boldsymbol{\beta}_i|\mathbf{x}_i,\mathbf{z}_i] = \boldsymbol{\Gamma} \boldsymbol{\Gamma} \boldsymbol{\Gamma}' = \boldsymbol{\Omega}. \tag{18}$$

The model is formulated with Γv_i rather than, say just v_i with covariance matrix Ω purely for convenience in setting up the estimation method. This is a random parameters formulation that appears elsewhere, for example, Greene (2002, 2005). The random effects model is a special case in which only the constant is random. The Mundlak (1978) and Chamberlain (1980) approach to modelling fixed effects is also accommodated by letting $\mathbf{z}_i = \bar{\mathbf{x}}_i$ in the equation for the overall constant term.

We are also interested in allowing the thresholds to vary across individuals. See, for example, King *et al.* (2004) for a striking demonstration of the pay-off to this generalisation. The thresholds are modelled randomly and non-linearly as

$$\mu_{ij} = \mu_{i,j-1} + \exp(\alpha_j + \delta' \mathbf{r}_i + \sigma_j w_{ij}), \quad w_{ij} \sim \mathbb{N}[0,1], \tag{19}$$

with normalisations and restrictions $\mu_{-1} = -\infty$, $\mu_0 = 0$, $\mu_J = +\infty$. For the remaining thresholds, we have (20).

$$\mu_{1} = \exp(\alpha_{1} + \boldsymbol{\delta}' \mathbf{r}_{i} + \sigma_{1} w_{j1}) = \exp(\boldsymbol{\delta}' \mathbf{r}_{i}) \exp(\alpha_{1} + \sigma_{1} w_{j1}),$$

$$\mu_{2} = \exp(\boldsymbol{\delta}' \mathbf{r}_{i}) [\exp(\alpha_{1} + \sigma_{1} w_{j1}) + \exp(\alpha_{2} + \sigma_{2} w_{j2})],$$

$$\mu_{j} = \exp(\boldsymbol{\delta}' \mathbf{r}_{i}) \left(\sum_{m=1}^{j} \exp(\alpha_{m} + \sigma_{m} w_{im}) \right), \qquad j = 1, \dots, J-1,$$

$$\mu_{J} = +\infty.$$

$$(20)$$

Alhough it is relatively complex, this formulation is necessary for several reasons:

- 1. It ensures that all of the thresholds are positive.
- 2. It preserves the ordering of the thresholds.
- 3. It incorporates the necessary normalisations.

Most importantly, it also allows observed variables and unobserved heterogeneity to play a role both in the utility function and in the thresholds. The thresholds, like the regression itself, are shifted by both observable (\mathbf{r}_i) and unobservable (w_{ij}) heterogeneity. The model is fully consistent, in that the probabilities are all positive and sum to one by construction. If $\delta = \mathbf{0}$ and $\sigma_j = 0$, then the original model is returned, with $\mu_1 = \exp(\alpha_1)$, $\mu_2 = \mu_1 + \exp(\alpha_2)$ and so on. Note that if the threshold parameters were specified as linear functions rather than as in (19), then it would not be possible to identify separate parameters in the regression function and in the threshold functions.

Finally, we allow for individual heterogeneity in the variance of the utility function as well as in the mean. This is likely to be an important feature of data on individual behaviour. The disturbance variance is allowed to be heteroscedastic, now specified randomly as well as deterministically. Thus,

$$Var[\varepsilon_i | \mathbf{h}_i, e_i] = \sigma_i^2 = \exp(\gamma' \mathbf{h}_i + \tau e_i)^2, \tag{21}$$

where $e_i \sim \mathbb{N}[0, 1]$. Let $\mathbf{v}_i = (v_{i1}, \dots, v_{iK})'$ and $\mathbf{w}_i = (w_{i1}, \dots, w_{i,J-1})'$. Combining all terms, the conditional probability of outcome j is

$$Prob[y_{i} = j | \mathbf{x}_{i}, \mathbf{z}_{i}, \mathbf{h}_{i}, \mathbf{r}_{i}, \mathbf{v}_{i}, \mathbf{w}_{i}, e_{i}]$$

$$= F \left[\frac{\mu_{ij} - \mathbf{\beta}_{i}' \mathbf{x}_{i}}{\exp(\mathbf{\gamma}' \mathbf{h}_{i} + \tau e_{i})} \right] - F \left[\frac{\mu_{i,j-1} - \mathbf{\beta}_{i}' \mathbf{x}_{i}}{\exp(\mathbf{\gamma}' \mathbf{h}_{i} + \tau e_{i})} \right], \tag{22}$$

where it is noted, once again, that both μ_{ij} and β_i vary with observed variables and with unobserved random terms. The log likelihood is constructed from the terms in (22). However, the probability in (22) contains the unobserved random terms, v_i , w_i , and e_i . The term that

enters the log likelihood function for estimation purposes must be unconditional on the unobservables. Thus, they are integrated out, to obtain the unconditional probabilities,

$$Prob[y_{i} = j | \mathbf{x}_{i}, \mathbf{z}_{i}, \mathbf{h}_{i}, \mathbf{r}_{i}]$$

$$= \int_{\mathbf{v}_{i}, \mathbf{w}_{i}, e_{i}} \left(F \left[\frac{\mu_{ij} - \mathbf{\beta}_{i}' \mathbf{x}_{i}}{\exp(\mathbf{\gamma}' \mathbf{h}_{i} + \tau e_{i})} \right] - F \left[\frac{\mu_{i,j-1} - \mathbf{\beta}_{i}' \mathbf{x}_{i}}{\exp(\mathbf{\gamma}' \mathbf{h}_{i} + \tau e_{i})} \right] \right)$$

$$\times f(\mathbf{v}_{i}, \mathbf{w}_{i}, e_{i}) \, d\mathbf{v}_{i} \, d\mathbf{w}_{i} \, de_{i}. \tag{23}$$

The model is estimated by maximum simulated likelihood. The simulated log likelihood function is given in (24):

 $\log L_S(\boldsymbol{\beta}, \boldsymbol{\Delta}, \boldsymbol{\alpha}, \boldsymbol{\delta}, \boldsymbol{\gamma}, \boldsymbol{\Gamma}, \boldsymbol{\sigma}, \tau)$

$$= \sum_{i=1}^{n} \log \frac{1}{M} \sum_{m=1}^{M} \left(F\left[\frac{\mu_{ij,m} - \mathbf{\beta}'_{i,m} \mathbf{x}_{i}}{\exp(\gamma' \mathbf{h}_{i} + \tau e_{i,m})} \right] - F\left[\frac{\mu_{i,j-1,m} - \mathbf{\beta}'_{i,m} \mathbf{x}_{i}}{\exp(\gamma' \mathbf{h}_{i} + \tau e_{i,m})} \right] \right). \tag{24}$$

The set $\mathbf{v}_{i,m}$, $\mathbf{w}_{i,m}$, $e_{i,m}$ is of M multivariate random draws for the simulation.³ This is the model in its full generality. Whether a particular data set will be rich enough to support this much parameterisation, particularly the elements of the covariances of the unobservables in Γ , is an empirical question that will depend on the application.

One is typically interested in estimation of parameters such as β in (24) to learn about the impact of the observed independent variables on the outcome of interest. This generalised ordered choice model contains four points at which changes in observed variables can induce changes in the probabilities of the outcomes: in the thresholds, μ_{ij} ; in the marginal utilities, β_i ; in the utility function, \mathbf{x}_i ; and in the variance, σ_i^2 . These could involve different variables or they could have variables in common. Again, demographics such as age, sex, and income, could appear anywhere in the model. In principle, then, if we are interested in all of these, we should compute all the partial effects,

$$\frac{\partial \operatorname{Prob}(y_i = j | \mathbf{x}_i, \mathbf{z}_i, \mathbf{r}_i, \mathbf{h}_i)}{\partial \mathbf{x}_i} = \operatorname{direct} \text{ of variables in the utility function,}$$

$$\frac{\partial \operatorname{Prob}(y_i = j | \mathbf{x}_i, \mathbf{z}_i, \mathbf{r}_i, \mathbf{h}_i)}{\partial \mathbf{z}_i} = \operatorname{indirect} \text{ of variables that affect the parameters } \boldsymbol{\beta},$$

$$\frac{\partial \operatorname{Prob}(y_i = j | \mathbf{x}_i, \mathbf{z}_i, \mathbf{r}_i, \mathbf{h}_i)}{\partial \mathbf{h}_i} = \operatorname{indirect} \text{ of variables that affect the variance of } \boldsymbol{\epsilon}_i,$$

$$\frac{\partial \operatorname{Prob}(y_i = j | \mathbf{x}_i, \mathbf{z}_i, \mathbf{r}_i, \mathbf{h}_i)}{\partial \mathbf{r}_i} = \operatorname{indirect} \text{ of variables that affect the thresholds.}$$

³We use Halton sequences rather than pseudo-random numbers. See Train (2003) for discussion.

The four terms (in order) are the components of the partial effects: (a) due directly to change in x_i ; (b) indirectly due to change in the variables z_i that influence β_i ; (c) due to change in the variables, h_i in the variance and; (d) due to changes in the variables r_i that appear in the threshold parameters, respectively. The probability of interest is

$$\operatorname{Prob}(y_{i} = j \mid \mathbf{x}_{i}, \mathbf{z}_{i}, \mathbf{h}_{i}, \mathbf{r}_{i})$$

$$= \int_{\mathbf{v}_{i}, \mathbf{w}_{i}, e_{i}} \begin{pmatrix} F\left[\frac{\mu_{ij} - (\boldsymbol{\beta} + \boldsymbol{\Delta}\mathbf{z}_{i} + \mathbf{L}\mathbf{D}\mathbf{v}_{i})'\mathbf{x}_{i}}{\exp(\gamma'\mathbf{h}_{i} + \tau e_{i})}\right] \\ -F\left[\frac{\mu_{i,j-1} - (\boldsymbol{\beta} + \boldsymbol{\Delta}\mathbf{z}_{i} + \mathbf{L}\mathbf{D}\mathbf{v}_{i})'\mathbf{x}_{i}}{\exp(\gamma'\mathbf{h}_{i} + \tau e_{i})}\right] \end{pmatrix} \times f(\mathbf{v}_{i}, \mathbf{w}_{i}, e_{i}) \, d\mathbf{v}_{i} \, d\mathbf{w}_{i} \, de_{i},$$

$$\mu_{ij} = \exp(\boldsymbol{\delta}'\mathbf{r}_{i}) \left(\sum_{m=1}^{j} \exp(\alpha_{m} + \sigma_{m}w_{im})\right), \qquad j = 1, \dots, J-1.$$
(25)

 $(\mathbf{LD})^*(\mathbf{LD})' = \gamma$. L is a lower triangular matrix with ones on the diagonal, **D** is a diagonal matrix, and **D**-squared is the diagonal matrix of Cholesky values of γ . If we let $\mathbf{Q} = \mathbf{D}$ -squared, then $\gamma = \mathbf{L}^*\mathbf{Q}^*\mathbf{L}'$. This is the Cholesky decomposition of γ . The set of partial effects is shown in equation set (26):

$$\frac{\partial \operatorname{Prob}(y_{i} = j | \mathbf{x}_{i}, \mathbf{z}_{i}, \mathbf{h}_{i}, \mathbf{r}_{i})}{\partial \mathbf{x}_{i}} = \int_{\mathbf{v}_{i}, \mathbf{w}_{i}, e_{i}} \left(\frac{1}{\exp(\gamma' \mathbf{h}_{i} + \tau e_{i})} \begin{cases} f \left[\frac{\mu_{ij} - \mathbf{\beta}_{i}' \mathbf{x}_{i}}{\exp(\gamma' \mathbf{h}_{i} + \tau e_{i})} \right] \\ -f \left[\frac{\mu_{i,j-1} - \mathbf{\beta}_{i}' \mathbf{x}_{i}}{\exp(\gamma' \mathbf{h}_{i} + \tau e_{i})} \right] \end{cases} (-\mathbf{\beta}_{i}) \right) \\
\times f(\mathbf{v}_{i}, \mathbf{w}_{i}, e_{i}) \, d\mathbf{v}_{i} \, d\mathbf{w}_{i} \, de_{i}, \tag{26a}$$

$$\frac{\partial \operatorname{Prob}(y_{i} = j | \mathbf{x}_{i}, \mathbf{z}_{i}, \mathbf{h}_{i}, \mathbf{r}_{i})}{\partial \mathbf{z}_{i}} = \int_{\mathbf{v}_{i}, \mathbf{w}_{i}, e_{i}} \left(\frac{1}{\exp(\gamma' \mathbf{h}_{i} + \tau e_{i})} \begin{cases} f \left[\frac{\mu_{ij} - \mathbf{\beta}_{i}' \mathbf{x}_{i}}{\exp(\gamma' \mathbf{h}_{i} + \tau e_{i})} \right] \\ -f \left[\frac{\mu_{i,j-1} - \mathbf{\beta}_{i}' \mathbf{x}_{i}}{\exp(\gamma' \mathbf{h}_{i} + \tau e_{i})} \right] \end{cases} (-\mathbf{\Delta}' \mathbf{x}_{i}) \right) \\
\times f(\mathbf{v}_{i}, \mathbf{w}_{i}, e_{i}) \, d\mathbf{v}_{i} \, d\mathbf{w}_{i} \, de_{i}, \tag{26b}$$

$$\frac{\partial \operatorname{Prob}(y_{i} = j | \mathbf{x}_{i}, \mathbf{z}_{i}, \mathbf{h}_{i}, \mathbf{r}_{i})}{\partial \mathbf{h}_{i}}$$

$$= \int_{\mathbf{v}_{i}, \mathbf{w}_{i}, e_{i}} \left\{ \begin{cases} f\left[\frac{\mu_{ij} - \mathbf{\beta}_{i}' \mathbf{x}_{i}}{\exp(\gamma' \mathbf{h}_{i} + \tau e_{i})}\right] \left(\frac{\mu_{ij} - \mathbf{\beta}_{i}' \mathbf{x}_{i}}{\exp(\gamma' \mathbf{h}_{i} + \tau e_{i})}\right) \\ -f\left[\frac{\mu_{i,j-1} - \mathbf{\beta}_{i}' \mathbf{x}_{i}}{\exp(\gamma' \mathbf{h}_{i} + \tau e_{i})}\right] \left(\frac{\mu_{i,j-1} - \mathbf{\beta}_{i}' \mathbf{x}_{i}}{\exp(\gamma' \mathbf{h}_{i} + \tau e_{i})}\right) \\ \times f(\mathbf{v}_{i}, \mathbf{w}_{i}, e_{i}) \, d\mathbf{v}_{i} \, d\mathbf{w}_{i} \, de_{i}, \tag{26c}$$

$$\frac{\partial \operatorname{Prob}(y_{i} = j | \mathbf{x}_{i}, \mathbf{z}_{i}, \mathbf{h}_{i}, \mathbf{r}_{i})}{\partial \mathbf{r}_{i}}$$

$$= \int_{\mathbf{v}_{i}, \mathbf{w}_{i}, e_{i}} \left\{ f\left[\frac{\mu_{ij} - \mathbf{\beta}_{i}' \mathbf{x}_{i}}{\exp(\gamma' \mathbf{h}_{i} + \tau e_{i})}\right] \left(\frac{\mu_{ij}}{\exp(\gamma' \mathbf{h}_{i} + \tau e_{i})}\right) \\ -f\left[\frac{\mu_{i,j-1} - \mathbf{\beta}_{i}' \mathbf{x}_{i}}{\exp(\gamma' \mathbf{h}_{i} + \tau e_{i})}\right] \left(\frac{\mu_{i,j-1}}{\exp(\gamma' \mathbf{h}_{i} + \tau e_{i})}\right) \\ \times f(\mathbf{v}_{i}, \mathbf{w}_{i}, e_{i}) \, d\mathbf{v}_{i} \, d\mathbf{w}_{i} \, de_{i}. \tag{26d}$$

Effects for particular variables that appear in more than one part of the model are added from the corresponding parts. Like the log likelihood function, the partial effects must be computed by simulation. If a variable appears only in \mathbf{x}_i , then this formulation retains both the 'parallel regressions' and 'single crossing' features of the original model. Nonetheless, the effects are highly nonlinear in any event. However, if a variable appears anywhere else in the specification, then neither of these properties will necessarily remain.

3.0 Empirical Application

The context of the application, using stated choice data from a larger study reported in Hensher (2006a, b), is an individual's choice amongst unlabelled attribute packages of alternative tolled and non-tolled routes for the car commuting trip in Sydney (Australia) in 2002. In this paper we are interested in one feature of the way in which individuals process attribute information, namely attribute inclusion or exclusion, given a maximum of five attributes per alternative. The dependent variable in the ordered choice model is the number of ignored attributes, or the number of attributes attended to from the full fixed set associated with each alternative package of route attributes. The utility function is defined over the attribute

information processed by each individual, with candidate influences on the each individual's decision heuristic including the dimensions of the choice experiment (for example, number of alternatives, range of attributes), the framing of the design attribute levels relative to a reference alternative (see below), an individual's socioeconomic characteristics, and attribute accumulation where attributes are in common units (see also Hensher, 2006b).

The establishment of attribute inclusion/exclusion (also referred to as preservation/non-preservation)⁴ in making choices in a stated choice (SC) context is often associated with design dimensionality and the so-called complexity of the SC experiment (Hensher, 2006a). It is typically implied that designs with more items to evaluate are more complex than those with less items⁵ (for example, Swait and Adamowicz, 2001a, b; Arentze et al., 2003), impose cognitive burden, and are consequently less reliable, in a behavioural sense, in revealing preference information. This is potentially misleading, since it suggests that complexity is an artefact of the quantity of information, in contrast to the relevance of information (Hensher, 2006b). In any setting where an individual has to process information on offer and make a choice, psychologists interested in human judgement theory have studied numerous heuristics that are brought to bear in aiding simplification of the decision task (Gilovich et al., 2002). The accumulating life experiences of individuals are also often brought to bear as reference points to assist in selectively evaluating information placed in front of them. These features of human processing and cognition are not new to the broad literature on judgement and decision-making, where heuristics are offered up as deliberative analytic procedures intentionally designed to simplify choice. The presence of a large amount of information, whether requiring active search and consideration or simply assessment when placed in front of an individual (the latter being the case in choice experiments), has elements of cognitive overload (or burden) that results in the adoption of rules to make processing manageable and acceptable (presumably implying that the simplification is worth it in terms of trading off the benefits and costs of a consideration of all information on offer or potentially available with some effort). It is not easy to distinguish between simplified processing because the context is of little interest or the effort is not worth it, versus a genuine interest in

⁴The paper is focused on attribute preservation and non-preservation; however it is important to recognise that one way in which the number of attributes are 'reduced', without attribute elimination, is by adding up common-metric attributes. Hence it is important that we consider it as well, and control for the possibility that some attributes are not eliminated but added up.

⁵Complexity also includes attributes that are lowly correlated, in contrast to highly correlated, the latter supporting greater ease of assessment in that one attribute represents other attributes.

Narrower than base

12

I	ne Suo-aesigns	s oj ine Overa	ii Design jor Five .	Attributes
Choice set of size	Number of alternatives	Number of attributes	Number of levels of attributes	Range of attribute levels
15	2	5	2	Wider than base
9	2	5	4	Base
6	3	5	4	Narrower than base

Table 1 The Sub designs of the Overall Design for Five Attributes

Note: Column 1 refers to the number of choice sets. The four rows represent the set of designs (see Appendix A). The number of alternatives does not include the reference alternative.

the task but with some ex-ante biases that translate into heuristics that capture how an individual desires to treat specific pieces of information. Either way, we see gains in investigating attribute processing and in time being able to separate real behavioural processing from processing for convenience (that lacks behavioural validity in respect to the choice of interest) given the task. Importantly, we suggest that the amount of information to process is less important than the relevance of the information, and indeed there are situations where so little information makes processing 'complex' in the sense that the decision-maker requires much more detail to define a choice of relevance.

The alternative attribute packages offered to individuals to evaluate are pivoted around the car-commuting experiences of sampled respondents. The use of a respondent's experience, embodied in a reference alternative, to derive the attribute levels of the experiment, is supported by a number of theories in behavioural and cognitive psychology, and economics, such as prospect theory, case-based decision theory, and minimum-regret theory (Starmer, 2000; Hensher, 2006b). Reference alternatives in SC experiments⁶ act to frame the decision context of the choice task within some existing memory schema of the individual respondents, and hence make preference-revelation more meaningful at the level of the individual.

Four stated choice sub-designs have been embedded in one overall design (Table 1). Each commuter evaluated one randomly assigned subdesign; however, across the full set of stated choice experiments, the designs differed in terms of the range and levels of attribute, the number of alternatives, and the number of choice sets. The combination of these dimensions of each design is often seen as the source of design 'complexity', and it is within this setting that we have varied the dimensions of an SC

⁶Hensher (2004), Rose et al. (2008) and Train and Wilson (2008) provide details of the design of pivotbased experiments.

experiment that each respondent is asked to evaluate, and through supplementary questions, established which attributes were 'ignored' in the evaluation and selection of an alternative.

Previous studies were used to identify candidate design dimensions. The five design dimensions are shown in Table 2. Five attributes were selected for each alternative, based on previous evidence (Hensher, 2001), to characterise the options: free flow time; slowed down time; stop/start time; variability of trip time; and total cost. Hensher (2006) explored how varying the number of attributes affects information processing, aggregating attributes according to four patterns, noting that aggregated attributes are combinations of existing attributes. We have selected a generic design (that is, unlabelled alternatives) to avoid confounding the effect of the number of alternatives with the labeling (for example, car, train). The attribute ranges are given in Table 1 with the sub-design dimensions shown in Table 2.

As a generic design, each of the alternatives, added as we move from two to three to four alternatives in a choice set (based on Table 1), are exactly the same; that is, for any two alternatives associated with a given design, we should not expect to find the parameter for an attribute (for example, 'free flow travel time') to be different for the set of non-reference alternatives. Therefore, we do not need the attribute 'free flow time alternative one' to be orthogonal to the attribute 'free flow time alternative two' and so on up to 'free flow time J-1 alternatives'. The designs are computergenerated. A preferred choice experiment design is one that maximises the determinant of the covariance matrix, which is itself a function of the estimated parameters. Knowledge of the parameters, or at least some priors (such as signs) for each attribute, from past studies, provides a useful input. We found that in so doing, the search eliminates dominant alternatives. The method used finds the D-optimality plan very quickly (Rose and Bliemer, 2007).

The *actual* levels of the attributes shown to respondents are calculated relative to those of the experienced reference alternative — a recent car commuter trip. The levels applied to the choice task differ depending on the range of attribute levels and the number of levels for each attribute. The design dimensions are translated into SC screens, illustrated in Figure 1. The range of the attribute levels vary *across* designs. Each sampled commuter is given a varying number of choice sets (or scenarios), but the number of alternatives remain fixed. Elicitation questions associated with attribute inclusion and exclusion are shown in Figure 2.

⁷This is an important point because we did not want the analysis to be confounded by extra attribute dimensions.

 Table 2

 The Attribute Profiles for the Design

		Base	Base range		Wider range	6		Narrow	Narrower range
Levels:	2	3	4	2	3	4	2	2 3	4
Free flow time Slow down time Stop/start time Uncertainty of travel time Total costs	±20 ±40 ±40 ±40 ±20	-20, 0, +20 -40, 0, +40 -40, 0, +40 -40, 0, +40 -20, 0, +20	-20, 0, +20 -20, -10, +10, +20 -40, 0, +40 -40, -20, +20, +40 -40, 0, +40 -40, -20, +20, +40 -40, 0, +40 -40, -20, +20, +40 -20, 0, +20 -20, -10, +10, +20	-20, +40 -30, +60 -30, +60 -30, +60 -20, +40	-20, +10, +40 -30, +15, +60 -30, +15, +60 -30, +15, +60 -20, +10, +40	-20, 0, +20, +40 -30, 0, +30, +60 -30, 0, +30, +60 -30, 0, +30, +60 -20, 0, +20, +40	±5 ±20 ±20 ±20 ±5	-5, 0, +5 -20, 0, +20 -20, 0, +20 -20, 0, +20 -5, 0, +5	-5, -2.5, +2.5, +5 -20, -2.5, +2.5, +20 -20, -2.5, +2.5, +20 -20, -2.5, +2.5, +20 -5, -2.5, +2.5, +5

Note: Units shown as percentages.

Figure 1
An Example of a Stated Choice Screen

	Details of Your Recent Trip	Alternative Road A	Alternative Road B	Alternative Road C
Time in free-flow (mins)	15	14	16	16
Time slowed down by other traffic (mins)	10	12	8	12
Time in Stop/Start conditions (mins)	5	4	6	4
Uncertainty in travel lime (mins)	+/- 10	+/- 12	+/- 8	+/- 8
otal Costs	\$ 2.00	\$ 2.10	\$ 2.10	\$ 1.90
If you take the same trip again, which road would you choose?	C Current Road	← Road A	← Road B	← Road C
If you could only choose to new roads, which would y	etween the ou choose?	C Road A	○ Road B	○ Road C

Figure 2
CAPI Questions on Attribute Relevance

. Please inc	ficate which of the following attributes you ign Time in free flow traffic	ored when con	sidering the choices you made in the 10 games
	Time slowed down by other traffic		
	Travel time variability		
	Total costs		
. Did you add	d up the components of: Travel time	C Yes	C No
important).	cimportance of the attributes in making the charmer in free flow traffic Time slowed down by other traffic Travel time variability	Tolices you mad	ie in the games (1 most important, 4 least
l. Are there ar	Total costs ny other factors that we have not included tha	twould have inf	fluenced the choices you made?

4.0 Empirical Analysis

Computer-aided personal interview (CAPI) surveys were completed in the Sydney metropolitan area in 2002.⁸ A stratified random sample was applied, based on the residential location of the household. Screening questions established eligibility in respect of commuting by car. Further details are given in Hensher (2006a). Final models are given in Table 3 for 2,562 observations.

The explanatory variables in the model were guided by the extant literature on heuristics and biases in choice and judgement (see Gilovich *et al.*, 2002), as well as empirical evidence from previous studies on attribute processing by Hensher (2006a, b). We selected candidate influences on the number of attributes actually processed (that is, deemed relevant) under three broad categories:

- (i) design dimensions of the choice experiment;
- (ii) framing around the reference or base alternative, in line with the theoretical argument promoted in prospect theory for reference points;
- (iii) the literature on heuristics that suggests that attribute packaging or attribute-accumulation⁹ is a legitimate rule for some individuals in stage 1 editing under prospect theory (Gilovich *et al.*, 2002).

The generalised ordered logit model has a preferred goodness of fit over the traditional ordered logit model. With four degrees of freedom difference, the likelihood ratio of 181.92 is statistically significant on any acceptable chi-squared test level. The generalised model has included a random parameter form for congestion time framing and has accounted for two systematic sources of variation around the mean of the random threshold parameter (that is, the accumulation of travel time and gender).

The evidence identifies a number of statistically significant influences on the number of attributes attended to, given the maximum number of attributes provided. The range of the attributes and the number of alternatives in the choice set condition mean attribute preservation, and the number of levels of an attribute has a systematic influence on the variance

⁸Interviews took between 20 and 35 minutes, with an interviewer present who entered an individual's responses directly into the CAPI instrument on a laptop.

⁹Accumulation, grouping, and aggregation are essentially the same constructs; namely where two or more attributes with a common metric unit are treated as combined attributes.

¹⁰The difference in the number of alternatives (from two to four, excluding the reference alternative) represents a range typically found in SC studies. The actual screens, with the reference alternative is in place, have between three and five alternatives. The number of alternatives is fixed per respondent but it varies across the sample.

 Table 3

 Ordered Logit Models (2,562 observations)

Attribute	Units	Ordered logit	Generalised ordered logit
Constant		2.9682 (4.17)	2.9504 (2.79)
Design dimensions Narrow attribute range Number of alternatives	1.0 Number	1.3738 (3.59) -0.9204 (-4.1)	1.4275 (2.35) -1.0205 (-2.87)
Framing around base alternative Free flow time for base minus SC alternative level	Minutes Minutes	0.0329 (4.02) -0.0083 (-1.80)	0.0599 (3.44)
Congested time for base minus SC alternative level	Minutes	-0.0083 (-1.80)	0.0761 (2.20)
Attribute packaging (or grouping) Adding travel time components	1.0	-0.7407 (-4.25)	-0.8700 (-3.33)
Variance decomposition Number of levels Free flow time for base minus SC alternative level	Number Minutes	0.1043 (2.35) -0.0164 (-2.75)	0.3357 (4.48) -0.0332 (-4.04)
Who pays $(1 = commuter personally)$	1.0	-0.3070 (5.74)	-0.3721 (-3.89)
Threshold parameters μ_1 μ_2 mean Standard deviation of μ_2 threshold		0 3.0973 (5.74)	0 0.8753 (3.71) 0.0767 (0.018)
parameter			0.0707 (0.010)
Threshold parameter decomposition Adding travel time components Gender (male = 1)	1.0 1.0		1.7447 (10.83) 0.3366 (2.80)
Standard deviation of random regression Congested time for base minus SC alternative level	parameters 1.0		0.2652 (2.48)
Count of choice responses			
	max # a	ttributes minus # ignored	obs
0		5-0	1,415
1 2		5-1 5-2	1,080 66
Log-likelihood		-1871.80	-1780.85

of the unobserved effects (or the error term). We framed the level of each attribute relative to that of the experienced car commute as:

- (i) free flow time for reference (or base) minus the level associated with an alternative in the SC design; and
- (ii) the congested travel time for the base minus the level associated with each SC alternative's attribute level.

The parameter estimates are statistically significant and negative, suggesting that the more that an SC attribute level ('free flow time' and 'congested time' (= slowed down plus stop/start time)) deviates from the reference alternative's level, the more likely that it is an individual will process an increased number of attributes. The attribute packaging effect for travel time has a negative parameter, suggesting that those individuals who add up components of travel time tend to preserve more attributes; indeed, aggregation is a way of simplifying the choice task without ignoring attributes. In the sample, 82 per cent of observations undertook some attribute packaging.

The evidence herein cannot establish whether an attribute reduction strategy is *strictly* linked to behavioural relevance, or to a coping strategy for handling cognitive burden, both being legitimate paradigms. It does, however, provide indications on what features of a specific choice experiment have an influence on how many attributes provided within a specific context are processed. It is likely that the evidence is application specific, but extremely useful when analysts compare the different studies and draw inferences about the role of specific attributes.

The threshold parameter has a statistically significant mean and two sources of systematic variation across the sample around the mean threshold parameter estimate. Across the sample, there were three levels of the ordered choice observed: level 0 is where all attributes are preserved; level 1 is where four of the five attributes were preserved; and level 3 is where three of the five attributes were preserved. No respondent preserved only one or two attributes. Hence, given three levels of the choice variable, there are two threshold parameters, one between levels 0 and 1 and one between levels 1 and 2 (see the explanation after equation (3)). As indicated in Section 2.1, a normalisation is required so that a constant can be identified. We set the threshold parameter for between levels 0 and 1 equal to zero (μ_1) and estimate the parameter between levels 1 and 2 (μ_2) . 11

We investigated an unconstrained random parameter normal distribution; however, the standard deviation parameter estimate was not statistically significant from zero. The evidence, however, justifies the inclusion of a non-fixed threshold parameter, with a higher mean estimate across the sampled population when an individual aggregates the travel time components and when they are male. This is an important finding,

¹¹Estimation of the threshold parameters is not a main object of fitting the ordered choice model *per se*. The flexibility of the threshold parameters is there to accommodate the variety of ways in which individuals will translate their underlying continuous preferences into the discrete outcome. The main objective of the estimation is the prediction of and analysis of the probabilities, for example, the partial effects. The threshold parameters do not have any interesting interpretation of their numerical values in their own right.

Table 4Marginal Effects Derived from Ordered Logit Models

	Ordered logit	Generalised ordered logit
Attribute	Average no. of attributes ignored	Average no. of attributes ignored
Design dimensions Narrow attribute range Number of alternatives	-0.4148, 0.3893, 0.0255 0.2779, -0.2608, -0.0171	-0.2502, 0.2242, 0.0259 0.1789, -0.1603, -0.0256
Framing around base alternative Free flow time for base minus SC alternative level Congested time for base minus SC alternative level	-0.0099, 0.0093, 0.0006 0.0025, -0.0024, -0.0002	-01017, 0.0094, 0.0011 -0.0134, 0.0119, 0.0014
Attribute packaging Adding travel time components	0.2237, -0.2099, -0.0137	0.1525, -0.1367, -0.0158
Variables in threshold Add travel time components Gender (male = 1)	_ _ _	0.0000, 0.06510, -0.06510 0.0000, 0.01785, -0.01785
Variance decomposition Number of levels Free flow time for base minus SC alternative level Who pays (1 = individual, 0 = a business)	-0.1104, 0.0249, 0.0856 -0.2386, 0.0537, 0.1849 0.0740, -0.0167, -0.0573	-0.01740, 0.0103, 0.0071 0.0026, -0.0015, -0.0010 0.0502, -0.0297, -0.0071

Note: The three marginal effects per attribute refer to the levels of the dependent variable.

since it justifies the new formulation of the threshold parameters in ordered choice models as behaviourally meaningful.

A direct interpretation of the parameter estimates is not informative, given the logit transformation of the choice-dependent variable (see equations (5) and (26)). We therefore provide the marginal (or partial) effects in Table 4 which have substantive behavioural meaning, defined as the derivatives of the choice probabilities (equation (25)). A marginal effect is the influence a one-unit change in an explanatory variable has on the probability of selecting a particular outcome, *ceteris paribus*. ¹² The marginal effects need not have the same sign as the model parameters. Hence, the statistical significance of an estimated parameter does not imply the same significance for the marginal effect.

We take a closer look at each model, discussing the evidence for design dimensions, framing around the base, attribute packaging, variance

¹²This holds for continuous variables only. For dummy (1,0) variables, the marginal effects are the derivatives of the probabilities given a change in the level of the dummy variable.

decomposition, and other effects. The magnitude and direction of influence is given in Table 4 for the marginal effects which have to be interpreted relative to each of the three levels of the number of attributes ignored.

In commenting on the marginal effects, it should be noted that, for the generalised ordered logit model, some attributes have more than one role; for example, the framing of free flow time is both a main effect influence as well as a source of variance decomposition (that is, systematic source of heterogeneity) for the unobserved variance; and the attribute accumulation for travel time is both a main effect and a systematic source of influence on the distribution of the random threshold parameter. The generalised ordered choice model (GOCM) takes all of these sources into account in identifying the marginal effects for each level of the choice variable. In contrast, where an attribute has multiple roles in the traditional ordered choice model (TOCM), the marginal effects are calculated separately. The marginal effects associated with variance decomposition in GOCM has two unique influences (that is, number of levels of an attribute and 'who pays for the trip', together with the framing around the base alternative for free flow time which is present elsewhere). 13

The dummy variable for 'narrow attribute range' has the highest marginal effect, although its influence is moderated in GOCM compared to TOCM. The probability of considering more (compared to less) attributes from the offered set decreases as an attribute's range narrows, ceteris paribus; that is, respondents tend to ignore more attributes when the difference between attribute levels is small. This result is perhaps due to the fact that evaluation of small differences is more difficult or perceptually less relevant than evaluation of large differences. An important implication is that if an analyst continues to include, in model estimation, an attribute across the entire sample that is *ignored by a respondent*, then there is a much greater likelihood of mis-specified parameter estimates in circumstances where the attribute range is narrower than wider.

The marginal effects for the narrow attribute range are positive when one (that is, 5-1) or two attributes (that is, 5-2) are ignored. Importantly, the positive effect is greater when one attribute is ignored than when two are ignored. This suggests that the probability of considering four or three attributes from the offered set increases as an attribute's range goes from narrow to non-narrow, *ceteris paribus*, but to a greater extent for four attributes. What we are observing across all three levels of the dependent variable is U- (or inverted U-) shaped response, which appears to be

¹³ For 'free flow time for base minus SC alternative level', we report this in variance decomposition to show its relatively small effect compared to the overall effect of this variable given in another row in the table.

the case for all attributes in GOCM. Thus, for the narrow attribute range we have the highest probability of preserving four attributes than of preserving three attributes, given that the probability of preserving all attributes is decreased. Given the observed profile of the sampled respondents preserving five, four, and three attributes (Table 2), where there are only 66 observations in the last category (compared to 1,415 and 1,080 in 5-0 and 5-1), we have greater confidence in the relative marginal effects of preserving all (that is, five) attributes and four attributes.

As we increase the 'number of alternatives' to evaluate (over the range of 2 to 4 plus the reference alternative), *ceteris paribus*, the importance of considering all attributes increases, as a way of making it easier to differentiate between the alternatives. This finding runs counter to some views; for example, that individuals will tend to ignore increasing amounts of attribute information as the number of alternatives increases. Our evidence suggests that the processing strategy is dependent on the nature of the attribute information, and not strictly on the quantity. The negative marginal effects for ignoring one and two attributes (or preserving four and three attributes) suggest that these rules are less likely to be adopted as the number of alternatives increases.

The theoretical argument promoted in prospect theory for reference points is supported by our empirical evidence. We have framed the level of each attribute relative to that of the experienced car commute trip as: (i) free flow time for current (or base) minus the level associated with an attribute and alternative in the SC design; and (ii) the congested travel time for the base minus the level associated with each SC alternative's attribute. The more that an SC attribute level deviates from the reference alternative's level, the more likely it is that an individual will process an increased number of attributes. This evidence was found for both the 'free flow time' and 'congested time' framing effects. Conversely, as the SC design attribute level moves closer to the reference alternative's level, individuals appear to use some approximation rule, in which closeness suggests similarity, and hence ease of eliminating specific attributes, because their role is limiting in differentiation.

Reference dependency not only has a direct (mean) influence on the number of attributes ignored; it also plays a role via its contribution to explaining heteroscedasticity in the variance of the unobserved effects. This has already been accounted for in the GOCM marginal effects for free flow time framing. It is separated out in the TOCM. The effect of widening the gap between the base and SC 'free flow time' reduces the heteroscedasticty of the unobserved effects across the respondents, increasing the acceptability of the constant variance condition when simpler models are specified.

1.42

1.14

.85

.57

.28

.00

-1.00

-50

.00

CONGD

Kernel density estimate for CONGD

Figure 3
Distribution of Preference Heterogeneity for Congested Time Framing

In GOCM, the congested time framing effect is represented by a distribution across the sample. The random parameter has a statistically significant standard deviation parameter estimate, resulting in a distribution shown in Figure 3. The range is from -0.857 to 1.257; hence, there is a sign change around the mean of 0.70833 and standard deviation of 0.2657. This results in the same mean marginal effect sign in GOCM as free flow time framing; however, when we treated congested time-framing as having a fixed parameter (in TOCM, where the standard deviation parameter was not statistically significant), the signs are swapped for all levels of the choice variable. The evidence from the GOCM is intuitively more plausible.

The attribute-accumulation rule in stage 1 editing under prospect theory is consistently strong for the aggregation of travel-time components. The positive marginal effect for the dummy variable 'adding three travel-time components' indicates that, on average, respondents who add up the time components, in assessing the alternatives, tend also to ignore more attributes. There is clear evidence that a relevant simplification rule is repackaging of the attribute set, where possible, through addition. This is not a cancellation strategy, but a rational way of processing the information content of component attributes, and then weighting this information (in some unobserved way) in comparing alternatives.

The characteristics of the respondent's proxy for other excluded contextual influences. A respondent's role in paying the toll was identified, through its influence on variance decomposition of the unobserved effects,

as a statistically significant socio-economic influence on the number of attributes considered. We have no priors on the likely sign of the influence on variance. The positive marginal effect for who pays suggests that where those pay for themselves (in contrast to a business paying) tends to result in a higher probability of preserving more attributes, although the influence is slightly less in GOCM compared to TOCM. This might mean that males do care more about the time/cost trade-off, in contrast to a situation where only time matters if someone else pays for the travel. Gender was a systematic source of influence on the threshold parameter, increasing its mean estimate for males.

5.0 Conclusions

The recognition of randomness in the threshold parameters in the presence of random parameters and the identification of systematic sources of heterogeneity in the mean threshold parameter estimate is an important extension of the existing ordered choice model. This paper has brought together all of the key contributions in the literature and extended them, in particular to ensure preservation of the ordering of thresholds in the context of random parameterisation of the thresholds (equations (16) to (20)).

The specific application herein, on the role that attributes play in choice-making in stated choice experiments, pivoted around a real market experience, has highlighted the role of random thresholds and decomposition, suggesting that the generalised empirical model is a rich behavioural addition to the literature on ordered choice modelling. We need, however, many studies in differing contexts before we can make general conclusions about the specific empirical evidence on sources of influence on the propensity for individuals to invoke specific attribute preservation heuristics.

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Appendix A

Designs for Five-Attributes

			4	Alternative 1				7	Alternative 2		
Block	Scenarios	Free flow time	Slowed down time	Stop/start time	Uncert of travel time	Total cost	Free flow time	Slowed down time	Stop/start time	Uncert of travel time	Total cost
_	1	1		0	1	1	0	1	0	0	-
_	2	0	_	0	_	0	-	0	_	_	0
_	3	_	_	0	_	_	0	_	0	_	-
_	4	0	0	_	_	0	_	_	0	0	1
_	5	0		0	0	_	_	0			1
_	9	0	0	0	_	_	0	0	0	0	1
_	7	0	_			_	_	0	0	0	1
_	8	1	_	_	_	0	0	_	0	_	1
_	6	0	0	_	_	0	0	_		0	1
_	10	0	0		0	0	0	0	0	0	0
_	11	0	_			_	0	0	-	0	1
_	12	0	0	_	0	0	_	0	0	0	0
1	13	1	_		0	-	0	0	-	_	1
_	14	1		0	0	_	0	0			0
_	15	1	0	0	0	_	1	1	1	0	1
2		0	0		0	0	0	0	0	_	0
2	2	1	_	_	_	0	0	_	0	0	0
2	33		_	_	_	_	_	_		_	0
2	4	0	_		0	0	_	0		0	0
2	5	1	0	0	0	_	_	0	1	_	0
2	9	1	0	0	0	1	_	0	0	0	0
2	7	1	_	0	0	0	0	0		_	0
2	8	0	_	0	0	_	_	_		0	1
2	6	1	1	_	_	0	0	0	0	0	1
2	10	1	0		0	0	0	_	0	_	0
2	11	1	_	0		_	0	0	-	1	1
2	12		0	0	_	0	_	0	0	_	
2	13	1	0	_	0	0	0	_	0		0
2	14	0	1	1	1	0	1	1	1	-	1
(15		0	0	0	0	0	-	0	0	0

				Alternative I				,	Alternative 2		
Block	Scenarios	Free flow time	Slowed down time	Stop/start time	Uncertainty of travel time	Total $cost$	Free flow time	Slowed down time	Stop/start time	Uncertainty of travel time	Total cost
_	1	1	0	0	з	3	3	3	1	1	-
1	2	2	0	3	2	_	0	1	0	1	2
1	3	1	1	С	2	7	С	0		0	3
1	4	2	33	-	33	7	0	1	2	2	3
1	5	2	1	2		_	-	c	0	0	0
1	9	2	_	_	0	0	С	2	0	33	1
1	7	С	0	2		3	2	2	33	С	7
1	8	0	3	2	0	-	3	2	1	2	0
1	6	0	0	3		_	33	3	2	2	0
2	1	1	33	3	1	3	7	7	2	0	7
2	2	0	0	0	2	7	_	2	2	_	0
2	3	0	ю	33	3	0	1	1	0	0	_
2	4	1	1	-	3	_	2	2	0	_	0
2	5	7	ю	0	2	33	1	7	3	0	ю
2	9	0	2	1	3	3	В	ю	3	0	7
2	7	ю	1	3	0	0	-	0	1	-	7
2	~	2	1	0	3	3	0	0	2	2	7
2	6	0	2	1	2	_	3	0	2	3	0

	1	1											
	Total cost	3	0	0	_	3	-	7	-	7	7	0	3
.3	Uncertainty of travel time	2	m	1	0	2	2	0	ю	2	3	0	П
Alternative 3	Stop/start time	0	m	3	3	1	2	2	0	ю	2	33	2
	Slowed down time	0	_	3	_	_	33	33	0	0	7	7	7
	Free Aow time	1	m	_	7	_	-	-	-	0	_	_	0
	Total	2	0	-	0	3	7	ж	7	-	_	_	3
. 2	Uncertainty of travel time	0	_	0	-	0	1	2	1	1	2	-	0
Alternative 2	Stop/start time	1	2	2	0	3	0	1	3	1	3	-	-
	Slowed down ime	1	0	_	7	3	0	7	7	_	0	Э	0
	Free flow time	3	7	0	3	0	0	7	0	-	ж	ж	3
	Total cost	1	_	7	0	7	0	ж	0	0	Э	Э	7
. 1	Uncertainty of travel time	3	2	3	3	3	0	8	2	0	-	3	7
Alternative I	Stop/start time	3	0	1	1	2	_	0	7	0	0	2	0
	Slowed down time	3	7	0	3	7	7	_	_	\mathcal{S}	Э	0	1
	Free flow time	2	0	7	0	3	7	0	\mathcal{C}	\mathcal{C}	2	2	7
	Scenarios	1	7	3	4	5	9	1	2	ю	4	5	9
	Block	1	_	_	_	_	_	7	7	7	2	2	2

			,	Alternative I				,	Alternative 2		
Block	Scenarios	Free flow time	Slowed down time	Stop/start time	Uncertainty of travel time	Total cost	Free flow time	Slowed down time	Stop/start time	Uncertainty of travel time	Total cost
_	_	2	-	0	0	2	0	2	0	0	2
	7	1	0	0	0	-	2	-	7	7	0
1	8	1	2	2	0	0	2	0	0	_	0
1	4	1	2	2	0	_	0	0	1	2	2
1	5	2	2	2		_				0	1
1	9	2	2	0	2	_	П	0		0	7
1	7	0	0	0		7	П	2	2	2	1
1	8	0	1	П		0	П	2	0	2	0
1	6	1	1	0	2	7	2	2	2	1	0
1	10	2	2		2	_	0	0	2	0	0
1	11	1	2	П	0	0	2	0	0	П	1
	12	0	2	0	0	0	2	П	П	2	0
2	П	2	0	П		2	2	0		П	1
2	2	0	0	П	2	_	2	2	2	0	0
2	т	0	-1	_	0	7	2	0	2		0
2	4	2	2	1	2	0	1	0	2	1	0
2	5	1	2	2	2	7		2			0
2	9	1	-1	0	2	_	0	0	2		7
2	7	1	1	2	2	0	0	2			_
2	∞	1	0	1	2	7	2	_	2	1	0
2	6	1	2	0	1	_	0	1	2	0	_
2	10	2	1	-	0	0	1	0	2	2	7
2	11	2	2	0	0	7	0	0	0	2	1
2	12	1	0	2	0	0	0	2	1	1	0

				Alternative 3					Alternative 4		
Block	Scenarios	Free flow time	Slowed down time	Stop/start time	Uncertainty of travel time	Total cost	Free flow time	Slowed down time	Stop/start time	Uncertainty of travel time	Total cost
	1		0		_	1	0		2	2	2
1	2	0	2	1	_	7	0	2	1	-	7
	3	0	2	7	2	2	0	1	1	2	1
1	4	П	2	2	2	7	2	П	0	П	1
1	5	2	2	2	1	7	0	0	0	2	7
1	9	1	0	1	0	7	0	1	2	1	1
1	7	2	-	1	0	7	2	1	1	0	7
1	~	2	0	2	0	_	2	0	2	0	1
1	6			0	2	7	0	0	_	0	_
1	10	1	1	0	1	0	2	2	1	2	1
1	11	0	1	2	2	0	2	0	0	0	7
1	12	1	0	2	1	-	1	0	7	1	1
2	1	0	2	2	2	0	1	1	0	0	0
2	2	1	1	0	1	7	1	-1	0	1	7
2	3	1	2	0	2	0	0	1	-	0	_
2	4	0	-	0	0	_	0	1	0	0	7
2	5	0	1	2	0	0	2	0	0	2	1
2	9	0	0	2	1	7	2	2	1	0	0
2	7	2	0	0	0	0	0	2	1	1	_
2	~	1	0	1	2	7	0	2	0	0	-
2	6	2	0	1	2	0	0	1	0		7
2	10	1	0	2	2	_	0	2	0	1	0
2	11	2	2	2	0	0	1	1	1	1	0
2	12	2	1	0	7	7	2	1	0	2	