CSE 361 HW-2

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1 3-2 Relative asymptotic growths

Indicate, for each pair of expressions (A, B) in the table below, whether A is O, Ω, or Θ of B. Assume $k \leq 1, \epsilon > 0, c > 1$ are constants. Your answer should be in the form of the table with "yes" or "no" written in each box.

A	B	O	Ω	Θ
$\mathrm{lg^k(n)}$	n^ϵ	yes	no	no
$\mathrm{n^k}$	$\mathrm{c^n}$	yes	no	no
$\sqrt{n}$	$\mathrm{n^{sin(n)}}$	no	no	no
$\mathrm{2^n}$	$\mathrm{2^{n/2}}$	no	yes	no
$\mathrm{n^{lg\,(c)}}$	$\mathrm{c^{lg\,(n)}}$	yes	yes	yes
$\mathrm{lg(n!)}$	$\mathrm{lg(n^n)}$	yes	yes	yes

2 3-3 Ordering by asymptotic growth rates

- Rank the following functions by order of growth; that is, find an arrangement $g_1, g_2, ..., g_{30}$ of the functions satisfying $g_1 \in \Omega(g_2), g_2 \in \Omega(g_3), ..., g_{29} \in \Omega(g(30)$. Partition your list into equivalence classes such that the functions $f(n)$ and $g(n)$ are in the same class if and only if $f(n) \in \Theta(g(n))$.

1
$\mathrm{n^{1/lg\,n}}$
$\mathrm{lg\ lg^*\ n}$
$\mathrm{lg^*\ lg\ n}$
$\mathrm{lg^*\ n}$

$2^{\lg n}$
ln ln n
$\sqrt{lgn}$
ln n
$\lg^2 n$
$2^{\sqrt{2lgn}}$
$\sqrt{n}$
$\sqrt{2}^{\lg n}$
$2^{\lg n}$
n
lg(n!)
n lg n
$4^{\lg n}$
n^2
n^3
(lg n)!
$n^{\lg \lg n}$
$(\lg n)^{\lg n}$
$(3/2)^n$
2^n
$n\ 2^n$
e^n
n!
(n+1)!
2^{2^n}
$2^{2^{n+1}}$

- Give an example of a single non-negative function f(n) such that for all functions $g_i(n)$ in part 1, f(n) is neither O(g(n)) nor Ω(g(n))

$n!!^{\sin(n)+1/2}$

3 3-4 Asymptotic Notation Properties

Prove or disprove each of the following conjectures:

3.1 f(n) ∈ O(g(n)) implies g(n) ∈ O(f(n))

False. n ∈ O(lg(n)) but lg(n) not ∈ O(n)

3.2 $f(n) + g(n) \in \Theta(\min(f(n),g(n)))$

False. $n + n^2$ not $\in \Theta(n)$

3.3 $f(n) \in O(g(n))$ implies $\lg(f(n)) \in O(\lg(g(n)))$

???

3.4 $f(n) \in O(g(n))$ implies $2^{f(n)} \in O(2^{g(n)})$

False. $f(n) = 2n$, $g(n) = n$. 2^{2n} not $\in O(2^n)$

3.5 $f(n) \in O((f(n))^2)$

False. $f(n) = 1/n$. $1/n$ not $\in O(1/n^2)$

3.6 $f(n) \in O(g(n))$ implies $g(n) \in \Omega(f(n))$

True by definition. f(n) <= C g(n) for positive C, therefore 1/c f(n) <= g(n)

3.7 $f(n) \in \Theta(f(n/2))$

2^n not <= C $2^{n/2}$ for big C

3.8 $f(n) + o(f(n)) \in \Theta(f(n))$

???