

Understanding the Energy Potential of Lithium-Ion Batteries: Definition and Estimation of the State of Energy

Katharina Lilith Quade,^{*[a, c]} Dominik Jöst,^[a, c] Dirk Uwe Sauer,^[a, b, c, d] and Weihan Li^[a, c]

An accurate estimation of the residual energy, i.e., State of Energy (*SoE*), for lithium-ion batteries is crucial for battery diagnostics since it relates to the remaining driving range of battery electric vehicles. Unlike the State of Charge, which solely reflects the charge, the *SoE* can feasibly estimate residual energy. The existing literature predominantly focuses on showcasing diverse methods with a gap in conducting in-depth analysis and comparison of the *SoE*. The scope of this work is to provide a comprehensive understanding of the *SoE* by discussing the feasibility and applicability of various definitions and estimation approaches from the literature. For the first time, we

classify existing *SoE* definitions, considering the differences between the inherent stored and usable energy. In the absence of a unified definition in the literature, we propose two physically feasible definitions. Based on the proposed definitions, we critically analyze the practical challenges of *SoE* estimation. Additionally, to examine the accuracy of the most suitable *SoE* definition, we conduct an experimental evaluation using a commercial NCA/C+Si cell. Compared to the State of stored Energy, we demonstrate that the State of Charge underestimates the residual energy by more than 3%, highlighting the importance of adequate *SoE* estimation.

Introduction

Motivation

The precise estimation of the remaining energy, the so-called State of Energy (*SoE*), is crucial in all sectors of electrified transportation, e.g., vehicles, trains, and ships.^[1–3] The *SoE* enables not only an efficient use of the complete battery system^[4] but also provides knowledge of the residual driving range and, therefore, mitigates the so-called range anxiety.^[3,5] In general, the *SoE* estimation can be divided into two categories: the estimation of the residual energy stored in a battery, *SoE_{stored}*, and the estimation of the residual usable energy, *SoE_{usable}*, which correlates with the remaining driving range of an electric vehicle. However, in literature, various *SoE* definitions exist for each category, making it challenging to compare existing *SoE* estimation algorithms. Traditionally, the

residual energy is estimated with the help of the State of Charge (*SoC*), meaning that the residual energy is incorrectly assumed to be constant for any *SoC* interval. To illustrate this, Figure 1(a) shows the residual energy and the differential energy with respect to the *SoC* in blue and green, respectively. The differential energy is calculated with the help of the differential watt-hour analysis (DWA), similar to the differential voltage analysis,^[6] based on the quasi open-circuit voltage (qOCV) of a tested, commercial NCA/C+Si battery cell. The values of the DWA are negative since the remaining energy decreases with decreasing *SoC*, showing that more energy per *SoC* can generally be drawn from a cell for higher *SoCs*. Therefore, although the *SoC* metric is commonly used for residual energy estimation, it cannot reflect the energy that can be drawn from a battery cell accurately.^[7] Another challenge that additionally occurs for residual usable energy estimation is

- [a] K. L. Quade, D. Jöst, Prof. Dr. D. U. Sauer, Dr. W. Li
Chair for Electrochemical Energy Conversion and Storage Systems,
Institute for Power Electronics and Electrical Drives (ISEA),
RWTH Aachen University
Campus-Boulevard 89, 52074 Aachen, Germany
E-mail: katharina.quade@isea.rwth-aachen.de
 - [b] Prof. Dr. D. U. Sauer
Institute for Power Generation and Storage Systems (PGS),
E.ON ERC, RWTH Aachen University
Mathieustrasse 10, 52074 Aachen, Germany
 - [c] K. L. Quade, D. Jöst, Prof. Dr. D. U. Sauer, Dr. W. Li
Jülich Aachen Research Alliance, JARA-Energy,
Templergraben 55, 52056 Aachen, Germany
 - [d] Prof. Dr. D. U. Sauer
Helmholtz Institute Münster (HI MS),
IEK 12, Forschungszentrum Jülich,
52425 Jülich, Germany
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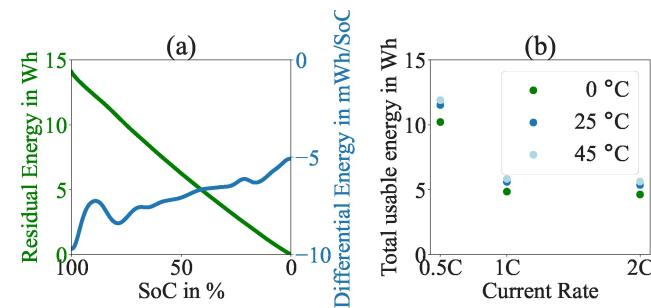


Figure 1. (a) Differential watt-hours analysis (DWA) of a commercial NCA/C+Si cell in green. The remaining stored energy is depicted in blue. Since all values of the DWA are negative, more energy per *SoC* can be discharged at higher *SoC* values. (b) Total usable energy of an NCA/C+Si cell for different temperatures and constant current rates during discharge. Generally, more energy can be discharged from the battery at higher temperatures and lower current rates. This is in line with findings from [9]. Both effects from (a) and (b) lead to errors during *SoC*-based range prediction.

that it is influenced by future operating factors such as temperature and current rate.^[4,8] Figure 1(b) shows the influence of operating factors by illustrating that the total usable energy increases for lower current rates and higher temperatures which is not reflected in the traditional methods.

In the existing literature, researchers have already proposed multiple definitions for the *SoE* metric but without examining the significance and applicability of these definitions. The lack of a clear and standardized definition of *SoE* makes it difficult to compare and evaluate different approaches for *SoE* estimation in real-world applications. Furthermore, there is an absence of a comprehensive comparison between the two distinct concepts: SoE_{stored} and SoE_{usable} . Given the importance of understanding these concepts for accurate residual energy estimation, it is surprising that the literature primarily focuses on highlighting improvements made in one of these concepts rather than providing a holistic analysis and comparison. Some authors, such as Hickey et al.,^[9] focus on the comprehensive comparison between the *SoE* and *SoC*, highlighting the advantages of the *SoE* over the *SoC* estimate for remaining energy. However, to the authors' best knowledge, there is no literature directly comparing the different energy concepts and focusing on the comprehensive understanding of the energy definition for each category.

Contributions of this work

To fill the aforementioned research gap, our work enhances the current understanding by providing an in-depth analysis of energy definition and the crucial factors that need to be taken into account when calculating the *SoE*. For the first time, we subsequently classify existing definitions into the two energy concepts leading to a clear understanding of the performance and comparability of residual energy estimation methods. As a novel contribution, we propose two *SoE* definitions that, unlike most previous publications, directly correlate with the battery's characteristics and avoid distortion,^[10] facilitating the implementation of a practical *SoE* algorithm. For the first time, we derive the mathematical relationship between residual stored energy and *SoC*. By using this Equation, future publications can extend their estimation methods based on existing *SoC* calculations. Most publications predominantly focus on improving *SoE* estimation methods and not on the direct comparison and comprehensive analysis of the metrics with the help of experiments. Thus, we conduct experiments and verify the advantages of adequate *SoE* estimation in contrast to traditional methods. With the characteristics of a commercial 3.35 Ah NCA/C+Si battery cell, we determined the *SoE(OCV)* and experimentally verified that the traditional method underestimates the residual energy significantly for the tested battery cell. To further distinguish the SoE_{stored} from SoE_{usable} experimentally, we conducted various constant current discharge experiments and highlighted the importance of the *SoE* metric for residual energy estimation, especially for higher currents.

Remainder of this work

The subsequent sections of this work are organized as follows: In the upcoming section, we briefly describe the theoretical background and the two different energy concepts. Then, we present *SoE* definitions in the existing literature for each energy concept. Building on that, we evaluate the strengths and weaknesses of these specific definitions and propose two feasible novel *SoE* definitions with experimental validation. We then critically discuss the challenges and opportunities of *SoE* estimation methods highlighting the effects that need to be considered for improving residual energy estimation. In the last section, we summarize the main contributions and the conclusions that can be drawn from this work.

Fundamentals

State of charge

To make an adequate estimation of the *SoE*, we first need to establish a definition of the *SoC*. There are several definitions of the *SoC* in literature.^[11,12] Each of them uses its own kind of nomenclature. To be able to make a meta-analysis, we define the *SoC* at time t as the ratio of the remaining charge $Q(t)$ to the maximum charge Q_{max} that can be drawn from the cell corresponding to the actual capacity within manufacturer specifications: $SoC(t) = \frac{Q(t)}{Q_{max}}$. The *SoC* is defined as a value between 0% and 100%. Since Q_{max} diminishes when the cell ages, the *SoC* for an aged cell still lies between 0% and 100%.

Energy concepts

An introduction to the different energy terms used in this work is essential for discussing the definitions and methods for *SoE* estimation. The used terms are schematically explained in Figure 2 as follows:

- When the cell is fully charged and has an *SoC* of 100%, the stored energy, E_{stored} , corresponds to the maximum stored energy $E_{max,stored}$. $E_{max,stored}$ is not entirely usable due to energy losses during operation $E_{max,dissipation}$. Thus, the maximum usable energy, $E_{max,usable}$, is less than the maximum stored energy $E_{max,stored}$. Since the cell is fully charged, the usable energy, E_{usable} , equals $E_{max,usable}$ and the dissipation energy, $E_{dissipation}$, equals $E_{max,dissipation}$.
- When the cell is further discharged, the stored energy, E_{stored} , and the usable energy, E_{usable} , decrease. The dissipation energy, $E_{dissipation}$, is less than the maximum dissipation energy $E_{max,dissipation}$ since energy has already been dissipated during operation ($E_{dissipated}$). It should be noted that E_{stored} is composed of E_{usable} and $E_{dissipation}$.
- When the cell is completely discharged and the *SoC* has dropped to 0%. The discharge current has approached 0 A, and all stored energy has been removed from the cell. E_{stored} as well as the usable energy E_{usable} are therefore 0 Wh. The

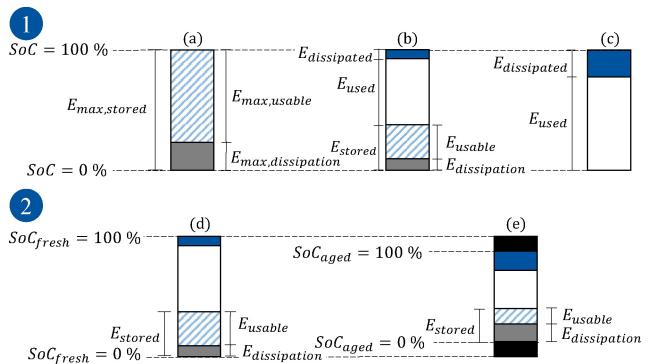


Figure 2. 1) Schematic depiction of the used energy terms. (a) The cell is fully charged, and the actually stored energy corresponds to the maximum stored energy. (b) During discharging, the stored and usable energy decrease due to operating losses. (c) The cell is fully discharged, and no energy is left. 2) Schematic depiction of the used energy terms during discharge under different aging conditions: d) Fresh cell e) Aged cell. Since the SoC takes into account the aging of the cell, the limits of the SoC for an aged cell are also between 0% and 100%.

value of the energy dissipated during the whole discharge process corresponds to $E_{max,dissipation}$.

Figure 2(d) and (e) further schematically explains the difference in the energy terms between a fresh and an aged cell during the discharge process. The available capacity for an aged cell is lower than that for a fresh cell. Since the available capacity of an aged cell is lower than that of a fresh cell, this results in a lower maximum stored energy of an aged cell compared to the maximum stored energy of a fresh cell. Additionally, it can be noted that for the aged cell, $E_{dissipation}$ and $E_{max,dissipation}$ are relatively larger due to an increase in internal resistance caused by progressive aging. If more losses occur during operation, less energy can be utilized, and E_{usable} further decreases.

Definitions of State of Energy

The SoE is not clearly defined in the existing literature. However, the basic idea is largely uniform: The SoE represents the energy state of a cell, i.e., the residual energy normalized to a specific reference energy value.

We assign the definitions from the literature roughly to two concepts and list them in Table 1 and Table 2. For each

Table 1. Definitions of the State of stored Energy in literature. We assign the definitions from the literature to the metric State of stored Energy if no losses are accounted for in the numerator. The column "Definition" shows the original definitions of the SoE from the paper. The column "Explanation of Variables" describes the variables as long as they are described in the respective manuscript. Only Equation (1.4) can be feasibly used for residual stored energy estimation.

Eq. No.	Definition	Explanation of Variables	Ref.
(1.1)	$\xi_{k+1} = \xi_k - p_{oc,k} \Delta t / E_a$	$p_{oc,k}$: pseudo power on the OCV E_a : nominal energy capacity	[10]
(1.2)	$z_{k+1} = z_k - p_{OC,k} \Delta t / E_a$	z : SoE $p_{OC,k}$: power on the OCV E_a : max available energy	[13]
(1.3)	$SoE_{k+1} = SoE_k - \frac{\eta_k U_{oc,k} i_k \cdot \Delta t}{E_a}$	k : indexes moment $k \Delta t$ Δt : sampling interval η : current efficiency U_{oc} : open-circuit voltage i : current E_a : maximum available energy of a full charged battery	[14]
(1.4)	$SoE(SoC) = \frac{E_{stored}(SoC)}{EC} = \frac{\int_{q(SoC=0\%)}^{q(SoC)} v_{Bat,OCV}(q) dq}{\int_{q(SoC=100\%)}^{q(SoC=0\%)} v_{Bat,OCV}(q) dq}$	E_{stored} : amount of stored energy EC : actual energy storage capacity $v_{Bat,OCV}$: battery open-circuit voltage	[15]
(1.5)	$SoE = \frac{E_{rem}}{E_{ava}}, E_{ava} = E_{rem} + E_{chge}$ $E_{rem} = \sum_{i=1}^n \int_{SOC_{t_i}}^{SOC_i} C_a^i U_{OCV}^i(SOC) dSOC$ $E_{chge} = \sum_{i=1}^n \int_{SOC_{t_i}}^{SOC_{t_i}^i} C_a^i U_{OCV}^i(SOC) dSOC$	E_{rem} : residual energy E_{ava} : maximum available energy E_{chge} : battery charge energy SoC_t : current State of Charge $SOC_{t_i}^i$: SoC of the i th cell, when battery reaches lower cut-off state at time t_i $SOC_{t_i}^i$: cell current SOC when battery reaches the upper cut-off voltage	[16]
(1.6)	$E_{PRDE}(t) = \sum_{i=1}^n C_N^i \int_{SOC_{t_i}}^{SOC_i} U_{OCV}^i(SOC) dSOC$	E_{PRDE} : battery pack remaining discharge energy C_N^i : maximum available capacity of the i th battery n : number of cells in the battery pack connected in series SOC_t^i : SOC of the i th battery at time t $SOC_{t_i}^i$: SOC of the i th battery when one of the batteries in the battery pack reaches the lower cut-off voltage	[17]

Table 2. Definitions of the State of usable Energy in literature. We assign the definitions from the literature to the metric State of usable Energy if losses are accounted for in the numerator. The column "Definition" shows the original definitions of the SoE from the paper. The column "Explanation of Variables" describes the variables as long as they are explained in the original manuscript. Only Equation (2.7) can be feasibly used for residual usable energy estimation.

Eq. No.	Definition	Explanation of Variables		Ref.
(2.1)	$Z_k = Z_{k-1} - \frac{\eta \Delta E_a}{E_a} = Z_{k-1} - \frac{\eta U_{t,k-1} I_{t,k} \Delta t}{E_a}$	Δt : sampling time ΔE_a : variation of battery energy during each sample time	E_a : available energy of battery η : energy efficiency of battery	[7]
(2.2)	$S_k = S_{k-1} - \frac{\eta_s \Delta E_a}{E_a} = S_{k-1} - \frac{\eta_s U_{t,k-1} I_{t,k} \Delta t}{E_a}$	Δt : sampling time s : SoE ΔE_a : variation of battery energy during two contiguous time samples	E_a : battery available energy η_s : battery energy efficiency	[19]
(2.3)	$SoE_k = SoE_{k-1} - \eta V_{t,k-1} I_{L,k-1} t_s / E_N$	η : energy efficiency E_N : nominal energy I_L : load current V_t : terminal voltage		[20, 21]
(2.4)	$SOE(t) = SOE(t_0) - \int_{t_0}^t \frac{U_t I_t dt}{E_n}$	U_t : terminal voltage of the battery I_t : load current of the battery E_n : total available energy of the battery t_0 : initial moment of discharge t : end moment of discharge		[4]
(2.5)	$SOE(k) = SOE(k-1) - \frac{\eta I_{k-1} V_{k-1} \Delta t}{E_N(T, \kappa)}$	η : coulombic efficiency I_{k-1} : current at sample time $k-1$ V_{k-1} : terminal voltage at sample time $k-1$ Δt : sample time $E_N(T, \kappa)$: total available energy T : temperature κ : discharge rate		[22]
(2.6)	$SoE\% = [1 + \frac{\int_{t_0}^{t_1} \eta_e V(t) I(t) dt}{E_n}] \cdot 100\%$	V : battery voltage I : battery current η_e : coulombic efficiency E_n : nominal energy		[9]
(2.7)	$E_{RDE} = \int_t^{t_{lim}} U_t \cdot I dt = \int_{Q_{cum}(t)}^{Q_{cum}(t_{lim})} U_t dQ$	U_t : battery terminal voltage $Q_{cum}(t)$: already cumulated discharge capacity t_{lim} : end-of-discharge time		[23]

concept, we then discuss in detail the representative definitions from the literature for *SoE*. The *SoE* definitions with the Equation numbers (1.1)–(1.6) aim to describe a theoretical energy state providing information about the energy inherently stored in a cell. Other authors define a practical energy state, which correlates with the energy that can be used, taking into account the dissipation energy. Those are the definitions with the Equation numbers (2.1)–(2.7). To overcome the weaknesses of existing definitions, we propose and analyze one definition for each concept that is physically feasible for *SoE* estimation and can be directly applied for online residual energy estimation.

Definitions of residual stored energy

Table 1 presents *SoE* definitions from the literature evaluated in this paper that do not intend to consider losses during the operation of the cell and are further described in the following.

Chang et al.^[10] define in Equation (1.1) the *SoE* as a state that relates to pseudo-power', p_{oc} , which is the product of the OCV and current. The losses that occur during operation are thus not taken into account, and the definition correlates with the energy that is stored in the battery. The *SoE* in this definition is normalized to the nominal energy, meaning that the *SoE* could be greater than 100%, indicating that more energy is stored than actually can be stored. In contrast to Equation (1.1), Xie et al.^[13] use the maximum available energy as a reference value that considers the influence of temperature and aging on the available energy, as shown in Equation (1.2). Therefore, losses during operation are considered. However, it is not advisable to include losses in the energy reference value while considering the residual stored energy. This results in a distortion of the *SoE*, and the *SoE* is not necessarily a value in the range of 0% and 100% and will be overestimated during charging and underestimated during discharging.

In Equation (1.3), Zhang et al.^[14] take the coulombic efficiency into account to cover the influences of charge losses on the SoE. Although this consideration is physically reasonable, it should be noted that usually, the coulombic efficiency of lithium-ion batteries can be approximated to the value of 100%.^[18] For other battery technologies, such as lead-acid batteries, coulombic efficiency needs to be considered. In this definition, the SoE is normalized to the maximum available energy depending on a constant temperature but varying current rates leading to a distortion of the SoE in the application.

Rubenbauer et al.^[15] define the SoE in Equation (1.4) as the ratio of stored energy to the actual energy storage capacity. Both the numerator and denominator correlate with the OCV. If the stored energy is summed up at all times, for example, during a complete discharge, this sum corresponds to the so-called energy capacity equivalent to the maximum stored energy. This ensures that the value of the SoE for a fully charged battery equals 100% and the value of the SoE for a fully discharged battery equals 0% and thus is applicable for online estimation.

Equation (1.5) defines the SoE for the battery pack as the ratio of the remaining energy to the maximum available energy. Both remaining energy and maximum available energy are a function of the OCV of the battery pack. It should be noted that the presented SoE definition considers the differences between the cells in a battery pack due to cell-to-cell variance. As the cells connected in series do not reach the lower cut-off voltage concurrently due to variances, the 'weakest' cell limits the total stored energy of the battery pack. Since the nominator and the energy reference value both correlate with the OCV, it is ensured that the SoE has a value between 0% and 100%. The definition presented by Zhang et al. reflects that losses due to operating conditions are neglected unless they are reflected in the OCV.

Instead of defining the SoE, in Equation (1.6), Zhang et al.^[17] define the battery pack remaining discharge energy E_{PRDE} as the cumulative energy of every cell to the moment one cell reaches its lower cut-off voltage. Similar to Equation (1.5), the remaining discharge energy is calculated for a battery pack consisting of various cells. Even though it is not clear how the maximum available capacity is defined, i.e., whether it is equal to the nominal capacity of the individual cell, the remaining discharge energy of the battery pack correlates with the OCV of the cells at time t . Thus, the definition feasibly describes the stored energy of a battery pack.

Definitions of residual usable energy

Table 2 presents SoE definitions from the literature that intend to consider losses during the operation of a cell. The definitions, specifications as well as explanation of the variables are the original terms used in the respective literature and are not further adapted.

Equations (2.1) and (2.2) both describe a discrete SoE that correlates with the terminal voltage and the instantaneous

current. The remaining energy is normalized to the available energy. By including the terminal voltage in the nominator, losses during cycling are considered, and therefore, they need to be considered in the energy reference value for a feasible definition of the SoE. However, it is not clear how the available energy is defined. Furthermore, the battery energy efficiency η is included, which is the ratio of the total discharge energy to the total charge energy. As described in [15] the battery energy efficiency already considers losses. Thus, the dissipation energy is considered twice in Equations (2.1) and (2.2) by taking into account energy efficiency and, additionally, the dissipation energy reflected in the terminal voltage.

In Equation (2.3), Wei et al.^[20] also include energy efficiency in their proposed SoE definition. In contrast to the previous definitions, the reference value represents the nominal energy; thus, the SoE is independent of future conditions. This definition makes the real-time estimation easier, but it does not reflect a feasible meaning of the SoE. By normalizing the usable energy to the nominal energy, the SoE may not reach the upper limit of 100% in operation. A more practical solution for the application would be to include aging in the nominal energy. Lai et al.^[4] define with equation (2.4) the State of usable Energy as a function of battery terminal voltage and current. The battery's total available energy considers different operating conditions such as temperature, aging, and current rate. However, it remains an issue that the dynamic variation of those conditions and their influence on the SoE are neglected. The varying temperature and current rate in operation may result in SoE values outside the feasible boundaries.

In Equation (2.5), Dong et al. present a discrete SoE definition that includes the coulombic efficiency.^[22] The energy reference value as a function of temperature and current rate represents the total available energy, which is defined as a product of the total available capacity and the OCV of a fully charged battery. Other environmental factors, such as aging, are not considered, whereas they are considered in the nominator by taking into account the terminal voltage that also reflects aging. In order to determine the total available energy, the authors suggest a case distinction for different temperatures and current rates. The dynamics of the total available energy are neglected.

Equation (2.6) describes the SoE as the ratio of usable energy to nominal energy. It should be noted that this definition only describes the SoE of a fully charged battery for a discharging process. With a slight modification of the definition, it can also be applied to batteries with any initial SoE. The losses are considered in the nominator of the proposed SoE definition, whereas they are neglected in the reference value with the normalization of the SoE to the nominal energy. Therefore, the SoE under this definition may be higher than the SoE that does not intend to consider losses during cycling. However, taking the nominal energy as a reference value makes a comparison of different SoE values easy.

In Equation (2.7), Liu et al.^[23] define the residual energy as the integration of the terminal voltage from the current time until the battery is completely discharged. By taking into account the terminal voltage, the dissipation energy is

Table 3. Advantages and disadvantages of the presented SoE definitions. In this Table, the existing definitions are assigned to the two concepts: SoE_{stored} and SoE_{usable} . Only Equation (1.4) for SoE_{stored} and Equation (2.7) for SoE_{usable} use the physically correct definition. Other definitions result in drifting SoE limits, which, if not accounted for, can lead to safety-critical operations.

SoE Definitions Evaluation					
No.	Pro	Stored Energy	No.	Pro	Usable Energy
		Con			Con
(1.1)	Allows rapid implementation and easy comparability	Nominal energy as reference value leading to drifting of SoE limits	(2.1), (2.2)	–	Losses accounted twice, available energy not apparent
(1.2)	–	Losses are considered in energy reference value	(2.3)	Easy real-time implementation	Static reference value, losses accounted twice
(1.3)	Consider coulombic efficiency	Operating conditions are considered in the reference value	(2.4), (2.5)	Account for temperature, aging and current	Not applicable for dynamic profiles
(1.4)	Use battery parameters also for reference value	–	(2.6)	Easy comparability	Only for fully charged battery, static reference value
(1.5)	Consider the cell-to-cell variance	No temperature dependency, efficiency is assumed to be 100	(2.7)	Clear, account for temperature, aging and current	–
(1.6)	Consider the cell-to-cell variance	Maximum available capacity not apparent			

considered. Therefore, this definition is feasible for determining the residual usable energy and highly applicable since the inherent correlation of the usable energy to the terminal voltage is reflected in the definition.

Discussion of existing and proposition of physically feasible SoE definitions

As presented, numerous definitions and concepts for the SoE exist in the literature. Table 3 summarizes the individual strengths and weaknesses for each presented SoE definition. It is visible that most existing definitions do not directly correlate with the remaining energy and rely on substantial simplifications. Moreover, the mismatch between the numerator and denominator in common SoE definitions belonging to different energy concepts leads to drifting limits and distorted SoE. For instance, (1.3) neglects operating conditions in the nominator while accounting for them in the denominator leading to SoE limits outside of bounds. To address the issue of SoE drifting, the nominator and denominator must refer to the same energy concept. To the best of the authors' knowledge, the existing literature does not currently provide a viable definition for each energy concept, even though a physically meaningful definition is imperative to develop efficient algorithms for residual energy estimation. To bridge this gap, in this section, we propose two novel definitions that are physically meaningful, mitigate bias, and apply to all battery types. Considering the battery's inherent characteristics and avoiding oversimplifications, these definitions are robust and reliable for SoE estimation. We verify the feasibility of the proposed definitions with the help of experiments. We then quantitatively compare our definition to the SoC showing the advantage of the proposed definition compared to the traditional methods.

State of stored energy

The State of stored Energy describes the ratio of the stored energy E_{stored} , which can ideally be discharged starting at time t , to the maximum stored energy $E_{max,stored}$. Since the charge amount that can be stored in the battery decreases, and the OCV curve shifts with progressive aging, the maximum stored energy decreases simultaneously. Similar to [15], SoE_{stored} is defined according to (1):

$$SoE_{stored}(t) = \frac{E_{stored}(t)}{E_{max,stored}} = \frac{\int_{Q(SoC=0)}^{Q(SoC_t)} U_{OCV}(Q)dQ}{\int_{Q(SoC=0)}^{Q(SoC=1)} U_{OCV}(Q)dQ} \quad (1)$$

In Figure 3, the OCV of the tested NCA/C+Si cell is depicted in blue, and the cell's terminal voltage based on a constant current discharging of the cell at 25 °C is depicted in green. E_{stored} and $E_{max,stored}$ both relate to the OCV, whereas the stored energy considers the voltage course between the current charge amount until no more energy or charge can be extracted and the SoC and the SoE reach 0%. The $E_{max,stored}$ for the tested cell, depicted in Figure 3, is 12.25 Wh and completely independent of discharge conditions. The SoE_{stored} does not include operational losses. Similar to the relationship between SoC and the OCV, a relationship between the SoE_{stored} and the OCV can be derived. For this purpose, it is helpful to discretize the continuous formulas for the SoC as shown in Equation (2) and for the SoE_{stored} as in Equation (3).

$$SoC_k = SoC_{k-1} + \frac{I_{L,k} \cdot \Delta t}{Q_{max}} \quad (2)$$

SoC_k describes the current state. SoC_{k-1} represents the previous SoC. The current at time k is $I_{L,k}$ and the maximum charge amount is denoted as Q_{max} . Δt represents the sampling time.

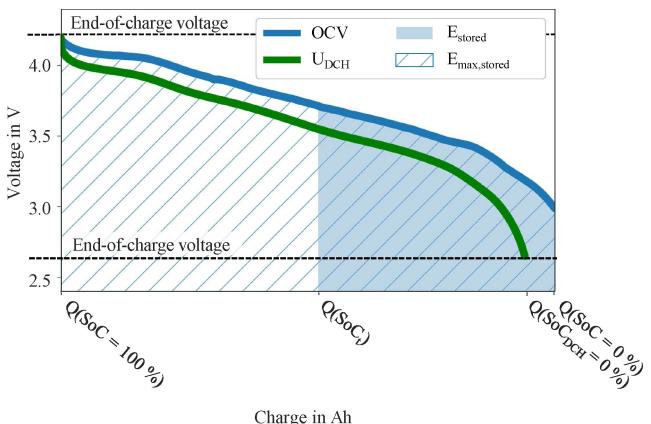


Figure 3. The stored energy is the integration of the OCV over charge. $E_{\text{stored}}(t)$ (blue area) is the energy from the actual time until no more energy is stored in the battery. The maximum stored energy (hatched area), $E_{\text{max,stored}}$, is the stored energy between the full and empty state of the cell. The specifications given by the manufacturer define the full and empty states. $Q_{\text{SoC}_{\text{DCH}}}$ considers that the end-of-discharge voltage is reached earlier for the given constant current discharge conditions. During the discharging process, the maximum stored energy of the tested cell is 12.25 Wh, differing from the maximum usable energy, which is 10.83 Wh for the given discharge conditions.

$$\text{SoE}_{\text{stored},k} = \text{SoE}_{\text{stored},k-1} + \frac{I_{L,k} \cdot U_{\text{OCV},k} \cdot \Delta t}{E_{\text{max,stored}}} \quad (3)$$

$\text{SoE}_{\text{stored},k}$ represents the current SoE , whereas $\text{SoE}_{\text{stored},k-1}$ describes the previous value. The current OCV is denoted as $U_{\text{OCV},k}$. $E_{\text{max,stored}}$ describes the maximum stored energy. By transforming Equation (2) and substituting it into equation (3), for the first time, we derive a direct, discrete mathematical relationship between the SoC and the $\text{SoE}_{\text{stored}}$ as seen in Equation (4).

$$\text{SoE}_{\text{stored},k} = \text{SoE}_{\text{stored},k-1} + \frac{Q_{\text{max}}}{E_{\text{max,stored}}} \cdot (\text{SoC}_k - \text{SoC}_{k-1}) \cdot U_{\text{OCV},k} \quad (4)$$

Equation (4) demonstrates that the stored residual energy is directly related to the SoC of a battery cell, meaning that the $\text{SoE}_{\text{stored}}$ can be determined after the diffusion processes have completely decayed by measuring the terminal voltage presenting a significant opportunity in the field of residual energy estimation. It should be mentioned that this relationship is influenced by progressing cell aging and varying temperatures since the ratio of maximum capacity and maximum stored energy changes.

Figure 4 shows the SoC and $\text{SoE}_{\text{stored}}$ as a function of the OCV. It is visible that, for a fixed OCV, the value of the SoE is lower than the SoC value associated with that voltage. In green, the deviation between SoC and $\text{SoE}_{\text{stored}}$, $\Delta \text{SoC} | \text{SoE}_{\text{stored}} = \text{SoC} - \text{SoE}_{\text{stored}}$, is depicted. The deviation of the SoC from the $\text{SoE}_{\text{stored}}$ amounts to a maximum of approximately 3% for the tested NCA/C+Si cell. It should be noted that the difference between SoC and $\text{SoE}_{\text{stored}}$ strongly depends on the OCV (SoC) relationship

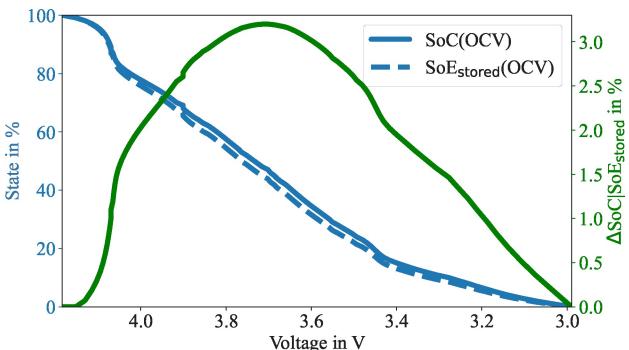


Figure 4. Comparison of $\text{SoC}(\text{OCV})$ and $\text{SoE}_{\text{stored}}(\text{OCV})$. The deviation is depicted in green. Visibly, a range prediction based on the $\text{SoC}(\text{OCV})$ would overestimate the range by over 3 %. This is mainly due to the fact that the SoC does not represent a linear Equation with respect to the OCV.

of a cell and varies for different cell chemistries. This is in line with the findings in [9]

State of usable energy

The State of usable Energy, $\text{SoE}_{\text{usable}}$, describes the ratio of the usable energy, E_{usable} , to the maximum usable energy, $E_{\text{max,usable}}$ and is defined at time t as a function of the environmental factors, depicted in this paper as λ , as in Equation (5):

$$\text{SoE}_{\text{usable}}(\lambda, t) = \frac{E_{\text{usable}}(\lambda, t)}{E_{\text{max,usable}}(\lambda)} = \frac{\int_{Q(\text{SoC}=0)}^{Q(\text{SoC}_t)} U_T(\lambda, Q) dQ}{\int_{Q(\text{SoC}=1)}^{Q(\text{SoC}=0)} U_T(\lambda, Q) dQ} \quad (5)$$

Figure 5 visualizes the definition of E_{usable} and $E_{\text{max,usable}}$. Since the discharge curve is smaller than the OCV curve for every time t and the remaining charge is less while discharging, the usable energy is significantly lower than the stored energy. For the given discharge process, the $E_{\text{max,usable}}$ is 10.83 Wh, and thus significantly lower than the $E_{\text{max,stored}}$ with 12.2525 Wh. As experimentally demonstrated by Hickey et al.,^[9] the difference between the remaining usable energy and the SoC and $\text{SoE}_{\text{stored}}$ value is strongly influenced by operating conditions, such as current rate, aging, and temperature. That is why, more energy could be used for a different power profile as some energy is still stored in the battery.

The terminal voltage can be represented by OCV and polarization voltage separately according to the law of thermodynamics.^[24] Whereas the OCV can be assumed to be approximately independent of operational factors, UP strongly depends on them.^[25] The mathematical transformation of Equation (5) allows the SoE to be represented differently in Equation (6).^[24,26]

$$\text{SoE}_{\text{usable}}(\lambda, t) = \frac{E_{\text{stored}}(t) - E_{\text{dissipation}}(\lambda, t)}{E_{\text{max,stored}} - E_{\text{max,dissipation}}(\lambda)} \quad (6)$$

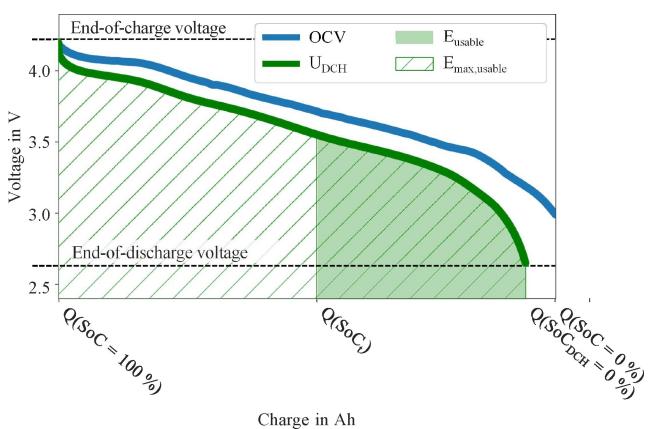


Figure 5. The usable energy calculated by integrating the terminal voltage over charge amount; $E_{\text{usable}}(t)$ (green area) is the energy from the actual time until no more energy can be used for the given power. The maximum usable energy (hatched area), $E_{\text{max,usable}}$, is the usable energy between the full and empty state of the cell. The specifications given by the manufacturer define the full and empty states. $Q_{\text{SoC}_{\text{DCH}}}$ considers that the end-of-discharge voltage is reached earlier for the given constant current discharge conditions. During the discharging process, the maximum stored energy of the tested cell is 12.25 Wh, differing from the maximum usable energy, which is 10.83 Wh for the given discharge conditions.

Methods for SoE Estimation

For the estimation of the SoE, well-known methods such as SoC estimation methods can be applied, which estimate the usable energy or the stored energy. This section presents and critically reviews existing methods specific to SoE estimation that are divided into power-integral methods, model-based methods, and data-driven methods.

Power-integral method

The simplest method for estimating the residual energy is the so-called power-integral method, first introduced by Mamadou et al.^[27,28] It is inspired by the Coulomb counting method for the SoC estimation^[11,29,30] and is an open-loop approach. If current and voltage can be measured precisely and an initial value is known, the SoE can be directly estimated via Equation (7).^[30]

$$\text{SoE}(k) = \text{SoE}(k-1) + \frac{I(k) \cdot U(k)}{E_{\text{ref}}} \quad (7)$$

Since the voltage and current values are affected by measurement deviations, estimation errors are accumulated over time, requiring a re-calibration in the application. Another shortcoming is that the initial SoE needs to be known accurately, and the relationship between the OCV and the SoE needs to be known precisely to adjust the initial value.^[30] Additionally, the reference value, E_{ref} , has to be feasibly chosen. To avoid bias for residual usable energy estimation, the reference value must consider internal and external influencing factors comprising future values and needs to be

corrected after a specific time.^[30] The advantage of this SoE estimation method is the simple implementation on a BMS.

Model-based methods

To overcome the weaknesses of the power-integral method, model-based methods can be applied. They usually employ a simplified battery model to connect the estimated parameters with the state.^[12,31] It is a two-step process, as shown in Figure 6. First, the parameters of a battery model are determined.

Second, the SoE is estimated with the help of a state estimation algorithm by comparing the estimated output of the battery model, the cell's terminal voltage, with the measured output. The model inputs the measured signals, such as current, voltage, and temperature. The corrected SoE is used as a new input for calculating the output in the next step. Model-based methods can be supported by predicting the operation of the vehicle with the help of navigation systems: If the future route is known, the terminal voltage can be predicted based on the model with current and temperature profiles as input. This is an excellent opportunity for model-based methods using operational strategy prediction to improve the estimation of SoE. The disadvantage is that an accurate model is needed to predict the terminal voltage precisely. In the following, direct model-based methods and Kalman filters for SoE estimation are introduced in more detail.

Direct model-based estimation

The SoE can be directly determined based on the estimated parameters of the battery model as proposed in [32] and [23]. Zhai et al.^[32] use a Thevenin model to determine the battery's terminal voltage and directly use their proposed definition of the SoE with the estimated terminal voltage as input. The model parameters are dependent on the temperature. Liu et al.^[23] also use a Thevenin model to estimate the terminal voltage based on SoC, resistance, and a given current profile.

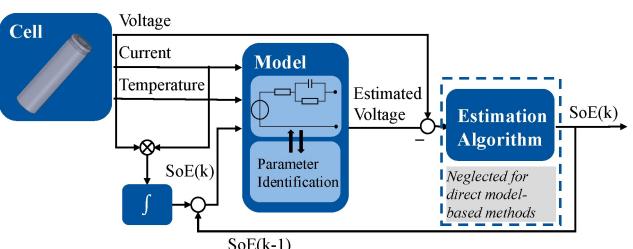


Figure 6. Schematics of a model-based estimation of SoE. Most SoE estimation algorithms use a cell model to estimate the voltage, which is then used for SoE estimation. The parameters of the model depend on current, temperature, and SoE. They are either identified with the help of a parameter identification algorithm or underlying look-up Tables. Based on the error between the measured and estimated terminal voltage, the SoE is adapted. Since direct model-based methods directly estimate the SoE from the cell voltage, there is no need for an additional estimation algorithm based on the voltage estimation error.

The remaining energy is calculated by integrating the product of predicted current and voltage at a specific future time point. Furthermore, Liu et al. analyze different model parameter updating routes.^[23] An advantage of the direct method based on a battery model is the comparably low computational cost since an additional algorithm that corrects the inherent error of the respective model is not applied. However, it should be noted that in the scope of the open-loop estimation, the accuracy of the SoE estimation solely depends on the accuracy of the underlying battery model,^[12] and the operating strategy needs to be known.

Kalman filters

In [8,14,26,33,34] the SoE is estimated based on the extended Kalman filter (EKF). In [33] Wang et al. estimate the State of stored Energy with a Thevenin model, whereas the model parameters are identified with the least-square algorithm.^[33] Zhang et al.^[14] also determine the SoE by estimating the OCV. The SoE is estimated based on a mathematical relationship between the SoE and the estimated OCV. The methods benefit from a low complexity but may suffer accuracy, especially for dynamic, real operations. Li et al.^[26,34] propose an SoE estimation method based on a physics-based fractional order model with variable solid-state diffusivity, which additionally takes electrolyte dynamics into account. The remaining energy loss is estimated based on load current, squared load current, SoC, and resistance. Since the dissipation energy is a function of internal resistance, current, and squared current, only two assumptions are required, which is especially applicable for online SoE estimation. Xia et al.^[8] additionally apply an adaptive noise correction for the parameter estimation of the underlying battery model, considering that the state parameters are highly dependent on temperature. The authors show that the error is comparably small, even for different temperatures and erroneous initial states. Since the terminal voltage can be estimated comparably accurately and robustly, the battery model is a starting point for different estimation methods. The authors use the battery model to co-estimate SoE and SoC, which is applicable for implementation on a BMS. Zhang et al.^[30] use an EKF and additionally address the challenge of the changing OCV and SoE relationship due to battery aging with the help of partial reconstruction of the specific relationship during operation. During operation, more reconstruction points are added to describe the OCV(SoE) relationship in a broader range allowing for the inherent consideration of aging in the underlying battery model.

To overcome any disadvantages of the EKF, such as a high estimation error of a highly non-linear system since an EKF only considers the first-order derivative of the Taylor expansion, the unscented Kalman filter (UKF) based on the unscented transformation is applied.^[35] In [16], Zhang et al. estimate the SoE with an extended Thevenin model. The parameters are estimated via Particle Swarm Optimization (PSO) since the accuracy is higher compared to the RLS method. The computation burden is acceptable because the PSO does not

run at each micro time length. Since a BMS is subject to noise that influences the accurate estimation of the SoE, Wei et al.^[20] lay a particular focus on the unbiased estimation of the model parameters with the help of the Bias Compensating RLS technique to increase the robustness of the estimation. A disadvantage of the method is the cost of a higher computational burden. Zhang et al.^[17] propose an SoE estimation method that can be used in a battery system considering cell inconsistencies. The method additionally takes into account the temperature of the specific cell. Chen et al.^[24] propose a two-step process for State of usable Energy estimation. First, they estimate the State of stored Energy with the help of a UKF. Then, they estimate the residual usable energy by considering the energy conversion efficiency. To calculate the energy conversion efficiency, the authors suggest predicting the future velocity with Markov chains. The predicted load current, as well as the State of stored Energy, can be mapped to the energy conversion efficiency. This approach takes advantage of the fact that the stored energy can be accurately estimated with known methods, such as the UKF. A disadvantage is that the approach requires experiments to show the relationship between energy conversion efficiency, load current, and State of stored Energy. This can lead to inaccuracies, especially with progressive aging.

To reduce the parameters of the filter, the central difference Kalman filter (CDKF) is introduced for SoE estimation by [13] Xi et al.^[13] propose a square root (SR) CDKF. Furthermore, the voltage hysteresis potential is considered in the battery model modeled with the help of a second voltage source in the Thevenin model. Since temperature strongly influences dissipation energy, the authors suggest a thermal evolution model to include temperature effects. The proposed model is especially interesting for cells with a high hysteresis effect. He et al.^[7] also apply a CDKF for the online SoE estimation. In contrast, they use an n-order hysteresis Gaussian model. The Genetic Algorithm determines the model parameters, which identifies an optimum parameter group for the model's coefficients. The authors state that the error for erroneous initial SoEs is comparably low. Since the computational burden is very high, it does not apply to real applications. Kalman filters can also be used as a first stage for so-called hybrid estimation methods. Wang et al. have reviewed such hybrid methods in [36]. Such methods take advantage of the fast convergence of KF methods on the one hand and the high adaptability over the lifetime of data-driven methods on the other hand. In [37] this method was demonstrated using an EKF and an artificial neural network. Combining both methods provided an absolute deviation from the reference value below 1%. Since this has yet to be demonstrated on SoE estimation, it potentially may be a promising method for future publications.

Data-driven methods

As highlighted by Zhang et al.,^[38] data-driven methods do not consider a battery cell's internal dynamics. These methods model the input and output relationship as a black box, relying

solely on the processed data.^[38] The three primary steps involved in the data-driven approach are the following:^[11,39] data collection and feature extraction, model training, and estimation. An advantage is that these approaches do not require a comprehensive understanding of the battery cell's electrochemical processes and internal mechanisms.^[38] Nonetheless, the high non-linearity of the cell that a simplified ECM cannot model can be captured. However, an extensive data set is needed,^[40,38] and the method's transferability to other cells is usually limited. The data quality also dramatically impacts the performance of the data-driven method.^[38] An advantage is that as cloud-based analytics advance, we will be able to save the data in the cloud, perform more in-depth analyses, and use the data to train the model.

Liu et al.^[41] use a back-propagation neural network (BPNN) to estimate the residual usable energy directly. The BPNN takes losses on the internal resistance, electrochemical reactions, and the change of the OCV during operation into account. In the input layer, current, temperature, and battery terminal voltage are used to estimate the *SoE* in the output layer. For training, data generated under various constant currents and temperatures are used. The performance is verified with a constant current discharge at a constant temperature. Dong et al.^[22] propose a hybrid method combining the neural network and Particle Filter (PF). They use a Wavelet Neural Network (WNN) with battery terminal voltage from the previous time step, the estimated *SoE*, current, and temperature as inputs and outputs of the battery terminal voltage at the current time step. The WNN is trained with various partially constant current profiles under constant temperature conditions. To verify the performance of the WNN, constant-current discharge data is used under dynamic temperature conditions, and dynamic discharge data is used under constant temperatures. The PF is then used to estimate the *SoE*.^[22] Both approaches are not validated for real driving or charging profiles; thus, their applicability in the real world is questionable. Ma et al.^[42] use a long short-term memory neural network to estimate *SoC* and *SoE* simultaneously for different and varying operating conditions. They test their algorithm for different temperatures, materials, and noise interference. The authors point out that their method is robust but also state that the network needs to be updated to consider progressive aging.

Opportunities and Challenges of *SoE* Estimation

The challenges and opportunities of *SoE* estimation differ significantly for each category: Whereas the challenges and opportunities for the *SoE_{stored}* are similar to those of the *SoC* estimation, the *SoE_{usable}* estimation additionally requires predicting the future operating strategy making an accurate estimation difficult.

An opportunity arises for estimating the *SoE_{stored}* when a well-performing estimation of the *SoC* is available.^[43] In this case, the *SoC* can be utilized to determine the residual stored

energy through a look-up Table. This already improves the energy estimation compared to the *SoC* without incorporating elaborate models and estimation methods, making it a very promising method for online *SoE* diagnostics since it does not have high computational complexity. While this method offers advantages, standalone *SoE_{stored}* algorithms encounter a significant challenge in precisely estimating the OCV. This challenge becomes particularly significant when dealing with different cell chemistries, mainly when hysteresis and a flat OCV occur. Therefore, developing reliable OCV estimation techniques to enhance the overall residual stored energy estimation accuracy becomes essential and requires further research. Additionally, combining different estimation methods into a hybrid approach is possible, achieving improved *SoE* estimation without substantially increasing computational complexity.

For instance, the power-integral approach can be complemented by OCV re-calibration, enhancing the accuracy of stored energy estimation while keeping the computational cost manageable. Furthermore, individual methods can be combined into a hybrid method to improve *SoE* estimation without significantly increasing the overall computational cost. For example, OCV re-calibration can potentially complement the power-integral approach without increasing the complexity of stored energy estimation.

In the scope of remaining usable energy estimation, the *SoE_{usable}* and *SoC* can be estimated simultaneously.^[8] Therefore, for model-based methods, only one battery model needs to be parameterized and online-updated to estimate the *SoC* and *SoE* reducing the complexity of the estimations.^[8] Since the *SoE_{usable}* requires the estimation of the cell's internal resistance, simultaneous estimation of *SoE_{usable}* and the State of Health (SoH) can potentially be performed. Additionally, capacity estimation has already been combined with *SoE* estimation in the literature. Long et al. develop in [44] a framework for simultaneous *SoH* and *SoE* estimation with a joint battery model. The *SoE* and the *SoH* are estimated with the EKF, demonstrating the potential of coupling existing model-based state estimation techniques coupled with *SoE* estimation. However, it remains challenging for residual usable energy estimation that the energy is not solely dependent on the characteristics of the cell but on the future operating strategy, which is unknown at present. To illustrate this challenge, four scenarios are shown schematically in Figure 7. For simplicity, in all four figures, a change in operation strategy is made at time *t* during a constant current discharge. In Figure 7(a), a higher absolute constant discharge current is applied at time *t*. This leads to a decrease in usable energy, depicted as *E_{usable,new}* compared to *E_{usable}*, the green area, when the operation strategy does not change. In Figure 7(b), the cell is discharged with a constant current of lower magnitude so that the remaining usable energy increases. It should be noted that a smaller current results in a larger remaining usable energy. Figure 7(c) shows the change in remaining usable energy when the cell is fully relaxed at time *t*. Relaxation does not change the remaining charge amount. However, the instantaneous voltage reaches the value of the OCV. If the cell is discharged with the same constant current, the voltage approaches the discharge voltage.

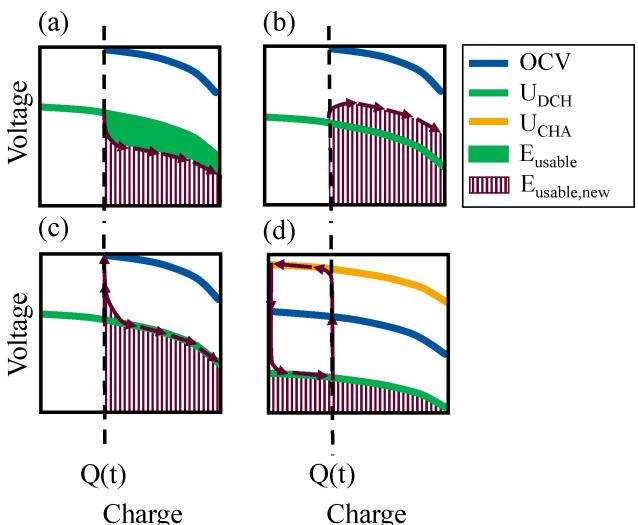


Figure 7. Schematics of the change in usable energy during constant current discharging and charging in dependency of different operating scenarios at time t . (a) Discharging with a higher absolute constant current. The usable energy decreases when the current rate increases (b) Discharging with a lower absolute constant current. The usable energy increases when the discharge current rate decreases. (c) Temporary relaxation of the cell. During relaxation, the charge amount in the battery cell does not change, but the terminal voltage converges to the OCV. When the discharge process continues, the voltage converges to the discharge voltage. (d) Charging with a constant current, then continuing the discharging process. Because of the instant charging of the battery cell, the voltage increases and charges the battery leading to more charge amount to the battery. When the discharge process continues, the terminal voltage decreases and converges back to the discharge voltage.

Since the voltage does not directly equal the value of the corresponding discharge voltage, the remaining usable energy is slightly greater. Figure 7(d) depicts a constant current charge at time t . The charge increases the remaining charge amount. If the cell is discharged again with the same constant current, the remaining usable energy is larger since, on the one hand, there is more charge. On the other hand, more energy is usable due to the slow approach of the voltage to the discharge voltage. The scenarios illustrate that the remaining usable energy is a function of the cell's properties and, moreover, of the operating conditions. Accurate estimation is only possible if the operating conditions are known until complete discharge, limiting the estimation by the given assumptions of the operating strategy. One possibility to tackle this issue is to weigh predicted average values, as Li et al.^[26] suggest, as well as in operation experienced values. Furthermore, the prediction of the future current with Markov chains based on real-driving data, as Chen et al.^[24] suggest, is feasible for online applications. However, one should keep in mind that the SoE_{usable} estimation requires a dynamic evaluation scale and, unlike other state estimates, due to the dependence of the estimate on future operations, it is specifically challenging to provide fixed error bounds. In general, SoE_{usable} estimation comes with higher computational costs compared to SoE_{stored} estimation since it does not solely rely on the specific battery's characteristics. However, in the case that the future operating strategy is known, i.e., the route of a battery electric vehicle, the estimation of maximum usable

energy can be accurate with the help of standard diagnostics methods. Time series-based field data are also particularly important in the context of SoE estimation. As cloud computing technology improves, we will be able to store and analyze this field data more efficiently in the cloud, using machine learning techniques to improve our ability to estimate the remaining usable energy. However, estimating the SoE from field data can be challenging due to small changes in DoD, noisy measurements, and fluctuating temperatures.

Conclusions

The SoE of a lithium-ion battery cell certainly is essential for residual energy estimation and has significant advantages compared to traditional metrics. This work analyzes common definitions and estimation methods for SoE estimation. We propose two novel definitions that are highly applicable to the implementation on a BMS and experimentally verify our findings. We discuss opportunities and challenges associated with residual energy estimation. The main contributions of this work can be summarized as follows:

1. Lucid classification of existing SoE definitions: For the first time, various SoE definitions from the literature are critically reviewed and classified into two concepts: the State of stored Energy and the State of usable Energy. The State of stored Energy correlates with the residual stored energy, therefore; neglecting the losses during operation. In contrast, the State of usable Energy correlates with the residual usable energy and considers internal and external influences. The latter is a critical state for the residual driving range of a battery electric vehicle. The classification of existing definitions enables a deeper understanding of the different aspects of energy within a battery cell and their implications and limitations for residual energy estimation.
2. Applicable, novel SoE definitions for BMS implementation: Based on the critical evaluation of the literature, two novel definitions for the SoE are proposed that are highly applicable to the implementation on a BMS. Unlike common SoE definitions, the proposed definitions directly relate to the battery characteristics and avoid distortion. Furthermore, these definitions address the specific requirements and challenges associated with SoE estimation for real-world applications, providing valuable guidance for algorithm development.
3. Experimental validation and quantitative analysis: We conducted various experiments to verify our results and show that for the tested NCA/C + Si battery cell, the difference between SoC and SoE_{stored} has an absolute deviation of 3%. Our experiments reinforce the accuracy and reliability of the proposed SoE definitions, contributing to their practical significance and emphasizing the advantage over traditional methods.
4. Direct comparison between SoC and SoE_{stored} : A mathematical relationship between the SoC and the residual stored has been presented for the first time. This relationship can be used to determine the remaining stored energy in the

presence of a sufficiently well-performing SoC estimator. This novel mathematical understanding will enhance the precision of residual energy estimation in practical applications with the help of SoC.

5. Comprehensive, critical analysis of opportunities and challenges of SoE estimation: We review implementations of a wide range of SoE estimation techniques and discuss opportunities and challenges of SoE estimation. Whereas the challenges for estimating the residual stored energy are similar to those for SoC estimation, one challenge for residual usable energy estimation remains that the accuracy depends not only on the cell's properties but also on the future operation strategy. Due to the inherent uncertainty associated with future operating conditions, evaluating the quality of the SoE_{usable} estimation within fixed error bounds is challenging and needs further research. However, the quality of the residual usable energy estimation can be improved by predicted current or temperature profiles that are updated during operation.

Overall, accurate estimation of the SoE has the potential to optimize the performance and efficiency of batteries, offering, as quantified, numerous advantages over traditional methods. In order to develop reliable approaches for estimating the residual energy, it is crucial to have a comprehensive understanding of the limitations and potential errors in the SoE definition. This work presents such an understanding, serving as a foundation for accurate and meaningful comparisons of experimental results in SoE estimation. Establishing this solid foundation paves the way for future advancements in the field of residual energy estimation.

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Conflict of Interests

The authors declare no conflict of interest.

Keywords: battery management system • electrochemistry • lithium • residual energy • state of energy

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