UT Rio Grande Valley November 7, 2021
PHYS-3305 Classical Mechanics Andreas Hanke

Problem Set 9 – due Friday, November 12 by 12:00 PM midnight

The Problem Set has **4 questions** on **2 pages**, with a total maximum credit of **30 points**. Please turn in well-organized, clearly written solutions (no scrap work). Questions 2 – 4 are taken from Chapter 8 of the textbook by Taylor.

Problem 1) Constant of motion of the central force problem [8 points]

Consider a two-body central force problem as described in Taylor, Section 8.1: Two bodies with masses m_1 and m_2 interact via a central force described by a potential energy of the form $U\left(\vec{\mathbf{r}}_1, \vec{\mathbf{r}}_2\right) = U\left(\left|\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2\right|\right)$; i.e., the potential energy only depends on the magnitude $\left|\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2\right|$ of the displacement vector $\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2$ between the two bodies. The Lagrange function is given by

$$\mathcal{L}\left(\vec{\mathbf{r}}_1, \vec{\mathbf{r}}_2, \frac{d\vec{\mathbf{r}}_1}{dt}, \frac{d\vec{\mathbf{r}}_2}{dt}\right) = \frac{1}{2} m_1 \frac{d\vec{\mathbf{r}}_1}{dt} \cdot \frac{d\vec{\mathbf{r}}_1}{dt} + \frac{1}{2} m_2 \frac{d\vec{\mathbf{r}}_2}{dt} \cdot \frac{d\vec{\mathbf{r}}_2}{dt} - U\left(\left|\vec{\mathbf{r}}_1 - \vec{\mathbf{r}}_2\right|\right) \qquad (1)$$

with $\vec{\mathbf{r}}_1 = \left(x_1, y_1, z_1\right)$ and $\vec{\mathbf{r}}_2 = \left(x_2, y_2, z_2\right)$.

a) Show that $\mathcal{L}$ is invariant under the coordinate transformation

$\vec{\mathbf{r}}_1 \to \vec{\mathbf{r}}_1\,' = \left(x_1 + s, y_1, z_1\right)$

$\vec{\mathbf{r}}_2 \to \vec{\mathbf{r}}_2\,' = \left(x_2 + s, y_2, z_2\right)$

What invariance property does the two-body central force problem thus have?

b) Due to the Noether theorem, the invariance property in a) implies the constant of motion

$$C = \sum_{i=1}^{6} \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \frac{dq_i\,'}{ds}\bigg|_{s=0} \quad \text{with } \left(q_1, q_2, q_3, q_4, q_5, q_6\right) = \left(x_1, y_1, z_1, x_2, y_2, z_2\right).$$

Find C and identify its physical meaning.

Hint: Only 2 terms contribute to the sum.

The remaining problems are taken from Chapter 8 of the textbook by Taylor.

Problem 8.1 (page 320) "Verify that the positions of two particles... " [6 points]

Problem 8.5 (page 321) "The momentum $\vec{\mathbf{p}}$ conjugate to the relative position ... " [8 points]

Problem 8.8 (page 321) "Two masses m_1 and m_2 move in a plane ... " [8 points]