UT Rio Grande Valley September 5, 2021
PHYS-3305 Classical Mechanics Andreas Hanke

Problem Set 2 – due Friday, September 17 by 12:00 PM midnight

The Problem Set has **5 questions** on **2 pages**, with a total maximum credit of **30 points**. Please turn in well-organized, clearly written solutions (no scrap work). Questions 2 and 3 are taken from the textbook.

Problem 1) Projectile [8 points]

Consider a projectile with position vector $\mathbf{r}(t) = wt\hat{\mathbf{x}} + \left(h - \frac{1}{2}gt^2\right)\hat{\mathbf{y}}$ where $g = 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2}$ is the acceleration of gravity, h is the initial height (at $t=0$), and w is the constant speed in x- direction.

a) Sketch the trajectory $\mathbf{r}(t)$ in the xy - plane.

b) Find the time t_f when the projectile hits the ground (at $y=0$) in terms of h and g.

c) Find the velocity vector $\mathbf{v}(t)$, the speed $v(t) = |\mathbf{v}(t)|$, and the unit vector $\hat{\mathbf{v}}(t) = \frac{\mathbf{v}(t)}{v(t)}$, in terms of w, g, and t.

d) Find the acceleration vector $\mathbf{a}$.

e) Show $\frac{dv}{dt} = a_{\parallel}(t)$ where $a_{\parallel}(t) = \mathbf{a}\cdot\hat{\mathbf{v}}(t)$ is the component of $\mathbf{a}$ in the direction of $\mathbf{v}(t)$. Explain this result in words. For some fixed time t, indicate $\mathbf{r}(t)$, $\mathbf{v}(t)$, $\mathbf{a}$, and $\mathbf{a}_{\parallel}(t)$ in your sketch from a)

(You don't need to write the argument (t) explicitly every time, but it's done here for clarity.)

Problem 2) Taylor, Problem 1.37 (page 39) "A student kicks a frictionless ... " [5 points]

Problem 3) Taylor, Problem 1.39 (page 39) "A ball is thrown ... " [5 points]

Problem 4) One-dimensional harmonic oscillator [6 points]

Consider a mass m with position $x(t)$ along the x- axis. The mass is subject to the harmonic force $F(x) = -kx$ with spring constant k (the minus sign indicates that $F(x)$ is a *restoring* force, pulling the mass back to the origin located at $x = 0$).

a) Use Newton's 2nd law, $F = m\dfrac{d^2x}{dt^2}$, to find the differential equation $\dfrac{d^2x}{dt^2} = -\dfrac{k}{m}x$.

b) Show that the differential equation from part a) is solved by

$$x(t) = a\cos(\omega t) + b\sin(\omega t) \quad ,$$

where a, b are constants, and find ω in terms of k and m.

Problem 5) Two-dimensional harmonic oscillator [6 points]

Consider a mass m with position vector $\vec{\mathbf{r}}(t) = r(t)\,\hat{\mathbf{r}}(t)$ in the xy- plane. The mass is connected by a spring with spring constant k to the origin $\mathbf{r} = 0$, resulting in the harmonic restoring force $\vec{\mathbf{F}}(t) = -k\,r(t)\,\hat{\mathbf{r}}(t)$ on the mass.

a) Write down Newton's second law, $\vec{\mathbf{F}}(t) = m\vec{\mathbf{a}}(t)$, in terms of two-dimensional polar coordinates (r,ϕ). Hint: Use the expression (1.47) for $\vec{\mathbf{a}}(t)$ in polar coordinates. You don't need to write the argument (t) explicitly every time; it's only done here for clarity.

b) Show that a particular solution of the differential equation found in part a) is obtained by setting $r = R =$const. and $\phi(t) = \omega t$ with $\omega=$const. (i.e. the particle is moving in a circular orbit with radius R and angular speed ω).

c) Find ω from part b) in terms of k and m, and compare the result with Problem 4).