Tutorial 1- MAC F311 Algebra 1

1. Determine all finite subgroups of $\mathbb{R}^*$, the group of nonzero real numbers under multiplication.

2. Let a be a group element of order n, and suppose that d is a positive divisor of n. Prove that $ord(a^d) = n/d$.

3. $U(15)$ has six cyclic subgroups. List them.

4. Prove that a group of even order must have an element of order 2.

5. Suppose G is a group that has exactly eight elements of order 3. How many subgroups of order 3 does G have?

6. Find a cyclic subgroup of order 4 in $U(40)$.

7. Find a non cyclic subgroup of order 4 in $U(40)$.

8. Let $H = \{a + bi \mid a, b \in \mathbb{R}, ab \geq 0\}$. Prove or disprove that H is a subgroup of $\mathbb{C}$ under addition.

9. Let $H = \{a + bi \mid a, b \in \mathbb{R}, a^2 + b^2 = 1\}$. Prove or disprove that H is a subgroup of $\mathbb{C}^*$ under multiplication. Describe the elements of H geometrically.

10. The smallest subgroup containing a collection of elements S is the subgroup H with the property that if K is any subgroup containing S then K also contains H. The notation for this subgroup is $\langle S \rangle$. In the group $\mathbb{Z}$, find:

    (a) $\langle 8, 14 \rangle$

    (b) $\langle m, n \rangle$

    (c) $\langle 12, 18, 45 \rangle$

    In each part, find an integer k such that the subgroup is $\langle k \rangle$.

11. Let G be a finite group with more than one element. Show that G has an element of prime order.
12. Let G be a finite Abelian group and let a and b belong to G. Prove that the set $\langle a, b \rangle = \{a^i b^j \mid i, j \in \mathbb{Z}\}$ is a subgroup of G.