# Summer II 2021 MATH 3352 Modern Geometry I
# Final Exam (20 points)

## Instruction

- Submit your answers to the following problems in a form of an electronic document (pdf is preferred) or a scanned copy of your handwriting.
- Show all your work on the problems to justify your answers.
- You may use any course materials or other resources to answer the problems. If any resources outside the course were used, provide a list of the external resources you referred to on the first page of your submission (e.g., title of textbooks, link to online resources, etc.).
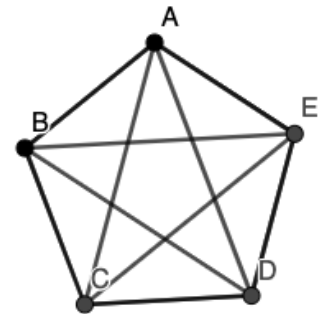- Do not copy and paste any existing materials or someone else's work in your answer.

## Problem 1 (4 points)

Consider a model with the interpretations of undefined terms S, L, and P as follows.

$S = \{A, B, C, D, E\}$

$L = \{all\ sets\ of\ two\ distinct\ points\ in\ S\}$

$P = \{all\ sets\ of\ three\ distinct\ points\ in\ S\}$



Determine which of the following postulates hold in this model. Justify your answer.

(a) Postulate 4: If two distinct points lie in a plane, then the line containing them lies in the plane.

(b) Postulate 5: If two distinct planes intersect, then their intersection is a line.

## Problem 2 (4 points)

If $P$ and $Q$ are on opposite sides of a line $l$, and $Q$ and $T$ are on same sides of $l$, what can you conclude? Justify your answer using the plane-separation postulate. (Hint: Use the plane-separation postulate)

## Problem 3 (4 points)

Prove that if two lines lie in the same plane, and are perpendicular to the same line, then they are parallel. (Hint: Use the proof by contradiction. If not, explain how it would contradict to the exterior angle theorem – the weak version).

## Problem 4 (4 points)

For each statement, indicate in which geometry or geometries the statement holds. (E: Euclidean, H: Hyperbolic, L: Elliptic, S: Spherical)

A. The angle sum of a triangle is greater than 180 degrees.

B. The exterior angle of a triangle is equal to the sum of the two opposite interior angles.

C. For two lines $l$, $m$, and a transversal $n$, if two corresponding angles are congruent, $l$ and $m$ are parallel.

D. All lines intersect at two distinct points.

## Problem 5 (4 points)

Explain how to measure an angle in Klein model. In your answer, you may include a sketch of an angle in Klein model as follows and indicate how to measure $\angle ABC$ in the sense of Klein model.