Algebra I Paul-Hermann Zieschang

Final Submit before December 8, 2021, 11:59 PM

1. Give the definition of a Galois pair. (All details required.)
2. What does one mean by a Galois element of a Galois pair? (You may (and should) refer to the notation which you have introduced in your answer to Question 1.)
3. Explain the Galois correspondence arising from a (commutative) field R as explicitly as possible. (Start with 'Let $\mathcal{S}$ denote the set of all subfields of R. The set $\mathcal{S}$ is ordered with respect to ....' Later: 'We define γ to be ....')
4. Let T be a (commutative) field, let S be a subfield of T, and let t be an element in T. Give the definition of the minimal polynomial of t over S. (All details required, the term *principal ideal* will appear.)
5. Let T be a (commutative) field, let S be a subfield of T. What does it mean for T to be finitely generated (algebraic, normal, separable, galois) over S. (*Not* all details needed.)
6. What do you think is the main result of the ten lectures, the result that all our efforts were aimed at.
7. *Bonus*: [Just short answers, no reasoning.] (i) What is the dimension of $\mathbb{Q}[\sqrt[3]{2}, \zeta]$ over $\mathbb{Q}$? (ii) What is the dimension of $\mathbb{Q}[\sqrt[4]{2}, i]$ over $\mathbb{Q}$? (iii) How many ideals does a field have? (iv) Is $\sqrt[13]{17}$ integral over $\mathbb{Z}$? (v) What is a complex number called that is not algebraic over $\mathbb{Q}$? (vi) Is each euclidean domain a principal ideal domain? (vii) Is each principal ideal domain a unique factorization domain?

You may use the notes, you may even quote word by word from the notes. You may also communicate with each other, but your answers should be in complete sentences and understandable for an interested reader.