# Homework 6

(You must justify ALL your claims unless otherwise stated)

## Problem 1

(a) Find the greatest integer, call it $m$, that can't be represented as the sum of multiples of 4 and 5.

(b) Prove that for all $n \geq m+1$, $n$ can be represented as the sum of multiples of 4 and 5.

## Problem 2

Prove the following statement using weak induction:

$$\forall n \geq 1,\ \frac{1 \cdot 3 \cdot 5 \ldots (2n+1)}{2 \cdot 4 \cdot 6 \ldots (2n+2)} \geq \frac{1}{2n+2}$$

## Problem 3

Prove the following statement using weak induction:

$$\forall n \in \mathbb{N},\ \sum_{k=1}^{n} k^3 = \left(\sum_{k=1}^{n} k\right)^2$$

[Hint: Do we have a formula for $\sum_{k=1}^{n} k$ that we can use? You don't need to re-prove statements that we previously proved in class]

## Problem 4

Recall that the Fibonacci numbers are defined by:

$$f_0 = 0,\ f_1 = 1,\ f_2 = 1,\ f_n = f_{n-1} + f_{n-2} \text{ for } n \geq 3$$

Show that: $f_1 + f_3 + f_5 + ... + f_{2n-1} = f_{2n}$ for all $n \geq 1$.

## Problem 5

Use strong induction to prove that: $\forall n \in \mathbb{N},\ 12|(n^4 - n^2)$.
[Hint: In your IS, write $n+1$ as $m+6$, where $m = n-5$. This means that the $n+1^{st}$ step uses the $n-5^{th}$ step. How many base cases will you need?]

## Problem 6

Prove that:

$$\forall n \geq 1, \prod_{k=1}^{n} \left(1 - \frac{1}{2^k}\right) \geq \frac{1}{4} + \frac{1}{2^{n+1}}$$