

Applied Machine Learning and Predictive Modelling 1

Dr. Luisa Barbanti and Dr. Matteo Tanadini
luisa.barbanti@hslu.ch

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1 Modelling non-linearities

Section 1

Modelling non-linearities

Given a Linear Model:

$$y = \beta_0 + \beta_1 \cdot x_1 + \beta_2 \cdot x_2 + \varepsilon$$

This model is said to be linear because **it is linear in its coefficients**. Indeed, the following **is** a Linear Model.

$$y = \beta_0 + \beta_1 \cdot x_1 + \beta_2 \cdot x_1^2 + \beta_3 \cdot \log(x_2) + \varepsilon$$

A model such as:

$$y = \beta_0 + \beta_1 \cdot x_1 + \beta_2 \cdot x_1^2 + \beta_3 \cdot \log(x_2) + \varepsilon$$

CAN model non-linear relationships. In **R** this model would be fitted as:

```
lm.non.lin.1 <- lm(y ~ x_1 + I(x_1^2) + log(x_2), data = someData)
```

Non-linear relationships

By including polynomials (e.g. $x_1 + x_1^2$) we **can** model non-linear relationships with a Linear Model.

This is also true for extensions of the Linear Model such as Generalised Linear Models.

- Linear models can model non-linear effects
- e.g. by adding quadratic terms
- $y = \beta_0 + \beta_1 \cdot x + \beta_2 \cdot x^2 + \varepsilon$
- **non-linear models are non-linear in their coefficients:**
- $y = \beta_0 + \beta_1 \cdot x^{\beta_2} + \varepsilon$

In alternative to polynomials we can use **GAMs**, which are an extension of the Linear Model.

Generalise Additive Models

GAMs come with advantages and disadvantages compared to e.g. polynomials. Here a very short selection of the most practice-relevant ones:

- + the degree of complexity must NOT be set by the user
- + the "estimated degrees of freedom" gives the user an indication of the complexity of a given smooth term
- + smooth terms can be visualised
- GAMs can run into computational issues (e.g. models that do not converge)
- the use of a quadratic term is simpler to explain than a GAM to a non-technical audience
- in order to fit and understand the results of a GAM some technical knowledge is required... and GAMs are a complex topic

Must know for a Data Scientist

A widespread wrong myth in data science is

"... we don't use Linear Models (or their extensions) because they cannot deal with non-linear relationships"

This is wrong!

Linear Models CAN MODEL NON-LINEAR RELATIONSHIPS.