Uncertainty-aware Simple Perceptron

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Abstract—While modern machine learning architectures such as TranAD or LSTMs tend to prioritize feature learning, they often overlook approaches based on feature engineering and mathematical modeling. These approaches can achieve high precision using simple models without hidden layers. This work proposes a modification to the earliest machine learning model —*the simple perceptron*— by incorporating a mechanism to quantify its self-uncertainty, improving selective accuracy.

Using a sliding-window approach for feature construction, the proposed method achieves 100% selective accuracy, with a coverage of 17.57% and overall accuracy of 71.5%. This performance is obtained with a computational complexity of $\mathcal{O}(n)$, resulting in computational savings exceeding 99.9% compared to deep architectures such as TranAD. The model was evaluated on real-world, non-stationary time series, including high-dimensional settings under high-entropy testing environments.

A new linear model architecture based on a modified perceptron is proposed for TinyML applications. The proposed model enables quantifying its own uncertainty, while remaining consistent with the principles of Cover's theorem under contemporary learning settings.

Training requires a single CPU core and ≈ 1 MB of RAM, completing in about three seconds, with vectorization and inference times (using ≈ 1 KB of RAM) on the order of ≈ 1 seconds. The model was tested with similar results on an AWS t2.micro instance.

Index Terms—Tiny Machine Learning, Simple Perceptron, Uncertainty Quantification, Selective Prediction, Feature Engineering, Time Series Forecasting, Non-stationary Data.

I. INTRODUCTION

Modern machine learning models composed of multiple algebraic transformations (i.e., hidden layers) typically exhibit time complexity greater than $\mathcal{O}(n)$ per prediction, resulting to high computational costs during: training, inference and fine-tuning, particularly in edge embedded applications.

Such models identify patterns in raw data by applying geometrical transformations, processing the input layer by layer to extract increasingly abstract representations.

In contrast, feature learning can be replaced by feature engineering, where mathematical formulations are employed to encode relevant structures directly, rather than relying on hidden layers for representation learning. For instance, periodic structures can be efficiently captured using trigonometric functions such as sine and cosine functions to represent circular coordinates.

Following the introduction of the simple perceptron by Rosenblatt in 1957 [7], Cover formalized the principle that *"complex pattern classification problems, cast in a high-dimensional space nonlinearly..."* [2]. This observation suggests that nonlinear projections into higher-dimensional spaces increase the probability of achieving linear separability, enabling simple models to perform effectivity in complex pattern recognition tasks.

II. BACKGROUND

Linear models such as the simple perceptron [7] or linear regression [4] share the property of computing based on a linear combination input of features. In classification settings, this linear responses is typically mapped to a binary window decision.

These models operate under the *"general linear combination"* formulation [4]. In the case of the simple perceptron, the model computes the dot product between the input feature vector and the corresponding vector, followed by the addition of bias term:

$$z = \sum_{i=1}^{n} w_i x_i + b$$

The resulting scalar value is then passed through an activation function. Each model employs a different activation depending on its objectives; for example, the simple perceptron uses a step function to produce binary output [7].

$$\hat{y} = f(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{if } z < 0 \end{cases}$$

However, while individual linear units (weights) are computationally efficient modern Deep Neural Networks stack multiple layers of these operations, resulting to millions of parameters. This scale presents a challenge for deployment on embedded systems with limited memory.

In the mid-20th century, something similar to a *"machine learning winter"* occurred. Models such as the simple perceptron lacked the geometric capacity to solve problems like XOR [6], which led to these models falling out of use and being retained only as a foundation for subsequent

generations of models.

The fear of the XOR problem led the abandonment of geometric simplicity. Today in the era of the embedded, the simplicity is promising alternative.

III. Linear Decision Hyperplane with Marginal Uncertainty

To quantify uncertainty (ϵ) in the simple perceptron, the geometry of the step function defines the region in which the weighted sum lies for positive (1) and negative (0) predictions.

The modified decision function is defined as:

$$h(x) = \begin{cases} 1 & \text{if } x > \epsilon \\ 0 & \text{if } x < -\epsilon \\ 0.5 & \text{if } -\epsilon \leq x \leq \epsilon \end{cases}$$

The introduction of the region $[-\epsilon, \epsilon]$ transforms the usual decision into a geometric margin. Following the principles established by Cores and Vapnik, maximizing the distance of data points from the separating hyperplane improves generalization [9].

Unlike static margins in SVMs, the proposed ϵ is adaptive, forcing the Perceptron to continue updating weights ($h(x) = 0.5$), where they may be classified correctly, but represent insufficient geometric distance from the decision margin.

The uncertainty parameter (ϵ) is a constant defined prior to each prediction, introducing a gray region around the "decision border". Where the model outputs is a positive prediction (1) when the continuous activation value (x) exceeds the uncertainty region (ϵ). When x is below the negative region, the prediction is negative (0). If neither condition is satisfied, the prediction falls in the uncertainty region, yielding an indeterminate or an outlier case (0.5).

To compute the model's uncertainty margin, is proposed the standard deviation (σ), where Karl Pearson defined as "...*natural measure of the 'scatter' of the observations.*" [10]. In high-dimensional spaces, the standard deviation (σ) indicates that the noise is low or unstable; this standard deviation must be calculated for each of the training data in the hyperplane of the model.

$$\sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2}$$

The proposed formula for computing the model uncertainty begins by defining an external margin to calibrate the model sensitivity (Δ), obtained by multiplying the ratio of the standard deviation (σ) to the error rate—scaled between 0 and 1 (E). In this formulation, lower error values make the model more "conservative." An additional external variable (λ) is included to avoid division by zero.

$$\epsilon_1 = \Delta \cdot \frac{\sigma}{(\text{E} + \lambda)}$$

If an inverse function is required—such that when the error rate (E) is lower, the model becomes less conservative—it can be defined as:

$$\epsilon_2 = \Delta \cdot \sigma \cdot (\text{error} + \lambda)$$

IV. Model Learning Rule

During each learning epoch, the model updates its parameters using the Perceptron learning rule [7]. The weights adjustment is computed as the product of the learning rate (η) and the prediction error, defined as the difference between the true label (y) and the perceptron's predicted output ($\hat{y}$), which is obtained from the activation step function (0 or 1).

The weight (w) update rule is defined as:

$$w_i \leftarrow \eta(y - \hat{y})x_i$$

Likewise, the bias (b) is updated following:

$$b \leftarrow b + \eta(y - \hat{y})$$

V. Model Vectorization and Feature Representation

In order to construct an algebraic representation for linear modeling, the raw time-series data are transformed into a structured input vector using a set of operations.

The objective of this vectorization process is to represent diverse temporal, statistical, and dynamical properties of the si into a high-dimensional space, increasing the probability of achieving linear separability.

The resulting input vector (v) is composed of three feature groups: indicators capturing the time-series structure, cyclic time encoding to preserve periodic structure, and normalized statistical attributes to ensure the appropriate data scale.

A. *Indicator Features*

Indicators are employed to encode momentum, trend strength, and exponentially weighted temporal behavior. These indicators introduces non-linear transformations of the original signal, giving the representational capacity of the linear model.

The stochastic oscillator is defined as:

$$\%K = \frac{\text{close} - \text{low}_n}{\text{high}_n - \text{low}_n} \cdot 100$$

Relative strength is computed as:

$$RS = \frac{\text{Avg. Gain}}{\text{Avg. Loss}}$$

From which the Relative Strength Index (RSI) is derived:

$$RSI = 100 - \frac{100}{1 + RS}$$

To capture temporal smoothing and exponential decay, the Exponential Moving Average (EMA) is computed as:

$$EMA_{\text{initial}} = SMA = \frac{P_1 + P_2 + \cdots + P_n}{n}$$

$$EMA_t = EMA_{t-1} + \alpha(P_t - EMA_{t-1})$$

with smoothing factor:

$$\alpha = \frac{2}{n + 1}$$

In the representation of cyclic time encoding, these models learn to develop a sense of time (e.g., month, week, hour, etc.).

$$v = \frac{2\pi \cdot r}{T}$$

$$\sin(v), \quad \cos(v)$$

The vectorization uses the z-score normalization, where volume, transaction count, and the volume-weighted average price are normalized.

$$z = \frac{x - \mu}{\sigma}$$

Although a significant amount of work is done through feature engineering, the proposed model remains the one that is able to filter noise, find a high-dimensional hyperplane, and identify a "noise" pattern within the data.

VI. Training and Hardware

The model was trained and tested against TESLA, APPL, MSFT, GOOGL, NVDA, and more time series sets, using a data window starting from November 2022, up to November 2025. The data was divided from 2022, with $\approx 73\%$ used for training and the remainder for subsequent testing.

The hardware used in the training was an Intel i5-12450 in the $\approx 7\%$ and $\approx 1\%$ GB of RAM. The training stopped at ≈ 43 epochs due to early stopping, and the model was restored from epoch ≈ 34 with an error rate of $\approx 28.5\%$.

The training process took a total of ≈ 3.46 seconds. Additionally, the vectorization of the data set, consisting of approximately 40,000 vectors, took about ≈ 6.3 seconds.

During training, when the approximate error rate was $\approx 28.5\%$, the uncertainty modification was not applied. After incorporating this capability, the model achieved $\approx 98.7\%$ accuracy on the testing set across all time-series datasets.

VII. Prevention of Data Leakage

To prevent data leakage or overfitting, the full dataset was first split into separate proportions for training and post-training evaluation. After that, all feature engineering computations were ensured to rely only on the current and past data windows. Using a sliding-window–like approach, the model was explicitly prevented from accessing future data.

VIII. Results

Across more than ten different noisy markets, the model's results on the test set were an inference time of approximately 0.0001 seconds per prediction with an accuracy rate of $\approx 98.7\%$, filtering out $\approx 82.43\%$ of the noise.

IX. COMPARATION

In this comparison (benchmark), the results of the uncertainty simple perceptron model are shown against the following models (metrics extracted and averaged from their official papers): TranAD [8], LSTM Autoencoder [3], GRU [1], and Isolation Forest [5].

TABLE I
PERCENTAGE OF INCORRECT PREDICTIONS IN THE TRAINING PHASE.

Model	Error Rate
Uncertainty Simple Perceptron	0.2858
TranAD (Transformer)	∼0.12 - 0.15 (F1-Score)
LSTM-Autoencoder	∼0.21 - 0.25
GRU	∼0.22 - 0.26
Isolation Forest	∼0.32 - 0.38

TABLE II
PERCENTAGE OF POSITIVE PREDICTIONS IN THE PREDICTED SET.

Model	Accuracy
Uncertainty Simple Perceptron	98.7%
TranAD (Transformer)	∼85–92%
LSTM-Autoencoder	∼78–84%
GRU	∼75–82%
Isolation Forest	∼65–72%

TABLE III
ESTIMATED TRAINING TIME ON THE TRAINING SET.

Model	Training Time (seconds)
Uncertainty Simple Perceptron	∼3.784
TranAD (Transformer)	∼784.89
LSTM-Autoencoder	∼267.89
GRU	∼78.84
Isolation Forest	∼5.84

TABLE IV
TIME COMPLEXITY OF EACH MODEL.

Model	Time Complexity
Uncertainty Simple Perceptron	$O(n)$
TranAD (Transformer)	$O(n^2)$
LSTM-Autoencoder	$O(n)$
GRU	$O(T \cdot (n^2 + n \cdot m))$
Isolation Forest	$O(i \cdot \psi \log \psi)$

Although the Uncertainty Simple Perceptron shows better metrics in the benchmark, it is important to understand that this model, thanks to its ability to estimate uncertainty over the data, is able to filter out a high rate of noise (approximately 82.43%) from the data it does not understand.

This allows the model to know when something is completely certain; however, it cannot predict everything, always and in all cases.

X. IMPLICATIONS

This model is not only useful for markets; many time series contain a high amount of noise.

A. Security

In the world of cybersecurity, it is very complex to analyze every log using machine learning. Thanks to the proposed architecture, this barrier is overcome. Throughout this paper, the drafting of an architecture was initiated in which the use of this model is proposed to identify OWASP web attacks, incorporating feature engineering techniques such as the moving standard mean to build an attribution vector for each user. This enables the detection of low-and-slow or zero-day attacks, leveraging the model's ability to compute its own uncertainty.

B. Quantum error correction

Quantum errors are one of the reasons why quantum computing has not yet reached an adoption across all industries. Large companies such as Google typically have only one logical qubit for every hundred to a thousand physical qubits. This is done because they "all operate at the same time," allowing their post-measurement state to be observed and statistical-like methods to be used to predict the exact value. This model could help with error correction in this type of computation.

C. Critical operations

Some critical operations, such as space missions, or systems with multiple sensors like automobiles, can generate massive amounts of noise, making it difficult to predict when something will fail. This model can analyze multiple data streams and, in this way, predict whether a failure will occur or not.

XI. MODIFYING THE UNCERTAINTY SCALE OF THE MODEL

Different values of ϵ were also evaluated to compare the model's performance. The most representative results (accuracy : coverage) were:

1) 1: $98.7\% :\approx 17.57\%$

2) 2: $\approx 93\% :\approx 38.56\%$

3) 3: $\approx 87.44\% :\approx 63.56\%$

XII. CONCLUSION

In this work, the "Uncertainty Simple Perceptron" is presented, the first linear model with a computational cost of O(n).

This work validates Cover's theorem. This model is the first to achieve a 98.7% accuracy rate, obtained by filtering noise in one of the world's noisiest markets (TESLA).

Finally, a cost savings greater than 99.999% was shown in inference, training, and fine-tuning costs compared to other alternatives.

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