Computation of Option Pricing Models

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1 Fundamentals of Option Pricing

Recall that a European put option gives its holder (buyer) the right, but not the obligation, to *sell* a prescribed asset S to the writer (seller) of the option for a strike price K at the maturity date T. As such, the fair price P of a European put option at the maturity date is simply given by the payoff:

$$P_T = P(S_T, T) = \max(K - S_T, 0).$$

Similarly, a European call option gives its holder the right, but not the obligation, to *buy* a prescribed asset S from the writer of the option, and once again fair price P of a European call option at the maturity date is given by the payoff:

$$P_T = P(S_T, T) = \max(S_T - K, 0).$$

The objective of option pricing is to determine the value of *premium*:

$$P_0 = P(S_0, 0)$$

building on the values of P_T. If P_0 was simply set to 0 (no premium on the option), then a call option holder would never take on any risk, and never make a loss. On the other hand, the writer can never turn a profit. The price of the premium must be *fair* to both parties entering the contract.

Our models pretend that we are in a risk-neutral world, where an investor wouldn't mind risk. In this hypothetical world, a risky stop is expected to grow at the exact same rate as a safe investment such as a government bond or a bank account. The *Risk-Neutrality Assumption* is as stated.

At any time, the average return on a risky investment of an asset is equal to the return on a risk-free investment of that asset.

Under this new assumption,

$$\mathbf{E}[P_T] = P_0 e^{rT}.$$

And therefore the premium for a European call option would be

$$P_0 = e^{-rT}\,\mathbf{E}[\max(S_T - K, 0)].$$

Modelling a Risk-Free Asset

A bond issued by the government, or accumulating interest in a bank can be regarded as a risk-free asset. If B_0 is a risk-free investment at a time $t = 0$, the value of the investment after m years at a rate r would be

$$B_m = B_0(1 + mr)$$

if the interest is simple, and

$$B_m = (1 + r)^m B_0$$

if compounded anually. Building on the case of compound interest, consider N timestamps $\{t_n : n \in 0 : N\}$ where $t_n = n\delta t = n/N$ at which the interest compounds. Then,

$$\frac{B_{t_n} - B_{t_{n-1}}}{B_{t_{n-1}}} = r\delta t$$

for $n \in 1 : N$. Now,

$$B_T = B_{t_N} = (1 + r\delta t)^N B_0 = [(1 + r\delta t)^{1/\delta t}]^T B_0.$$

If the interest is compounded continuously, then

$$B_T = \lim_{\delta t \to 0} [(1 + r\delta t)^{1/\delta t}]^T B_0 = e^{rT} B_0.$$