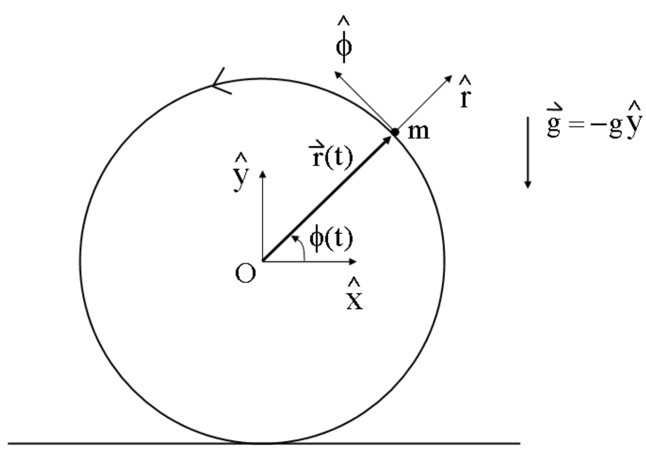
UT Rio Grande Valley — September 18, 2021
PHYS-3305 Classical Mechanics — Andreas Hanke

Problem Set 3 – due Friday, September 24 by 12:00 PM midnight

The Problem Set has **4 questions** on **2 pages**, with a total maximum credit of **30 points**. Please turn in well-organized, clearly written solutions (no scrap work). Question 2 is taken from the textbook.

Problem 1) Looping the loop [8 points]

Consider a roller coaster car of mass m moving inside a circular loop-the-loop section of radius R of a roller coaster, as shown in the figure. The two forces on the car are the gravitational force $\vec{\mathbf{F}}_g = m\vec{\mathbf{g}} = -mg\hat{\mathbf{y}}$ and the normal force $\vec{\mathbf{F}}_n = -F_n\,\hat{\mathbf{r}}$ of the tracks. Here $F_n \geq 0$, so that the tracks push the car towards the center O of the loop.



a) Find the components F_r and F_ϕ of the total force $\vec{\mathbf{F}} = \vec{\mathbf{F}}_g + \vec{\mathbf{F}}_n$ on the car.

b) Write down Newton's 2nd law for the car in polar coordinates.
Hint: Use Taylor, Eq. (1.48) with $r = R = \text{const.}$

c) Assume that at the top of the loop, i.e., for $\phi = 90°$, the normal force $\vec{\mathbf{F}}_n$ on the car vanishes. Find the corresponding speed v_{top} of the car at the top of the loop. Argue why this value of v_{top} is the minimum speed of the car at the top to stay on the track.

Hint: Set $F_n = 0$ in Newton's 2nd law for the component F_r and use $v = R\dot{\phi}$.

Problem 2) Taylor, Problem 2.15 (page 75) “Consider a projectile launched …” [8 points]

Problem 3) Velocity selector (Wien filter) [6 points]

Consider a particle with charge q moving with constant velocity $\vec{\mathbf{v}} = v\,\hat{\mathbf{x}}$ along the x - axis. The particle is moving in perpendicular electric and magnetic fields, $\vec{\mathbf{E}} = E\,\hat{\mathbf{y}}$ and $\vec{\mathbf{B}} = B\,\hat{\mathbf{z}}$. Find the speed v for which the total electromagnetic force on the particle, $\vec{\mathbf{F}} = q\left(\vec{\mathbf{E}} + \vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$, is zero (so that the particle keeps moving along a straight line). Hint: Use $\hat{\mathbf{x}} \times \hat{\mathbf{z}} = -\hat{\mathbf{y}}$.

Problem 4) Charged particle in a magnetic field [8 points]

Consider a particle with charge q and mass m moving in a circular orbit with radius R in the xy - plane, described by the position vector $\vec{\mathbf{r}}(t) = R\,\hat{\mathbf{r}}(t)$ with $\hat{\mathbf{r}}(t) = \cos(\omega t)\,\hat{\mathbf{x}} + \sin(\omega t)\,\hat{\mathbf{y}}$ and constant angular speed ω. The particle is moving in a constant magnetic field $\vec{\mathbf{B}} = -B\,\hat{\mathbf{z}}$ parallel to the z - axis (that is, $\vec{\mathbf{B}}$ is pointing <u>into</u> the xy - plane = paper plane).

a) Find the unit vector $\hat{\phi}(t)$ and show: $\frac{d}{dt}\hat{\mathbf{r}}(t) = \omega\,\hat{\phi}(t)$ and $\frac{d}{dt}\hat{\phi}(t) = -\omega\,\hat{\mathbf{r}}(t)$.

b) Show that the acceleration of the particle is $\vec{\mathbf{a}}(t) = -\omega^2 R\,\hat{\mathbf{r}}(t)$.

c) Show that the magnetic force $\vec{\mathbf{F}}(t) = q\vec{\mathbf{v}}(t) \times \vec{\mathbf{B}}$ on the particle is $\vec{\mathbf{F}}(t) = -q\omega RB\,\hat{\mathbf{r}}(t)$.
Hint: Show first that $\vec{\mathbf{v}}(t) = R\omega\,\hat{\phi}(t)$ and $\hat{\phi}(t) \times \hat{\mathbf{z}} = \hat{\mathbf{r}}(t)$.

d) Use Newton's 2nd law, $\vec{\mathbf{F}}(t) = m\,\vec{\mathbf{a}}(t)$, to show that $\omega = \frac{qB}{m}$ (ω is called cyclotron frequency)

(You don't need to write the argument (t) explicitly every time; it's only done here for clarity.)