Research Statement

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I work at the intersection of applied mathematics and machine learning (ML) theory. Leveraging tools from randomized algorithms, high-dimensional probability, and statistical learning theory, my research seeks to *develop a unified theoretical and algorithmic framework for computation- and sample-efficient methods in high-dimensional learning problems*.

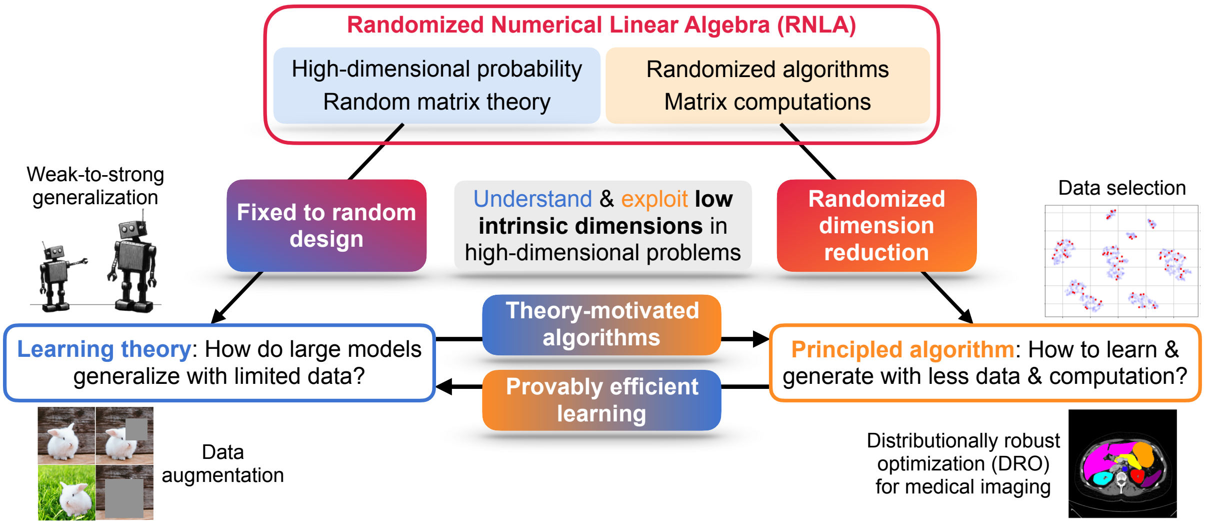

Randomized Numerical Linear Algebra (RNLA)
High-dimensional probability
Random matrix theory
Randomized algorithms
Matrix computations
Weak-to-strong generalization
Fixed to random design
Understand & exploit low intrinsic dimensions in high-dimensional problems
Randomized dimension reduction
Data selection
Learning theory: How do large models generalize with limited data?
Theory-motivated algorithms
Provably efficient learning
Principled algorithm: How to learn & generate with less data & computation?
Data augmentation
Distributionally robust optimization (DRO) for medical imaging

Figure 1: Three synergistic pillars of my research: **randomized numerical linear algebra** (Section 1), **learning theory** (Section 2), and **principled algorithms** (Section 3).

A central theme of my work is a modern manifestation of Occam's razor: conditioned on informative priors like large data matrices and foundation models, the simplest hypothesis is usually the best. *Low intrinsic dimensions* of high-dimensional problems brought by such virtue of simplicity pave the way toward understanding and designing learning algorithms. Building on this central thread, Figure 1 outlines three synergistic pillars of my research:

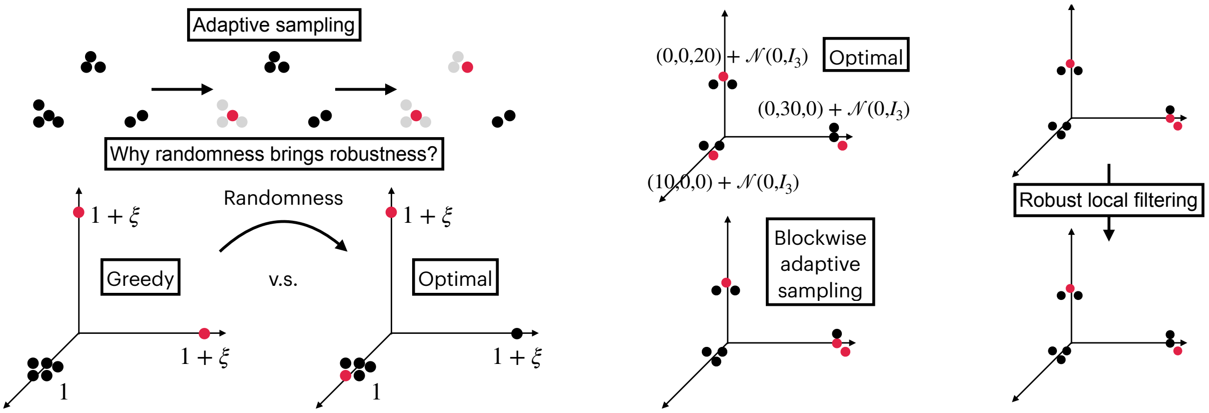
- **Randomized numerical linear algebra (RNLA)** (Section 1): I design fast and robust randomized algorithms for matrix low-rank approximations, including the *CUR/interpolative decomposition (ID)* [2, 8, 13] and *singular value decomposition (SVD)* [1, 5]. The mathematical and algorithmic foundations of RNLA have been instrumental in my work on understanding and designing learning algorithms (*e.g.*, [7, 9]) in Sections 2 and 3.
- **Learning theory** (Section 2): I develop theoretical underpinnings for when and why large models generalize with limited data in various learning paradigms like *data augmentation* [16], *knowledge distillation* [3], *weak-to-strong generalization* [9, 12], and *chain-of-thought* [15]. By unveiling the mechanisms behind these sample-efficient learning paradigms, my research in learning theory casts light on the algorithmic designs in Section 3 (*e.g.*, [4, 6, 7]).
- **Principled algorithms** (Section 3): I design robust and provable algorithms for computation- and sample-efficient learning, *e.g.*, *data selection* [6, 7, 14], *distributionally robust optimization* with applications to medical imaging [4], *structured pruning* of large language models (LLMs) [11], and *randomized time integration* for differential equations [10].

Sections 1 to 3 will focus on one representative topic in each pillar.

1 Randomized Numerical Linear Algebra (RNLA)

Data-driven algorithms are typically bottlenecked by the prohibitive costs of processing and storing large-scale data matrices. Real-world high-dimensional problems frequently exhibit latent low-dimensional structures [38], inducing different notions of *low intrinsic dimensions* [18, 34, 36] and motivating dimensionality reduction methods like low-rank approximations. My research in RNLA leverages two core randomization techniques—*sampling* [8] and *sketching* [2, 5, 13]—to exploit such low intrinsic dimensions and design fast, accurate, and robust algorithms for low-rank approximations.

1.1 Robust Blockwise Adaptive Sampling for Interpolative Decomposition (ID). *ID* is a low-rank approximation that selects a subset of rows/columns as a basis. It has wide applications in numerical analysis and ML, including rank-structured matrix compression [29], low-rank adaptation (LoRA) [30], and data selection [6, 7]. Although polynomial-time heuristics with near-optimal accuracy (*e.g.*, [22, 26]) exist for the NP-hard optimal subset selection problem [24], the unprecedented scales of modern computational challenges call for efficiency beyond asymptotic complexities, *e.g.*, *hardware efficiency* on parallel processors and *rank-adaptiveness* supporting adaptive selection with automatic termination.


Adaptive sampling
Why randomness brings robustness?
$1+\xi$
Randomness
Greedy
v.s.
Optimal
$1+\xi$
1
$1+\xi$
$1+\xi$
1
$(0,0,20)+\mathcal{N}(0,I_3)$
Optimal
$(0,30,0)+\mathcal{N}(0,I_3)$
$(10,0,0)+\mathcal{N}(0,I_3)$
Blockwise adaptive sampling
Robust local filtering

(a) Adaptive sampling and its robustness (b) Suboptimality of blockwise adaptive sampling (c) Fix with RBRP

Figure 2: Illustrations of RBRP for row (column) subset selection. Dots denote n rows in $\mathbb{R}^{n\times d}$; red dots are rows selected by ID; and gray dots are rows removed by adaptive updates.

With Professor Gunnar Martinsson's research group at UT Austin, I developed *robust blockwise random pivoting (RBRP)* [8], a randomized ID algorithm that is hardware-efficient, rank-adaptive, and robust to adversarial inputs, with state-of-the-art accuracy and efficiency. Figure 2 illustrates the key ideas of RBRP: (a) Adaptive sampling [22] achieves near-optimal accuracy by combining adaptiveness and randomness, circumventing adversarial inputs like Figure 2(a). (b) However, the sequential nature of adaptive sampling compromises hardware efficiency, while blockwise adaptive sampling is vulnerable to inputs like Figure 2(b). (c) RBRP resolves this tension through *robust local filtering* that removes redundant points in each block with negligible additional cost (Figure 2(c)), achieving hardware efficiency, rank-adaptiveness, and robustness.

1.2 Sketching-based Low-rank Approximations with Posterior Guarantees. Alternative to sampling, *sketching* [28, 40] via Johnson-Lindenstrauss transforms [32] randomly projects high-dimensional data to a low-dimensional subspace while preserving essential information. Sketching has been thoroughly studied for low-rank approximations with a priori guarantees [28, 40]. However, data-agnostic a priori guarantees from concentration bounds are often too pessimistic for practical error estimation, while data-dependent *posterior* guarantees are much less explored. My Ph.D. work [1, 2, 5] with Professors Gunnar Martinsson and Yuji Nakatsukasa (Oxford) developed and analyzed sketching-based randomized algorithms for ID, CUR, and SVD with tight and efficiently computable posterior error guarantees.

2 Learning Theory

A central question of ML theory is *when and why large models generalize in the overparametrized regime* (*i.e.*, with fewer samples $n < d$ than the parameter count d). Benign overfitting [37] provides a canonical insight: when the essential features of data concentrate in a subspace of low dimension $k_* \ll n$, overparametrized models can fit the n samples while generalizing well by diluting noise in the $(d - k_*)$-dimensional subspace orthogonal to essential features. My research in learning theory explains the mechanisms behind various sample-efficient learning paradigms whose low intrinsic dimensions k_* arise implicitly from the simplicity bias [9, 12] or explicitly from structure-aware objectives [3, 16].

2.1 Intrinsic Dimensions of Fine-tuning Explain Weak-to-Strong (W2S) Generalization. Post-training alignment adapts pre-trained models to downstream tasks using limited data, commonly through highly overparametrized *fine-tuning*. The sample efficiency of fine-tuning is empirically justified by its low intrinsic dimensions [17]. Conditioned on the simple dynamics of fine-tuning in the kernel regime [31, 35], low intrinsic dimensions emerge from the simplicity bias of optimizers like gradient descent [41]. Low intrinsic dimensions provide fresh perspectives for assorted post-training schemes, like W2S generalization [20].

As contemporary ML models outperform human in many domains, an imminent question of *superalignment* asks whether stronger models can still learn from weaker human supervision. W2S generalization [20] gives a positive answer: a strong pre-trained student fine-tuned with pseudo-labels generated by a weaker teacher often outperforms its teacher. Collaborating with undergraduates Yicheng Li (NYU Shanghai), Yunai Li (SJTU), Professors Jason D. Lee (UC Berkeley), and Qi Lei (NYU), I developed a theoretical framework to explain W2S from a variance reduction perspective, via the low intrinsic dimension of fine-tuning [9]. We model fine-tuning the strong student f_s and weak teacher f_w as regression problems over high-dimensional pre-trained features concentrated in low-dimensional subspaces, $\mathcal{V}_s, \mathcal{V}_w$ of dimensions d_s, d_w, respectively. Assuming that the pre-trained features of both teacher and student are sufficiently expressive for the downstream task (*i.e.*, negligible bias), we characterize the dominant variance of W2S precisely: when fine-tuning the strong student with $N \gtrsim d_s$ pseudo-labels generated by the weak teacher fine-tuned with $n \gtrsim d_w$ noisy labels with variance σ^2,

$$\mathbf{Var}(f_{\mathrm{w2s}}) \asymp \sigma^2 \Bigg(\underset{\text{Var. in } \mathcal{V}_s \cap \mathcal{V}_w}{\boxed{\frac{d_{s\wedge w}}{n}}} + \underset{\textbf{W2S}}{\boxed{\frac{d_s}{N}}} \underset{\text{Var. in } \mathcal{V}_w \setminus \mathcal{V}_s}{\boxed{\frac{d_w - d_{s\wedge w}}{n}}} \Bigg), \tag{1}$$

where $d_{s\wedge w} \in [0, \min\{d_s, d_w\}]$ measures $\mathcal{V}_s, \mathcal{V}_w$ overlap through their geodesic distance. (1) implies that variance of the teacher is inherited by the student in $\mathcal{V}_s \cap \mathcal{V}_w$, while reduced by a factor of d_s/N in the subspace of discrepancy $\mathcal{V}_w \setminus \mathcal{V}_s$. W2S generalization emerges from such variance reduction when the student and teacher are sufficiently different (*i.e.*, small $d_{s\wedge w}$).

2.2 Structure-awareness Unveils Intrinsic Dimensions. Besides simplicity biases, structure-aware objectives can explicitly induce low intrinsic dimensions and enable sample-efficient learning. With Shuo Yang, Kevin Miller, Professors Rachel Ward, Inderjit Dhillon, Sujay Sanghavi (UT Austin), and Qi Lei, I provided theoretical underpinnings for the sample efficiency of two such structure-aware learning schemes: *data augmentation consistency regularization* [16] and *relational knowledge distillation* [3].

3 Principled Learning Algorithms

The algorithmic and theoretical foundations in Sections 1 and 2 have been instrumental in my work on efficient, robust, and provable algorithms for large-scale learning problems.

3.1 Efficient Data Selection for Fine-tuning. The theoretical insights in Section 2.1 suggest that sample efficiency of fine-tuning stems from its low intrinsic dimensions. A pre-trained foundation model is an "Occam's razor" that concentrates high-dimensional downstream features in low-dimensional subspaces. However, downstream data are often redundant, while labels tend to be noisy and expensive. This calls for efficient strategies to find high-quality data subsets, labeling and fine-tuning on which preserve the generalization of full-data training.

With Professor Qi Lei's research group at NYU, I developed a scalable data selection framework, *Sketchy Moment Matching (SkMM)* [7], that achieves near-optimal sample complexities provably and state-of-the-art performance empirically. SkMM controls both bias and variance induced by data selection in two stages: (i) Leveraging algorithmic tools from RNLA, bias is controlled via *gradient sketching* that explores the fine-tuning parameter space of high dimension d for an informative subspace $\mathcal{S}$ of intrinsic dimension $\dim(\mathcal{S}) \ll d$. (ii) Variance is reduced over the low-dimensional $\mathcal{S}$ via *moment matching* that accelerates classical optimal experimental design methods through continuous relaxations [7] or RNLA-inspired heuristics based on adaptive sampling [6]. Theoretically, fine-tuning on n samples selected by SkMM preserves the fast-rate generalization $O(\dim(\mathcal{S})/n)$, with sample complexities independent of d. Algorithmically, SkMM runs in linear time $O(Nd)$ for a full data size N. The computational bottleneck is a single epoch of backpropagation for gradient sketching that is parallelizable on modern hardware and can be further accelerated with structured sketching (*e.g.*, [23]). Empirically, SkMM demonstrates state-of-the-art computation- and sample-efficiency against a broad spectrum of baselines on assorted regression and classification tasks.

3.2 Applications in Scientific ML and Beyond. Benefits of randomization extend beyond efficiency to other desiderata like *stability*. Collaborating with Professor Benjamin Peherstorfer's research group at NYU, I developed a randomized time integration scheme for solving evolution problems like dynamical systems and partial differential equations (PDEs) using nonlinear parametrizations like neural networks [10]. Our randomized time integrator uses sketching as an efficient regularization that provably stabilizes the poorly conditioned least-squares problems that arise from nonlinear parametrization. In addition, my learning theory work in Section 2 have inspired practical *distributionally robust algorithms*. For example, my Ph.D. work [4] with Professor Rachel Ward's research group at UT Austin introduced a scalable distributionally robust optimization algorithm for reliable medical image segmentation under concept shifts.

4 Ongoing and Future Research Agenda

An overarching goal of my research is to build mathematical underpinnings and algorithmic improvements for high-dimensional learning problems by bridging them with theoretical and algorithmic wisdom in RNLA. My future research will be built around the three synergistic pillars in Figure 1, extending foundations and insights in Sections 1 to 3 to emerging challenges in large-scale learning[1].

4.1 RNLA in Large-Scale Optimization and Learning. Large-scale training of ML models involves two core questions: (i) how to optimize the model with low per-iteration costs and fast convergence, and (ii) whether the algorithm steers the model toward global minima that

[1] Sections 4.2 and 4.3 are the themes of my MSCA-PF proposal, "SIMPLE-LLMs: Simplicity-Inspired Mechanistic Principles for Learning and Emergence in LLMs", hosted by Professor Nicolas Boullé at Imperial College.

generalize well? Toward the first question, low/mixed-precision arithmetic is becoming a de facto choice for reducing per-iteration costs in large-scale training [25]. This reinvigorates a classical theme in numerical linear algebra: *numerical stability*. Motivated by our finding in [10] that randomization stabilizes poorly conditioned problems efficiently, an exciting future direction is using randomization to design fast and stable matrix-preconditioned stochastic optimizers, like variants of Shampoo [27] and Muon [33], for low-precision training. A parallel but equally important direction posed by the second question is to understand the learning dynamics of different preconditioned optimizers. Bridging my experience in RNLA and learning theory, I plan to explore these two directions jointly, pursuing fast and stable preconditioned optimizers with favorable generalization.

4.2 Sample Efficiency of In-context Learning (ICL). Alternative to fine-tuning, ICL has emerged as a powerful post-training alignment scheme that adapts pre-trained LLMs to downstream tasks with only a few in-context demonstrations during inference, without updating model parameters [19]. A canonical insight exploited in Sections 2.1 and 3.1 is that sample efficiency of post-training alignment stems from the "Occam's razor" brought by rich knowledge encoded in LLMs. This motivates an exciting question: *what is the "Occam's razor" of ICL, and how does it emerge from LLMs?* I envision quantifying the sample efficiency of ICL through intrinsic low-complexity structures unveiled by LLMs, and designing concise and reliable contexts for ICL leveraging such mechanisms.

4.3 Efficient Matrix Computations during Post-training. Besides sample efficiency, "Occam's razor" in post-training further facilitate progress toward *computational and memory efficiency*. (i) For fine-tuning, the presence of low intrinsic dimensions explains the empirical success of *low-rank adaptation (LoRA)* [30], which explicitly enforces "simplicity" of fine-tuning updates by constraining them to low-rank matrices. This opens up exciting opportunities to bridge the understanding of training dynamics from learning theory with algorithmic building blocks from RNLA, exploring alternative low-complexity structures for LoRA that steer fine-tuning toward better generalization and efficiency. (ii) For inference of LLMs, the attention mechanism is a computational bottleneck that scales quadratically with the context length [39]. Motivated by common low-complexity patterns in attention matrices [42], sparse/low-rank attention approximations have been exploited to alleviate the computational and memory burden [21]. My ongoing collaboration with a group of applied mathematicians led by Professors Laura Grigori (EPFL), Anna Ma (UCI), and Deanna Needell (UCLA), initiated during the IPAM RNLA Workshop at UCLA, pursues lightweight LLM inference leveraging fast matrix computations from RNLA.

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