Fixed points for the frozen velocity limit $\longleftrightarrow u_1^* = 0$

$$\beta_{g_1} = g_1(-y + 2\gamma_D - 2\gamma_v),$$

$$\beta_{g_2} = g_2(-\epsilon - \gamma_{g_2}),$$

$$\beta_{u_2} = -u_2\gamma_{u_2},$$

$$\beta_{u_1} = u_1(-\eta + \gamma_D),$$

 $\beta_a = -a\gamma_a,$

anomalous dimensions

$$\gamma_{D} = \frac{g_{1}}{4(1+u_{1})} \left[3 + \alpha \frac{u_{1}-1}{u_{1}+1} + \frac{4\alpha a(1-a)}{(1+u_{1})^{2}} \right] + \frac{g_{2}}{8},
\gamma_{a} = (1-2a) \left[\frac{g_{1}\alpha(1-a)}{2(1+u_{1})^{3}} + \frac{g_{1}u_{2}}{4a(1+u_{1})} \left(3 + \alpha - \frac{2\alpha}{1+u_{1}} \right) + \frac{g_{2}}{8a} \right],
\gamma_{u_{2}} = \frac{g_{1}(1-2u_{2})}{4(1+u_{1})} \left[3 + \alpha \frac{u_{1}-1}{u_{1}+1} + \frac{2\alpha a(1-a)}{u_{2}(1+u_{1})^{2}} \right] - \frac{g_{2}}{8},
\gamma_{g_{2}} = -\frac{3g_{1}}{2(1+u_{1})} + \frac{g_{1}\alpha}{1+u_{1}} \left[\frac{(1-2a)^{2}}{2} + \frac{1-3a(1-a)}{1+u_{1}} + \frac{2a(1-a)u_{1}}{(1+u_{1})^{2}} \right] - \frac{3g_{2}}{2},
\gamma_{v} = \frac{g_{1}\alpha}{4(1+u_{1})^{2}} \left[\frac{4a(1-a)}{1+u_{1}} - 1 \right] + \frac{g_{1}u_{2}}{2(1+u_{1})} \left[3 + \frac{\alpha u_{1}}{1+u_{1}} \right].$$

anomalous dimensions at fixed point $u_1^* = 0$

$$\begin{split} \gamma_D^* &= \frac{g_1^*}{4} \bigg[3 - \alpha + 4\alpha a^* (1 - a^*) \bigg] + \frac{g_2^*}{8}, \\ \gamma_a^* &= (1 - 2a^*) \bigg[\frac{g_1^* \alpha (1 - a^*)}{2} + \frac{g_1^* u_2^* (3 - \alpha)}{4a^*} + \frac{g_2^*}{8a^*} \bigg], \\ \gamma_{u_2^*} &= \frac{g_1^* (1 - 2u_2^*)}{4} \bigg[3 - \alpha + \frac{2\alpha a^* (1 - a^*)}{u_2^*} \bigg] - \frac{g_2^*}{8}, \\ \gamma_{g_2^*} &= -\frac{3g_1^*}{2} + \frac{g_1^* \alpha}{2} \bigg[10(a^*)^2 - 10a^* + 3 \bigg] - \frac{3g_2^*}{2}, \\ \gamma_v &= \frac{g_1^* \alpha}{4} \bigg[4a^* (1 - a^*) - 1 \bigg] + \frac{3g_1^* u_2^*}{2}. \end{split}$$

Fixed points

• FP I

$$g_1^*=0,\quad g_2^*=0,\quad u_1^*=0,\quad a^* \text{ not fixed },\quad u_2^* \text{ not fixed }$$

$$\Omega_1=-\epsilon,\quad \Omega_2=-\eta,\quad \Omega_3=-y$$

• FP II

$$g_1^* = 0, \quad g_2^* = \frac{2\epsilon}{3}, \quad u_1^* = 0, \quad a^* = \frac{1}{2}, \quad u_2^* = 0$$

$$\Omega_1 = \frac{\epsilon}{12}, \quad \Omega_2 = \frac{\epsilon}{6}, \quad \Omega_3 = \epsilon, \quad \Omega_4 = \frac{\epsilon}{6} - y, \quad \Omega_5 = \frac{\epsilon}{12} - \eta$$

• FP III

$$g_1^* = \frac{2(3-\alpha)y}{9}, \quad g_2^* = 0, \quad u_1^* = 0, \quad a^* = \frac{1}{2}, \quad u_2^* = \frac{\alpha}{2(\alpha-3)}$$

$$\Omega_1 = \frac{\alpha-3}{6}y, \quad \Omega_2 = \frac{3-\alpha}{6}y - \eta, \quad \Omega_3 = y, \quad \Omega_4 = \frac{\alpha^2 - 9\alpha + 18}{18} - \epsilon$$

• FP IV

$$u_1^* = 0, \quad g_2^* = 0,$$

 $y = g_1^* \left(\frac{3}{2} - 3u_2^*\right), \quad 2a^*(1 - a^*)\alpha + (3 - \alpha)u_2^* = 0$

• FP V

$$g_1^* \neq 0$$
, $g_2^* \neq 0$, $u_1^* = 0$, $a^* = \frac{1}{2}$, $u_2^* \neq 0$

 g_1^*, g_2^* and u_2^* are determined from equations (explicit results can be obtained with mathematica)

$$\begin{split} \frac{3g_1^*}{2} - 3g_1^*u_2^* + \frac{g_2}{4} &= y, \\ \frac{g_1^*}{4}(6 - \alpha) + \frac{3g_2^*}{2} &= \epsilon \\ \frac{g_1^*(u_2^*)^2}{2}(3 - \alpha) + \frac{g_1^*u_2^*}{4}(2\alpha - 3) - \frac{\alpha g_1^*}{8} + \frac{g_2^*u_2^*}{8} &= 0 \end{split}$$

• FP VI

$$g_1^* = \frac{2(\epsilon - y)}{2\alpha - 9}, \quad g_2^* = \frac{12\epsilon + 8y(\alpha - 6)}{2\alpha - 9}, \quad u_1^* = 0, \quad a^*(1 - a^*) = \frac{\epsilon(\alpha - 6) + 3y(5 - \alpha)}{2\alpha(\epsilon - y)}, \quad u_2^* = 1$$

$$\Omega_1, \quad \Omega_2, \quad \Omega_3, \quad \Omega_4,$$

$$\Omega_5 = \frac{(\alpha - 6)\epsilon + 3(5 - \alpha)y}{2(2\alpha - 9)} - \eta$$