Homework 1

Problem 1

Prove the following proposition:
Let $a, b, c \in \mathbb{Z}$. If a divides b and a divides c, then a divides $b - c$.

Problem 2

(a) Let A be the set of all ordered pairs of integers whose sum is negative. Write A in set-builder notation.

(b) Let B be the set of all ordered pairs of real numbers which lie on the graph of the function $f(x) = \sin(x^2 + 1)$. Write B in set-builder notation.

(c) Let $C = \{\dots, -14, -10, -6, -2, 2, 6, 10, 14, \dots\}$. Write C in set-builder notation.

Problem 3

Determine with proof whether the following propositions are true or false.

(a) If n is an even integer, then n^2 is an even integer.

(b) All prime numbers are odd integers.

(c) All odd integers are prime numbers.

Problem 4

Write each of the following sets in list notation.

(a) $D = \{n \in \mathbb{N} : 2 \text{ divides } n \text{ or } 3 \text{ divides } n\}$

(b) $E = \{n \in \mathbb{N} : 2 \text{ divides } n \text{ and } 3 \text{ divides } n\}$

(c) $F = \{n \in \mathbb{Z} : 3 \text{ is the only prime that divides } n\}$

Problem 5

For fixed $n \in \mathbb{N}$, let p represent the proposition 'n is even', let q represent the proposition 'n is prime' and let r represent the proposition '$n = 2$'. Translate each of the following propositional formulae into a plain English sentence—note that your sentence will include the variable n.

(a) $(p \wedge q) \Rightarrow r$;

(b) $p \Rightarrow ((\neg q) \vee r)$;

(c) $(q \wedge (\neg r)) \Rightarrow (\neg p)$;

(d) $(p \wedge (\neg q)) \vee (\neg r)$.

Problem 6

Prove that there is an irrational real number x such that $x^{\sqrt{2}} \in \mathbb{Q}$.
[Hint: we know that $\sqrt{2} \notin \mathbb{Q}$ is true, but we don't know whether $\sqrt{2}^{\sqrt{2}} \in \mathbb{Q}$ is true or false. However we *do* know that $\sqrt{2}^{\sqrt{2}} \in \mathbb{Q} \vee \sqrt{2}^{\sqrt{2}} \notin \mathbb{Q}$ is true. Consider doing a proof by cases.]