Likert Scale Dual Response in Conjoint Analysis

Prachi Bhalerao Dan Yavorsky Geoffery Zheng

2024-07-09

Write the abstract here ...

1 Introduction

Lorem ipsum

2 Model

2.1 Set Up

Consumer $i = 1, \ldots, N$ derives utility from good $j \in \mathcal{J}_i = \{0, \ldots, J_i\}$ with utility u_{ij} given by

$$u_{ij} = h(\mathbf{x}_j, \boldsymbol{\beta}_i) + \eta_{ij}.$$

where $\mathbf{x}_j$ is a length-P column vector of good characteristics, $\boldsymbol{\beta}_i$ is a length-P column vector of consumer-specific taste parameters, and η_{ij} encapsulates factors known to the consumer but unobserved by the researcher. We take $h(\mathbf{x}_j, \boldsymbol{\beta}_i) = \mathbf{x}'_j \boldsymbol{\beta}_i$ but this specification is not required.

The "zero"-th (or "outside") good is special and is associated with a zero vector of good characteristics ($\mathbf{x}_0 = \mathbf{0}$) such that $h(\mathbf{x}_0, \boldsymbol{\beta}_i) = 0$. Consumers observe $\mathbf{x}_j$ and η_{ij} for all "inside" goods ($j > 0; j \in \mathcal{J}_i$), but they *do not* observe η_{i0}.

The consumer first reports her preferred inside good, which is given by

$$j_i^* = \arg \max_{j>0;\, j \in \mathcal{J}_i} u_{ij}.$$

The consumer then reports a value y_i on a discrete qualitative scale $w \in \mathcal{W} = \{1, \ldots, W\}$ to reflect the probability p_i that she would purchase j^*, i.e., that she prefers good j_i^* to the outside good $j = 0$.

Apportion the range of F into a number W of intervals $\phi_w = [\alpha_{w-1}, \alpha_w)$, where $\alpha_0 = 0$ and $\alpha_W = 1$. We assume that consumers share the definition of the qualitative scale and thus understand these intervals, whereas α_w for $0 < w < W$ are unobserved by the researcher and are to be estimated.

Let u_i^* denote the utility of good j_i^* for consumer i. Consumers know u_i^*, but do not know η_{i0}, and thus the purchase probability from the consumer's perspective given by

$$p_i = \Pr\left(\eta_{i0} < u_i^*\right) = F\left(u_i^*\right).$$

The act of reporting the interval $y_i = w$ into which p_i falls, reveals that $p_i \in [\alpha_{w-1}, \alpha_w)$ and is therefore equivalent to reporting that

$$u_i^* \in \left[F^{-1}\left(\alpha_{w_{i-1}}\right),\ F^{-1}\left(\alpha_{w_i}\right)\right],$$

2.2 Probability Specification

We model $\eta_{ij} \overset{iid}{\sim} \text{Gumbel}\,(0,1)$ with η_{ij} assumed to be independent of both $\mathbf{x}_j$ and η_{ik} for $k \neq j$.[1]

From the consumer's perspective,

$$p_i = \Pr\left(\eta_{i0} < u_i^*\right) = F_{\text{Gumble}(0,1)}\left(u_i^*\right).$$

and $\Pr(y_i = w)$ is given by

$$\Pr(y_i = w) = F_{\text{Gumbel}(0,1)}\left(F^{-1}_{\text{Gumbel}(0,1)}\left(\alpha_w\right)\right) - F_{\text{Gumbel}(0,1)}\left(F^{-1}_{\text{Gumbel}(0,1)}\left(\alpha_{w-1}\right)\right) = \alpha_w - \alpha_{w-1}.$$

From the researcher's perspective, the unobserved likelihood of purchase is

$$p_i = \Pr\left(u_i^* - \eta_{i0} > 0\right) = \Pr\left(\varepsilon_i > 0\right) = 1 - \Pr\left(\varepsilon_i < 0\right)$$

where $\varepsilon_i = u_i^* - \eta_{i0}$. As is well known,[2] ε_i follows a Gumbel distribution with location parameter $\overline{\mu}_i$ and scale parameter 1, where

$$\overline{\mu}_i = \ln\left(\sum_{j \in \mathcal{J}_i} \exp\left(\mathbf{x}_j{}'\boldsymbol{\beta}_i\right)\right). \tag{1}$$

and so the unobserved choice probability is

$$p_i = 1 - F_{\text{Gumble}(\overline{\mu}_i,1)}\left(\varepsilon_i\right).$$

[1]This can be motivated by a framework in which $u_0 = 0$ and $u_{ij} = \mathbf{x}_j{}'\boldsymbol{\beta}_i + \eta_{ij} - \eta_0$ for $j > 0; j \in \mathcal{J}_i$. Here, η_0 captures the consumer's uncertainty about their future tastes. Given that utilities are ordinal and η_0 is a common shock, it plays no role in the choice among the most preferred inside good $j^* \in \mathcal{J}_i$.

[2]See, e.g., (McFadden 1981) or (Cardell 1997).

[TODO: add a line/paragraph to connect p_i above to $\Pr\left(u_i^* \in \phi_{w(i)}\right)$ below.]

Therefore, the probability that consumer i reports $y_i = w$ is

$$\begin{aligned}\Pr\left(u_i^* \in \phi_{w(i)}\right) &= F_{\text{Gumbel}\left(\overline{\mu},1\right)}\left(F^{-1}_{\text{Gumbel}(0,1)}\left(\alpha_{w_i}\right)\right) - F_{\text{Gumbel}\left(\overline{\mu},1\right)}\left(F^{-1}_{\text{Gumbel}(0,1)}\left(\alpha_{w_i-1}\right)\right) \\ &= \left(\alpha_{w_i}\right)^{\exp(\overline{\mu})} - \left(\alpha_{w_i-1}\right)^{\exp(\overline{\mu})}.\end{aligned}$$

We note that, while the parametrization of $\eta_0 \sim \text{Gumbel}\left(0,1\right)$ preserves symmetry among the $J+1$ goods and is thus a natural choice, the framework can easily accommodate an alternative distribution for η_0. For example, one could use an affine function of individual characteristics to accommodate individual-level variation in the propensity to prefer the outside good.

3 Simulation Study

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4 Empirical Analysis

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5 Discussion

Differs from (Brazell et al. 2006)

6 Conclusion

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References

Brazell, Jeff D., Christopher G. Diener, Ekaterina Karniouchina, William L. Moore, Válerie Séverin, and Pierre-Francois Uldry. 2006. "The No-Choice Option and Dual Response Choice Designs." *Marketing Letters* 17 (4): 255–68. https://doi.org/10.1007/s11002-006-7943-8.

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