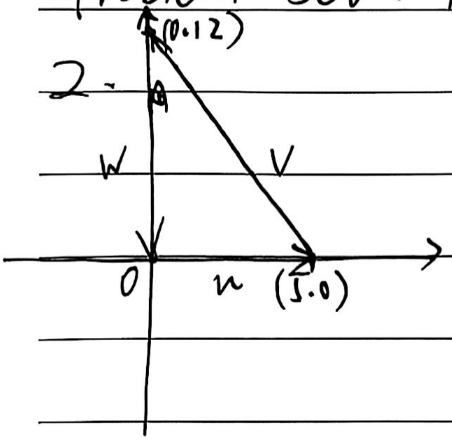
- Problem Set 1.1

2.



$\vec{u} = (5,0)$

$\vec{v} = (-5,12)$

$\vec{w} = (0,-12)$

$\vec{u}+\vec{v}+\vec{w} = \vec{0}$

$|\vec{u}| = 5 \quad |\vec{v}| = \sqrt{5^2+12^2} = 13 \quad |\vec{w}| = 12$

3. (a) 同共线.

所有线性组合在一条线上

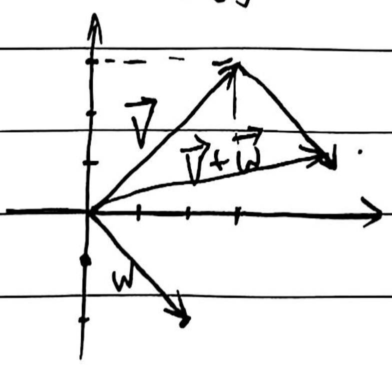
(b) 独立.

在三维 $\mathbb{R}^3$ 中的 plane

(c) 独立.

all of $\mathbb{R}^3$

5. $2\vec{v} = \begin{bmatrix} 6 \\ 6 \end{bmatrix} \quad \vec{v} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \quad \vec{w} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$



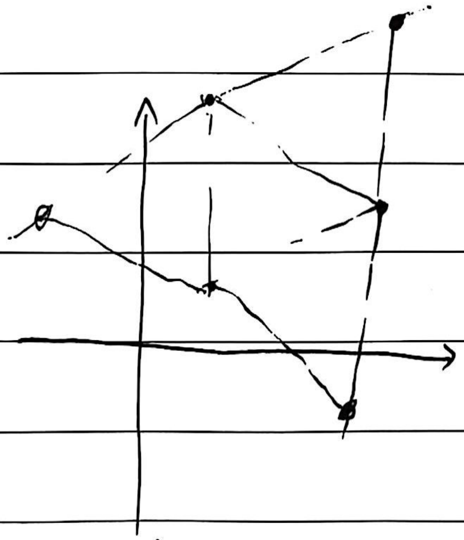
7.

$$\vec{u}+\vec{v}+\vec{w} = \begin{bmatrix} 1+(-3)+2 \\ 2+1+(-3) \\ 3+(-2)+(-1) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

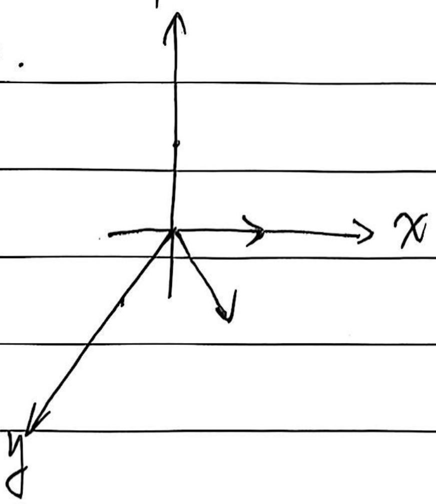
$$2\vec{u}+2\vec{v}+\vec{w} = \begin{bmatrix} 2-6+2 \\ 4+2-3 \\ 6-4-1 \end{bmatrix} = \begin{bmatrix} -2 \\ 3 \\ 1 \end{bmatrix}$$

$\because \vec{w} = c\vec{u}+d\vec{w}$. $c = -1$. $v = -1$

11.



13.



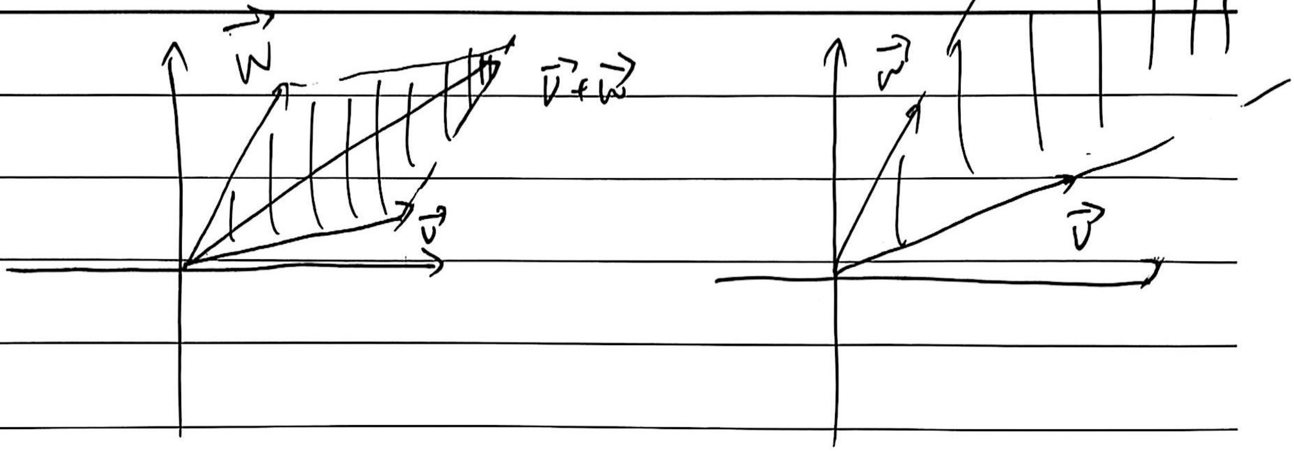
z always equals 0

so. the plane $z=0$

17.

$$\begin{aligned} c+2d&=14 \\ 2c+d&=8 \end{aligned} \Rightarrow \begin{aligned} c&=2 \\ d&=4 \end{aligned}$$

19.



23. no.

a line.

24. ∵ 2^4 ∵ every can be either 1, −1

volume = 1. (is volume defined ?)

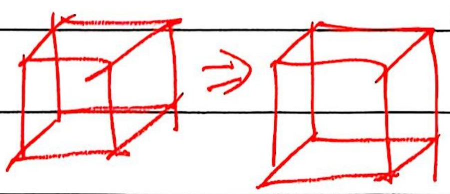
for every corner 4 edges

$\frac{4\times 16}{2} = 32$ edges. e.g. (1, 0, 0, 0)

~~4 edges for a~~ 3D face ? 10 ?

25: ~~$y+2z=0$ $y=2$ $z=-1$ $x=-3$~~

~~$y=4$ $z=-2$ $x=-6$~~



平移.

每一个面都会产生一个新"3D"面. 原有两3D面.

⇒ 8 3D面

亦或是自由度的思想、一个3D面 ⇒ 固定一个自由度.

25.

$x\vec{u}+y\vec{v}+z\vec{w}=\vec{b}$

$\Rightarrow \begin{cases} x+2y+z=0 \\ 3x+7y+5z=1 \end{cases}$ $y+2z=1$

$y=1\ z=0.\quad x=-2$

$y=-1\ z=1\quad x=1$

不一定. ~~①~~ ~~量~~ 全共线. 0 种方案.

26. $\vec{v}\cdot\vec{w}$ are on the same line

(1, 0, 0, 0)
(0, 1, 0, 0)
(0, 0, 1, 0)
(0, 0, 0, 1)

27.

$2c-d+0=1$

$-c+2d-e=0$

$0-d+2e=0$

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} c \\ d \\ e \end{bmatrix}=\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$3d-2e=1$
$-d+2e=0$

$4e=1\quad e=\frac{1}{4}.\quad d=\frac{1}{2}.\quad c=\frac{3}{4}$

$c=\frac{3}{4}.\ d=\frac{1}{2}\quad e=\frac{1}{4}$

Problem Set 1.2.

2.

$\|u\| = \sqrt{(-6)^2+8^2} = 10$

$\|v\| = \sqrt{3^2+4^2} = 5$

$\|w\| = \sqrt{1^2+2^2} = \sqrt{5}$

3. $\hat{v} = \frac{\vec{v}}{\|v\|} = \begin{bmatrix} \frac{3}{5} \\ \frac{4}{5} \end{bmatrix}$

$\hat{w} = \frac{\vec{w}}{\|w\|} = \begin{bmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{5}}{5} \\ \frac{2\sqrt{5}}{5} \end{bmatrix}$

$\cos\theta = \hat{v} \cdot \hat{w} = \cancel{\frac{3\sqrt{5}}{25} + \frac{8\sqrt{5}}{25} = \frac{11\sqrt{5}}{25}}$ $\frac{2\sqrt{5}}{5}$

$0^\circ: \vec{a} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$ $90^\circ: \vec{b} = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$ 180° $\vec{c} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$

5. $\hat{v} = \frac{\vec{v}}{\|v\|} = \left(\frac{\sqrt{10}}{10}, \frac{3\sqrt{10}}{10}\right)$

$\vec{w} = \frac{\vec{w}}{\|w\|} = \left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$

$U_1 = \left(\frac{3\sqrt{10}}{10}, \frac{\sqrt{10}}{10}\right)$

$U_2 = \left(-\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0\right)$

$\left\|\begin{pmatrix} a_1 \\ c_1 \end{pmatrix}\right\|$

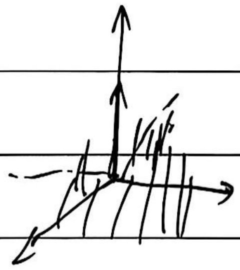
7.
(a) $\cos\theta = \frac{\vec{v}\cdot\vec{w}}{||v||\,||w||} = \frac{1}{2\cdot 1} = \frac{1}{2}$ $\theta = \frac{\pi}{3}$

(b) $\cos\theta = \frac{\vec{v}\cdot\vec{w}}{||v||\,||w||} = \frac{4-2-2}{||v||\,||w||} = 0.$ $\theta = \cancel{90^\circ}\ \frac{\pi}{2}$

(c) $\cos\theta = \frac{\vec{v}\cdot\vec{w}}{||v||\,||w||} = \frac{2}{2\cdot 2} = \frac{1}{2}.$ $\theta = \frac{\pi}{3}$

(d) $\cos\theta = \frac{\vec{v}\cdot\vec{w}}{||v||\,||w||} = \frac{-5}{\sqrt{10}\cdot\sqrt{5}} = -\frac{\sqrt{2}}{2}$ $\theta = \frac{3\pi}{4}$

11.
钝角 obtuse angle



~~all the area in~~

$z < 0$

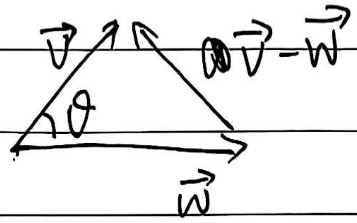
13. $+\ +\ -\ -$ $a = (1, 1, -1, -1)$
$+\ -\ +\ -$ $b = (1, -1, 1, -1)$
$-\ +\ +\ -$ $c = (-1, 1, 1, -1)$

17. $\cancel{\sqrt{4^2+2^2} = 2\sqrt{5}}$ $\vec{v}+\vec{w} = (3, 4)$

$4^2+2^2 = 20.$ $3^2+4^2 = 25$

$(-1)^2+(2)^2 = 5.$ $20+5 = 25$

19.



$v^2+w^2-2|v||w|\cdot\cos\theta = |\vec{v}-\vec{w}|^2$

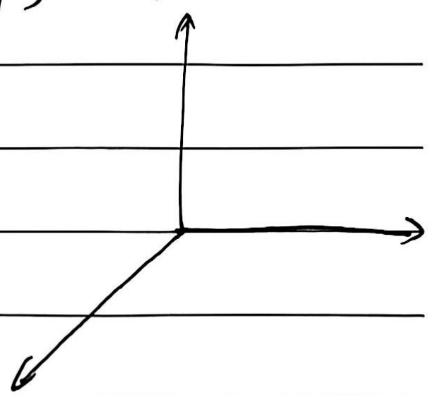
23. (a) $(\vec{v}-\vec{w})^2 = v^2+w^2-2|v||w|\cos\theta$ $(\vec{v}+\vec{w})^2 = v^2+w^2+2|v||w|\cos\theta$

if $|\cos\theta|>1$ $(x_1, y_1)\cdot(x_2, y_2)$

$\cos\theta = \dfrac{x_1x_2+y_1y_2}{\sqrt{x_1^2+y_1^2}\cdot\sqrt{x_2^2+y_2^2}}$ 由 Cauchy

$(x_1^2+y_1^2)(x_2^2+y_2^2) \geq (x_1x_2+y_1y_2)^2$ 即证.

26. Yes.



$\Leftrightarrow$ $\mathbb{R}^3$ 中最多两两有n个钝角.

4个很显然.

但并不会 prove

先取一个 不妨设为z轴

则剩下三个 $z<0$ $\Rightarrow$ $(z分量)^2>0$

又投影到 x, y 平面上最多三个

$\Rightarrow$ 4个

27.

$+ \ + \ + \ +$

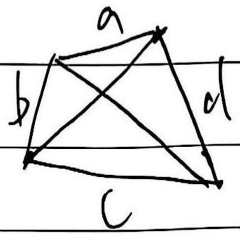
$+ \ + \ - \ -$

$+ \ - \ + \ -$

$+ \ - \ - \ +$

28. code. skip

29.



不妨设 $\|v_1\| < \|v_2\| < \|v_3\| < \|v_4\|$

~~obvio~~ 取三条，则要求 $\|v_1\| + \|v_2\| + \|v_3\| > \|v_4\|$

此时一定能构造出一个四边形.

反之. $\|v_1\| + \|v_2\| + \|v_3\| < \|v_4\|$ 时一定不行

Problem Set 1.3

2.

$a+2b+3c=0$ ①

$4a+5b+6c=0$ ②

$7a+8b+9c=0$ ③

$\Rightarrow$ ~~$a+c=0$~~

~~$7a+c=0$~~

①② . $-3b-6c=0$

$b=-2c$

$a=1.\ b=-2.\ c=1$

$a=1.\ b=-2.\ c=1$

3. ~~B.P~~ span B = plane. $\mathbb{R}^2$

span C = plane. $\mathbb{R}^2$

5.

$$Ax = 1\begin{bmatrix}2\\4\\0\end{bmatrix} + 2\begin{bmatrix}1\\2\\1\end{bmatrix} + 5\begin{bmatrix}2\\4\\0\end{bmatrix} = \begin{bmatrix}14\\28\\2\end{bmatrix}$$

$$By = 4\begin{bmatrix}1\\1\\1\end{bmatrix} + 4\begin{bmatrix}0\\1\\1\end{bmatrix} + 10\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}4\\8.\\18\end{bmatrix}$$

$$Iz = \begin{bmatrix}z_1\\z_2\\z_3\end{bmatrix}$$

7.

(a) $\begin{bmatrix}1&0\\0&1\\0&0\end{bmatrix}$ $\begin{bmatrix}0&1\\1&0\\1&1\end{bmatrix}$

(b) $\begin{bmatrix}1&0\\9&8\end{bmatrix}$ $\begin{bmatrix}-1&0\\0&0\\0&0\end{bmatrix}$

(c) No

11.

$A = \begin{bmatrix} a_1 & a_2 \end{bmatrix}$ $B = \begin{bmatrix} a_1 & a_2 & a_3 \end{bmatrix}$

(a) Yes.

(b). maybe

(c) no.

13.

$A = \begin{bmatrix} 2 & 3 & 5 \\ -1 & -2 & -3 \\ -1 & -1 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -2 & -1 \\ -2 & -1 & -3 \\ 1 & 3 & 4 \end{bmatrix}$

usually the same plane

17 rank. zero

$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$

rank one

$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

rank 2.

$\begin{bmatrix} 1 & 2 \\ 1 & 2 \\ 1 & 2 \end{bmatrix}$ $\begin{bmatrix} 1 & -3 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

rank 3 X

~~comib~~ combinations of two lines at most

$\mathbb{R}^2$. $\mathbb{R}^4$ is impossible

18.

~~a k.~~

~~c.~~

$$\cancel{\begin{bmatrix} kc & c \\ kd & d \end{bmatrix}}$$

$$\begin{bmatrix} kc & kd \\ c & d \end{bmatrix} \Rightarrow a = \frac{c}{d} \cdot b$$

$$c = \frac{c}{d} b$$

23

$$\begin{bmatrix} 1 & 1 & 0 \\ 3 & 2 & 1 \\ 7 & 4 & 3 \end{bmatrix} \quad c = 3$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \quad c = -1$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 2 & 1 & 5 \\ 3 & 3 & 6 \end{pmatrix} \quad c = 0.$$

$$\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \quad \begin{bmatrix} -2 & 1 \\ 4 & -2 \end{bmatrix} \quad c = \pm 2$$