第十一章

电路的频率响应

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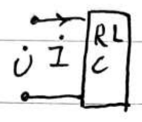
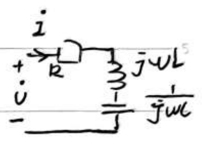
11.2 RLC串联电路的谐振

1. 谐振的定义：含RLC的一端口电路，在特定条件下出现端口电压电流同相位的现象时，称电路发生了谐振



$\frac{\dot{U}}{\dot{I}} = Z = R$ 发生谐振

2. 串联谐振的条件 $Z = R + j(\omega L - \frac{1}{\omega C}) = R + jX$



当 $X=0 \Rightarrow \omega_0 L = \frac{1}{\omega_0 C}$ 时，谐振

$\omega_0 = \frac{1}{\sqrt{LC}}$ 谐振角频率 $f_0 = \frac{1}{2\pi\sqrt{LC}}$ 谐振频率

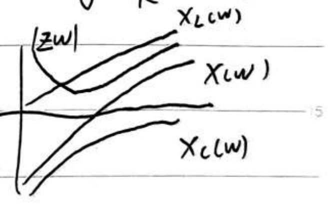
串联电路实现谐振：(1) L C不变，改 ω (2) 电源f不变，改L/C

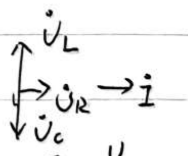
3. RLC串联电路谐振的特点

阻抗的频率特性 $Z = R + j(\omega L - \frac{1}{\omega C}) = |Z(\omega)| \angle \varphi(\omega)$

$|Z(\omega)| = \sqrt{R^2+X^2}$ 幅频特性 $\varphi(\omega) = tg^{-1}\frac{X}{R}$ 相频特性

容性区	电阻性	感性区
$\omega < \omega_0$	$\omega = \omega_0$	$\omega > \omega_0$
$X(j\omega) < 0$	$X(j\omega) = 0$	$X(j\omega) > 0$
$\varphi(j\omega) < 0$	$\varphi(j\omega) = 0$	$\varphi(j\omega) > 0$
$R < \|Z(j\omega)\|$	$Z(j\omega_0) = R$	$R < \|Z(j\omega)\|$
$\lim_{\omega\to 0}\|Z(j\omega)\| = \infty$		$\lim_{\omega\to\infty}\|Z(j\omega)\| = \infty$





(1) 谐振时 $\dot{U}$ 与 $\dot{I}$ 同相，$Z=R$，$|Z|_{min}$，$I, U_{R max}$，$I_0 = \frac{U}{R}$

(2) LC上电压大小相等，相位相反，串联总电压为0.（电压谐振）

$\dot{U}_L + \dot{U}_C = 0$，LC短路 $\dot{U}_R = \dot{U}$ $|\dot{U}_L| = |\dot{U}_C| = QU$

品质因数 $Q = \frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{\rho}{R}$ ρ：特性阻抗

(3) 谐振中出现过电压 $\rho = \omega_0 L = 1/\omega_0 C >> R$ 时

$Q >> 1$，$U_L = U_C = QU >> U$

(4) 谐振中的功率

$P = UI\cos\varphi = UI = RI_0^2 = U^2/R$

电源向电路输送电阻消耗的功率，电阻功率达max

$Q = UI\sin\varphi = Q_L + Q_C = 0$ $Q_L = \omega_0 L I_0^2$ $Q_C = -\omega_0 L I_0^2$

(5) 谐振时的能量关系

$W_C = \frac{1}{2}CU_C^2 = \frac{1}{2}LI_m^2\cos^2\omega_0 t$ 电场能量

$W_L = \frac{1}{2}Li^2 = \frac{1}{2}LI_m^2\sin^2\omega_0 t$ 磁场能量

电场 — 电磁互换

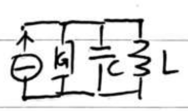
W总不变，为 CQ^2U^2

$Q = \frac{\omega_0 L}{R} = \omega_0\frac{LI_0^2}{RI_0^2} = 2\pi\frac{LI_0^2}{RI_0^2 T_0} = 2\pi\frac{\text{电磁场总储能}}{\text{谐振T内电路耗能}}$

Q反映电磁振荡程度，Q↑能量↑

4. 并联谐振电路

1. G、C、L并联



$Y = G + j(\omega C - \frac{1}{\omega L})$

$\omega_0 = \frac{1}{\sqrt{LC}}$ ①入端导纳为纯电导，|Y|最小，电压最大

② LC上电流大小相等，相位相反，总电流为0.（电流谐振）

$\dot{I}_C = \dot{U}j\omega_0 C = j\omega_0 C\frac{\dot{I}_S}{G} = jQ\dot{I}_S$ $\dot{I}_L = jQ\dot{I}_S$

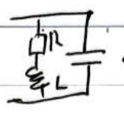
品质因数 $Q = \frac{\omega_0 C}{G} = \frac{1}{\omega_0 GL} = \frac{1}{G}\sqrt{\frac{C}{L}}$

④能量 $W = W_C + W_L = LQ^2I_S^2$

③功率 $P = U^2G$ $|Q_L| = |Q_C| = \omega_0 CU^2 = \frac{U^2}{\omega_0 L}$

5. 线圈谐振

实际电感线圈总是存在电阻，



(1) 谐振条件

$\omega_0 = \sqrt{\frac{1}{LC} - (\frac{R}{L})^2}$ $R < \sqrt{\frac{L}{C}}$ 时可谐振

$\because R << \omega L$ $\therefore Y \approx \frac{R}{(\omega L)^2} + j(\omega C - \frac{1}{\omega L})$ $\omega \approx \frac{1}{\sqrt{LC}}$

等效为

$G \approx \frac{R}{(\omega_0 L)^2}$

$Q = \frac{\omega_0 C}{G} = \frac{\omega_0 C}{R/(\omega_0 L)^2} = \frac{\omega_0^3 CL^2}{R} = \frac{\omega_0 L}{R}$

(2) 谐振特点

① 谐振时阻抗大 $Z(\omega_0) = R_0 = \frac{R^2+(\omega_0 L)^2}{R} \approx \frac{L}{RC}$

② 电流一定时，端电压较高 $U_0 = I_0\frac{L}{RC}$

③ 支路电流是总电流Q倍，设 $R << \omega L$ $I_L \approx I_C = QI_0 >> I_0$

本笔记在https://github.com/dydcyy-gh/study-notes开源