

Practice Problems for Lectures 9-11

(not to be submitted)

Problem 1

Which one of the following statements is the true one? Prove your answer.

(a) $\mathcal{P}(\mathbb{N}) \subseteq \bigcup_{n \in \mathbb{N}} \mathcal{P}([n])$

(b) $\bigcup_{n \in \mathbb{N}} \mathcal{P}([n]) \subseteq \mathcal{P}(\mathbb{N})$

Problem 2

Let A be a set. Evaluate the following:

(a) $\bigcup_{X \in \mathcal{P}(A)} X$

(b) $\bigcap_{X \in \mathcal{P}(A)} X$

Problem 3

For each of the following, decide if the given description is a well-defined function. Justify your answer.

(a) $f : \mathbb{Q} \rightarrow \mathbb{Q}$, defined by $f(x) = \frac{1}{x+3}$, for all $x \in \mathbb{Q}$.

(b) $g : \mathbb{R} \rightarrow \mathbb{R}$, defined by: for $x \in \mathbb{R}$

$$g(x) = \begin{cases} x^2 + 1, & \text{if } x \geq -2 \\ x + 5, & \text{if } x < 0 \end{cases}$$

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(c) $h : \mathbb{R} \times \mathbb{R} \rightarrow \{-2, -1, 0, 1, 2\}$, defined by: for $(x, y) \in \mathbb{R} \times \mathbb{R}$

$$h(x, y) = \begin{cases} 1, & \text{if } x > y \\ -1, & \text{if } y > x \\ 0, & \text{if } x = y \end{cases}$$

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Problem 4

Let $f : \mathbb{N} \rightarrow \mathbb{Q}$, $g : \mathbb{Q} \rightarrow \mathbb{R}$, $h : \mathbb{N} \rightarrow \mathbb{Z}$, and $k : \mathbb{Z} \rightarrow \mathbb{R}$ defined by

$$f(x) = \frac{1}{x^2 + 1}, \text{ for all } x \in \mathbb{N}$$

$$g(x) = \begin{cases} \frac{1}{x}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}, \text{ for all } x \in \mathbb{Q}$$

$$h(x) = x^4, \text{ for all } x \in \mathbb{N}$$

$$k(x) = \begin{cases} \sqrt{x} + 1, & \text{if } x \geq 0 \\ \sqrt{-x} + 1, & \text{if } x < 0 \end{cases}, \text{ for all } x \in \mathbb{Z}$$

Prove that $g \circ f = k \circ h$, using the extensionality axiom for functions.