Introduction to Numerical Semigroups

Mini-Course — Summer 2023

M, 10 to 11:30 AM via Zoom

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1 Instructor Information

Dr. Dylan C. Beck, Visiting Assistant Professor of Mathematics at Baker University

- Discord: https://discord.gg/XtaKgwEynC (Enroll here to discuss the mini-course.)
- email: Dylan.Beck@BakerU.edu (Capitalization is used for clarity.)
- pronouns: he / him / his

- textbook: *Numerical Semigroups* by J.C. Rosales and P.A. García-Sánchez
- virtual classroom: Click to access our virtual classroom via Zoom. (passcode: 044163)
- web page: https://dylan-c-beck.github.io/mini-course.html

2 Course Information

2.1 Course Description

We will develop some fundamental concepts and necessary tools to study numerical semigroups. Explicitly, we will discuss basic properties and invariants of numerical semigroups, e.g., multiplicity, Frobenius number, pseudo-Frobenius numbers, Apéry set, gaps, and genus (to name a few).

2.2 Course Objectives

Our primary goal throughout this mini-course is to produce a short manuscript containing many of the principal results in the theory of numerical semigroup; the document will contain relevant definitions, propositions, theorems, and proofs to support our work with the aim that this manuscript may serve as reference for future research in numerical semigroups. Exploration and inquiry-based learning will be implemented with discussion, examples, and questions in lieu of lectures.

2.3 Course Prerequisites

Each student must have a keen interest in developing critical thinking skills, improving mathematical writing and communication, and investigating subtle mathematical structures in a definition-proof style. Existing experience with proof-writing, basic set theory, and elementary number theory (e.g., divisibility properties of integers, the Division Algorithm, Bézout's Identity, and Euclid's Lemma) will benefit interested students; however, we will work throughout the course to develop these skills.

2.4 Student Expectations

Communication between students in the course and the instructor will occur primarily in the virtual classroom and Discord; however, each student should regularly check their email for course updates and supplementary materials as well as the mini-course web page for notes and examples.

Collaboration between students and the instructor is critical for the success of this mini-course. Consequently, each student must be willing to devote time both collectively in virtual meetings and independently to thinking about the course materials. Even more, each student must work to become familiar with the mathematical typesetting language LaTeX. Contributions to the manuscript must be typeset in LaTeX and provided to the instructor to be compiled in an OverLeaf document.

Outside of weekly course meetings, students should expect to spend (at least) two hours preparing materials and studying for every hour spent in class. Unlike in high school, students that do not understand the material covered should not assume that the instructor will repeat material until it is understood and mastered; rather, each student is expected and encouraged to ask questions as they occur throughout the course. Certainly, all students should devote time to studying course materials independently, but if that does not work, students should consider discussing their concerns with the instructor. Do not hesitate to ask questions, as this course is cumulative.