Homework 9

(You must justify ALL your claims unless otherwise stated)

Problem 1

(a) Prove that $5n + 3$ and $3n + 2$ are relatively prime for all $n \in \mathbb{N}$.

(b) Prove that if a and b are relatively prime integers then $\gcd(a + b, a - b) = 1$ or 2.

Problem 2

Prove the following:

(a) For all positive integers a, b, c, $\gcd(a, bc) | \gcd(a, b) \cdot \gcd(a, c)$

(b) For all positive integers a, b, c, if $\gcd(a, b)$ and $\gcd(a, c)$ are relatively prime then $\gcd(a, bc) = \gcd(a, b) \cdot \gcd(a, c)$.

Problem 3

Let $a, b \in \mathbb{Z}$ with $b \neq 0$ and suppose that a has remainder 1 when divided by b. Prove that a^n has remainder 1 when divided by b for all $n \in \mathbb{N}$.

Problem 4

Determine which of the following equations have integer solutions $(x, y) \in \mathbb{Z}^2$:

1. $465x + 4920y = 1$
2. $54585x - 4920y = 75$
3. $496185x + 54585y = -10745$