第五章

振动

第五章　机械振动

机械振动：回来往复运动

振动：任一物理量在某个确定数值附近周期性变化

波动：振动状态在空间的传播（第六章）　振动 → 波动

简谐振动叠加 → 简谐运动

5.1 简谐振动

弹簧振子　胡克定律 $\vec{F} = -k\vec{x}$　恢复力

$F = ma = -kx \quad a = -\frac{k}{m}x \quad 令 \sqrt{\frac{k}{m}} = \omega$

$\therefore a + \omega^2 x = 0 \quad \frac{d^2x}{dt^2} + \omega^2 x = 0$ 解方程

$x = A\cos(\omega t + \varphi)$

单摆 $\omega = \sqrt{\frac{g}{l}}$　复摆 $\omega = \sqrt{\frac{mbg}{l}}$　$\theta = \theta_m \cos(\omega t + \varphi)$

$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \varphi) \quad v_{max} = \omega A$

$a = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \varphi) \quad a_{max} = \omega^2 A$

$$\begin{cases} \vec{F} = -k\vec{x} & \frac{d^2x}{dt^2} + \omega^2 x = 0 & x = A\cos(\omega t + \varphi) \\ \vec{M} = -k\vec{\theta} & \frac{d^2\theta}{dt^2} + \omega^2 \theta = 0 & \theta = \theta_m \cos(\omega t + \varphi) \end{cases}$$

满足其一即为简谐

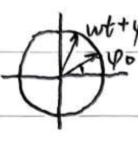
弹簧振子频率 $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$　周期 $T = 2\pi\sqrt{\frac{m}{k}}$

φ 初相位　$v_m = \omega A$ 速度幅　$a_m = \omega^2 A$ 加速度幅

加速度与位移反相位

相位差 $\Delta\varphi = \varphi_2 - \varphi_1$　时间差 $\frac{\varphi_2 - \varphi_1}{\omega}$

旋转矢量法（逆时针）



旋转矢量在 X 轴的投影点 M 的运动规律

$x = A\cos(\omega t + \varphi)$　M 的运动为简谐振动 $T = \frac{2\pi}{\omega}$

回复力 $F = -kx$　平衡位置 $x = 0$　$\frac{d^2x}{dt^2} = -\omega^2 x$ 力学描述

$x = A\cos(\omega t + \varphi) \quad v = -A\omega\sin(\omega t + \varphi) \quad a = -\omega^2 A\cos(\omega t + \varphi) = -\omega^2 x$

简谐振动的能量

动能 $E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \varphi)$

势能 $E_p = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t + \varphi)$　$\omega^2 = \frac{k}{m}$

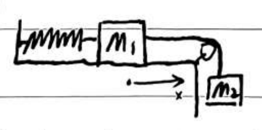
$E_k + E_p = \frac{1}{2}kA^2 = \frac{1}{2}mv_m^2$

动势能变化频率是弹簧振子振动频率的两倍

f 一定，谐振动总能量与振幅平方成正比

$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} = \pm\omega\sqrt{A^2 - x^2}$

例



$m_1 = m_2 = m$　弹性系数 k，自然伸长时释放

求 x_{max}　v_{max}　T

m_1 初始位置为原点　$\frac{1}{2}(2m)\left(\frac{dx}{dt}\right)^2 + \frac{1}{2}kx^2 - mgx = 0$　①

$\frac{dx}{dt} = 0$ 时，$x_{max} = \frac{2mg}{k}$

对①式求导 $\frac{d^2x}{dt^2} + \frac{k}{2m}x - \frac{g}{2} = 0$　$\frac{d^2x}{dt^2} = 0, \frac{dx}{dt} = max$

此时 $x = \frac{mg}{k}$　$\therefore v_{max} = \left(\frac{dx}{dt}\right)_{max} = \left(\frac{mg^2}{2k}\right)^{\frac{1}{2}}$

$A = \frac{mg}{k}$，$v_{max} = \omega A$　$\omega = 2\pi\sqrt{\frac{2m}{k}}$

5.2 简谐振动的叠加

1. 同一直线上同频率合成

$x_1 = A_1\cos(\omega t + \varphi_1) \quad x_2 = A_2\cos(\omega t + \varphi_2)$

$\vec{A} = \vec{A_1} + \vec{A_2} \quad A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos(\varphi_2 - \varphi_1)}$

$\varphi = \arctan\frac{A_1\sin\varphi_1 + A_2\sin\varphi_2}{A_1\cos\varphi_1 + A_2\cos\varphi_2}$

2. 同一直线上 n 个同频率简谐振动合成

振幅相等，均为 a，初相位分别为 0, δ, 2δ, 3δ …

$A_{合} = a\frac{\sin\frac{n\delta}{2}}{\sin\frac{\delta}{2}}$　初相位 $\varphi = \frac{n-1}{2}\delta$

$x = A\cos(\omega t + \varphi) = a\frac{\sin\frac{n\delta}{2}}{\sin\frac{\delta}{2}}\cos\left(\omega t + \frac{n-1}{2}\delta\right)$

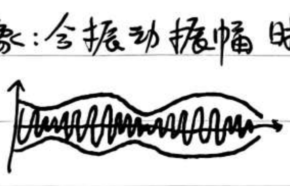
3. 同一直线上两个频率相近的简谐振动合成

$\vec{A_2}$ 相对 $\vec{A_1}$ 转动角速度 $\omega_2 - \omega_1$

两矢量重合：合振动振幅极大，反向极小

$x = \underbrace{2A\cos\frac{\omega_2 - \omega_1}{2}t}_{振幅}\cos\left(\frac{\omega_2 + \omega_1}{2}t + \varphi\right)$

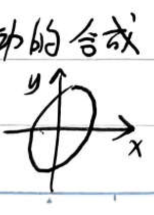
拍现象：合振动振幅时强时弱



$T = \frac{2\pi}{\omega_2 - \omega_1} \quad f = \frac{\omega_2 - \omega_1}{2\pi}$

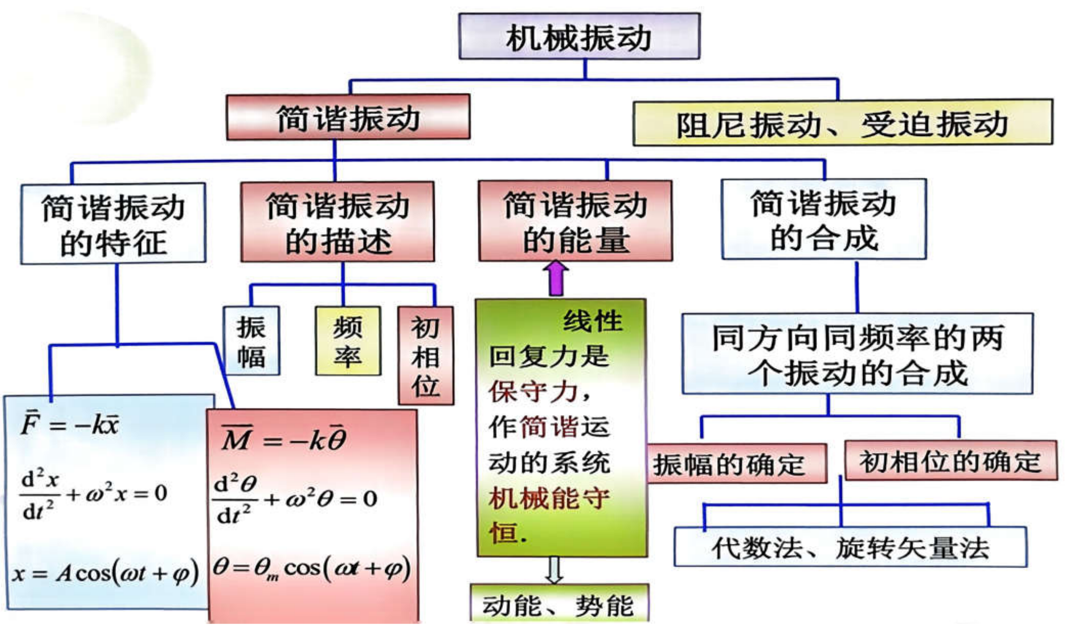
4. 相互垂直的简谐振动的合成

$\begin{cases} x = A_1\cos(\omega t + \varphi_1) \\ y = A_2\cos(\omega t + \varphi_2) \end{cases}$



$\frac{x^2}{A_1^2} + \frac{y^2}{A_2^2} - \frac{2xy}{A_1A_2}\cos(\varphi_2 - \varphi_1) = \sin^2(\varphi_2 - \varphi_1)$ 椭圆

5. 简谐运动的分解



本笔记在https://github.com/dydcyy-gh/study-notes开源