Full name	Andrew ID

21-127 Test 1 (Practice)

Wednesday, 15 February 2023

Please read the following instructions carefully before the test begins.

Before the test

- Do not open the test until instructed to do so.
- Write your full name and Andrew ID in the boxes at the top of this page.
- Place your Carnegie Mellon University ID card face-up in front of you.
- Turn off your electronic devices (e.g. phone, tablet, laptop, calculator), and store any devices, notes or books out of sight (e.g. in a closed bag).

During the test

- Write clearly and legibly with a pen or pencil that is dark enough to be readable when scanned.
- You must justify all answers and claims with mathematical proof, unless otherwise specified.
- If you continue a solution on one of the extra pages (pages 13–14), you should clearly indicate in your solution the page number where it is continued.
- You may not use notes, books, other reference materials, calculators or electronic devices on this test.
- You may not communicate with others or attempt to look at other students' work during the test.
- If you require assistance, please raise your hand and wait for a proctor to come to you.
- If you need to leave the classroom (e.g. to use the bathroom), please raise your hand, show your CMU ID card to a proctor, and leave your belongings in the classroom.
- If you finish the test with 5 minutes or more remaining, you may turn in your test and leave the classroom discreetly; otherwise, please remain seated until the test ends.

After the test

- Stop working immediately when you are instructed to do so.
- Turn in all 15 pages of this test; if you tore out any pages, put them back in their correct positions.

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1. (a) Write the definition of the empty set $\varnothing$ using a logical formula **[5]**

(b) For each of the following, name one element from the set. If it is not possible, **prove** that it is the empty set. **[10]**

(i) $\mathscr{P}(\mathbb{Z} \setminus \mathbb{Q})$

(ii) $\bigcap_{n=1}^{\infty} \left(0, \frac{1}{n}\right)$

(ii) $\{x \in \mathbb{R} : \exists n \in \mathbb{Z}, (x = \sqrt{n} \wedge \exists k \in \mathbb{Z}, n = 4k-1)\}$

(iii) $\{x \in \mathbb{Q} : x^2 + 4x + 4 = 0\}$

Page 4 of 15 (Q1)

More space for (Q1)

2. (a) State the extensionality axiom **[5]**

(b) Let A, B be two arbitrary sets. Prove that $A \setminus B = A \setminus (A \cap B)$ **[10]**

Page 6 of 15 (Q2)

More space for (Q2)

3. (a) Define the set of all rational numbers in set-builder notation **[5]**

(b) Let a, b, c be integers. Prove that **[10]**

$$a|b \wedge a|c \Rightarrow \forall s,t \in \mathbb{Z},\ a|(sb+tc)$$

Page 8 of 15 (Q3)

More space for (Q3)

4. Give an example of logical formulae $p(x), q(x)$ and a set S such that the following are not logically equivalent. Justify your answer with a proof. **[10]**

- $\exists x \in S, (p(x) \wedge q(x))$
- $(\exists x \in S, p(x)) \wedge (\exists x \in S, q(x))$

More space for (Q4)

5. Consider the proposition $\varphi : \exists x \in \mathbb{Z}, \forall y \in \mathbb{Z}(y = x \vee y^2 > x^2)$

(a) Determine whether φ is True or False. Justify your answer with a proof. **[8]**

(b) Write $\neg\varphi$ in maximally negated form. **[7]**

Page 12 of 15 (Q5)

More space for (Q5)

If you use this page to continue a solution to a question, please clearly indicate on the first page of your solution where it is continued (this is page 13).

If you use this page to continue a solution to a question, please clearly indicate on the first page of your solution where it is continued (this is page 14).

Initials

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