COLLEGE OF NATURAL & HEALTH SCIENCES

COURSE SYLLABUS

MATH 543 Modern Geometry (3 credits)

Instructor: Dr. Jeff King
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Office Hours: by appointment

Course Meeting Times: 06/03-27/19 MTWR 9:00-11:45 am via Zoom
06/10-13/19 MTWR 9:00-12:00pm in Ross 2275

"The royal road to knowledge,
it is easy to express:
to err, and err, and err again,
but less, and less, and less."

This course is about geometry, but it is also a course about learning to develop and express your own mathematical ideas. It will emphasize ideas and imagination in addition to techniques and calculations. We will be investigating not only the planar Euclidean geometry that you have probably previously studied and may have even taught, but also geometry on spheres, cones, pool tables, and other surfaces. We will try to imagine what geometry would be like for someone living on each of these surfaces. We will be studying these geometries not only because they are useful and can tell us surprising things about the world in which we live, but also because they are beautiful and fascinating subjects.

This course may be very different from most other math courses that you have taken. High school and introductory college math courses usually focus on teaching methods of doing computations: mathematics as finding the correct answer. In this course, we are going to focus on another kind of mathematics: mathematics as a way of thinking about and trying to understand the world. We'll try to understand how people decide what is true, and how they reason about mathematics and geometry in particular. We are going to focus on the process as much as the results of mathematical thought. Thus problem solving, reasoning and proof, and oral and written communication of mathematical ideas will all be vital components of the class.

This course will require a willingness to invest significant amounts of effort grappling with developing your own ideas. In this course, as in the real world, you will be the ultimate arbiter of what you believe to be true. Deciding for your self what is true is not easy, but it can be very rewarding.

Structure of the course: The course will be divided into two interrelated strands, in order to divide up our long days into different kinds of activities. The first strand will be to work our way through David Clark's *Euclidean Geometry: A Guided Inquiry Approach*. This book is designed to allow us to work our way through the content of Euclidean Geometry from an axiomatic point of view. As we do this, you will be working on solving problems and proving theorems from this book outside of class time, and then I will be randomly calling on members of the class to present their work on these problems in class each day. The class will then decide as a whole if we think the work presented is correct. I will also be asking each of you to keep a solutions journal, in which you write up your solutions to each problem/theorem after we have discussed it in class. This part of the class is designed to give you a deep understanding of Euclidean Geometry, particularly the parts of Euclidean Geometry that are taught at the secondary level. (In fact, this textbook has been used to teach some high school geometry classes.)

The second strand of the course will be to work on a series of group projects. These group projects will focus on some aspects of Euclidean Geometry not covered by the text, as well as some non-Euclidean geometries. Working with non-Euclidean Geometries will give us the chance to experience what it is like to make sense of new geometric spaces and to decide for ourselves what is true there–something that we can't do very well with Euclidean geometry, since you probably already know quite a bit about it.

Assignments: Each strand of the course will have its own assignments. For the first strand, you will be responsible for working through the next set of problems/theorems each day in order to be prepared to present them in class, and you will also be responsible for writing up solutions to each problem in your solutions journal. I will be awarding points for each presentation made in class, and I will collect your solutions journal periodically and will randomly select problems to grade from the journal.

For the second strand, I will be asking you to write up and turn in the solutions to the group projects we work on. These solutions will go through a process of revision: I will make comments on them, and then you will have an opportunity to revise them. You should explore each question and write out your thinking in a way that can be shared with others. Focus on your own ideas and understandings, and turn in whatever your thinking is on a question, even if only to say, "I do not understand such and such" or "I am stuck here." Be as specific as possible. Conjecture. Use pictures. Respond to my comments and questions. These assignments will be graded on how close to a complete correct answer you are, and you can continue to revise them until you have them completely correct.

I encourage you to make revisions as soon as you can after getting feedback from me. We will have a lot of different things going on, and if you let revisions pile up, they may become overwhelming. I will always try to have commented drafts back to you within two class meetings. Group projects may be revised as many times as necessary. Revisions can be turned in up until our final class, but, again, I encourage you to revise them and try to get them completed as soon as possible.

I will also occasionally ask you to post to the discussion group on Canvas. Participation in Canvas discussions will contribute to your class participation grade.

There will also be a final group assignment in lieu of a final exam. This extended project about geometry on the cone will give you the opportunity to look at a new geometric model and to go through the complete mathematical process of exploring a new geometric space, collecting data about it, making conjectures about it, proving these conjectures, and then explaining your discoveries to others. You will submit a written paper describing what you have discovered on the last day of class, when you will also present your findings to the rest of the class.

The University's official policy is that students should expect to spend at least two hours working outside of class for each hour they spend in class. Because this class is being offered in an intensive four week format, it will require an intensive time commitment outside of class as well. We will be meeting for 12 hours each week, so you should expect to spend at least 24 hours a week working outside of class. A significant portion of this time will be needed to prepare for the presentations that will be made each day in class, so you should expect to have to spend time working on this class outside of our meeting time every day.

Texts: The main text for this course will be *Euclidean Geometry: A Guided Inquiry Approach* by David Clark. This will be supplemented by other course materials that you will be given as the class progresses.

More about Strand One: For this strand of the course, we will be working our way through the class textbook. We will spend approximately the first half of each of our class meetings presenting and working our way through material from this book. I expect that we will get through the first four chapters of the book, although this will depend on exactly how fast or slow the class presentations go.

In this strand, we will be following a method of learning mathematics pioneered by the mathematician R. L. Moore at the University of Texas at Austin, and sometimes referred to as "The Modified Moore Method." The hallmark of this method is that the students (you!) are responsible for creating all of the mathematics we will be discussing. This will allow you to develop your own deep conceptual understanding of the fundamental mathematics underlying the secondary geometry curriculum.

The core of this strand is the sequence of definitions, problems to be solved, and theorems to be proven, all of which can be found in our textbook. You will be solving the problems and proving the theorems, and then presenting your solutions to the class. Here is how this part of the class will work:

1. Presentation points will be awarded for solving Problems and proving Theorems and then presenting the solutions to the class.
2. Problems and Theorems should be viewed as *conjectures*—that is, they may be either true or false. They can be solved by showing which category they fall in. In other words, you solve a problem or theorem by giving a convincing argument that it is true (or false). If the Problem or Theorem is not correct as stated, you should try to see if there is a way to fix it.
3. You will earn presentation points by presenting a correct solution to an unsolved problem to the class. A solution is correct if it is unanimously judged by the class (including me) to be completely convincing. A correct solution presented to the class normally earns 10 points, although solutions may be worth fewer or more points for problems that are easier or harder than average.
4. Presentation points may be awarded for other reasons, such as for contributing a significantly different alternative proof, or proposing a conjecture that gets made into one of the course problems.
5. The intent of this course is for you to solve the problems yourself. You may discuss the problems you are working on with other members of the class as long as neither of you has already solved the problem. If you have solved a problem you can try to help another student arrive at the solution themselves, but you MAY NOT just give them your solution.
6. We will start presentations each day from where we left off the day before. It is up to you to judge how far we are likely to get each day and to keep far enough ahead in the problem sequence that you will be prepared for any problems or theorems you are called on to present.
7. You will also need to keep a solutions journal in which you write up your solutions to every problem from the textbook. Your solutions journal for each chapter will be collected only after the whole chapter has been presented in class and we have had a chance to discuss all of it. The intent of the solutions journal is to leave you with a complete explication of the fundamental content of the secondary geometry curriculum, written in your own words.
8. I encourage you to type up all of your solutions in Microsoft Word or in the mathematical typesetting system LaTeX, though scans of handwritten solutions will also be accepted. However, typing your solutions will make it much easier to revise them after they have been presented in class. I will be collecting your solutions journals after we have completed each chapter and awarding homework points for them. Some problems may not be presented in class, and only turned in through your journals. However, any problem that any of us would like to see discussed in class will be discussed in class.
9. Each time I collect your solutions journal, I will pick approximately two problems at random to grade. Thus, you need to make sure that all of your solutions are written up to the quality of something you are turning in to be graded. Since you will be turning in problems after they have been discussed in class, you should be able to do an excellent job of writing them up. Problems that are completely clear and correct will be awarded 10 points, and problems that are not completely clear and correct will have points deducted as appropriate. For example, one point might be deducted for a problem with minor typos, misstatements, or missing explanation; two to three points might be deducted for a minor conceptual flaw in the reasoning or significant missing explanation; five to eight points might be deducted for major conceptual flaws in the reasoning and explanation; and all ten points will be deducted for problems that are completely missing or contain no significant correct work.
10. The *only* texts that you may consult in solving these problems are the official course materials and your own notes. You should not consult any outside textbooks or other materials unless I have specifically

given you permission. Solutions to many of the problems that we will be covering can be found in other textbooks, online, *etc.*, but if you consult these books, you will be depriving yourself of the understanding that will come from solving the problems yourself.

11. You should start working on these problems right away, and have worked at least through the end of Problem 19 prior to our first class meeting.
12. Class presentation grades will be determined by how many presentation points you have at the end of the course. I can't tell you in advance how many points will correspond to a certain grade, but if I feel that anyone with a certain number of points has been doing A level work in their presentations, for example, then everyone with at least that many points will get an A for their presentation grade.

More about Strand Two: We will spend the second half of each of our class meetings discussing and working on group problems in small groups. These group problems can be found in the course packet available on the course web page. This strand is intended to give you the experience of working though unfamiliar problems, many of them within the unfamiliar setting of some non-Euclidean geometries. In this way, your experiences with the projects in this strand may mirror some of the experiences your students have when encountering new ideas in your own classes. In order to do this, we need to look at non-Euclidean geometries, since most of you are already very familiar with Euclidean geometry.

For most of these projects, we will spend a couple of days working on them in class in small groups in breakout rooms in Zoom, as well as discussing them as a class. Each group will then need to write up and submit their solutions to the problems through Canvas. Solutions should be typed and submitted as a .pdf or Microsoft Word document, and will typically be initially due several days after we have finished working on them in class. These solutions, which can be revised as many times as needed, will be graded on how close they are to being 100% complete and correct. Each time I receive a solution from a group, I will annotate it with comments and return it to the group through Canvas. My intention is to always return submitted projects within two classes of the time they are submitted. My comments will indicate any aspects of your write-ups that need to be improved for it to be 100% complete and correct. I will also assign a grade to each draft that will indicate the percentage of the way to a complete and correct solution that I think you have come. Because you will have the ability to keep revising each assignment until it is 100% complete and correct, there is no reason that every person in the class couldn't get 100% for their overall group project grade.

When you receive a draft of a project back, you shouldn't think of the grade as a traditional evaluation of the quality of your work; rather, you should think of it as indicating how much more work is needed for a complete solution and understanding. Because we will be working on difficult problems in unfamiliar spaces, it won't be uncommon for first drafts to need a lot of further work to be 100% complete and correct. So, for example, a 75% grade on a draft shouldn't be thought of as an C; rather, it means that the draft has addressed about 3/4 of what it needs to be 100% complete and correct, and the remaining quarter will hopefully be fixed in future drafts.

Technological Requirements: Because this is an online course, you will need access to a computer with a camera from which to log onto Canvas and Zoom. All class assignments will be submitted electronically through Canvas as Microsoft Word or .pdf files. You will need a USB headset, as experience has shown that Zoom works best when everyone is using a headset. We will also be using several pieces of software throughout the course. These are all freely available online, but will require you to install them. Some of them are JAVA applets that will require you to have a working version of JAVA installed on your computer. Software that we will be using will include GeoGebra, Spherical Easel, CDEG, and non-Euclid. These are all available as links from Canvas.

Course Materials: Because this is a geometry course, it will also require some physical materials. You will need paper, a ruler and compass, tracing paper, tape, and string. You will also want to have a sphere of some kind to look at when we study spherical geometry. Tennis balls work well, as do the plastic spheres often available at craft shops.

Group Work: We will often work in groups in this course. Whenever a group hands in a written assignment, they are required to include the names of those who participated fully, and only those names. Your name on an assignment certifies that you participated equally in the project. It is dishonest to turn in work that is not solely and equitably the creation of the team members. You should not include on the report the name of someone who started but did not finish, or who did not contribute their share. If you are having issues with the way a group is working, please let me know about it.

Outside Sources: A central aim of this course is to help you learn to develop your own ideas about mathematical questions. You therefore should **NEVER** consult any reference materials outside of the course texts in answering questions for this course. This includes materials found on the internet. The ideas that you present should be your own. When you are working on problems to present from the textbook, you may discuss the problems you are working on with other members of the class as long as neither of you has already solved the problem. If you have solved a problem you can try to help another student arrive at the solution themselves, but you MAY NOT just give them your solution.

Office Hours: I will be available every day right after class on Collaborate for office hours. Other office hours will be by appointment. Please do make appointments! I can generally be available on Collaborate before class as well if you let me know in advance. I am also happy to talk to people on the phone. My cell phone number is listed above and you are always welcome to try calling me during reasonable hours. If you want to be sure that I will be available, you can send me an email or talk to me to arrange a time during class.

Attendance: Even though this is an online class, it will be highly interactive. It is absolutely vital for a class like this that you are at all of our class meetings. Because our class time will be spent in student presentations, small group work, and class discussions, it will be challenging to get caught back up after missing any part of class. You will find it to be strongly in your best interest to make every effort to attend every class and to arrive on time.

Initial Assignment: Prior to our first class meeting, you should obtain all required materials for the course; work on the problems from the textbook at least through Problem 19; install GeoGebra on the computer you will be using for the course and explore it a bit; and write an introduction for yourself on the Canvas discussion group. Your introduction should say something about who you are; where you are teaching; and your past experiences with geometry as a student or teacher, whether good or bad.

Co-Requisites/Prerequisites: None

Catalog Course Description: A survey of both traditional Euclidean geometry and contemporary geometries, in which applications of geometry are integrated into the study of the mathematical structure of geometrical systems.

Course Objectives:
After completing this course, students should be able to:

1. Derive and explain geometric arguments and proofs in written and oral form.
2. Decide whether or not geometric arguments given by others are correct.
3. Understand the ideas underlying the typical secondary geometry curriculum well enough to explain them to their own students and use them to inform their own teaching.
4. Understand and explain the relationship between models, axioms, and theorems, and how knowledge about different models can tell us about what theorems can be proven from a given set of axioms.
5. Work effectively in groups, and evaluate when group projects might be appropriate and effective in their own classrooms.

Outline of Course Content:
The course will consist of two pieces:

1. We will work our way through David Clark's *Euclidean Geometry: A Guided Inquiry Approach.* This book gives an axiomatic presentation of Euclidean Geometry in which students prove and present their proofs of the basic facts of Euclidean Geometry at a level that is rigorous but accessible to high school students. (This book has, in fact, been used for some high school classes.)
2. We will also simultaneously work on group projects exploring other aspects of Euclidean and non-Euclidean Geometry.

Topics that will be covered will include:

1. Congruence and Symmetries
2. Euclid's *Elements*
3. The relationship between Axioms, Theorems, Proofs, and Models
4. Spherical Geometry
5. Proofs about areas
6. Proofs about Angles
7. Geometry on the Cone

Grades: Grades will be computed as follows:

- 40% Class Presentations
- 10% Class Participation
- 15% Solution Journals
- 20% Group Projects
- 15% Final Project

Method of Evaluation: +/-letter graded, standard breakdown.

Required Text:
David Clark, *Euclidean Geometry: A Guided Inquiry Approach*, American Mathematical Society, 2012.

UNC's Policies: UNC's policies and recommendations for academic misconduct will be followed. For additional information, please see the Dean of Students website, Student Handbook link http://www.unco.edu/dos/handbook/index.html. Academic Misconduct will not be tolerated, and may result in penalties that range from failing the assignment in question to failing the class as a whole.

Disability Support Services: Any student requesting disability accommodation for this class must inform the instructor giving appropriate notice. Students are encouraged to contact Disability Support Services at (970) 351-2289 to certify documentation of disability and to ensure appropriate accommodations are implemented in a timely manner.

Things may change: It is possible that some of the policies outlined in this syllabus and/or on the class Canvas page may have to be changed at some point during the class, at the discretion of the instructor. If any of these policies are changed, you will be clearly notified.