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Abstract

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1 Introduction

2 Description of the models. Field theoretic formulation

In the Langevin formulation the models are defined by stochastic differential equations for the order parameter $\psi = \psi(t, \mathbf{x})$:

$$\partial_t \psi_a = \lambda_0 \left\{ (-\tau_0 + \partial^2)\psi_a - V(\psi) \right\} + \zeta = 0, \tag{1}$$

where $\partial_t = \partial/\partial t$, ∂^2 is the Laplace operator, $\lambda_0 > 0$ is the kinematic (diffusion) coefficient and $\tau_0 \propto (T - T_c)$ is the deviation of the temperature (or its analog) from the critical value. The Gaussian random noise $\zeta = \zeta(t, \mathbf{x})$ with the zero mean is specified by the pair correlation function:

$$\langle \zeta(t, \mathbf{x}) \zeta(t', \mathbf{x}') \rangle = 2\lambda_0 \delta(t - t') \delta^{(d)}(\mathbf{x} - \mathbf{x}') \tag{2}$$

d is the dimension of the $\mathbf{x}$ space. Here and below, the bare (unrenormalized) parameters are marked by the subscript "o". Their renormalized analogs (without the subscript) will appear later on. The nonlinearity has the form $V(\psi) = g_0 R_{abc} \psi_b(x) \psi_c(x)/2$; g_0 being the coupling constant. Summations over repeated indices are always implied. The tensor R_{abc} is expressed in the terms of the set of $n + 1$ vectors e_α.

$$R_{abc} = \sum_\alpha e_a^\alpha e_b^\alpha e_c^\alpha, \tag{3}$$

where the e_a^α satisfy

$$\sum_{\alpha=1}^{n+1} e_a^\alpha = 0, \quad \sum_{\alpha=1}^{n+1} e_a^\alpha e_b^\alpha = (n+1)\delta_{ab}, \quad \sum_{\alpha=1}^{n} e_a^\alpha e_a^\beta = (n+1)\delta^{\alpha\beta} - 1. \tag{4}$$

Where n is dimension of hyperspace. Using (4) one can obtain that contraction of two and three tensors R have forms

$$R_{abc}R_{abd} = R_1\delta_{cd}, \quad R_{abc}R_{cde}R_{efa} = R_2R_{bdf}, \tag{5}$$

where

$$R_1 = (n+1)^2(n-1), \quad R_2 = (n+1)^2(n-2). \tag{6}$$

This stochastic problem (1), (2) can be reformulated as field theoretic model of the doubled set of fields $\Phi = \{\psi, \psi^\dagger\}$ with action functional

$$\mathcal{S}(\psi, \psi^\dagger) = \psi^\dagger_a \left(-\partial_t + \lambda_0\partial^2 - \lambda_0\tau_0\right)\psi_a + \lambda_0\psi^\dagger_a\psi^\dagger_a - g_0R_{abc}\lambda\psi^\dagger_a\psi_b\psi_c/2. \tag{7}$$

Here, $\psi^\dagger = \psi^\dagger(t, \mathbf{x})$ is the auxiliary "response field" and the integrations over the arguments of the fields are implied, for example

$$\psi^\dagger\partial_t\psi = \int dt \int d\mathbf{x}\psi^\dagger(t,\mathbf{x})\partial_t\psi(t,\mathbf{x}).$$

The field theoretic formulation means that the statistical averages of random quantities in the original stochastic problems can be represented as functional integrals over the full set of fields with weight $\exp\mathcal{S}(\Phi)$, and can therefore be viewed as the Green functions of the field theoretic models with actions (7). In particular, the linear response function of the problems (1), (2) is given by the Green function

$$G = \langle\psi^\dagger(t,\mathbf{x})\psi(t',\mathbf{x}')\rangle = \int\mathcal{D}\psi^\dagger\int\mathcal{D}\psi \;\; \psi^\dagger(t,\mathbf{x})\psi(t',\mathbf{x}')\,\exp\mathcal{S}(\psi,\psi^\dagger) \tag{8}$$

of the corresponding field theoretic model.

The model (7) corresponds to the standard Feynman diagrammatic technique with two bare propagators $\langle\psi\psi^\dagger\rangle_0$, $\langle\psi\psi\rangle_0$ and triple vertex $\sim \psi^\dagger\psi^2$. In the frequency-momentum representation the propagators have the forms

$$\begin{aligned}\langle\psi\psi^\dagger\rangle_0(\omega,k) &= \frac{1}{-\mathrm{i}\omega + \lambda_0\left(k^2+\tau_0\right)},\\ \langle\psi\psi\rangle_0(\omega,k) &= \frac{2\lambda_0}{\omega^2 + \lambda_0^2\left(k^2+\tau_0\right)^2}\end{aligned} \tag{9}$$

The Galilean invariant coupling with the velocity field $\mathbf{v} = \{v_i(t,\mathbf{x})\}$ for the compressible fluid ($\partial_i v_i \neq 0$) can be introduced by the replacement

$$\partial_t\psi_b \to \partial_t\psi_b + a_0\,\partial_i(v_i\psi_b) + (a_0-1)(v_i\partial_i)\psi_b = \nabla_t\psi_b + a_0(\partial_i v_i)\psi_b \tag{10}$$

in (1). Here $\nabla_t \equiv \partial_t + v_i\partial_i$ is the Lagrangian derivative, a_0 is an arbitrary parameter and $\partial_i = \partial/\partial x_i$.

In the real problem, the field $\mathbf{v}(t,\mathbf{x})$ satisfies the Navier–Stokes equation. We will employ the rapid-change model [16], where the velocity obeys a Gaussian distribution with zero mean and the correlation function

$$\langle v_i(t,\mathbf{x})v_j(t',\mathbf{x}')\rangle = \delta(t-t')\,D_{ij}(\mathbf{r}), \quad \mathbf{r}=\mathbf{x}-\mathbf{x}' \tag{11}$$

with

$$D_{ij}(\mathbf{r}) = \mathbf{D_0}\int_{\mathbf{k}>\mathbf{m}} \frac{\mathbf{dk}}{(\mathbf{2}\pi)^{\mathbf{d}}}\,\frac{\mathbf{1}}{\mathbf{k}^{\mathbf{d}+\xi}}\,\{\mathbf{P_{ij}}(\mathbf{k})+\alpha\mathbf{Q_{ij}}(\mathbf{k})\}\,\exp(\mathrm{i}\mathbf{kr}). \tag{12}$$

Here $P_{ij}(\mathbf{k})=\delta_{ij}-k_ik_j/k^2$ and $Q_{ij}(\mathbf{k})=k_ik_j/k^2$ are the transverse and the longitudinal projectors, $k\equiv|\mathbf{k}|$ is the wave number, $D_0>0$ is an amplitude factor and $\alpha>0$ is an arbitrary parameter. The case $\alpha=0$ corresponds to the incompressible fluid ($\partial_i v_i=0$), while the limit $\alpha\to\infty$ at fixed αD_0 corresponds to the purely potential velocity field. The exponent $0<\xi<2$ is a free parameter which can be viewed as a kind of Hölder exponent, which measures "roughness" of the velocity field; the "Kolmogorov" value is $\xi=4/3$, while the "Batchelor" limit $\xi\to 2$ corresponds to smooth velocity. The cutoff in the integral (12) from below at $k=m$, where $m\equiv 1/\mathcal{L}$ is the reciprocal of the integral turbulence scale $\mathcal{L}$, provides IR regularization. Its precise form is unimportant; the sharp cutoff is the simplest choice for the practical calculations.

The action functional for the full set of fields $\Phi=\left\{\psi,\psi^\dagger,\mathbf{v}\right\}$ become

$$\begin{aligned}\mathcal{S}(\Phi) = \psi_a^\dagger\left\{-\nabla_t+\lambda_0\left(\partial^2-\tau_0\right)-a_0(\partial_i v_i)\right\}\psi_a + \\ +\lambda_0\psi_a^\dagger\psi_a^\dagger - \frac{g_0\lambda_0}{2}R_{abc}\psi_a^\dagger\psi_b\psi_c+\mathcal{S}(\mathbf{v})\end{aligned} \tag{13}$$

This is obtained from (7), by the replacement (10) and adding the term corresponding to the Gaussian averaging over the field $\mathbf{v}$ with the correlator (11), (12):

$$\mathcal{S}(\mathbf{v}) = -\frac{\mathbf{1}}{\mathbf{2}}\int\mathbf{dt}\int\mathbf{dx}\int\mathbf{dx}'\mathbf{v_i}(\mathbf{t},\mathbf{x})\mathbf{D}_{\mathbf{ij}}^{-\mathbf{1}}(\mathbf{r})\mathbf{v_j}(\mathbf{t},\mathbf{x}'), \tag{14}$$

where

$$D_{ij}^{-1}(\mathbf{r}) \propto \mathbf{D_0^{-1}}\mathbf{r}^{-\mathbf{2d}-\xi}$$

is the kernel of the inverse linear operation for the function $D_{ij}(\mathbf{r})$ in (12).

In addition to (9), the Feynman diagrams for the model (14) involve the propagator $\langle vv\rangle_0$ specified by the relations (11), (12) and the new vertex

$$\psi^\dagger v_i V_i\psi \equiv -\psi^\dagger\left\{(v_i\partial_i)\psi + a_0(\partial_i v_i)\right\}\psi. \tag{15}$$

In the diagrams it corresponds to the vertex factor

$$V_i = -\mathrm{i}k_i - \mathrm{i}a_0 q_i, \tag{16}$$

where k_i is the momentum argument of the field ψ and q_i is the momentum of v_i.

Actual expansion parameter in the perturbation theory of the model is $u_0 = g_0^2$, so for the full model, the role of the coupling constants (expansion parameters in the perturbation theory) is played by the three parameters

$$u_0 = g_0^2 \sim \Lambda^{6-d}, \qquad w_0 = D_0/\lambda_0 \sim \Lambda^{\xi}, \qquad w_0 a_0 \sim \Lambda^{\xi}. \tag{17}$$

The last relations, following from the dimensionality considerations (more precisely, see the next section), define the typical UV momentum scale Λ. From the relations (17) it follows that the interactions $\psi^\dagger \psi_a \psi_a$ become logarithmic (the corresponding coupling constant u_0 becomes dimensionless) at $d = 6$. Thus for the single-charge problems (7), the value $d = d_c = 6$ is the upper critical dimension, and the deviation $\varepsilon = 4 - d_c$ plays the part of the formal expansion parameter in the RG approach: the critical exponents are nontrivial for $\varepsilon > 0$ and are calculated as series in ε.

The additional interactions $\sim \psi_a^\dagger v \partial \psi_a$ of the full model (14) become logarithmic at $\xi = 0$. The parameter ξ is not related to the spatial dimension and can be varied independently. However, for the RG analysis of the full problems it is important that all the interactions become logarithmic at the same time. Otherwise, one of them would be weaker than the others from the RG viewpoint and it would be irrelevant in the leading-order IR behaviour. As a result, some of the scaling regimes of the full model would be overlooked. In order to study all possible scaling regimes and the crossovers between them, we need a genuine three-charge theory, in which all the interactions are treated on equal footing. Thus we will treat ε and ξ as small parameters of the same order, $\varepsilon \propto \xi$. Instead of the plain ε expansion in the single-charge models, the coordinates of the fixed points, critical dimensions and other quantities will be calculated as double expansions in the ε–ξ plane around the origin, that is, around the point in which all the coupling constants in (17) become dimensionless. Similar situation was encountered earlier in various models of turbulence and complex critical behaviour, e.g. [12, 13, 14, 15, 20].

3 Canonical dimensions, UV divergences and the renormalization

It is well known that the analysis of UV divergences is based on the analysis of canonical dimensions ("power counting"); see e.g. [1, 2]. Dynamical models of the type (7), in contrast to static ones, have two independent scales: the time scale T and the length scale L. Thus the canonical dimension of any quantity F (a field or a parameter) is characterized by two numbers, the frequency dimension d_F^ω and the momentum dimension d_F^k, defined such that $[F] \sim [T]^{-d_F^\omega}[L]^{-d_F^k}$. These dimensions are found from the obvious normalization conditions

$$d_k^k = -d_{\mathbf{x}}^k = 1, \ d_k^\omega = d_{\mathbf{x}}^\omega = 0, \ d_\omega^k = d_t^k = 0, \ d_\omega^\omega = -d_t^\omega = 1$$

and from the requirement that each term of the action functional be dimensionless (with respect to the momentum and frequency dimensions separately). Then, based on d_F^k and d_F^ω, one can introduce the total canonical dimension $d_F = d_F^k + 2d_F^\omega$ (in the free theory, $\partial_t \propto \partial^2$), which plays in the theory of renormalization of dynamical models the same role as the conventional (momentum) dimension does in static problems; see Chap. 5 of [2]. The canonical dimensions for the models (14) are given in table 1, including renormalized parameters (without subscript "o"), which will be introduced soon.

Table 1: Canonical dimensions of the fields and parameters in the model (14).

F	ψ	$\psi^\dagger$	v	λ_0, λ	τ_0, τ	m, μ, Λ	g_0^2	ω_0	$g^2, \omega, \alpha, a_0, a$
d_F^k	$\frac{d-2}{2}$	$\frac{d+2}{2}$	-1	-2	2	1	$2-d$	ξ	0
d_F^ω	0	0	1	1	0	0	2	0	0
d_F	$\frac{d-2}{2}$	$\frac{d+2}{2}$	1	0	2	1	$6-d$	ξ	0

As already discussed in the end of the previous section, the full model is logarithmic (all the coupling constants are simultaneously dimensionless) at $d = 6$ and $\xi = 0$. Thus the UV divergences in the Green functions manifest themselves as poles in $\varepsilon = 6 - d$, ξ and, in general, their linear combinations.

The total canonical dimension of an arbitrary 1-irreducible Green function $\Gamma = \langle \Phi \cdots \Phi \rangle_{1-\text{ir}}$ is given by the relation [2]

$$d_\Gamma = d_\Gamma^k + 2d_\Gamma^\omega = d + 2 - N_\Phi d_\Phi, \tag{18}$$

where $N_\Phi = \{N_\psi, N_{\psi^\dagger}, N_v\}$ are the numbers of corresponding fields entering into the function Γ, and the summation over all types of the fields is implied. The total dimension d_Γ in logarithmic theory (that is, at $\varepsilon = \xi = 0$) is the formal index of the UV divergence $\delta_\Gamma = d_\Gamma|_{\varepsilon=\xi=0}$. Superficial UV divergences, whose removal requires counterterms, can be present only in those functions Γ for which δ_Γ is a non-negative integer.

From table 1 and (18) we find

$$\delta_\Gamma = 8 - 2N_\psi - 4N_{\psi^\dagger} - N_v. \tag{19}$$

In dynamical models, the 1-irreducible diagrams without the fields $\psi^\dagger$ vanish, and it is sufficient to consider the functions with $N_{\psi^\dagger} \geq 1$. In our model we have Galilean symmetry and symmetry of tensor $R_{acc} = 0$ from (3),(4). With these restrictions, the analysis of the expressions (19) shows that in model, superficial UV divergences can be present in the following 1-irreducible functions:

$$\langle \psi^\dagger \psi^\dagger \rangle \quad (\delta = 0) \quad \text{with the counterterms} \quad \psi^\dagger \psi^\dagger,$$

$$\langle \psi^\dagger \psi \rangle \quad (\delta = 2) \quad \text{with the counterterms} \quad \psi^\dagger \partial_t \psi,\ \psi^\dagger \partial^2 \psi,\ \psi^\dagger \psi,$$

$$\langle \psi^\dagger \psi \psi \rangle \quad (\delta = 0) \quad \text{with the counterterms} \quad \psi^\dagger \psi \psi,$$

$$\langle \psi^\dagger \psi v \rangle \quad (\delta = 1) \quad \text{with the counterterms} \quad \psi^\dagger (v\partial) \psi,\ \psi^\dagger (\partial v) \psi.$$

All the remaining terms are present in the corresponding action functional (14), so that our models are multiplicatively renormalizable. The Galilean symmetry also requires that the counterterms $\psi^\dagger \partial_t \psi$ and $\psi^\dagger (v\partial)\psi$ enter the renormalized action only in the form of the Lagrangian derivative $\psi^\dagger \nabla_t \psi$, imposing no restriction on the Galilean invariant term $\psi^\dagger (\partial v)\psi$.

We thus conclude that the renormalized actions can be written in the forms

$$\begin{aligned} \mathcal{S}^R(\Phi) = \psi_a^\dagger \left\{ -Z_1 \nabla_t + \lambda \left(Z_2 \partial^2 - Z_3 \tau \right) - a Z_6 (\partial_i v_i) \right\} \psi_a + \\ + \lambda Z_5 \psi_a^\dagger \psi_a^\dagger - g R_{abc} \mu^\varepsilon \lambda Z_4 \psi_a^\dagger \psi_b \psi_c / 2 + \mathcal{S}(\mathbf{v}) \end{aligned} \tag{20}$$

for the model with $\mathcal{S}(\mathbf{v})$ from (14).

Here λ, τ, g, u and a are renormalized analogs of the bare parameters (with the subscripts "o") and μ is the reference mass scale (additional arbitrary parameter of the renormalized theory). The renormalization constants Z_i absorb the poles in ε and ξ and depend on the dimensionless parameters u, w, α and a. Expression (21) can be reproduced by the multiplicative renormalization of the fields $\psi \to \psi Z_\psi$, $\psi^\dagger \to \psi^\dagger Z_{\psi^\dagger}$ and the parameters:

$$\begin{aligned} g_0 = g\mu^{\varepsilon/2} Z_g, \quad u_0 = g\mu^\varepsilon Z_u, \quad w_0 = w\mu^\xi Z_w, \\ \lambda_0 = \lambda Z_\lambda, \quad \tau_0 = \tau Z_\tau, \quad a_0 = a Z_\tau. \end{aligned} \tag{21}$$

Since the last term $\mathcal{S}(\mathbf{v})$ given by (14) is not renormalized, the amplitude D_0 from (12) is expressed in renormalized parameters as $D_0 = w_0 \lambda_0 = w \lambda \mu^\xi$, while the parameters m and α are not renormalized: $m_0 = m$, $\alpha_0 = \alpha$. Owing to the Galilean symmetry, the both terms in the covariant derivative ∇_t are renormalized with the same constant Z_1, so that the velocity field is not renormalized, either. Hence the relations

$$Z_w Z_\lambda = 1, \quad Z_m = Z_\alpha = Z_v = 1. \tag{22}$$

Comparison of the expressions (14) and (21) gives the following relations between the renormalization constants Z_1–Z_6 and (21):

$$\begin{aligned} Z_1 = Z_\psi Z_{\psi^\dagger}, \quad Z_2 = Z_1 Z_\lambda, \quad Z_3 = Z_2 Z_\tau, \\ Z_4 = Z_\lambda Z_{\psi^\dagger}^2, \quad Z_5 = Z_\lambda Z_g Z_\psi^3 Z_{\psi^\dagger}, \quad Z_6 = Z_1 Z_a \end{aligned} \tag{23}$$

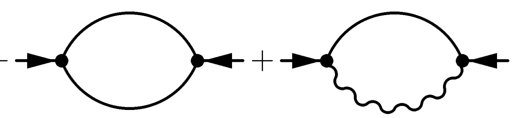
for the Potts model. Resolving these relations with respect to the renormalization constants of the fields and parameters gives

$$Z_\lambda = Z_1^{-1} Z_2, \quad Z_\tau = Z_2^{-1} Z_3, \quad Z_a = Z_1^{-1} Z_6, \quad Z_u = Z_1^{-1} Z_2^{-3} Z_4^2 Z_5, \qquad Z_\psi = Z_1^{1/2} Z_2^{1/2} Z_5^{-1/2}, \quad Z_\psi^\dagger = Z_1^{1/2} Z_2^{-1/2} Z_5^{1/2} \tag{24}$$

for the model, where we passed to the coupling constant $u = g^2$ with $Z_u = Z_g^2$.

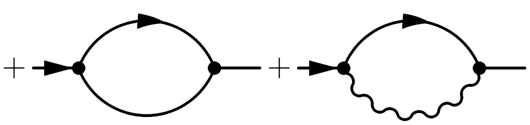
The renormalization constants can be found from the requirement that the Green functions of the renormalized model (21), when expressed in renormalized variables, be UV finite (in our case, be finite at $\varepsilon \to 0$, $\xi \to 0$). The constants Z_1–Z_6 are calculated directly from the diagrams, then the constants in (21) are found from (25). In order to find the full set of constants, it is sufficient to consider the 1-irreducible Green functions which involve superficial divergences; In the one-loop approximation, they are given in this equations.

$$\langle \psi^\dagger \psi^\dagger \rangle_{1-ir} = 2\lambda Z_5 +$$



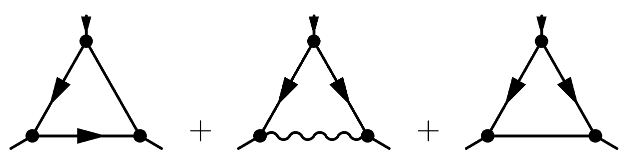
(25)

$$\langle \psi \psi^\dagger \rangle_{1-ir} = Z_1 i\omega - \lambda \left(Z_2 k^2 - Z_3 \tau \right) +$$



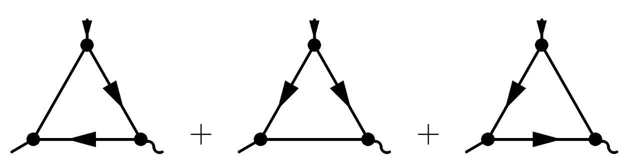
(26)

$$\langle \psi \psi \psi^\dagger \rangle_{1-ir} = -g\lambda \mu^{\frac{\varepsilon}{2}} Z_4 +$$



(27)

$$\langle \psi \psi^\dagger v \rangle_{1-ir} = -a_0 k_i Z_6 +$$



(28)

The solid lines with arrows denote the propagator $\langle \psi\psi^\dagger\rangle_0$, the arrow points to the field $\psi^\dagger$. The solid lines without arrows correspond to the propagator $\langle\psi\psi\rangle_0$ and the wavy lines denote the velocity propagator $\langle vv\rangle_0$ specified in (11). The external ends with incoming arrows correspond to the fields $\psi^\dagger$, the ends without arrows correspond to ψ. The triple vertex with one wavy line corresponds to the vertex factor (16).

All the diagrammatic elements should be expressed in renormalized variables using the relations (21)–(24). In the one-loop approximation, the Z's in the bare terms should be taken in the first order in $u = g^2$ and w, while in the diagrams they should simply be replaced with unities, $Z_i \to 1$. Thus the passage to renormalized variables in the diagrams is achieved by the simple substitutions $\lambda_0 \to \lambda$, $\tau_0 \to \tau$, $g_0 \to g\mu^{\varepsilon/2}$ and $w_0 \to w\mu^{\xi}$.

In practical calculations, we used the minimal subtraction (MS) scheme, in which the renormalization constants have the forms $Z_i = 1+$ only singularities in ε and ξ, with the coefficients depending on the completely dimensionless renormalized parameters u, w, a and α. And results for z_i have forms:

$$Z_1 = 1 - \frac{uR_1}{2\varepsilon}, \quad Z_2 = 1 - \frac{uR_1}{3\varepsilon} - \frac{w}{6\xi}(5+\alpha), \quad Z_3 = 1 - \frac{2R_1 u}{\varepsilon},$$
$$Z_4 = 1 - \frac{2R_2 u}{\varepsilon} - \frac{w\alpha a^2}{\xi}, \quad Z_5 = 1 - \frac{uR_1}{2\varepsilon} - \frac{w}{\xi}\alpha(a-1)^2,$$
$$Z_6 = 1 - \frac{uR_1}{\varepsilon 2a}(4a-1). \tag{29}$$

To simplify the resulting expressions, we passed in (29) to the new parameters

$$u \to u/128\pi^3, \quad w \to w/64\pi^3;$$

here and below they are denoted by the same symbols u and w. The parameters R_1 and R_2 are related to the dimension n of the order parameter by the expression (6). Although we are especially interested in the cases $n = 0$ and $n = -1$, for completeness the coefficients R_1 and R_2 in what follows are assumed to be arbitrary.

These relations will play an important role in the analysis of the fixed points of the model (14) in the next sections.

4 RG functions and RG equations

Let us recall an elementary derivation of the RG equations; detailed exposition can be found in monographs [1, 2]. The RG equations are written for the renormalized Green functions $G^R = \langle\Phi\cdots\Phi\rangle_R$, which differ from the original (unrenormalized) ones $G = \langle\Phi\cdots\Phi\rangle$ only by normalization (due to rescaling of the fields) and choice of parameters, and therefore can equally be used for analyzing the critical behaviour.

The relation $\mathcal{S}^R(\Phi, e, \mu) = \mathcal{S}(\Phi, e_0)$ between the bare (14) and renormalized (21) action functionals results in the relations

$$G(e_0, \dots) = Z_{\psi}^{N_\psi} Z_{\psi^\dagger}^{N_{\psi^\dagger}} G^R(e, \mu, \dots). \tag{30}$$

between the Green functions. Here, as usual, N_ψ and $N_{\psi^\dagger}$ are the numbers of corresponding fields entering into G (we recall that in our models $Z_v = 1$); $e_0 = \{\lambda_0, \tau_0, u_0, w_0, a_0, m_0, \alpha_0\}$ is the full set of bare parameters and $e = \{\lambda, \tau, u, w, a, m, \alpha\}$ are their renormalized counterparts (we recall that $\alpha_0 = \alpha$ and $m_0 = m$); the dots stand for the other arguments (times/frequencies and coordinates/momenta).

We use $\widetilde{\mathcal{D}}_\mu$ to denote the differential operation $\mu\partial_\mu$ for fixed e_0 and operate on both sides of the equation (30) with it. This gives the basic RG differential equation:

$$\left\{\mathcal{D}_{RG} + N_\psi \gamma_\psi + N_{\psi^\dagger}\gamma_{\psi^\dagger}\right\} G^R(e, \mu, \dots) = 0, \tag{31}$$

where $\mathcal{D}_{RG}$ is the operation $\widetilde{\mathcal{D}}_\mu$ expressed in the renormalized variables:

$$\mathcal{D}_{RG} \equiv \mathcal{D}_\mu + \beta_u \partial_u + \beta_w \partial_w + \beta_a \partial_a - \gamma_\lambda \mathcal{D}_\lambda - \gamma_\tau \mathcal{D}_\tau. \tag{32}$$

Here we have written $\mathcal{D}_x \equiv x\partial_x$ for any variable x, the anomalous dimensions γ are defined as

$$\gamma_F \equiv \widetilde{\mathcal{D}}_\mu \ln Z_F \quad \text{for any quantity } F, \tag{33}$$

and the β functions for the dimensionless couplings u, w and a are

$$\begin{aligned} \beta_u &\equiv \widetilde{\mathcal{D}}_\mu u = u\,(-\varepsilon - \gamma_u), \\ \beta_w &\equiv \widetilde{\mathcal{D}}_\mu w = w\,(-\xi - \gamma_w), \\ \beta_a &\equiv \widetilde{\mathcal{D}}_\mu a = -a\gamma_a, \end{aligned} \tag{34}$$

where the second equalities come from the definitions and the relations (21). The fourth β function

$$\beta_\alpha = \widetilde{\mathcal{D}}_\mu \alpha = -\alpha\gamma_\alpha \tag{35}$$

vanishes identically due to (41) and for this reason does not appear in the subsequent relations.

The anomalous dimension corresponding to a given renormalization constant Z_F is readily found from the relation

$$\gamma_F = (\beta_u\partial_u + \beta_w\partial_w + \beta_a\partial_a)\ln Z_F \simeq -\left(\varepsilon\mathcal{D}_u + \xi\mathcal{D}_w\right)\ln Z_F. \tag{36}$$

In the first relation, we used the definition (33), expression (32) for the operation $\widetilde{\mathcal{D}}_\mu$ in renormalized variables, and the fact that the Z's depend only on the completely dimensionless coupling constants u, w and a. In the second (approximate) relation, we only retained the leading-order terms in the β functions (34), which is sufficient for the first-order approximation. The leading-order expressions (29) for the renormalization constants have the form

$$Z_F = 1 + \frac{u}{\varepsilon} A_F(a,\alpha) + \frac{w}{\xi} B_F(a,\alpha). \tag{37}$$

Substituting (37) into (36) leads to the final UV finite expressions for the anomalous dimensions:

$$\gamma_F = -uA_F(a,\alpha) - wB_F(a,\alpha) \tag{38}$$

for any constant Z_F. This gives

$$\begin{gathered}\gamma_1 = R_1 u/2, \quad \gamma_2 = R_1 u/3 + w(5+\alpha)/6, \quad \gamma_3 = 2R_1 u, \\ \gamma_4 = 2R_2 u + w\alpha a^2, \quad \gamma_5 = uR_1/2 + w\alpha(a-1)^2, \quad \gamma_6 = uR_1(4a-1)/2a.\end{gathered} \tag{39}$$

The multiplicative relations (24) between the renormalization constants result in the linear relations between the corresponding anomalous dimensions:

$$\begin{gathered}\gamma_\lambda = \gamma_2 - \gamma_1, \quad \gamma_\tau = \gamma_3 - \gamma_2, \\ \gamma_a = \gamma_6 - \gamma_1, \quad \gamma_u = -\gamma_1 - 3\gamma_2 + 2\gamma_4 + \gamma_5, \\ 2\gamma_\psi = \gamma_1 + \gamma_2 - \gamma_5, \quad 2\gamma_\psi^\dagger = \gamma_1 - \gamma_2 + \gamma_5.\end{gathered} \tag{40}$$

Along with (39), these relations give the final first-order explicit expressions for the anomalous dimensions of the fields and parameters. The exact relations (22) result in

$$\gamma_w = -\gamma_\lambda, \quad \gamma_m = \gamma_\alpha = \gamma_v = 0. \tag{41}$$

5 Attractors of the RG equations and scaling regimes for the Potts model

It is well known that possible asymptotic regimes of a renormalizable field theoretic model is determined by the asymptotic behaviour of the system of ordinary differential equations for the so-called invariant (running) coupling constants

$$\mathcal{D}_s \bar{g}_i(s,g) = \beta_i(\bar{g}), \quad \bar{g}_i(1,g) = g_i, \tag{42}$$

where $s = k/\mu$, k is the momentum, $g = \{g_i\}$ is the full set of coupling constants and $\bar{g}_i(s,g)$ are the corresponding invariant variables. As a rule, the IR ($s \to 0$) and UV ($s \to \infty$) behaviour of such system is determined by fixed points g_{i*}. The coordinates of possible fixed points are found from the requirement that all the β functions vanish:

$$\beta_i(g_*) = 0, \tag{43}$$

while the type of a given fixed point is determined by the matrix

$$\Omega_{ij} = \partial\beta_i/\partial g_j|_{g=g^*} : \tag{44}$$

for an IR attractive fixed points (which we are interested in here) the matrix Ω is positive, that is, the real parts of all its eigenvalues are positive. In our models, the fixed points for the full set of couplings u, w, a, α should be determined by the equations

$$\beta_{u,w,a,\alpha}(u_*, w_*, a_*, \alpha_*) = 0, \tag{45}$$

with the β functions defined in the preceding section. However, in our models the attractors of the system (42) involve, in general, two-dimensional surfaces in the full four-dimensional space of couplings. First, the function (35) vanishes identically, so that the equation $\beta_\alpha = 0$ gives no restriction on the parameter α. It is then convenient to consider the attractors of the system (42) in the three-dimensional space u, w, a; their coordinates, matrix (44) and the critical exponents will, in general, depend on the free parameter α. What is more, in this reduced space the attractors will be not only fixed points, but also lines of fixed points, which can be conveniently parametrized by the coupling a. Although the general pattern of the attractors appears rather similar for the both models, it is instructive to discuss them separately.

The one-loop expressions for the β functions in the model (21) are easily derived from the definitions (34), relations (41) and (41), and explicit expressions (39):

$$\begin{aligned} \beta_u &= u\left[-\varepsilon + Ru + w(5+\alpha)/2 - w\alpha f(a)\right], \\ \beta_w &= w\left[-\xi + R_1 u/6 + w(5+\alpha)/6\right], \\ \beta_a &= uR_1(1-3a)/2, \end{aligned} \tag{46}$$

where the function $f(a) = 2a^2 + (a-1)^2$ achieves the minimum value $f(1/3) = 2/3$ at $a = 1/3$. And $R = R_1 - 4R_2$. For the set (46) the equations (45) have the following four solutions:

(1) The line of Gaussian (free) fixed points: $u_* = w_* = 0$, a_* arbitrary.

(2) The point $w_* = 0$, $u_* = \varepsilon/R$, $a_* = 1/3$, corresponding to the pure Potts model (turbulent advection is irrelevant).

(3) The line of fixed points

$$u_* = 0, \quad w_* = 6\xi/(5+\alpha), \quad a_* \text{ arbitrary}, \tag{47}$$

corresponding to the passively advected scalar without self-interaction.

(4) The most nontrivial fixed point, corresponding to the new regime (universality class), both the advection and the self-interaction are relevant:

$$u_* = \frac{[\varepsilon(5+\alpha) - \xi(15-\alpha)]}{6\Delta}, \quad w_* = \frac{[-\varepsilon R_1/6 + R\xi]}{\Delta}, \quad a_* = 1/3. \tag{48}$$

Where $\Delta = [5(R + R_1/2) + \alpha(R - R_1/6)]/6$ We recall that α is treated as a free parameter, which the coordinates of the fixed points can depend on.

Admissible fixed point must be IR attractive and satisfy the conditions $u_* > 0$, $w_* > 0$, which follow from the physical meaning of these parameters.

For the point (1) we have

$$\Omega_u = -\varepsilon, \quad \Omega_w = -\xi, \quad \Omega_a = 0,$$

so it is admissible for $\varepsilon < 0$, $\xi < 0$ (region I in fig.[1-8]). Vanishing of the element Ω_a reflects the fact that the parameter a_* for the point I is arbitrary, or, in other words, the point is degenerate. The point (2) can be physical only if $R > 0, R_1 < 0$ (and any α) and is IR stable if

$$\varepsilon > 0, \quad \xi < -R_1\varepsilon/6R$$

(region II in fig.[1-8]).

The physical region of stability passive scalar point (region III in fig.[1-8]) can be described by this equations with restriction on α:

$$\begin{gathered} -(5+\alpha)\varepsilon + (15-\alpha)\xi > 0\xi > 0 \\ (a-1/3)^2 < \frac{-(5+\alpha)\varepsilon + (15-\alpha)\xi}{18+\alpha\xi}, \end{gathered} \tag{49}$$

Thus the both inequalities (21), (22) are satised in a sector in the upperleft quadrant; the lower bound is (21) and the upper bound is (22). From (23) it follows that, when changes from 0 to , the coecient D(C A)/A(D B) changes from R1 /6R to (R + R1 /6)/2R. The coecient D/B = (5 +)/(15) changes from R1 /6R to 1. Thus for = 0 the domain has zero width, and when grows it is getting wider.

The point (4), depending on the R, R_1, have three cases:

IV-1. $R > -R_1/2$, $R_1 < 0$

The point physical and IR attractive when

$$\xi > -\frac{R_1\varepsilon}{6R} \quad \xi < \frac{5+\alpha}{15-\alpha}\varepsilon \quad \alpha > 0. \tag{50}$$

So for small $\alpha < 15$, the physical region is the sectior in the upper-right quadrant in the ε–ξ plane, bounded by the ray $\xi = -R_1\varepsilon/6R$ from below and $\xi < (5+\alpha)/(15-$

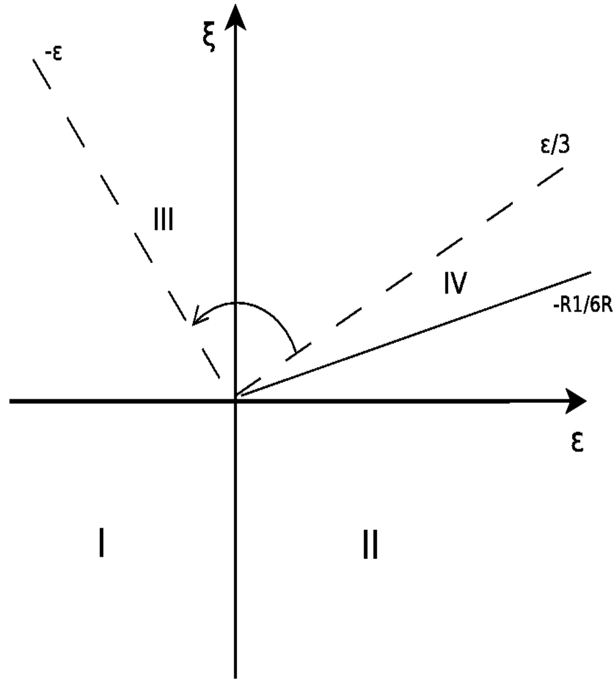


Figure 1: case:1

α) from above. When α grows, the upper ray $\xi < (5+\alpha)/(15-\alpha)$ rotates counter clockwise and moves to the upper-left quadrant. For case IV-1 α changes from 0 to ∞ and the ray changes from $\xi = \varepsilon/3$ to $\xi = -\varepsilon$ exactly like the boundary of the point III (fig.[1]).

IV-2. $-R_1/2 > R > 0$
The point describes by the same expressions (50), but α changes from α_0 to ∞, where $\alpha_0 = -5(R + R_1/2)/(R - R_1/6) > 0$. In this case $-R1/6R > 1/3$, and when $\alpha = 0$, regions II and III are crossed (fig.[2]).

When α grows boundaries of point II rotates counter clockwise and when $\alpha = \alpha_0$ boundaries of points II and III are coincides (fig.[3]).

For $\alpha > \alpha_0$ regions II and III aren't crossed and between them we have IV regime like in the case 1 (fig.[4]).

IV-3. $0 > R > R_1/6$
For this parameters we have no II regime. In this case regime of stability IV point

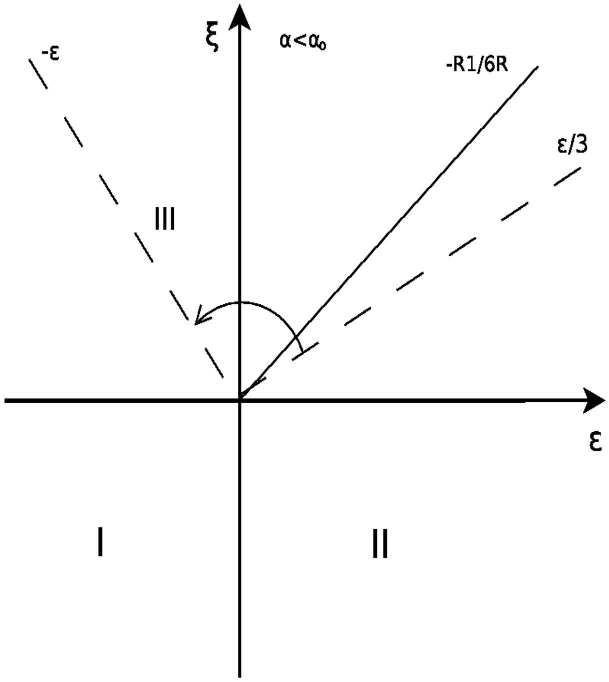


Figure 2: case:2

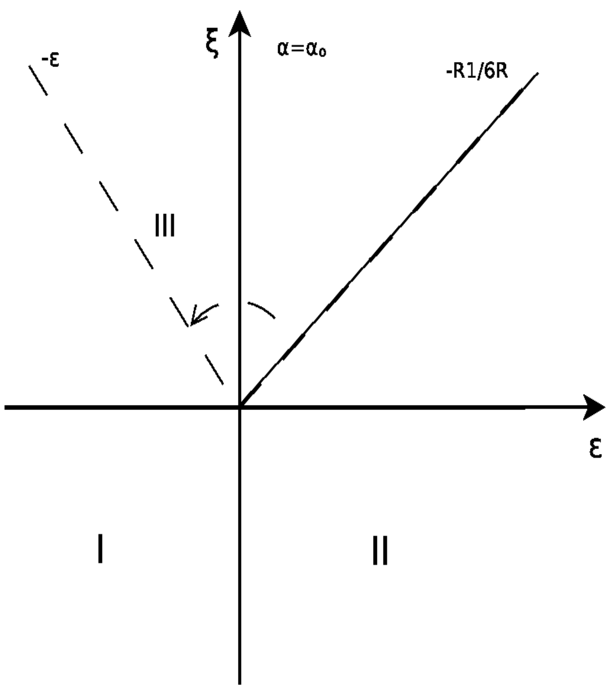


Figure 3: case:2

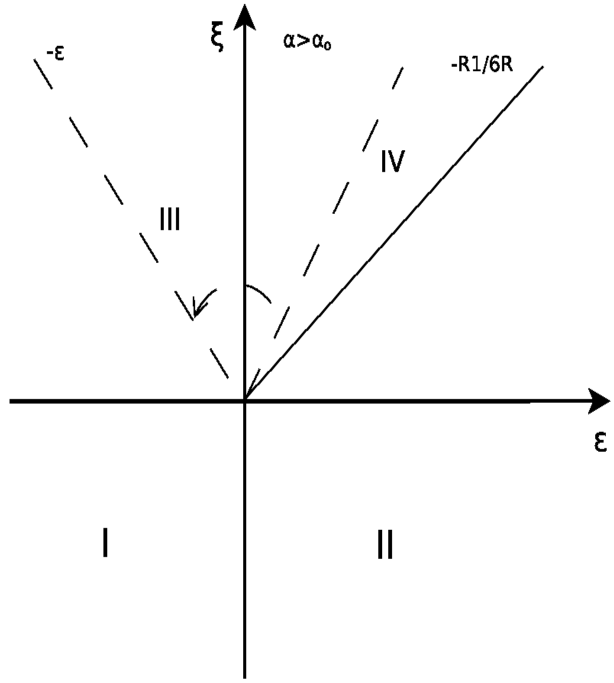


Figure 4: case:2

describes by this inequalities:

$$\xi < \varepsilon(5+\alpha)/(15-\alpha), \tag{51}$$

$$\xi < \varepsilon\frac{(5+\alpha)(R+R_1/6)}{(5-\alpha)2R}, \tag{52}$$

$$\alpha > \alpha_0. \tag{53}$$

But now $\alpha_0 < 0$ so the both inequalities (51), (52) are satisfied in a sector in the upperleft quadrant; the lower bound is (51) and the upper bound is (52). When α changes from α to ∞, the coefficient in (51) changes from $R1/6R$ to $(R+R1/6)/2R$. The coefficient $(5+\alpha)/(15-\alpha)$ changes from $R1/6R$ to 1. Thus for $\alpha = \alpha_0$ the domain has zero width, and when α grows it is getting wider (fig.[5,6,7]).

In the one-loop approximation (46), all the boundaries between the regions of admissibility are given by straight rays; there are neither gaps nor overlaps between the different regions. Due to higher-order corrections, the boundaries between the regions II and IV and between the regions III and IV can change and become curved. However, it can be argued that no gaps nor overlaps can appear between them to all orders. The similar situation was encountered earlier in the crossover between the long-range and short-range regimes of the Gribov model; see [21]. Here we only note that, in the present case, it is important that the models with $u = 0$ or $w = 0$ are "closed with respect to renormalization" so that the functions β_u for $w = 0$ and

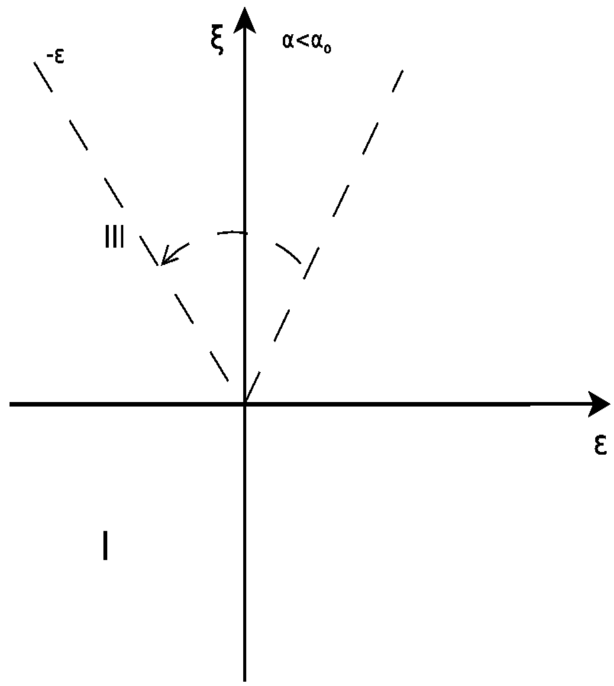


Figure 5: case3

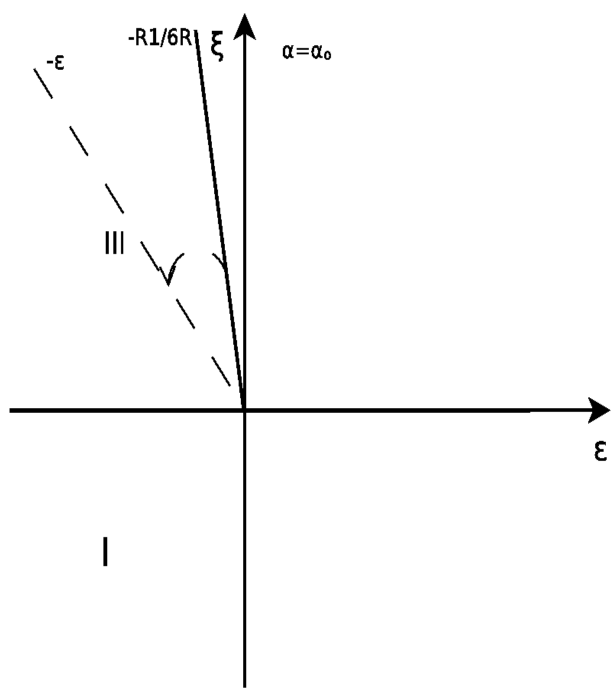


Figure 6: case:3

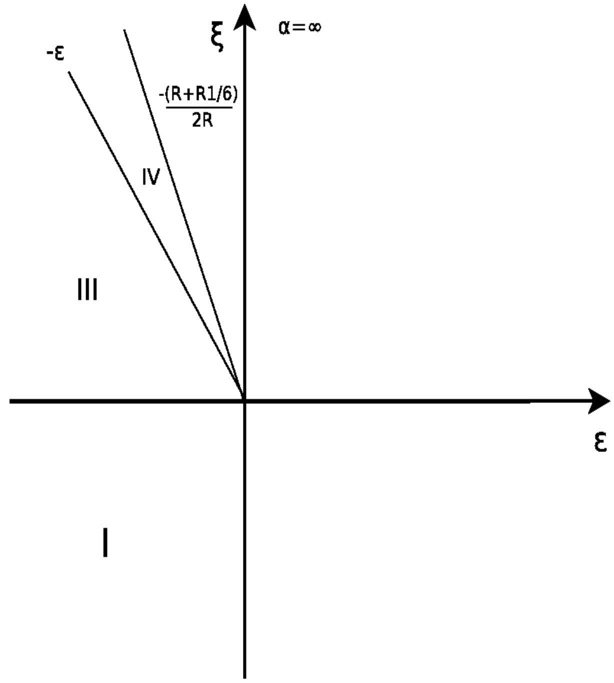


Figure 7: case:3

β_w for $u = 0$ coincide with the β functions of the Gribov and rapid-change models to all orders of the perturbation theory.

It remains to note that the location of the boundary between the regions III and IV depends on the parameter α. For the $\alpha = 0$, it is given by the ray $\xi = \varepsilon/2$, in agreement with the result, derived earlier in [14] for the incompressible case. When α increases, the ray rotates counter clockwise, and in the limit $\alpha \to \infty$ it approaches the vertical ray $\varepsilon = 0$, $\xi > 0$.

5.1 Scaling regimes for the model A

The one-loop expressions for the β functions in the model (**??**), (**??**) are easily derived from the definitions (34), relations (41), (**??**) and (41), and explicit expressions (**??**):

$$\beta_u = u\left[-\varepsilon + 3u + w(3+\alpha)/2 - w\alpha f(a)\right],$$
$$\beta_w = w\left[-\xi + w(3+\alpha)/4\right],$$
$$\beta_a = u(4a-1)/4, \tag{54}$$

where the function $f(a) = 3a^2 + (a-1)^2$ achieves the minimum value $f(1/4) = 3/4$ at $a = 1/4$.

(1) The line of Gaussian (free) fixed points: $u_* = w_* = 0$, a_* arbitrary. IR attractive for $\varepsilon < 0$, $\xi < 0$.

(2) The point $w_* = 0$, $u_* = \varepsilon/3$, $a_* = 1/4$, corresponding to the pure model A (turbulent advection is irrelevant). This point is IR attractive for $\xi < 0$, $\varepsilon < 0$.

(3) The line of fixed points

$$u_* = 0, \quad w_* = 4\xi/(3+\alpha), \quad a_* \text{ arbitrary}, \tag{55}$$

corresponding to the passively advected scalar without self-interaction. It involves the interval

$$(a_* - 1/4)^2 < \frac{1}{16\alpha\xi}\left[-\varepsilon(3+\alpha) + \xi(6-\alpha)\right], \tag{56}$$

which is IR attractive for $\xi > 0$, $\varepsilon < \xi(6-\alpha)/(3+\alpha)$. This interval becomes infinite for $\alpha \to 0$ and tends to the finite value $-(\varepsilon+\xi)/16\xi$ for $\alpha \to \infty$ (note that the right hand side of the inequality (56) is positive within the region of IR stability).

(4) The fixed point, corresponding to the new universality class, where both the advection and the self-interaction are relevant:

$$w_* = 4\xi/(3+\alpha), \quad u_* = \frac{\varepsilon(3+\alpha) - \xi(6-\alpha)}{3(3+\alpha)}, \quad a_* = 1/4. \tag{57}$$

This point is IR attractive for $\xi > 0$, $\varepsilon > \xi(6-\alpha)/(3+\alpha)$.

When α increases, the boundary between the regions of stability of the regimes (3) and (4) rotates in the upper half-plane ε–ξ counter clockwise from the ray $\xi = \varepsilon/2$ ($\alpha \to 0$) to $\xi = -\varepsilon$ ($\alpha \to \infty$). The regions of IR stability of the fixed points (1)–(4) in the ε–ξ plane are shown in figure **??** for some value of $\alpha < 6$, when the boundary between the regions III and IV lies in the right upper quadrant; for $\alpha > 6$ it moves to the left upper quadrant.

6 Critical scaling and critical dimensions

Existence of IR attractors of the RG equations implies existence of self-similar (scaling) behaviour of the Green functions in the IR range. In this critical scaling all the IR irrelevant parameters (λ, μ and the coupling constants) are fixed and the IR relevant parameters (times/frequencies, coordinates/momenta, τ and the fields) are dilated. In dynamical models, critical dimensions Δ_F of the IR relevant quantities F are given by the relations

$$\Delta_F = d_F^k + \Delta_\omega d_F^\omega + \gamma_F^*, \qquad \Delta_\omega = 2 - \gamma_\lambda^*, \tag{58}$$

with the normalization condition $\Delta_k = 1$; see e.g. [2] for more detail. Here $d_F^{k,\omega}$ are the canonical dimensions of F, given in table 1, and γ_F^* is the value of the corresponding anomalous dimension (33) at the fixed point: $\gamma_F^* = \gamma_F(u_*, w_*, a_*)$. This gives $\Delta_\psi = d/2+\gamma_\psi^*$, $\Delta_{\psi^\dagger} = d/2+\gamma_{\psi^\dagger}^*$ for the Gribov model, $\Delta_\psi = d/2-1+\gamma_\psi^*$, $\Delta_{\psi^\dagger} = d/2+1+\gamma_{\psi^\dagger}^*$ for the model A and $\Delta_\omega = 2+\gamma_\tau^*$ for the both models. Substituting the coordinates of the fixed points from sections **??** and 5.1 into the explicit one-loop expressions (**??**), (**??**) for the anomalous dimensions and using the exact relations (41)–(**??**), one obtains the leading-order expressions for the critical dimensions. They are summarized in tables **??** and **??** for the Gribov model and model A, respectively.

The results for the Gaussian points (1) are exact; all the results for the fixed points (2) have corrections of order ε^2 and higher. The result $\Delta_\omega = 2-\xi$ for the fixed points (3) and (4) is also exact, as follows from the general relations $\gamma_\lambda = \gamma_w$ in (41) and $\Delta_\omega = 2-\gamma_\lambda^*$ in (58), and the identity $\gamma_w^* = \xi$, which is a consequence of the fixed-point equation $\beta_w = 0$ with β_w from (34) for any fixed point with $w_* \neq 0$. The other results for the fixed points (4) have higher-order corrections in ε and ξ.

The result $\Delta_\tau = 2-\xi$ for the fixed points (3) is also exact: for $u=0$, the 1-irreducible function $\langle\psi^\dagger\psi\rangle$ is given by the only one-loop diagram, which gives rise only to the counterterm $\psi^\dagger\partial^2\psi$; cf. [17] for the pure Kraichnan's model. Hence the exact relations $Z_1 = Z_3 = 1$ for $u=0$. Then from the relations (24) and (41) it follows $\gamma_\tau = -\gamma_\lambda = \gamma_w$ which gives $\gamma_\tau^* = -\xi$ for the fixed points with $w_* \neq 0$.

Fixed point (3) in table **??** illustrates the general fact that the dimensions Δ_ψ and $\Delta_{\psi^\dagger}$ for the Gribov model interchange under the transformation $a \to 1-a$, as a consequence of the exact symmetry (**??**), (**??**), which results in the relations (**??**) for the renormalization constants. It remains to note that for any fixed point the critical dimension of the velocity field is given by the exact relation $2\Delta_v = -\xi + \Delta_\omega$ following from the form of its pair correlation function (11), (12).

7 Discussion and conclusions

We studied effects of turbulent mixing on the critical behaviour, with special attention paid to *compressibility* of the fluid. Two representative models of dynamical critical behaviour were considered: the model A, which describes relaxational dynamics of a non-conserved order parameter in an equilibrium critical system, and the strongly non-equilibrium Gribov model, which describes spreading processes in a reaction-diffusion system. The turbulent mixing was modelled by the Kazantsev–Kraichnan "rapid-change" ensemble: time-decorrelated Gaussian velocity field with the power-like spectrum $\propto k^{-d-\xi}$. The both stochastic problems can be reformulated as multiplicatively renormalizable field theoretic models, which allows one to apply the field theoretic renormalization group to the analysis of their IR behaviour.

We showed that, depending on the relation between the spatial dimension d and the exponent ξ, the both models exhibit four different critical regimes, associated with four possible fixed points of the RG equations. Three fixed points correspond to known regimes: (1) Gaussian fixed point; (2) critical behaviour typical of the original model without mixing (that is, model A or Gribov model); (3) scalar field without self-interaction, passively advected by the flow (the nonlinearity in the order parameter in the original dynamical equations appears unimportant). The most interesting fourth point corresponds to a new type of critical behaviour (4), in which the nonlinearity and turbulent mixing are both relevant, and the critical exponents depend on d, ξ and the compressibility parameter α.

Practical calculations of the critical exponents and the regions of stability for all the regimes were performed in the one-loop approximation of the RG, which corresponds to the leading order of the double expansion in two parameters ξ and $\varepsilon = 4-d$. It has shown that, for the both models, compressibility enhances the role of the nonlinear terms in the dynamical equations. The region in the ε–ξ plane, where the new nontrivial regime (4) is stable (the corresponding fixed point is positive and IR attractive), is getting much wider as the degree of compressibility increases. In its turn, turbulent transfer becomes more efficient due to combined effects of the mixing and the nonlinear terms.

Let us illustrate these general statements by the example of a cloud of particles, randomly walking in a nearly critical turbulent medium. The mean-square radius $R(t)$ of a cloud of such particles, started from the origin at zero time, is related to the linear response function (8) in the time-coordinate representation as follows:

$$R^2(t) = \int d\mathbf{x}\, x^2\, G(t,\mathbf{x}), \quad G(t,\mathbf{x}) = \langle \psi(t,\mathbf{x})\psi^\dagger(0,\mathbf{0})\rangle, \quad x = |\mathbf{x}|. \tag{59}$$

For the response function, the scaling relations of the preceding section give the following IR asymptotic expression:

$$G(t,\mathbf{x}) = x^{-\Delta_\psi - \Delta_{\psi^\dagger}}\, F\left(\frac{x}{t^{1/\Delta_\omega}}, \frac{\tau}{t^{\Delta_\tau/\Delta_\omega}}\right), \tag{60}$$

with some scaling function F. Substituting (60) into (59) gives the desired scaling expression for the radius:

$$R^2(t) = t^{(d+2-\Delta_\psi-\Delta_{\psi^\dagger})/\Delta_\omega}\, f\left(\frac{\tau}{t^{\Delta_\tau/\Delta_\omega}}\right), \tag{61}$$

where the scaling function f is related to F in (60) as

$$f(z) = \int d\mathbf{x}\, x^{2-2\Delta_\psi}\, F(x,z).$$

Directly at the critical point (assuming that the function f is finite at $\tau = 0$) one obtains from (61) the power law for the radius:

$$R^2(t) \propto t^{\Omega}, \quad \Omega \equiv (d+2-\Delta_\psi-\Delta_{\psi^\dagger})/\Delta_\omega = (2-\gamma^*_\psi-\gamma^*_{\psi^\dagger})/\Delta_\omega; \tag{62}$$

the last equality following from the relations (58). For the Gaussian fixed points the usual diffusion law $R(t) \propto t^{1/2}$ is recovered. For the regimes (3) the exact result $R(t) \propto t^{1/(2-\xi)}$ is derived, which for the Kolmogorov value $\xi = 4/3$ gives $R(t) \propto t^{3/2}$ in agreement with Richardson's "4/3 law" $dR^2/dt \propto R^{4/3}$ for a passively advected scalar impurity. For the other two fixed points the exponents in (61), (62) are given by infinite series in ε (points 2) or ε and ξ (points 4); the first-order approximations are readily obtained from the results given in tables **??** and **??**.

For the case of incompressible fluid ($\alpha = 0$), the most realistic values $d = 2$ or 3 and $\xi = 4/3$ lie in the region of stability of the passive scalar regime (3), so that the spreading of the cloud is determined completely by the turbulent transfer and is described by the power law (62) with the exact exponent $\Omega^{(3)} = 2/(2-\xi)$. As the degree of compressibility α increases, the boundary between the regions of stability of the regimes (3) and (4) in the ε–ξ plane moves such that the region of stability of the regime (4) is getting wider (see the discussion in section 5). When α becomes large enough, these physical values of d and ξ necessarily fall into the region of stability of the new regime (4), the advection and the nonlinearity become both important, and the crossover in the critical behaviour occurs. The new exponent in (62) can be represented in the form

$$\Omega^{(4)} = \Omega^{(3)} + \delta\Omega, \quad \delta\Omega = -(\gamma^*_\psi + \gamma^*_{\psi^\dagger})/(2-\xi) \tag{63}$$

(we recall that $\Delta_\omega = (2-\xi)$ exactly for the both regimes (3) and (4)). For the Gribov model, substituting the one-loop expressions for the dimensions γ^*_ψ and $\gamma^*_{\psi^\dagger}$ from the table **??** into (63) gives

$$\delta\Omega = \frac{(3+\alpha)\varepsilon - 6\xi}{3(5+2\alpha)(2-\xi)}. \tag{64}$$

Straightforward analysis of the expression (64) shows that, within the region of stability of the regime (4), the quantity $\delta\Omega$ is positive and grows monotonically with α. Thus the spreading of the cloud becomes faster in comparison with the pure turbulent transfer, due to combined effects of the mixing and the nonlinear terms, and accelerates as the degree of compressibility increases. For the model A, the exponents Ω in (62) for the regimes (3) and (4) coincide in the one-loop approximation, but the main quantitative conclusion remains the same: compressibility enhances the role of the nonlinear terms and leads to the widening of the region of stability of the full-scale critical regime.

Further investigation should take into account conservation of the order parameter, its feedback on the dynamics of the advecting velocity field (mode-mode coupling in the spirit of the model H of critical dynamics), non-Gaussian character and finite correlation time of the velocity statistics. This work remains for the future.

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References

References

[1] Zinn-Justin J 1989 *Quantum Field Theory and Critical Phenomena* (Oxford: Clarendon)

[2] Vasil'ev A N 2004 *The Field Theoretic Renormalization Group in Critical Behavior Theory and Stochastic Dynamics* (Boca Raton: Chapman & Hall/CRC)

[3] Halperin B I and Hohenberg P C 1977 *Rev. Mod. Phys.* **49** 435; Folk R and Moser G 2006 *J. Phys. A: Math. Gen.* **39** R207

[4] Hinrichsen H 2000 *Adv. Phys.* **49** 815; Ódor G 2004 *Rev. Mod. Phys.* **76** 663

[5] Janssen H-K and Täuber U C 2004 *Ann. Phys. (NY)* **315** 147

[6] Ivanov D Yu 2003 *Critical Behaviour of Non-Idealized Systems* (Moscow: Fizmatlit) [in Russian]

[7] Khmel'nitski D E 1975 *Sov. Phys. JETP* **41** 981; Shalaev B N 1977 *Sov. Phys. JETP* **26** 1204; Janssen H-K, Oerding K and Sengespeick E 1995 *J. Phys. A: Math. Gen.* **28** 6073

[8] Satten G and Ronis D 1985 *Phys. Rev. Lett.* **55** 91; 1986 *Phys. Rev.* A **33** 3415

[9] Onuki A and Kawasaki K 1980 *Progr. Theor. Phys.* **63** 122; Onuki A, Yamazaki K and Kawasaki K 1981 *Ann. Phys.* **131** 217; Imaeda T, Onuki A and Kawasaki K 1984 *Progr. Theor. Phys.* **71** 16

[10] Beysens D, Gbadamassi M and Boyer L 1979 *Phys. Rev. Lett* **43** 1253; Beysens D and Gbadamassi M 1979 *J. Phys. Lett.* **40** L565

[11] Ruiz R and Nelson D R 1981 *Phys. Rev.* A **23** 3224; **24** 2727; Aronowitz A and Nelson D R 1984 *Phys. Rev.* A **29** 2012

[12] Antonov N V, Hnatich M and Honkonen J 2006 *J. Phys. A: Math. Gen.* **39** 7867

[13] Antonov N V and Ignatieva A A 2006 *J. Phys. A: Math. Gen.* **39** 13593

[14] Antonov N V, Iglovikov V I and Kapustin A S 2009 *J. Phys. A: Math. Theor.* **42** 135001

[15] Antonov N V, Ignatieva A A and Malyshev A V 2010 E-print LANL arXiv:1003.2855 [cond-mat]; to appear in *PEPAN (Phys. Elementary Particles and Atomic Nuclei, published by JINR)* **41**

[16] Falkovich G, Gawędzki K and Vergassola M 2001 *Rev. Mod. Phys.* **73** 913

[17] Antonov N V 2006 *J. Phys. A: Math. Gen.* **39** 7825

[18] Antonov N V, Hnatich M and Nalimov M Yu 1999 *Phys. Rev.* E **60** 4043

[19] van Kampen N G 2007 *Stochastic Processes in Physics and Chemistry, 3rd ed.* (Amsterdam: North Holland)

[20] Sak J 1973 *Phys. Rev.* B **8** 281; Honkonen J and Nalimov M Yu 1989 *J. Phys. A: Math. Gen.* **22** 751 Janssen H-K 1998 *Phys. Rev.* E **58** R2673; Antonov N V 1999 *Phys. Rev.* E **60** 6691; 2000 *Physica* D **144** 370

[21] Janssen H-K, Oerding K, van Wijland F and Hilhorst H J 1999 *Eur. Phys. J.* B **7** 137; Janssen H-K and Stenull O 2008 *Phys. Rev.* E **78** 061117