MATH 3352 Modern Geometry I

Course Project 2 (10 pt)

INSTRUCTION

You will investigate a proof of the Exterior Angle Theorem (EAT) below that requires GEX constructions and Screencast presentation. Consider the following statement of EAT from Chapter 2.1.

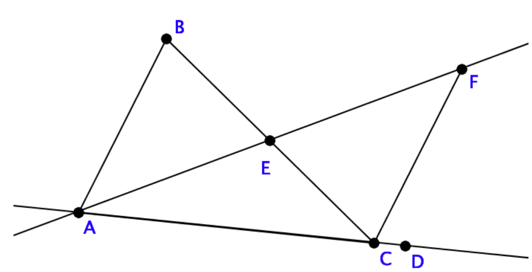
> **Exterior Angle Theorem (EAT)**
> For a given triangle, if the one of the sides is extended, then the exterior angle produced is greater than either of the two interior and opposite angles.

In high school geometry, the "exterior angle theorem" usually means that the exterior angle in a triangle equals the sum of the two opposite interior angles of the triangle. Notice that it is stronger than the statement above in EAT with respect to the extent to which the theorem can tell us about the properties of exterior angle. In fact, the conclusion in high school version provides more information and implies the conclusion in EAT. In this regard, EAT can be considered a weak version of the high school version.

On the other hand, EAT is also considered a primitive version of the theorem in the sense that we can see how it evolves while developing the foundations of Euclidean geometry in our class by introducing postulates. We can deduce the stronger version once we introduce all the postulates of Euclidean geometry, but EAT may be the best what we can draw from a given set of postulates at certain point. In the following sequence of the problems, we will investigate how a proof of EAT may or may not work in different geometries.

Consider the following construction that illustrates a proof of EAT.

Proof) For a given triangle ΔABC, extend the side $\overline{AC}$ and pick an arbitrary point D on the line $\overleftrightarrow{AC}$ such that $A - C - D$. There is the midpoint of the side $\overline{BC}$. (Let's call the midpoint E.)



On the line $\overleftrightarrow{AE}$, there is a point F such that $\overline{AE} \cong \overline{EF}$ and $A - E - F$. Then, we have two congruent triangles ΔAEB and ΔFEC. Then the angle $\angle EBA$ and $\angle ECF$ are congruent. Since $\angle BCD$ is greater than $\angle BCF$, $\angle BCD$ is greater than $\angle CBA$.

This proof illustrates how to use construction of congruent triangles in a particular way showing that the exterior angle $\angle BCD$ is greater than $\angle BAC$. Using a similar construction on the other side of the triangle, we can also prove that $\angle BCD$ is greater than $\angle CBA$. But it is known that EAT does not hold for an arbitrary triangle in spherical geometry, thus the previous proof may not work for some triangles on sphere.

SUBMISSION

You will submit a robust construction of proof for EAT in spherical geometry and a screencast presentation explaining when the construction in spherical geometry works or fails.

1. GEX Construction of EAT Proof in Spherical Geometry
Note that this construction works in Euclidean geometry (even without Euclid's fifth postulate) but does not work in spherical geometry. In this problem, provide a GEX construction illustrated in the previous page in spherical geometry.

2. Screencast Presentation of Counter-Example in Spherical Geometry
After creating your construction above, look for such cases when the proof does not work. As you drag around stretch them far apart the three vertices of ΔABC and stretch them far apart, you will see in some cases that $\angle BCD$ is NOT greater than $\angle CBA$. Create a screencast presentation to show such cases and explain "when" the construction works or not. You need to be specific in describing when the construction fails in terms of what would be a condition that the construction works or fails.