

Full name

Andrew ID

21-127 Test 2 (Practice)

Wednesday, 22 March 2023

Please read the following instructions carefully before the test begins.

Before the test

- Do not open the test until instructed to do so.
- Write your full name and Andrew ID in the boxes at the top of this page.
- Place your Carnegie Mellon University ID card face-up in front of you.
- Turn off your electronic devices (e.g. phone, tablet, laptop, calculator), and store any devices, notes or books out of sight (e.g. in a closed bag).

During the test

- Write clearly and legibly with a pen or pencil that is dark enough to be readable when scanned.
- You must justify all answers and claims with mathematical proof, unless otherwise specified.
- If you continue a solution on one of the extra pages (pages 12–13), you should clearly indicate in your solution the page number where it is continued.
- You may not use notes, books, other reference materials, calculators or electronic devices on this test.
- You may not communicate with others or attempt to look at other students' work during the test.
- If you require assistance, please raise your hand and wait for a proctor to come to you.
- If you need to leave the classroom (e.g. to use the bathroom), please raise your hand, show your CMU ID card to a proctor, and leave your belongings in the classroom.
- If you finish the test with 5 minutes or more remaining, you may turn in your test and leave the classroom discreetly; otherwise, please remain seated until the test ends.

After the test

- Stop working immediately when you are instructed to do so.
- Turn in all 14 pages of this test; if you tore out any pages, put them back in their correct positions.

Do not write on this page

1. (a) Write the definition of an injective function [5]

(b) For each of the following functions, decide if the given function is well-defined. If it is, [10]
no need for justification. If it is not, explain why it is not well-defined

(i) $f : \mathcal{P}(\mathbb{N}) \rightarrow \mathbb{N}$, where $f(A) = |A|$, for all $A \in \mathcal{P}(\mathbb{N})$.

(ii) $g : \mathbb{N} \times \mathbb{N} \rightarrow \mathbb{N}$, where $g(m, n) = 4m + 5n$, for all $(m, n) \in \mathbb{N} \times \mathbb{N}$

(c) For the well-defined function(s) above, decide if they are injective [5]

Page 4 of 14 (Q1)

More space for (Q1)

2. (a) Write the definition of a right inverse of a function [5]

(b) Let $f : X \rightarrow Y$ be a function. Prove that, if f has a right inverse, then f is surjective. [10]

Page 6 of 14 (Q2)

More space for (Q2)

3. (a) Write down the recursive definition of $n!$, for all $n \in \mathbb{N}$ [5]
(b) Prove that $n^3 + 2n$ is divisible by 3 for all integers $n \in \mathbb{N}$. [10]

Page 8 of 14 (Q3)

More space for (Q3)

4. (a) Define what it means for a relation to be antisymmetric [5]
- (b) Let $X = \{1, 2, 3\}$. [10]
- Give the graph of a nonempty relation R on X that is transitive and symmetric but not reflexive. If no such relation exists, prove that it is not possible
 - Give the graph of a nonempty relation S on X that is reflexive and symmetric but not transitive. If no such relation exists, prove that it is not possible

More space for (Q4)

5. (a) Let $f : X \rightarrow Y$ be a function and let $A \subseteq X$. Define the image of A under f . [5]
- (b) Give an example of a function $f : X \rightarrow Y$ and a subset $A \subseteq X$ such that $f^{-1}[f[A]] \neq A$. [7]
- (c) Let $f : X \rightarrow Y$ be an injective function. Prove that $f^{-1}[f[A]] = A$, for all $A \subseteq X$. [8]

Page 11 of 14 (Q5)

More space for (Q5)

If you use this page to continue a solution to a question, please clearly indicate on the first page of your solution where it is continued (this is page 12).

If you use this page to continue a solution to a question, please clearly indicate on the first page of your solution where it is continued (this is page 13).

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