User Manual: Non-Linear Beam Solver

Technical Documentation

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Abstract

This document serves as the user manual for the Non-Linear Beam Solver web tool. It outlines the operational procedures, input definitions, underlying physical simulation principles, and the mathematical derivation of the numerical methods used to solve for secondary bending in fastened joints.

Contents

1 Introduction

The **Non-Linear Beam Solver** is a browser-based engineering utility designed to analyze **secondary bending** in fastened joints (such as lap joints or hardpoints) under axial tension or compression.

Unlike simple linear beam calculators, this tool accounts for **Geometric Non-Linearity**. When a joint with an offset (eccentricity) is pulled, it deflects. As it deflects, the internal moment arms change, altering the stiffness and internal forces. This interaction is critical for determining the true stress state in aerospace and structural connections.

2 Quick Start Guide

1. **Open the Tool:** Load the HTML file in any modern web browser.
2. **Define Geometry:** Input the sheet thicknesses (t_{lower}, t_{upper}) and the total length of the modeled section (L).
3. **Define Loads:** Set the maximum axial load (P_{max}) and the location/load-transfer fractions of the fasteners.
4. **Run Simulation:** Click the **Run Simulation** button.
5. **Analyze Results:**
   - View the "Bending / Direct Stress Ratio" to assess the severity of secondary bending.
   - Inspect the interactive charts to visualize the deflected shape and moment distribution.

3 Input Parameters

3.1 Material & Geometry

- **Young's Modulus (E):** The stiffness of the material (e.g., 10.5×10^6 psi for Aluminum).
- **Beam Length (L):** The total length of the section being analyzed.
- **t_{lower} / t_{upper}:**
  - t_{lower}: The thickness of the main sheet being analyzed (the sheet starting at $s = 0$).
  - t_{upper}: The thickness of the overlapping sheet (strap/splice).

  Note: These values control the bending stiffness (I) and the eccentricity (moment arm) of the applied loads.

3.2 Loads & Fasteners

- **Max Initial Load (P_{max}):** The total axial load applied at the start of the beam ($s = 0$).
- **Fasteners:** A text area defining where load is transferred out of the lower sheet into the upper sheet.
  - **Format:** `Position, Fraction` (one per line).
  - **Example:**

```
3.0, 0.35  // At s=3.0, 35% of P_max is transferred
4.0, 0.30  // At s=4.0, 30% of P_max is transferred
```

The solver calculates the moment generated by this transfer based on the offset distance between the sheet neutral axes.

3.3 Solver Settings

- **Nodes (N):** The number of segments to discretize the beam. Higher numbers (e.g., 200) increase accuracy but increase computation time.
- **Steps:** Number of load increments. For highly non-linear problems, ramping the load helps convergence.
- **Stress Offset:** The distance *before* the first fastener where the critical stress check is performed (to avoid singularity points).
- **Stiffness Transition Offset:** The distance *before* the first fastener where the beam stiffness effectively doubles (representing the physical overlap start).
- **Fix Rotation at $s = L$:**
  - **Unchecked:** The end is free to rotate.
  - **Checked:** The end is clamped ($\theta = 0$). Used to simulate symmetry in the middle of a joint.

4 Simulation Reality & Physics

This tool simulates the physical behavior known as the **Beam-Column effect** with eccentric loading. A schematic of the solver's physical model is shown in Figure 1.

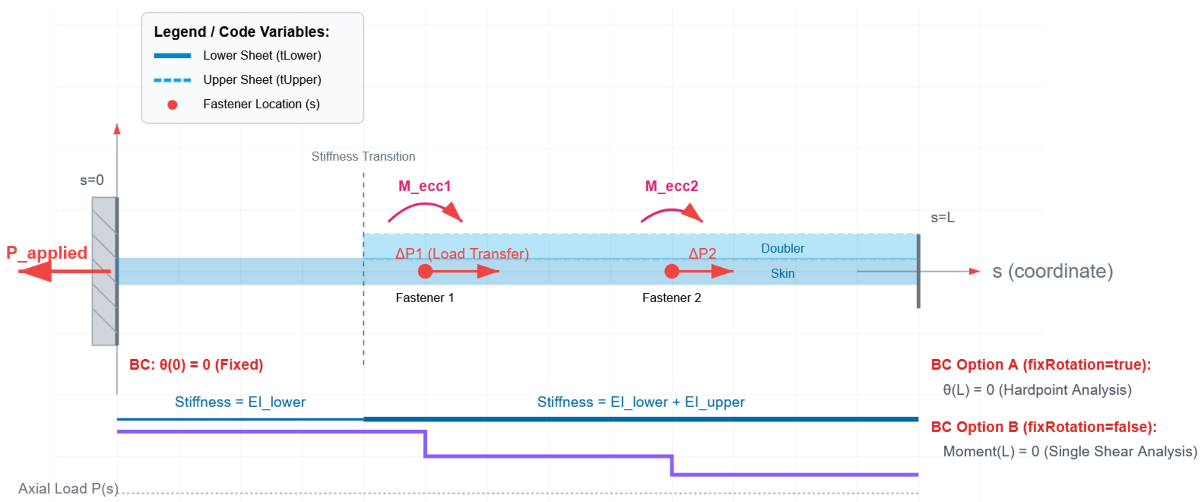

Figure 1: **Schematic of the Physical Model.** This diagram illustrates the boundary conditions at $s = 0$, the stiffness transition where the doubler begins, the moment eccentricity (M_{ecc}) generated by load transfer at fasteners, and the two available boundary condition options at $s = L$.

1. **Variable Stiffness:** The code assumes the beam starts as a single sheet (stiffness of t_{lower}) and transitions to a double stack (stiffness of $t_{\text{lower}} + t_{\text{upper}}$) near the fasteners. This mimics the physical overlap of a lap joint.

2. **Load Transfer & Eccentricity:** In a lap joint, when load moves from one sheet to another via a fastener, it creates a **Moment**. The force is applied at a distance equal to half the combined thickness:

$$e = \frac{t_{\text{lower}} + t_{\text{upper}}}{2}$$

3. **Geometric Non-Linearity:** As the beam bends, the axial load P is no longer perfectly aligned with the neutral axis of the deflected beam. The solver calculates the equilibrium in the *deformed* state.

5 Assumptions and Limitations

1. **2D Plane Analysis:** The beam is modeled as a 1D line element in 2D space. Out-of-plane bending or twisting is not considered.
2. **Elastic Material:** The material follows Hooke's Law ($\sigma = E\epsilon$). Plasticity or material yielding is not modeled.
3. **Euler-Bernoulli Beam Theory:** Plane sections remain plane. Shear deformation is ignored (valid for long, slender beams).
4. **Point Fasteners:** Fasteners are modeled as point sources of load transfer and moment application. Fastener flexibility/tilting is not explicitly modeled.
5. **Large Rotation, Small Strain:** The solver uses $\sin(\theta)$ terms allowing for moderate rotations, but assumes strains remain small (length of the beam ds does not change significantly).

A Mathematical Derivation

The code solves the governing differential equation for a beam element under axial and transverse loading using the **Finite Difference Method** and a **Newton-Raphson** iterative solver.

A.1 The Governing Equation

Consider a beam element of length ds with bending stiffness EI. Let θ be the rotation of the beam cross-section relative to the horizontal. From equilibrium of moments on a deformed element:

$$M(s) = EI\frac{d\theta}{ds} \tag{1}$$

The equilibrium equation involving the internal bending moment (M), the axial load (P), and an externally applied moment density (m) arising from the fastener eccentricity is:

$$\frac{dM}{ds} - P\sin(\theta) = 0 \tag{2}$$

Substituting the constitutive law ($M = EI \cdot \theta'$):

$$\frac{d}{ds}\left(EI\frac{d\theta}{ds}\right) - P\sin(\theta) = m \tag{3}$$

Assuming EI is constant over a small segment, we arrive at the governing differential equation:

$$EI\frac{d^2\theta}{ds^2} - P\sin(\theta) - m = 0 \tag{4}$$

Where:

- $EI\frac{d^2\theta}{ds^2}$ is the resistance to bending.
- $P\sin(\theta)$ is the transverse component of the axial load due to rotation (Geometric Non-linearity).
- m is the distributed moment applied by the fasteners.

A.2 Finite Difference Discretization

We discretize the beam into N nodes. For an internal node i, the second derivative is approximated as:

$$\theta_i'' \approx \frac{\theta_{i-1} - 2\theta_i + \theta_{i+1}}{ds^2} \tag{5}$$

The residual equation F_i (which must equal 0 for equilibrium) at node i becomes:

$$F_i = \frac{EI_i}{ds^2}(\theta_{i-1} - 2\theta_i + \theta_{i+1}) - P_i\sin(\theta_i) - m_i = 0 \tag{6}$$

A.3 Newton-Raphson Solution

Because of the $\sin(\theta)$ term, the system is non-linear. We solve for the vector $\boldsymbol{\theta}$ using Newton-Raphson iteration.

First, we calculate the Jacobian matrix (J), where $J_{ij} = \frac{\partial F_i}{\partial \theta_j}$.

- Diagonal term ($j = i$):

$$J_{i,i} = \frac{-2EI}{ds^2} - P_i \cos(\theta_i)$$

- Off-diagonal terms ($j = i \pm 1$):

$$J_{i,i\pm1} = \frac{EI}{ds^2}$$

The solution is updated iteratively until the norm of the residual vector $|F|$ is below the tolerance threshold (10^{-6}):

$$\Delta\boldsymbol{\theta} = -J^{-1}\mathbf{F} \tag{7}$$

$$\boldsymbol{\theta}_{\text{new}} = \boldsymbol{\theta}_{\text{old}} + \Delta\boldsymbol{\theta} \tag{8}$$

A.4 Calculation of Outputs

Once $\theta(s)$ is solved, the physical outputs are derived:

- **Deflection (x, y):**

$$x(s) = \int \cos(\theta)\, ds, \quad y(s) = \int \sin(\theta)\, ds$$

- **Moment (M):**

$$M(s) = EI\frac{d\theta}{ds}$$

- **Stress Ratio:**

$$\sigma_{\text{total}} = \frac{P}{A} \pm \frac{Mc}{I}, \quad \text{Ratio} = \frac{\sigma_{\text{bending}}}{\sigma_{\text{direct}}}$$