Linear Models 1: Cats Lab

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Applied Machine Learning and Predictive Modelling 1, HS25 (HSLU)

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1 Import packages

```
## Import data sets
import pandas as pd
import matplotlib.pyplot as plt
import seaborn as sns
import statsmodels.formula.api as smf
import numpy as np
import statsmodels.api as sm
import matplotlib.patches as mpatches
```

2 Load data set

```
## Load dataset
d_cats = pd.read_csv("../../Datasets/Cats.csv")
## Rename col names because "." causes troubles
d_cats.rename(columns = {'Body.weight': 'Body_weight',
                         'Heart.weight': 'Heart_weight'},
              inplace = True)
## Inspect structure
print(d_cats.info())
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 144 entries, 0 to 143
Data columns (total 3 columns):
    Column
 #
            Non-Null Count Dtype
                 144 non-null object
144 non-null float64
0
    Sex
    Body_weight 144 non-null
1
    Heart_weight 144 non-null float64
dtypes: float64(2), object(1)
memory usage: 3.5+ KB
None
```

print(d_cats.head())

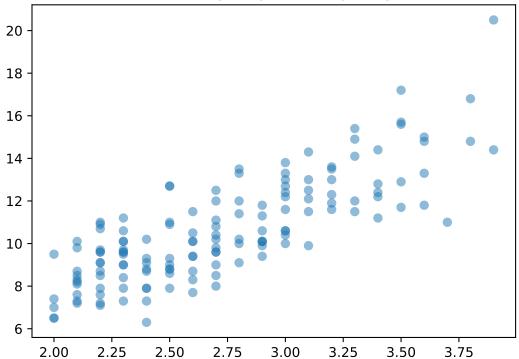
```
Sex Body_weight Heart_weight
  F
            2.0
                       7.0
            2.0
                        7.4
  F
1
  F
                       9.5
2
           2.0
3 F
            2.1
                       7.2
            2.1
                       7.3
```

3 Graphical Analysis

```
## Clean figure object
plt.clf()
## Scatterplot of Heart_weight vs. Body_weight
plt.scatter(d_cats['Body_weight'], d_cats['Heart_weight'], alpha = 0.5)
plt.title("Heart weight against body weight")

## If you want to change axes name
# plt.xlabel("Body weight")
# plt.ylabel("Heart weight")
```

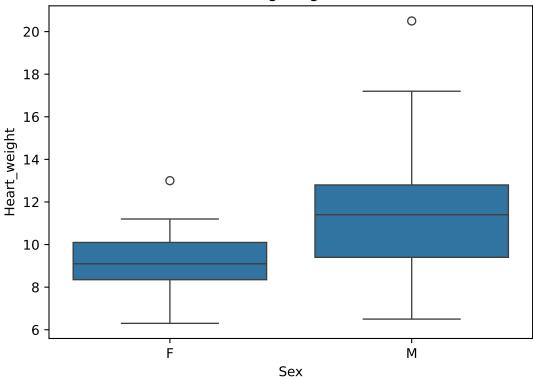
Heart weight against body weight



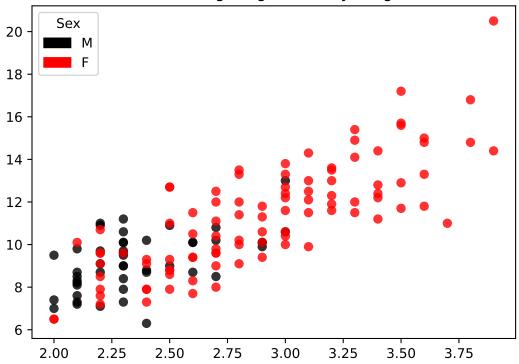
```
## Clean figure object
plt.clf()
## Boxplot of Heart_weight by Sex
sns.boxplot(x = 'Sex', y = 'Heart_weight', data = d_cats)
plt.title("Heart weight against sex")

## If you want to change y label
# plt.ylabel("Heart weight")
```

Heart weight against sex

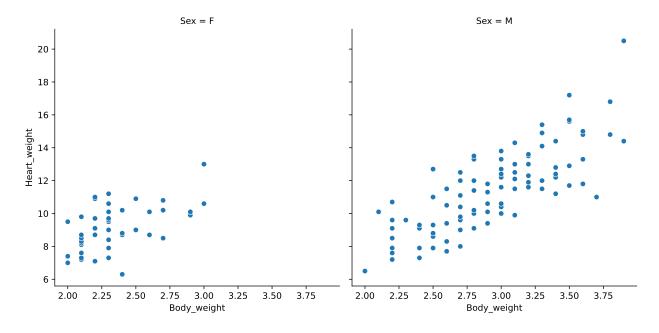


Heart weight against body weight



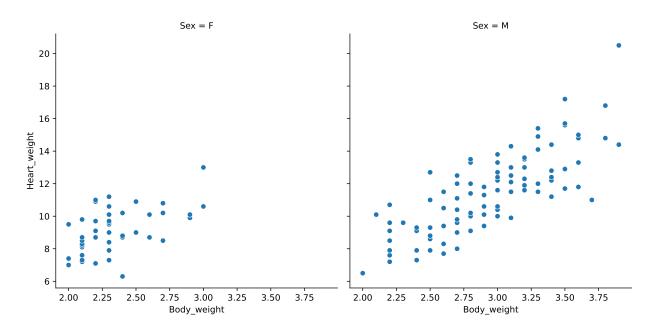
```
## Clean figure object
plt.clf()

## Scatterplot with panelling by Sex
g = sns.FacetGrid(d_cats, col = 'Sex', height = 5, aspect = 1)
g.map(sns.scatterplot, 'Body_weight', 'Heart_weight')
```

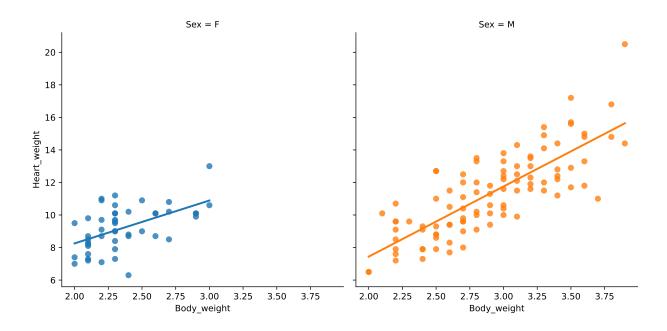


```
## If you want to color the points by groups
# g.set_titles("{col_name}")

## If you want to change the axes name
# g.set_axis_labels("Body weight", "Heart weight")
```



```
## If you want to color the points by groups
# g.set_titles("{col_name}")
## If you want to change the axes name
# g.set_axis_labels("Body weight", "Heart weight")
```



4 Fitting models

```
## Fit a linear model: Hwt ~ Bwt
lm_cats = smf.ols('Heart_weight ~ Body_weight', data = d_cats).fit()
print(lm_cats.summary())
```

OLS Regression Results

Dep. Variable: Heart_w		Heart_weight	R-squ	ared:		0.647	
Model:		OLS		Adj. R-squared:		0.644	
Method:		Least Squares		F-statistic:		259.8	
Date:	Thu	hu, 11 Sep 2025 Prob (F-statistic):			6.97e-34		
Time:		14:40:04	Log-L	ikelihood:		-257.06	
No. Observation	ons:	144	AIC:			518.1	
Df Residuals:		142	BIC:			524.1	
Df Model: 1			-				
Covariance Type: nonrobus		nonrobust	;				
	coef	std err	t	P> t	[0.025	0.975]	
Intercept	-0.3567	0.692	-0.515	0.607	-1.725	1.012	
Body_weight	4.0341	0.250	16.119	0.000	3.539	4.529	
Omnibus: 4.637			Durbi	======== n-Watson:		1.580	

```
      Prob(Omnibus):
      0.098
      Jarque-Bera (JB):
      4.217

      Skew:
      0.408
      Prob(JB):
      0.121

      Kurtosis:
      3.191
      Cond. No.
      17.8
```

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```
## coefficients
print(lm_cats.params)

Intercept    -0.356662
Body_weight    4.034063
dtype: float64

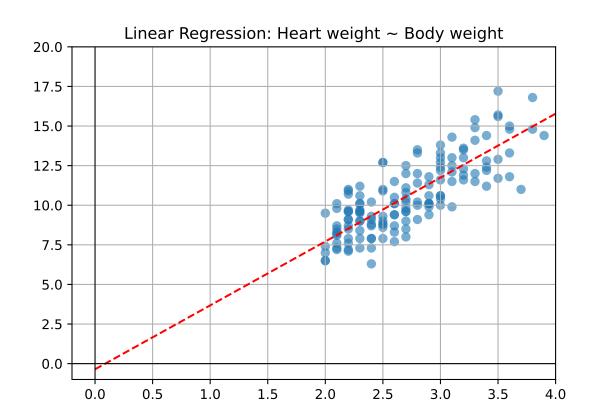
## Clean figure object
plt.clf()

## Scatter plot for the observed data
plt.scatter(d_cats['Body_weight'], d_cats['Heart_weight'], alpha = 0.6)

## Set limits for the plot
plt.ylim(-1, 20)
```

```
plt.xlim(-0.2, 4)
```

```
plt.grid(True)
## Create data set for regression line
lo = 0
hi = 4
n = 20
x = [(hi - lo) / (n - 1) * i for i in range(n)]
x_{array} = np.array(x)
intercept, slope = lm_cats.params
## Plot regression line
plt.plot(x_array, intercept + slope * x_array,
         color = "red", linestyle = "dashed")
plt.axvline(0, color = "black", linewidth = 0.8)
plt.axhline(0, color = "black", linewidth = 0.8)
plt.title("Linear Regression: Heart weight ~ Body weight")
## If you want to change axes name
# plt.xlabel("Body weight")
# plt.ylabel("Heart weight")
```

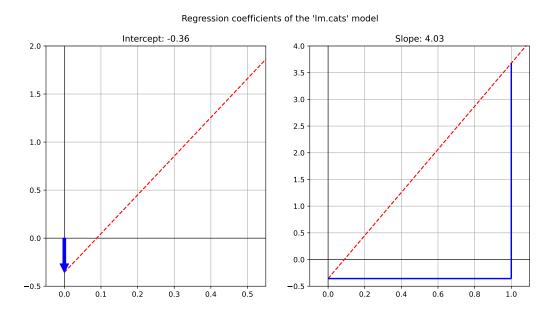


```
## Let's zoom in and concentrate on the two regression coefficients
## Divide the plotting region into two
fig, axes = plt.subplots(1, 2, figsize = (12, 6))
## Set common title
fig.suptitle("Regression coefficients of the 'lm.cats' model")
### First plot: Intercept ###
ax1 = axes[0]
ax1.set_title(f"Intercept: {intercept:.2f}")
#############################
## 2) Add grid and regression line
ax1.grid(True)
ax1.axhline(0, color = "black", linewidth = 0.8)
ax1.axvline(0, color = "black", linewidth = 0.8)
ax1.plot(x_array, intercept + slope * x_array,
        color = "red", linestyle = "dashed")
#############################
## 3) Add arrow for intercept
ax1.annotate("", xy = (0, intercept), xytext = (0, 0),
            arrowprops = dict(facecolor = 'blue',
            edgecolor = 'blue', linewidth = 1))
## zoom in
\# ax1.set\_xlim(-0.01, 0.2)
# ax1.set_ylim(-0.5, 2)
ax1.set_xlim(-0.05, 0.55)
```

```
ax1.set_ylim(-0.5, 2)
```

....

Plot it plt.show()



4.1 Including the Sex predictor

```
lm_cats_2 = smf.ols('Heart_weight ~ Body_weight + Sex', data = d_cats).fit()
print(lm_cats_2.summary())
```

OLS Regression Results

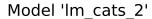
Dep. Variable:	Heart_weight		R-squared:			0.647		
Model:		OLS Adj. R-squared:			0.642			
Method:	Lea	st Squares	F-stat	F-statistic:				
Date:	Thu, 1	1 Sep 2025	Prob (<pre>Prob (F-statistic):</pre>		1.37e-32		
Time:		14:40:05	Log-Li	kelihood:	-257.02			
No. Observations	:	144	AIC:	AIC:		520.0		
Df Residuals:		141	BIC:	BIC:		529.0		
Df Model:		2						
Covariance Type:		nonrobust						
==============		:======:		========				
	coef s	td err	t	P> t	[0.025	0.975]		
Intercept -	 0.4150	0.727	-0.571	0.569	-1.853	1.023		

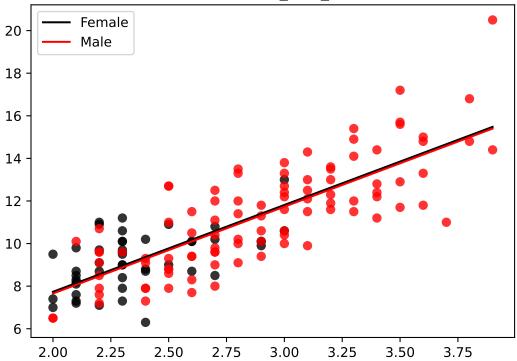
Sex[T.M]	-0.0821	0.304	-0.270	0.788	-0.683	0.519
Body_weight	4.0758	0.295	13.826	0.000	3.493	4.659
=========	========	:=======:		========	========	======
Omnibus:		4.665	Durbin	-Watson:		1.581
Prob(Omnibus)	:	0.097	Jarque-Bera (JB):			4.245
Skew:		0.410	Prob(JB):			0.120
Kurtosis:		3.192	Cond.	No.		19.6
=========	========	.=======			========	======

Notes:

[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

```
## Clean figure object
plt.clf()
## Visualise regression lines for each sex
## color points based on sex
colors = {'F': 'black', 'M': 'red'}
## Plot the data
plt.scatter(d_cats['Body_weight'],
            d_cats['Heart_weight'],
            c = d_cats['Sex'].map(colors),
            alpha = 0.8)
plt.title("Model 'lm_cats_2'")
# plt.xlabel("Body_weight")
# plt.ylabel("Heart_weight")
## Add regression lines for each sex
intercept_female = lm_cats_2.params['Intercept']
slope2 = lm_cats_2.params['Body_weight']
## Regression line for female
plt.plot(d_cats['Body_weight'], intercept_female + slope2 * d_cats['Body_weight'],
        color = "black", label = "Female")
## Intercept for male
intercept_male = intercept_female + lm_cats_2.params['Sex[T.M]']
## Regression line for male
plt.plot(d_cats['Body_weight'],
         intercept_male + slope2 * d_cats['Body_weight'],
         color = "red", label = "Male")
plt.legend(loc = 'upper left')
```





```
print("Female intercept:", intercept_female)
```

Female intercept: -0.41495262809572275

```
print("Male intercept:", intercept_male)
```

Male intercept: -0.49704946327762756

```
## Generate the summary of the linear model as a string
lm_summary = lm_cats_2.summary().as_text()

## Split the summary into lines and extract the desired lines
## (example indices for similar output)
summary_lines = lm_summary.split("\n")
## Adjust indices as necessary for the specific content
desired_lines = summary_lines[11:18]

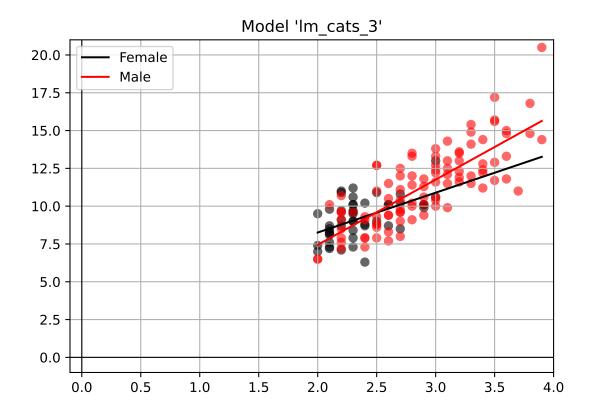
# Print the extracted lines
for line in desired_lines:
    print(line)
```

```
coef std err t P>|t| [0.025 0.975]
```

Intercept	-0.4150	0.727	-0.571	0.569	-1.853	1.023	
Sex[T.M]	-0.0821	0.304	-0.270	0.788	-0.683	0.519	
Body_weight	4.0758	0.295	13.826	0.000	3.493	4.659	

4.2 Include Sex:Body_weight interaction

```
lm_cats_3 = smf.ols('Heart_weight ~ Body_weight * Sex', data = d_cats).fit()
## Clean current figure
plt.clf()
# Visualise regression with interaction
plt.scatter(d_cats['Body_weight'], d_cats['Heart_weight'],
            c = d_cats['Sex'].map(colors), alpha = 0.6)
plt.title("Model 'lm_cats_3'")
plt.xlim(-0.1, 4)
plt.ylim(-1, 21)
plt.grid(True)
plt.axhline(0, color = "black", linewidth = 0.8)
plt.axvline(0, color = "black", linewidth = 0.8)
# Add regression lines
intercept = lm_cats_3.params['Intercept']
slope = lm_cats_3.params['Body_weight']
# For females
plt.plot(d_cats['Body_weight'], intercept + slope * d_cats['Body_weight'],
color = "black", label = "Female")
# For males
slope_male = slope + lm_cats_3.params['Body_weight:Sex[T.M]']
intercept_male = intercept + lm_cats_3.params['Sex[T.M]']
plt.plot(d cats['Body weight'],
         intercept_male + slope_male * d_cats['Body_weight'],
         color = "red", label = "Male")
plt.legend(loc = 'upper left')
```



```
## Regression coefficients
coefficients_interaction = lm_cats_3.params
print("Regression Coefficients:")
```

Regression Coefficients:

print(coefficients_interaction)

Intercept 2.981312 Sex[T.M] -4.165400 Body_weight 2.636414 Body_weight:Sex[T.M] 1.676265

dtype: float64

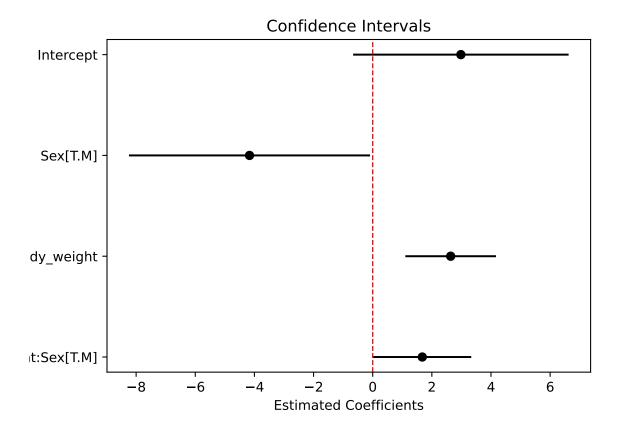
```
## Confidence intervals
confidence_intervals_interaction = lm_cats_3.conf_int()
print("\nConfidence Intervals:")
```

Confidence Intervals:

```
print(confidence_intervals_interaction)
```

```
Intercept
                     -0.662080 6.624705
Sex[T.M]
                     -8.241601 -0.089199
Body_weight
                      1.102414 4.170414
Body_weight:Sex[T.M] 0.020827 3.331702
## Clean figure object
plt.clf()
## Visualise confidence intervals
coef_names = lm_cats_3.params.index[::-1]
coef_values = lm_cats_3.params[::-1]
conf_int_values = confidence_intervals_interaction[::-1]
plt.errorbar(coef_values, range(len(coef_values)),
             xerr = [coef_values - conf_int_values[0],
             conf_int_values[1] - coef_values],
             fmt = 'o', color = 'black')
plt.axvline(0, linestyle = "dashed", color = "red", linewidth = 1)
plt.yticks(range(len(coef_values)), coef_names)
```

```
plt.xlabel("Estimated Coefficients")
plt.title("Confidence Intervals")
```

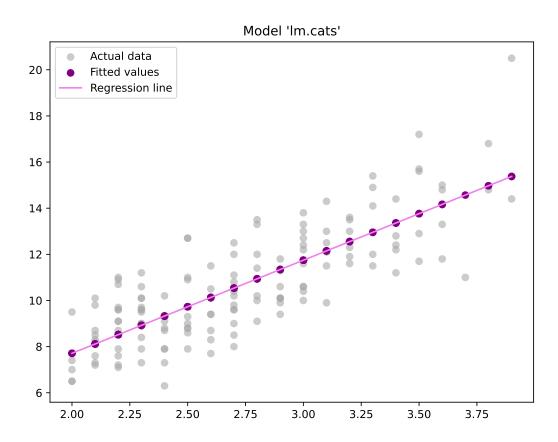


5 Appendix

```
## Model with no interaction
print("Model with no interaction formula:")
Model with no interaction formula:
print(lm_cats_2.model.formula)
Heart_weight ~ Body_weight + Sex
r_squared_no_interaction = lm_cats_2.rsquared
print("\nR-squared (no interaction):")
R-squared (no interaction):
print(r_squared_no_interaction)
0.6468035413329445
## Model with interaction
print("\nModel with interaction formula:")
Model with interaction formula:
print(lm_cats_3.model.formula)
Heart_weight ~ Body_weight * Sex
r_squared_interaction = lm_cats_3.rsquared
print("\nR-squared (with interaction):")
R-squared (with interaction):
print(r_squared_interaction)
0.6566329434116425
# Adjusted R-squared for the model with no interaction
adj_r_squared_no_interaction = lm_cats_2.rsquared_adj
print("Adjusted R-squared (no interaction):")
```

Adjusted R-squared (no interaction):

```
print(adj_r_squared_no_interaction)
0.6417936624866032
# Adjusted R-squared for the model with interaction
adj_r_squared_interaction = lm_cats_3.rsquared_adj
print("\nAdjusted R-squared (with interaction):")
Adjusted R-squared (with interaction):
print(adj_r_squared_interaction)
0.6492750779133205
5.1 Fitted values
## Get fitted values for the model
fitted_cats = lm_cats.fittedvalues
# Display the first few fitted values (similar to head() in R)
print("\nFirst few fitted values:")
First few fitted values:
print(fitted_cats.head())
0
    7.711463
    7.711463
1
    7.711463
    8.114869
     8.114869
dtype: float64
## Clean figure object
plt.clf()
## Plot the actual data points (Heart_weight vs. Body_weight)
plt.figure(figsize = (8, 6))
plt.scatter(d_cats['Body_weight'], d_cats['Heart_weight'],
            color = 'darkgray', label = "Actual data", alpha = 0.6)
## Plot the fitted values (predicted values) from the model (lm_cats)
plt.scatter(d_cats['Body_weight'], fitted_cats,
            color = 'purple', label = "Fitted values", zorder = 5)
## Add the regression line
```



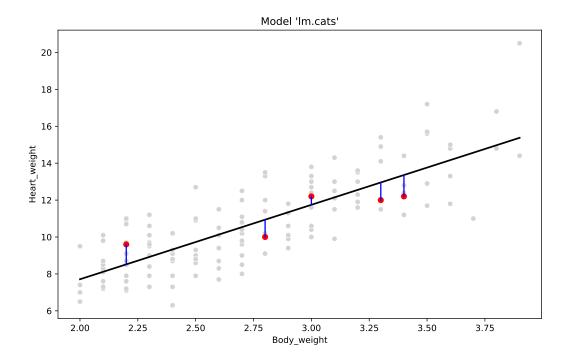
5.2 Residuals

```
# Residuals are the difference between the actual values and the fitted values
resid_cats = d_cats['Heart_weight'] - lm_cats.fittedvalues

# Length of the residuals
len_resid_cats = len(resid_cats)

# First few residuals
head_resid_cats = resid_cats.head()
```

```
# Output the results
print("Length of residuals:", len_resid_cats)
Length of residuals: 144
print("First few residuals:\n", head_resid_cats)
First few residuals:
    -0.711463
1
  -0.311463
    1.788537
    -0.914869
   -0.814869
dtype: float64
# Set seed for reproducibility
np.random.seed(20)
# Sample 5 random indices from the range 1 to 144 (Python uses 0-indexing)
sample_indices = np.random.choice(range(144), size = 5, replace = False)
# Get the corresponding residuals and fitted values using the sampled indices
sample_residuals = resid_cats.iloc[sample_indices]
sample_fitted = fitted_cats.iloc[sample_indices]
# Output the results
print("Sampled residuals:\n", sample_residuals)
Sampled residuals:
      -0.938713
126
    -1.159151
      1.081724
103
       0.454474
     -0.955744
dtype: float64
print("Sampled fitted values:\n", sample_fitted)
Sampled fitted values:
88
       10.938713
126
      13.359151
       8.518276
56
       11.745526
103
       12.955744
121
dtype: float64
## Clean figure object
plt.clf()
# Create the plot
```

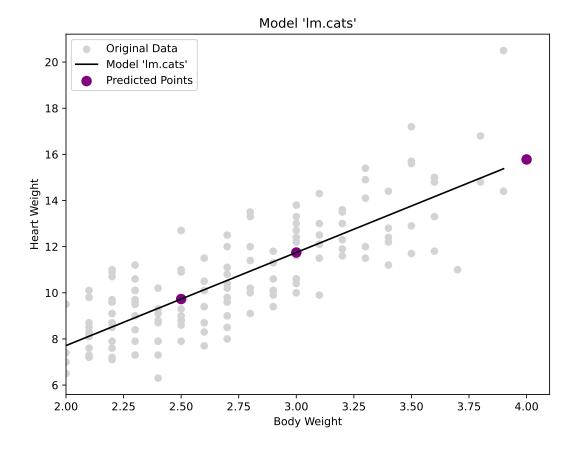


5.3 Predicted values

```
## Clean figure object
plt.clf()
## Fit the model
```

```
lm_cats = smf.ols('Heart_weight ~ Body_weight', data = d_cats).fit()
## Plot the data with model fit
plt.figure(figsize = (8, 6))
plt.scatter(d_cats['Body_weight'], d_cats['Heart_weight'],
            color = 'lightgray', label = 'Original Data')
plt.plot(d_cats['Body_weight'], lm_cats.fittedvalues,
         color = 'black', label = "Model 'lm.cats'")
plt.title("Model 'lm.cats'")
plt.xlabel('Body Weight')
plt.ylabel('Heart Weight')
## New data and predictions
new_data_cats = pd.DataFrame({'Body_weight': [4, 2.5, 3]})
pred_new_cats = lm_cats.predict(new_data_cats)
## Add predictions to the plot
plt.scatter(new_data_cats['Body_weight'], pred_new_cats,
            color = 'purple', s = 75, label = 'Predicted Points')
## Adjust plot limits and show
plt.xlim(2, 4.1)
```

plt.legend()



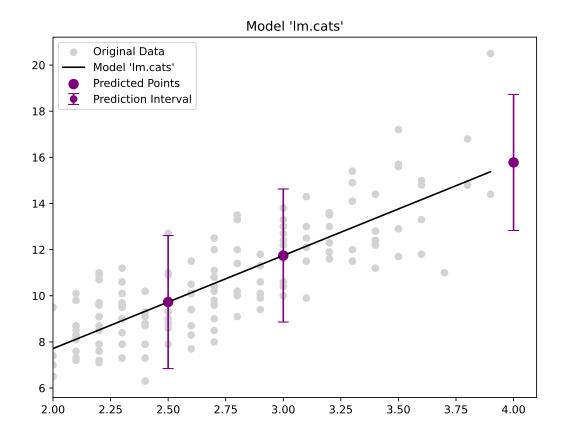
```
## Predictions for new data
new_data_cats = pd.DataFrame({'Body_weight': [4, 2.5, 3]})
pred_new_cats = lm_cats.predict(new_data_cats)
print(pred_new_cats)
```

0 15.779588 1 9.728494 2 11.745526 dtype: float64

```
## Predictions with confidence intervals
# pred_new_cats_ci = lm_cats.get_prediction(new_data_cats).summary_frame()
pred_new_cats_ci = lm_cats.get_prediction(new_data_cats).conf_int(alpha = 0.05)
print(pred_new_cats_ci)
[[15.10432673 16.45484999]
[ 9.46490162  9.99208701]
 [11.46995403 12.02109729]]
## Predictions with confidence intervals
# pred_new_cats_ci = lm_cats.get_prediction(new_data_cats).summary_frame()
pred_new_cats_ci = lm_cats.get_prediction(new_data_cats).conf_int(alpha = 0.05,
                                                                  obs = True)
print(pred_new_cats_ci)
[[12.83018038 18.72899634]
 [ 6.84535188 12.61163675]
 [ 8.86126338 14.62978794]]
## Clean figure object
plt.clf()
## Plot the data with model fit
plt.figure(figsize = (8, 6))
```

```
plt.scatter(d_cats['Body_weight'], d_cats['Heart_weight'],
            color = 'lightgray', label = 'Original Data')
plt.plot(d_cats['Body_weight'], lm_cats.fittedvalues,
         color = 'black', label = "Model 'lm.cats'")
plt.title("Model 'lm.cats'")
# plt.xlabel('Body Weight')
# plt.ylabel('Heart Weight')
## New data and predictions
new_data_cats = pd.DataFrame({'Body_weight': [4, 2.5, 3]})
pred_new_cats = lm_cats.predict(new_data_cats)
## Compute prediction intervals
pred_new_cats_ci = lm_cats.get_prediction(new_data_cats).conf_int(alpha = 0.05,
                                                                  obs = True)
lower_bounds = pred_new_cats_ci[:, 0]
upper_bounds = pred_new_cats_ci[:, 1]
## Add predictions to the plot
plt.scatter(new_data_cats['Body_weight'], pred_new_cats,
            color = 'purple', s = 75, label = 'Predicted Points')
## Add prediction intervals as error bars
plt.errorbar(new_data_cats['Body_weight'], pred_new_cats,
             yerr=[pred_new_cats - lower_bounds, upper_bounds - pred_new_cats],
             fmt='o', color='purple', capsize=5, label='Prediction Interval')
## Adjust plot limits and show
plt.xlim(2, 4.1)
```

plt.legend()



5.4 Treatment contrasts

```
## Set the first 10 observations to 'Unknown'
d_cats.loc[:9, 'Sex'] = 'unknown'
```

5.5 Changing the reference level

```
## Change the reference level (set 'M' as the reference)
## Before change
d_cats['Sex'] = pd.Categorical(d_cats['Sex'], categories = ['M', 'F', 'unknown'])
```

```
## After change
d_cats['Sex'] = d_cats['Sex'].cat.reorder_categories(['M', 'F', 'unknown'],
                                                     ordered = True)
lm_cats_Newsex2 = smf.ols('Heart_weight ~ Sex', data = d_cats).fit()
## Regression coefficients
coefficients = lm_cats_Newsex2.params
print("Regression Coefficients:")
Regression Coefficients:
print(coefficients)
Intercept
                 11.322680
Sex[T.F]
                 -1.771329
Sex[T.unknown]
                 -3.412680
dtype: float64
     How are regression coefficients estimated?
print(f"Formula: {lm_cats.model.formula}")
Formula: Heart_weight ~ Body_weight
print("\nCoefficients:")
Coefficients:
print(lm_cats.params)
Intercept
             -0.356662
Body_weight
              4.034063
dtype: float64
## Create dummy variables for 'Sex'
X = pd.get_dummies(d_cats[['Body_weight', 'Sex']], drop_first = True)
## # Add constant for intercept
X = sm.add_constant(X)
X_matrix = X.values
print("Shape of X (design matrix):", X_matrix.shape)
```

Shape of X (design matrix): (144, 4)

```
print("\nFirst few rows of X (design matrix):")

First few rows of X (design matrix):

print(X_matrix[:5]) # Display the first 5 rows

[[1.0 2.0 False True]
  [1.0 2.0 False True]
  [1.0 2.1 False True]
  [1.0 2.1 False True]
  [1.0 2.1 False True]]
```