INSA de Toulouse
Département GMM
BE - Processus de Poisson et Application en actuariat et fiabilité - 5 ModIA
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Large deviation principle - A probabilistic counterpart to ruin probabilities

Keywords :   insurance, Hawkes processes, thinning method

Context of the project

The main goal of this project is to study the ruin probability in the the Cramér-Lundberg model; and its probabilistic counterpart known as the Large Deviation Principle for the study of rare events. Recall the classical Cramér-Lundberg model for modelling the risk process (wealth) as :

$$R_t = u + ct - \sum_{i=1}^{N_t} Y_i, \quad t \geq 0. \tag{1}$$

In this model it is assumed that the claim sizes Y_i's and the counting process N are independent. We will prove during the lecture that there exists a critical value of premium c (known as the Net Profit Condition) which prevents the almost-sure ruin of the wealth process in infinite time. But how does the ruin probability behave in term of the initial premium u if the NPC is satisfied ? We will see that one way to answer this question is to translate it in terms of "rare events" analysis using the so-called Large Deviation Principle in probability theory.

The main purpose of this project is thus to study the ruin probability from a theoretical point of view using this probabilistic tool; and from a numerical point of view by providing simulations of this ruin probability.

Work to be performed

The work asked to the students goes in two different directions.

Theoretical results

First it is asked to the students to study the Large Deviation Principle and how it relates to the ruin probability analysis.

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Numerical results

The second types of results is to perform numerical simulations for the ruin probability in order to check the optimality of the theoretical bounds. The numerical simulations, will be presented in a Notebook Python (only).

References

[1] S. Asmussen. Tail Probabilities for Non-Standard Risk and Queueing Processes with Subexponential Jumps. *Applied Probability Trust*, 31(2):422–447, 1999.

[2] F. Aurzada and M. Buck. Ruin probabilities in the Cramér–Lundberg model with temporarily negative capital. *European Actuarial Journal*, 10:261–269, 2020.