

# Mathematical Induction – Fibonacci Sequence Problems

For practice (precalculus / first-proofs level)

Let the sequence  $\{F_n\}$  be defined by

$$F_1 = 1, \quad F_2 = 1, \quad F_{n+2} = F_n + F_{n+1} \quad \text{for all } n \geq 1.$$

1. Prove, by induction on  $n$ , that for every integer  $n \geq 1$ ,

$$\sum_{k=1}^n F_k = F_{n+2} - 1.$$

2. Prove, by induction on  $n$ , that for every integer  $n \geq 1$ ,

$$\sum_{k=1}^n (F_k)^2 = F_n F_{n+1}.$$

3. Prove, by induction on  $n$ , that for every integer  $n \geq 1$ ,

$$\sum_{k=1}^n k F_k = n F_{n+2} - F_{n+3} + 2.$$

4. Prove, by induction on  $n$ , that for every integer  $n \geq 2$ ,

$$F_{n+1} F_{n-1} - (F_n)^2 = (-1)^n.$$

5. Prove, by induction on  $n$ , that for every integer  $n \geq 1$ ,

$$F_1 + 2F_2 + 3F_3 + \cdots + nF_n = F_n F_{n+2}.$$