EFFECTS OF TURBULENT TRANSFER ON THE CRITICAL BEHAVIOUR

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Abstract. Critical behaviour of two systems, subjected to the turbulent mixing, is studied by means of the field theoretic renormalization group. The first system, described by the equilibrium model A, corresponds to relaxational dynamics of a non-conserved order parameter. The second one is the strongly nonequilibrium reaction-diffusion system, known as Gribov process or directed percolation process. The turbulent mixing is modelled by the stochastic Navier-Stokes equation with random stirring force with the correlator $\propto \delta(t-t')p^{4-d-y}$, where p is the wave number, d is the space dimension and y the arbitrary exponent. It is shown that, depending on the relation between y and d, the systems exhibit various types of critical behaviour. In addition to known regimes (original systems without mixing and passively advected scalar field), existence of new strongly nonequilibrium universality classes is established, and the corresponding critical dimensions are calculated to the first order of the double expansion in y and $\varepsilon = 4 - d$ (one-loop approximation).

1. Introduction

Various systems of very different physical nature exhibit interesting singular behaviour in the vicinity of their critical points. Their correlation functions acquire self-similar form with universal critical dimensions: they depend only on few global characteristics of the system (like symmetry or space dimension). Quantitative description of critical behaviour is provided by the field theoretic renormalization group (RG). In the RG approach, possible types of critical regimes (universality classes) are associated with infrared (IR) attractive fixed points of renormalizable field theoretic models. Most typical equilibrium phase transitions belong to the universality class of the O_n-symmetric ψ^4 model of an n-component scalar order parameter. Universal characteristics of the critical behaviour depend only on n and the space dimension d and can be calculated within various systematic perturbation schemes, in particular, in the form of expansions in $\varepsilon = 4 - d$ or $1/n$; see the monographs [1, 2] and the literature cited therein.

Aleksandr Nikolaevich Vasiliev made valuable contribution to the development of field theoretic methods and their application in the theory of critical behaviour and theory of turbulence. His work in this field is summarized in the three monographs [2, 3, 4]. The most remarkable specific achievements are probably the calculation of Fisher's exponent η in the O_n-symmetric ψ^4 model to the order $1/n^3$ [5] and the third-order calculation of the anomalous exponents in Kraichnan's model of turbulent advection [6]. In the present paper, we apply the field theoretic RG and generalized ε expansion to the problem of the effects of turbulent transfer on various types of critical behaviour.

Over the past few decades, constant interest has been attracted by the spreading processes and corresponding nonequilibrium phase transitions; see e.g. the review papers [7, 8] and the literature cited therein. Spreading processes are encountered in physical, chemical, biological and ecological systems: autocatalytic reactions, percolation in porous media, epidemic diseases and so on. The transitions between the fluctuating (active) and absorbing (inactive) phases, where all the fluctuations cease entirely, are especially interesting as examples of nonequilibrium critical behaviour.

It has long been realized that the behaviour of a real critical system is extremely sensitive to external disturbances, gravity, impurities and turbulent mixing; see the monograph [9] for the general discussion and references. What is more, some disturbances (randomly distributed impurities or turbulent mixing) can produce completely new types of critical behaviour with rich and rather exotic properties.

These issues become even more important for nonequilibrium phase transitions, because the ideal conditions of a "pure" stationary critical state can hardly be achieved in real chemical or biological systems, and the effects of various disturbances can never be completely excluded. In particular, intrinsic turbulence effects cannot be avoided in chemical catalytic reactions or forest fires. One can also speculate that atmospheric turbulence can play important role for the spreading of an infectious disease by flying insects or birds. Effects of different kinds of regular and turbulent flows on the critical behaviour were studied in [10]–[17].

In a number of papers [14]–[17], critical behaviour of various systems, subjected to the turbulent mixing, was studied by means of the field theoretic RG. As a rule, the turbulence was modelled by the time-decorrelated Gaussian velocity field with the velocity correlation function of the form $\langle vv \rangle \propto \delta(t-t')\, p^{-d-\xi}$, where p is the wave number and $0 < \xi < 2$ is a free parameter with the real ("Kolmogorov") value $\xi = 4/3$. This "Kraichnan's rapid-change model" has attracted enormous attention recently because of the insight it offers into the origin of intermittency and anomalous scaling in fully developed turbulence; see the review paper [18] and references therein. The RG approach to that problem is reviewed in [19]. In the context of our study it is especially important that Kraichnan's ensemble allows one to easily model anisotropy of the flow [16] and compressibility of the fluid [17], which appears much more difficult if the velocity is described by the full-scale dynamical equations.

However, the Gaussianity and vanishing correlation time are drastic simplifications of the real situation, and it is desirable to investigate effects of turbulent mixing, caused by more realistic velocity fields. In this paper, we study effects of a strongly non-Gaussian velocity field with finite correlation time, governed by a stochastic dynamical equation. More precisely, we employ the stochastic Navier–Stokes equation for an incompressible velocity, with random stirring force with the correlator $\propto p^{4-d-y}$, where y is the arbitrary exponent with the physical ("Kolmogorov") value $y = 4$. The RG approach to this model is reviewed in [2, 4].

Two representative cases of dynamical critical behaviour are considered: equilibrium critical dynamics of a non-conserved order parameter with ψ^4-type Hamiltonian, and the nonequilibrium system near its transition point between the absorbing and fluctuating states. The former model corresponds to critical fluid systems (binary mixtures or liquid crystals), and the latter describes the spreading processes in reaction-diffusion systems, belongs to the universality class known as Gribov process or directed percolation process, and is equivalent (up to the Wick rotation) to the Reggeon field theory [7, 8].

It is shown that, depending on the relation between y and d, the both systems exhibit various types of critical behaviour, associated with different IR attractive fixed points of the RG equations. In addition to known asymptotic regimes (like equilibrium critical dynamics without mixing or passively advected scalar without self-interaction), existence of new, strongly nonequilibrium, types of critical behaviour (universality classes) is established, and the corresponding domains of stability in the y–d plane and the critical dimensions are calculated to the leading order of the double expansion in y and $\varepsilon = 4 - d$, which corresponds to the one-loop approximation of the RG.

2. Description of the models

In the Langevin formulation the models are defined by stochastic differential equations for the order parameter $\psi = \psi(t, \mathbf{x})$:

$$\partial_t \psi = \lambda_0 \left\{ (-\tau_0 + \partial^2)\psi - u_0 \psi^3/3! \right\} + \zeta \tag{2.1}$$

for the model A and

$$\partial_t \psi = \lambda_0 \left\{ (-\tau_0 + \partial^2)\psi - g_0 \psi^2/2 \right\} + \zeta \sqrt{\psi} \tag{2.2}$$

for the Gribov process. Here, $\partial_t = \partial/\partial t$, ∂^2 is the Laplace operator, g_0 and $u_0 > 0$ are the coupling constants, $\lambda_0 > 0$ is the kinematic (diffusion) coefficient and $\tau_0 \propto (T - T_c)$ is the deviation of the temperature (or its analog) from the critical value. The Gaussian random noise $\zeta = \zeta(t, \mathbf{x})$ with zero average is specified by the pair correlation function:

$$\langle \zeta(t, \mathbf{x}) \zeta(t', \mathbf{x}') \rangle = 2\lambda_0 \delta(t - t') \delta^{(d)}(\mathbf{x} - \mathbf{x}') \tag{2.3}$$

for the model A and

$$\langle \zeta(t, \mathbf{x}) \zeta(t', \mathbf{x}') \rangle = g_0 \lambda_0 \, \delta(t - t') \delta^{(d)}(\mathbf{x} - \mathbf{x}') \tag{2.4}$$

for the Gribov process; d being the dimension of the $\mathbf{x}$ space. The factor $\sqrt{\psi}$ in the noise term of (2.2) guarantees that in the absorbing state the fluctuations cease entirely. The expressions for the correlators differ only by normalization: the factor $2\lambda_0$ in (2.3) is dictated by the fluctuation-dissipation relation and ensures the correspondence to the static ψ^4 model, while $g_0\lambda_0$ in (2.4) provides the simple form of the symmetry that exists in the field theoretic formulation of the Gribov model; see eq. (3.5) below. The subscript "0" marks the bare (unrenormalized) parameters; their renormalized analogs (without the subscript) will appear later.

For incompressible fluid, the Galilean covariant coupling with the transverse (due to the incompressibility condition $\partial_i v_i = 0$) velocity field $\mathbf{v} = \{v_i(t, \mathbf{x})\}$ is introduced by the substitution

$$\partial_t \to \nabla_t = \partial_t + v_i \partial_i \tag{2.5}$$

in (2.1) and (2.2), where $\partial_i = \partial/\partial x_i$ and ∇_t is the Lagrangian (Galilean covariant) derivative. We will employ the velocity field satisfying the NS equation with a random stirring force

$$\nabla_t v_k = \nu_0 \partial^2 v_k - \partial_k \mathcal{P} + f_k, \tag{2.6}$$

where ∇_t is the Lagrangian derivative (2.5), $\mathcal{P}$ and f_k are the pressure and the transverse random force per unit mass. We assume for f a Gaussian distribution with zero average and correlation function

$$\langle f_i(x) f_j(x') \rangle = \frac{\delta(t-t')}{(2\pi)^d} \int_{p\geq m} d\mathbf{p}\, P_{ij}(\mathbf{p})\, \mathcal{D}_f(p)\, \exp\left\{ i\mathbf{p}\,(\mathbf{x}-\mathbf{x}')\right\}, \tag{2.7}$$

where $P_{ij}(\mathbf{p}) = \delta_{ij} - p_i p_j/p^2$ is the transverse projector and $\mathcal{D}_f(p)$ is some function of $p = |\mathbf{p}|$ and model parameters. The momentum $m = 1/\mathcal{L}$, the reciprocal of the integral turbulence scale $\mathcal{L}$, provides IR regularization (its precise form is unessential; the sharp cutoff is the simplest choice for the practical calculations).

The standard RG formalism is applicable to the problem (2.6), (2.7) if the correlation function of the random force is chosen in the power form

$$\mathcal{D}_f(p) = D_0\, p^{4-d-y}, \tag{2.8}$$

where $D_0 > 0$ is the positive amplitude factor and the exponent $0 < y \leq 4$ plays the role of the RG expansion parameter, analogous to that played by $\varepsilon = 4 - d$ in models of critical behaviour. Its physical value is $y = 4$: with the appropriate choice of the amplitude, the function (2.8) for $y \to 4$ turns to the δ function, $\mathcal{D}_f(p) \propto \delta(\mathbf{p})$, which corresponds to the injection of energy to the system owing to interaction with the largest turbulent eddies; for a more detailed discussion of this point see e.g. [2, 4].

3. Field theoretic formulation and renormalization

The stochastic problems (2.1)–(2.4) can be reformulated as field theoretic models of the doubled set of fields $\Psi = \{\psi, \psi^\dagger\}$ with action functional

$$\mathcal{S}_A(\psi,\psi^\dagger) = \psi^\dagger\left(-\partial_t + \lambda_0\partial^2 - \lambda_0\tau_0\right)\psi + \lambda_0\psi\psi^\dagger - \frac{u_0}{3!}\psi^\dagger\psi^3 \tag{3.1}$$

for the model A and

$$\mathcal{S}_G(\psi,\psi^\dagger) = \psi^\dagger(-\partial_t + \lambda_0\partial^2 - \lambda_0\tau_0)\psi + \frac{g_0\lambda_0}{2}\left\{(\psi^\dagger)^2\psi - \psi^\dagger\psi^2\right\} \tag{3.2}$$

for the Gribov model. Here, $\psi^\dagger = \psi^\dagger(t,\mathbf{x})$ is the auxiliary "response field" and the integrations over the arguments of the fields are implied, for example

$$\psi^\dagger\partial_t\psi = \int dt \int d\mathbf{x}\psi^\dagger(t,\mathbf{x})\partial_t\psi(t,\mathbf{x}).$$

The stochastic problem (2.6)–(2.8) corresponds to the field theoretic model with the action

$$\mathcal{S}_{NS}(v',v) = v' D_v v'/2 + v'\left\{-\nabla_t + \nu_0\partial^2\right\} v, \tag{3.3}$$

where D_v is the correlation function (2.7) and all the needed integrations and summations over the vector indices are understood. The auxiliary vector field $\mathbf{v}' = \{v'_i(t,\mathbf{x})\}$ is also transverse, $\partial_i v'_i = 0$, which allows one to omit the pressure term in the action functional (3.3).

The field theoretic formulation means that statistical averages of random quantities in the original stochastic problems can be represented as functional integrals over the full set of fields with the weight $\exp \mathcal{S}(\Phi)$, and can therefore be viewed as the Green functions of the field theoretic models with actions (3.1)–(3.6). In particular, the linear response function of the stochastic problems (2.1)–(2.4) is given by the Green function

$$G = \langle \psi^\dagger(t,\mathbf{x})\psi(t',\mathbf{x}')\rangle = \int \mathcal{D}\psi^\dagger \int \mathcal{D}\psi \ \ \psi^\dagger(t,\mathbf{x})\psi(t',\mathbf{x}') \exp \mathcal{S}(\psi,\psi^\dagger) \tag{3.4}$$

of the corresponding field theoretic models.

The model (3.2) is symmetric with respect to the transformation

$$\psi(t,\mathbf{x}) \to \psi^\dagger(-t,-\mathbf{x}), \quad \psi^\dagger(t,\mathbf{x}) \to \psi(-t,-\mathbf{x}), \quad g_0 \to -g_0. \tag{3.5}$$

Reflection of the constant g_0 is in fact unimportant because the actual expansion parameter in the perturbation theory of the model is $u_0 = g_0^2$. The model (3.1) is symmetric with respect to the reflection of the fields $\psi \to -\psi$, $\psi^\dagger \to -\psi^\dagger$. These symmetries survive the inclusion of the velocity field.

The full-scale models are described by the action functionals

$$\mathcal{S}^F_{A,G}(\Psi) = \mathcal{S}_{NS}(v',v) + \mathcal{S}_{A,G}(\psi,\psi^\dagger,v), \tag{3.6}$$

where $\Psi = \{\psi,\psi^\dagger,v,v'\}$ is the full set of fields and the substitution (2.5) is made in the functionals (3.1) and (3.2). For these models, the full set of coupling constants ("charges") involves the three parameters

$$u_0 \sim \Lambda^{4-d}, \quad w_0 = D_0/\nu_0^3 \sim \Lambda^y, \quad e_0 = \lambda_0/\nu_0, \tag{3.7}$$

where Λ is some typical UV momentum scale. The ratio e_0 is not an expansion parameter in the perturbation theory, but it should also be treated as an additional coupling constant because it is dimensionless and the renormalization constants and RG functions depend on it.

From the relations (3.7) it follows that the interactions $(\psi^\dagger)^2\psi$ and $\psi^\dagger\psi^2$ in (3.2) and $\psi^\dagger\psi^3$ in (3.1) become logarithmic (the corresponding coupling constant u_0 becomes dimensionless) at $d = 4$. Thus for the single-charge problems (3.1), (3.2), the value $d = 4$ is the upper critical dimension, and the deviation $\varepsilon = 4 - d$ plays the part of the formal expansion parameter in the RG approach: the critical exponents are nontrivial for $\varepsilon > 0$ and can be calculated as series in ε. The vertex term $v'(v\partial)v$ in (3.3) and the additional interactions $\psi^\dagger(v\partial)\psi$ in the full models (3.6) become logarithmic at $y = 0$. The parameter y is not related to d and can be varied independently. However, for the RG analysis of the full problems it is important that all the interactions become logarithmic at the same time. Otherwise, one of them would be weaker than the others from the RG viewpoints and it would be irrelevant in the leading-order IR behaviour. As a result, some of the scaling regimes of the full model would be lost.

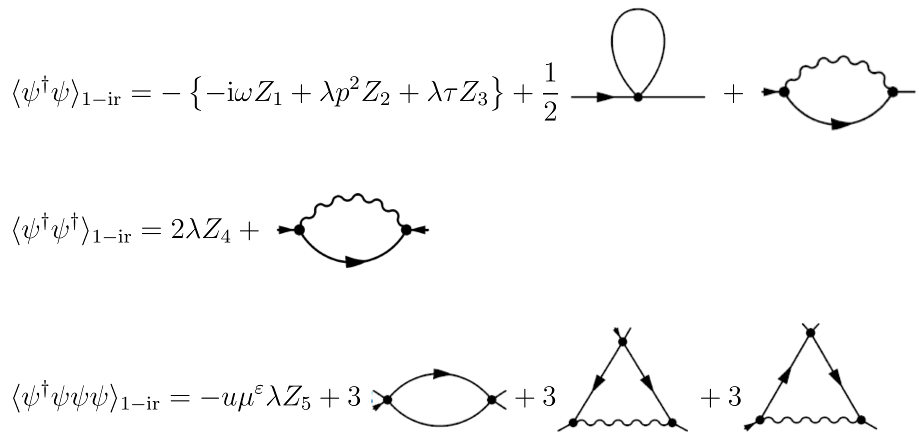

Figure 1. One-loop approximation for the relevant 1-irreducible Green functions in the model (3.8).

In order to study all possible scaling regimes and the crossovers between them, we need a genuine three-charge theory, in which all the interactions are treated on equal footing. Thus we will treat ε and y as small parameters of the same order, $\varepsilon \propto y$. Instead of the plain ε expansion in the single-charge models, the coordinates of the fixed points, critical dimensions and other quantities will be calculated as double expansions in the ε–y plane around the origin, that is, around the point in which all the coupling constants in (3.7) become dimensionless.

The analysis based on the dimensionality considerations and the symmetries of the full-scale models (3.6) shows that they are multiplicatively renormalizable. The role played by the symmetries is very important: in particular, the Galilean invariance requires that the counterterms $\psi^\dagger\partial_t\psi$ and $\psi^\dagger(v\partial)\psi$ enter the renormalized action only in the form of invariant combination $\psi^\dagger\nabla_t\psi$. It also shows that the counterterm $\psi^\dagger\psi v^2$, absent in the unrenormalized actions (3.6) and allowed by the dimension, is in fact forbidden. Thus, all the UV divergences (having the form of singularities at ε and $y \to 0$) can be absorbed into a finite set of renormalization constants Z_i. The renormalized action functionals have the forms:

$$\mathcal{S}^R_A = \mathcal{S}^R_{NS} + \psi^\dagger\left\{-Z_1\partial_t + Z_2\lambda\partial^2 - Z_3\lambda\tau\right\}\psi + Z_4\lambda\psi\psi^\dagger - Z_5\frac{u\mu^\varepsilon\lambda}{3!}\psi^\dagger\psi^3 \quad (3.8)$$

for the model A and

$$\mathcal{S}^R_G = \mathcal{S}^R_{NS} + \psi^\dagger\left\{-Z_1\partial_t + Z_2\lambda\partial^2 - Z_3\lambda\tau\right\}\psi + Z_4\frac{g\mu^{\varepsilon/2}\lambda}{2}\left\{(\psi^\dagger)^2\psi - \psi^\dagger\psi^2\right\} \quad (3.9)$$

for the Gribov model, where $\mathcal{S}^R_{NS}$ is obtained from (3.3) by the substitution $\nu_0 \to \nu Z_\nu$ and $w_0\nu_0^3 \to w\mu^y\nu^3$ (the nonlocal term with the random force correlator in (3.3) is not renormalized). Here and below τ, u and so on are renormalized analogs of the bare parameters (with the subscripts "0") and μ is the reference mass scale (additional arbitrary parameter of the renormalized theory).

The one-loop calculation of the renormalization constants Z_i is easily performed: in fact, in this approximation there are no new Feynman diagrams in comparison to the models (3.1)–(3.3) and the passive scalar case. More precisely, all the new diagrams appear UV finite for an incompressible fluid and give no contribution to Z_i. This fact is illustrated for the model A in figure 1, where all 1-irreducible Green functions, needed for the calculation of the renormalization constants, are shown in the one-loop approximation. The solid lines with arrows denote the bare propagator $\langle\psi\psi^{\dagger}\rangle_0$, the arrow pointing to the field $\psi^{\dagger}$. The solid lines without arrows correspond to the propagator $\langle\psi\psi\rangle_0$ and the wavy lines denote the velocity propagator $\langle vv\rangle_0$. The external ends with incoming arrows correspond to the fields $\psi^{\dagger}$, the ends without arrows correspond to ψ. The quartic vertex with one incoming arrow corresponds to the interaction $-u\mu^{\varepsilon}\lambda\psi^{\dagger}\psi^3/3!$, while the triple vertex with one wavy line corresponds to $-\psi^{\dagger}(v\partial)\psi$. Due to the transversality of the velocity field, the derivative at the latter vertex can also be moved onto the field $\psi^{\dagger}$ using integration by parts: $-\psi^{\dagger}(v\partial)\psi = \psi(v\partial)\psi^{\dagger}$. Thus in any diagram involving n external vertices of this type, the factor p^n with n external momenta p will be taken outside the corresponding integrals. This reduces the dimension of the integrand by n units and can make it UV convergent. In the case at hand, this proves the UV finiteness of the last two diagrams in the function $\langle\psi^{\dagger}\psi\psi\psi\rangle_{1-ir}$ and the only diagram in $\langle\psi^{\dagger}\psi^{\dagger}\rangle_{1-ir}$ (which otherwise would be logarithmically divergent). Since in our models the scalar field is passive (no feedback on the velocity statistics), the constant Z_{ν} in the full model (3.6) is the same as in (3.3), but with the substitution $d = 4$.

In the minimal subtraction scheme the one-loop expressions for the constants Z_i contain only simple poles in ε and y and have the forms:

$$Z_1 = Z_4 = 1, \quad Z_3 = 1 + \frac{u}{\varepsilon}, \quad Z_2 = 1 - \frac{w}{4ye(e+1)}, \quad Z_5 = 1 + \frac{3u}{\varepsilon} \tag{3.10}$$

for the model A,

$$\begin{aligned} &Z_1 = 1 + \frac{u}{4\varepsilon}, \quad Z_3 = 1 + \frac{u}{2\varepsilon}, \quad Z_4 = 1 + \frac{u}{\varepsilon}, \\ &Z_2 = 1 + \frac{u}{8\varepsilon} - \frac{w}{4ye(e+1)} \end{aligned} \tag{3.11}$$

for the Gribov model and

$$Z_{\nu} = 1 - \frac{w}{12y} \tag{3.12}$$

for the both cases (in order to simplify the coefficients, the factor $1/16\pi^2$ is absorbed into the constants u and w).

4. Fixed points and scaling regimes

The RG equations for our multiplicatively renormalized models (3.8), (3.9) are derived in a standard fashion, similar to that for the analogous models with Kraichnan's velocity field [17], and we do not present them here.

It is well known that possible IR scaling regimes of a renormalizable field theoretic model are associated with IR attractive fixed points of the corresponding RG equations;

see e.g. [1, 2]. For a given point, the Green functions demonstrate self-similar (scaling) asymptotic behaviour in the IR range, with definite critical dimensions Δ_F of all fields and parameters F of the model. The coordinates g_{i*} of the fixed points are found from the requirement that the β-functions, corresponding to all renormalized couplings g_i, vanish. The type of a fixed point is determined by the matrix $\Omega_{ik} = \partial\beta_i/\partial g_k$, where β_i is the full set of β-functions and g_k the full set of couplings. For an IR attractive fixed point the matrix Ω is positive, i.e., the real parts of all its eigenvalues are positive.

In our case, $g_i = \{u, w, e\}$. Admissible fixed point must be IR attractive for some values of y and ε and satisfy the conditions $u_*, w_*, e_* > 0$, which follow from the physical meaning of these parameters. The functions β_i, calculated in the one-loop approximation from the renormalization constants (3.10)–(3.12), have the forms:

$$\beta_u = u\left\{-\varepsilon + 3u + \frac{w}{2e(e+1)}\right\}, \quad \beta_e = w\left\{\frac{e}{12} + \frac{w}{4(e+1)}\right\} \tag{4.1}$$

for the model A,

$$\beta_u = u\left\{-\varepsilon + \frac{3u}{2} + \frac{w}{2e(e+1)}\right\}, \quad \beta_e = w\left\{\frac{e}{12} - \frac{w}{4(e+1)}\right\} - \frac{ue}{8} \tag{4.2}$$

for the Gribov model and

$$\beta_w = w\left\{-y + w/4\right\} \tag{4.3}$$

for the both models, with higher-order corrections in u and w.

The analysis of the functions (4.1), (4.3) reveals four admissible fixed points of the model A:

(1) The Gaussian (free) fixed point: $u_* = w_* = 0$, e_* arbitrary. This point is IR attractive for $y < 0$, $\varepsilon < 0$. The critical dimensions are found exactly:

$$\Delta_\psi = d/2 - 1, \quad \Delta_{\psi^\dagger} = d/2 + 1, \quad \Delta_\omega = \Delta_\tau = 2.$$

(2) The point $u_* = 0$, $w_* = 4y$, $2e_* = -1 + \sqrt{13}$ (the positive root of the equation $e(e+1) = 3$), corresponding to the passively advected scalar without self-interaction: the vertex $\psi^\dagger\psi^3$ in (3.1) is IR irrelevant in the sense of Wilson. This point is IR attractive for $y > 0$, $y > 3\varepsilon/2$. The critical dimensions are also known exactly:

$$\Delta_\psi = d/2 - 1, \quad \Delta_{\psi^\dagger} = d/2 + 1, \quad \Delta_\omega = \Delta_\tau = 2 - y/3.$$

(3) The point $w_* = 0$, $u_* = \varepsilon/3$, e_* arbitrary, corresponding to the pure A model: the turbulent advection is IR irrelevant.‡ This point is IR attractive for $y < 0$, $\varepsilon > 0$. The critical dimensions for this regime depend only on ε:

$$\Delta_\psi = 1 - \varepsilon/2, \quad \Delta_{\psi^\dagger} = 3 - \varepsilon/2, \quad \Delta_\omega = 2, \quad \Delta_\tau = 2 - \varepsilon/3,$$

with the higher-order corrections, known up to ε^4 for $\Delta_{\omega,\psi^\dagger}$ [20] and ε^5 for the others [1, 2].

‡ This becomes obvious if, by rescaling the fields, the coupling constant w_0 is placed in front of the interaction terms $\psi'(v\partial)\psi$, which is more familiar for the field theory. We do not do it, however, in order to retain the natural form of the covariant derivative (2.5).

(4) The most interesting point $w_* = 4y$, $u_* = \varepsilon/3 - 2y/9$, $2e_* = -1 + \sqrt{13}$, IR attractive for $y > 0$, $y < 3\varepsilon/2$. It corresponds to a new full-scale nonequilibrium universality class, in which both the self-interaction and turbulent mixing are relevant. Here, the critical dimensions are calculated in the form of double series in ε and y. The one-loop expressions read:

$$\Delta_\psi = 1 - \frac{4(\varepsilon + y)}{3}, \quad \Delta_{\psi^\dagger} = 3 - \frac{4(\varepsilon + y)}{3}, \quad \Delta_\tau = 2 - \varepsilon + \frac{y}{3}, \quad \Delta_\omega = 2 - \frac{y}{3}. \tag{4.4}$$

The last dimension is exact, the others have higher-order corrections in ε and y.

For the Gribov case, the analysis of the functions (4.2), (4.3) reveals *five* admissible fixed points:

(1) The Gaussian point: $u_* = w_* = 0$, e_* arbitrary, attractive for $y < 0$, $\varepsilon < 0$. Here, the critical dimensions are:

$$\Delta_\psi = \Delta_{\psi^\dagger} = d/2, \quad \Delta_\omega = \Delta_\tau = 2.$$

(2) The point $u_* = 0$, $w_* = 4y$, $2e_* = -1 + \sqrt{13}$, attractive for $y > 0$, $y > 3\varepsilon/2$. It corresponds to the passively advected scalar without self-interaction: the vertices $(\psi^\dagger)^2\psi$ and $\psi^\dagger\psi^2$ in (3.2) are irrelevant. The critical dimensions are:

$$\Delta_\psi = \Delta_{\psi^\dagger} = d/2, \quad \Delta_\omega = \Delta_\tau = 2 - y/3.$$

(3) The point $w_* = 0$, $u_* = 2\varepsilon/3$, $e_* = \infty$, IR attractive for $y < 0$, $\varepsilon > 0$. It corresponds to the pure Gribov process (turbulent advection is irrelevant). The critical dimensions depend only on ε:

$$\Delta_\psi = \Delta_{\psi^\dagger} = 2 - 7\varepsilon/12, \quad \Delta_\tau = 2 - \varepsilon/4, \quad \Delta_\omega = 2 - \varepsilon/12, \tag{4.5}$$

with known corrections of order ε^2 [8].

(4) The full-scale point, corresponding to a new universality class: $w_* = 4y$, $u_* = 4\varepsilon/5 - 8y/15$, $2e_* = -1 + \sqrt{1 + 40y/(4y - \varepsilon)}$. This point is IR attractive for $y > \varepsilon/4$, $y < 3\varepsilon/2$. The dimensions are calculated as double series in ε and y with the one-loop expressions:

$$\Delta_\psi = \Delta_{\psi^\dagger} = 2 - \frac{3\varepsilon}{5} + \frac{y}{15}, \quad \Delta_\tau = 2 - \frac{y + \varepsilon}{5}, \quad \Delta_\omega = 2 - \frac{y}{3} \text{ (exact)}. \tag{4.6}$$

(5) The point $u_* = 2\varepsilon/3$, $w_* = 4y$, $e_* = \infty$, IR attractive for $y > \varepsilon/4$, $\varepsilon > 0$. This point requires a careful interpretation. Although the value of w_* at this point is nontrivial (and the velocity field is therefore non-Gaussian), the turbulent mixing is nevertheless irrelevant. Indeed, straightforward analysis of the Green functions with the scalar fields ψ, $\psi^\dagger$, for example the function (3.4), shows that the Feynman diagrams involving the velocity field vanish in the limit $e = \lambda/\nu \to \infty$, while the diagrams without the velocity are independent of e and remain finite. Thus from the physics viewpoints this regime is similar to (3) and corresponds to the pure Gribov process, and the corresponding critical dimensions indeed coincide with (4.5).

In figure 2 we show the domains in the ε–y plane, where the fixed points listed above are IR attractive. The plot corresponds to the Gribov case; for the model A, the domain (5) is absent, and the boundary between the domains (3) and (4) is given by

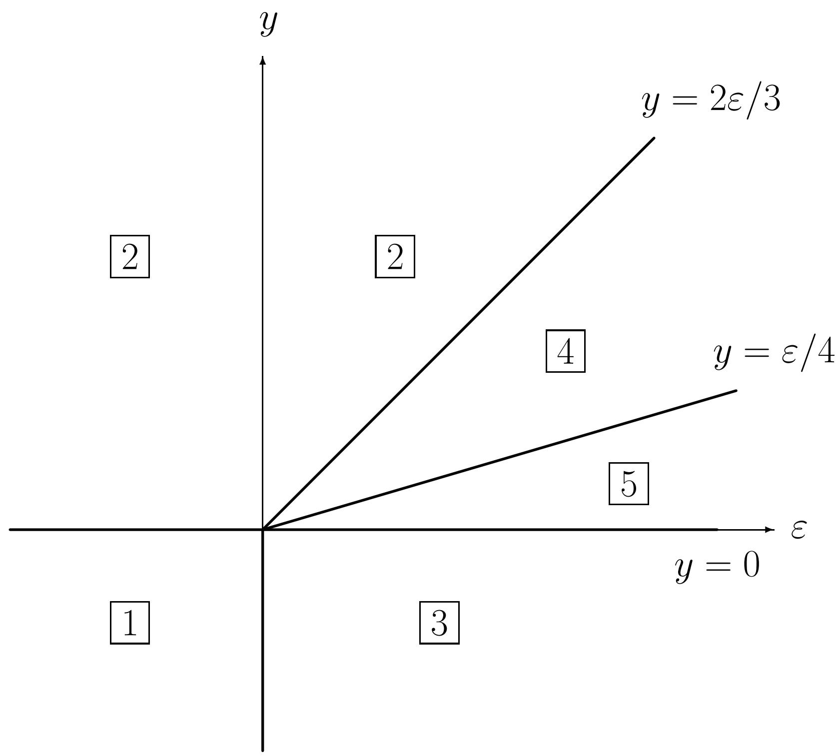

Figure 2. Domains of IR stability of the fixed points in the models (3.6). The numbers in boxes correspond to the fixed points (1)–(5) in the text. For the model A, the domain (5) is absent, and the boundary between (3) and (4) is given by the ray $y = 0$, $\varepsilon > 0$.

the ray $y = 0$, $\varepsilon > 0$. In the one-loop approximation, all the boundaries of the domains are given by straight lines; there are neither gaps nor overlaps between the domains. Due to higher-order corrections to the functions (4.1)–(4.3), the boundaries between the domains (2), (4) and (5) can change and become curved. It can be argued, however, that no gaps nor overlaps can appear between them to all orders; cf. [17, 21]. It is important here that the special cases $u = 0$ or $w = 0$ of the full models are "closed with respect to renormalization" in the sense that the functions β_u for $w = 0$ coincide with the β functions of the Gribov model or model A, while the functions $\beta_{w,e}$ for $u = 0$ coincide with their counterparts in the passive scalar model to all orders of the perturbation theory. It is also not impossible that the absence of the regime (5) for the model A is an artefact of the one-loop approximation, and it will appear on the two-loop level due to nontrivial contributions to the renormalization constants $Z_{1,4}$ in (3.10).

5. Conclusion

Effects of turbulent mixing on the critical behaviour were studied. Two representative models of dynamical critical behaviour were considered: the model A, which describes relaxational dynamics of a non-conserved order parameter in an equilibrium critical system, and the strongly nonequilibrium Gribov model, which describes spreading processes in a reaction-diffusion system. The turbulent mixing was modelled by the stochastic Navier-Stokes equation with random stirring force with the prescribed correlation function $\propto \delta(t-t')p^{4-d-y}$. The original stochastic problems can be reformulated as multiplicatively renormalizable field theoretic models, which allows one to apply the field theoretic RG to the analysis of their IR behaviour. We showed that, depending on the relation between the spatial dimension d and the exponent y, the models exhibit different critical regimes, associated with possible IR attractive fixed points of the RG equations. For the both models, the most interesting point corresponds to a new type of critical behaviour, in which the nonlinearity and turbulent mixing are both relevant, and the critical dimensions depend on the two parameters d and y. Practical calculations of the dimensions and the domains of IR stability for all the regimes were performed in the one-loop approximation of the RG, which corresponds to the leading order of the double expansion in y and $\varepsilon = 4 - d$.

From the dimensions of the coupling constants (3.7) one could expect that the full-scale regime (4) must take place when y and ε are both positive, but the careful RG analysis has shown that the domains of its IR stability is in fact much narrower: for the Gribov model, in the one-loop level it reduces to the sector $\varepsilon/4 < y < 2\varepsilon/3$, while for the model A one obtains $0 < y < 2\varepsilon/3$. This effect leads to interesting physical prediction: in contrary to what could be naively anticipated, the most realistic spatial dimensions $d = 2$ or 3 and the Kolmogorov exponent $y = 4$ for the fully developed turbulence lie in the domain of IR stability of the passive-scalar regime. For the Gribov case this means that the spreading of the agent is completely determined by the turbulent transfer. For the equilibrium model A, this is reminiscent of the observation made in [10, 11] (however, for a conserved order parameter and non-random velocities) that the critical fluctuations are suppressed by the motion of the fluid and the behaviour of the system becomes close to the mean-field limit in a strong shear flow; see also discussion in [13].

It is interesting to compare our results with those, obtained earlier in [17], where the turbulence was modelled by Kraichnan's ensemble – the time-decorrelated Gaussian velocity field with the correlator $\propto \delta(t-t')\, p^{-d-\xi}$. It turns out, that the number and the character of the critical regimes (free theory, passive scalar, ordinary phase transition and the new full-scale regime) are the same for the both ensembles. (For the Gribov case, the single passive-scalar regime for Kraichnan's ensemble corresponds to the set of two regimes (3) and (5) in the Navier-Stokes model.) What is more, in the one-loop approximations the domains of IR stability in the y–ε plane and the explicit expressions for the critical dimensions coincide for the two ensembles. (To compare the results for the two different ensembles, one has to identify $y = 3\xi$, because $\xi = 4/3$ for Kraichnan's ensemble and $y = 4$ for the Navier-Stokes case correspond to Kolmogorov's velocity spectrum.) Such agreement allows one to conclude that Kraichnan's ensemble, in spite of its relative simplicity, may serve as acceptable model of turbulent mixing.

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