UT Rio Grande Valley October 30, 2021
PHYS-3305 Classical Mechanics Andreas Hanke

Problem Set 8 – due Friday, November 5 by 12:00 PM midnight

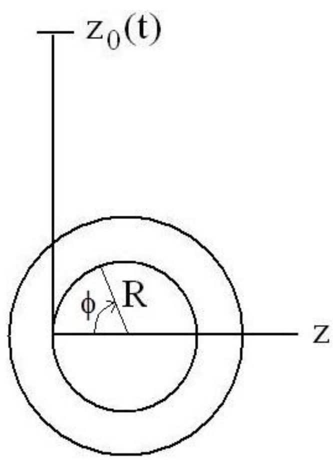
The Problem Set has **5 questions** on **2 pages**, with a total maximum credit of **30 points**. Please turn in well-organized, clearly written solutions (no scrap work). Questions 1, 3 – 5 are taken from Chapter 7 of the textbook by Taylor.

Problem 1) Yoyo (1) [6 points]

Solve Taylor, Problem 7.14 "Figure 7.12 shows a crude model of a yoyo. ... " (page 283). Solve the problem as posed in the question, i.e., using the vertical distance x as generalized (adapted) coordinate. Note that the yoyo is subject to the gravitational force of the Earth, i.e., the potential energy is $U(x) = -mgx$ (with x directed vertically downward, as shown in Figure 7.12).

Problem 2) Yoyo (2) [6 points]

Reconsider the yoyo of Problem 1, but with the following modification: assume that the point at which the string is suspended is not fixed but is given by an external vertical driving $z_0(t)$ (describing an up and down motion of your hand holding the string). The vertical position of the yoyo is $z(t)$ where z is directed vertically downward (see figure below).



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a) Find the Lagrange function $\mathcal{L}(z, \dot{z}, \dot{\phi})$ of the yoyo <u>without</u> the constraint due to the fact that the yoyo is connected to the string.

Hint: The total kinetic energy of the yoyo is $T = \frac{1}{2} m \dot{z}^2 + \frac{1}{2} I \dot{\phi}^2$ (with $I = \frac{1}{2} m R^2$) and the potential energy is $U = -mgz$.

b) Identify the constraint function $c(z, \phi, t) = 0$ describing the fact that the yoyo is connected to the string.

c) Find the Euler Lagrange (EL) equations for the coordinates $z(t)$ and $\phi(t)$.

What is the physical meaning of the Lagrange multiplier λ?

d) Eliminate the Lagrange multiplier λ to find a single equation of motion for $z(t)$.

e) Verify that for $z_0(t) = 0$ the equation of motion for $z(t)$ reduces to the EL equation for $x(t)$ found in Problem 1.

The remaining problems are taken from Chapter 7 of the textbook by Taylor.

Problem 7.21 (page 284) "The center of a long frictionless rod ... " [6 points]

Hint: In this problem the potential energy U is zero, so that there is only kinetic energy T.

Problem 7.23 (page 284) "A small cart (mass m) is mounted ... " [6 points]

Hint: In this problem the kinetic energy T and thus the Lagrange function $\mathcal{L} = T - U$ depends explicitly on time t due to the externally driven oscillation of the large cart, i.e., $\mathcal{L} = \mathcal{L}(x, \dot{x}; t)$.

Problem 7.35 (page 287) "Figure 7.16 is a bird's-eye view ... " [6 points]

Hint: In this problem the potential energy U is zero, so that there is only kinetic energy T.