Evaluating the Limit $\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta}$

We have already proved the existence of this limit in class earlier in the semester; however, if you are interested in reviewing the proof on your own, you may do so by using the following link. ▸ Link

Trigonometric Limits

Ex.: Evaluate the following limit.

$$\lim_{x \to 0} \frac{\sin^2(3x)}{x \sin(5x)}$$

Trigonometric Limits

Ex.: Evaluate the following limit.

$$\lim_{x\to 0} \frac{\sin^2(3x)}{x\sin(5x)} = \lim_{x\to 0} \left(\frac{3\sin(3x)}{3x}\right)\left(\frac{3\sin(3x)}{3x}\right)\left(\frac{5x}{5\sin(5x)}\right)$$

Trigonometric Limits

Ex.: Evaluate the following limit.

$$\begin{aligned}
\lim_{x\to 0} \frac{\sin^2(3x)}{x\sin(5x)} &= \lim_{x\to 0} \left(\frac{3\sin(3x)}{3x}\right)\left(\frac{3\sin(3x)}{3x}\right)\left(\frac{5x}{5\sin(5x)}\right) \\
&= \lim_{x\to 0} (3)\left(\frac{\sin(3x)}{3x}\right)(3)\left(\frac{\sin(3x)}{3x}\right)\left(\frac{1}{5}\right)\left(\frac{5x}{\sin(5x)}\right)
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&= \frac{9}{5}
\end{aligned}$$

Total Distance vs. Position

- Like the name suggests, the position function gives the position of a body on the real line it travels about.
- Later, in Calculus III, the position function shows the position of a body by its coordinates in the plane. But for now, we stay in 1D.
- On the other hand, the total distance traveled is what your odometer or pedometer shows.
- We find the total distance by calculating the positive difference of the positions on the intervals created by the starting, ending, and turning points (where velocity is zero or DNE). We then add those together.
- Use this link to explore why this is the case. Link

Curve Sketching

Ex.: Draw a sketch of the graph of the function with the properties

1. $f(-1) = f(1) = 0$;
2. $f'(x) > 0$ on $(-\infty, -1) \cup (1, \infty)$;
3. $f'(x) < 0$ on $(-1, 1)$;
4. $f''(x) < 0$ on $(-\infty, 0) \cup (2, \infty)$;
5. $f''(x) > 0$ on $(0, 2)$; and
6. $\lim\limits_{x \to \infty} f(x) = 2$.

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For this type of the problems, use a table for intervals of increasing / decreasing and concave up / down. Put any points given and asymptotes on the plane, and then, connect the pieces using the table.

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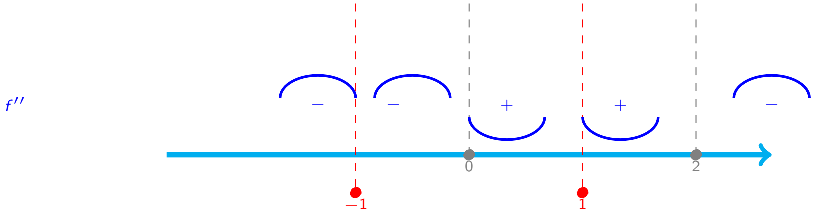

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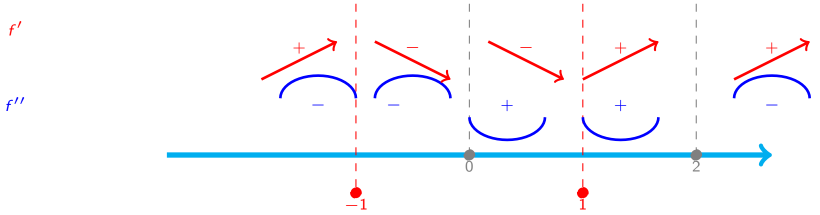

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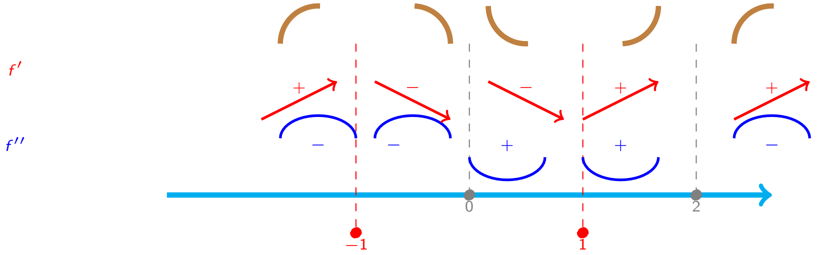

iClicker Question 1

Using Logarithmic Differentiation

Compute the derivative of $y = \dfrac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}$.

(a.) $\left(\frac{8x}{x^2+1} - \frac{6}{2x+1} - \frac{15}{3x-1}\right)\frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}$

(b.) $\frac{8x(x^2+1)^3}{6(2x+1)^2 \cdot 15(3x-1)^4}$

(c.) $\frac{4}{x^2+1} - \frac{3}{2x+1} - \frac{5}{3x-1}$

(d.) $\frac{8x}{x^2+1} - \frac{6}{2x+1} - \frac{15}{3x-1}$

iClicker Question 2

Using Logarithmic Differentiation

Compute the derivative of $y = [\cos(x)]^{\sqrt{x}}$.

(a.) $-\sin(x)\sqrt{x} \cdot [\cos(x)]^{\sqrt{x}-1}$

(b.) $\left(\frac{\ln\cos(x)}{2\sqrt{x}} - \sqrt{x}\tan(x)\right)[\cos(x)]^{\sqrt{x}}$

(c.) $\frac{\ln\cos(x)}{2\sqrt{x}} - \sin(x) \cdot \frac{\sqrt{x}}{\cos(x)}$

(d.) $\frac{\ln\cos(x)}{2\sqrt{x}} + \frac{\sqrt{x}}{\cos(x)}$

iClicker Question 3

Using Logarithmic Differentiation

Compute the derivative of $y = [\arctan(x)]^{\arcsin(x)}$.

(a.) $\frac{\ln\arctan(x)}{\sqrt{1-x^2}} + \frac{\arccos(x)}{x^2+1}$

(b.) $\frac{\ln\arctan(x)}{\sqrt{1-x^2}} + \frac{\arcsin(x)}{(x^2+1)\arctan(x)}$

(c.) $\left(\frac{\ln\arctan(x)}{\sqrt{1-x^2}} + \frac{\arcsin(x)}{(x^2+1)\arctan(x)}\right)[\arctan(x)]^{\arcsin(x)}$

(d.) $\frac{\arcsin(x)[\arctan(x)]^{\arcsin(x)-1}}{x^2+1}$

Horizontal Tangent Line

Find a point on $y = [\ln(x + 4)]^2$ with a horizontal tangent line.

(a.) $(-4, 0)$

(b.) $(e - 4, 1)$

(c.) $(e - 4, \frac{2}{e})$

(d.) $(-3, 0)$