Hints for Problem Set 11

Problem 2) Inertia tensor for mass points at the corners of a cube

Taylor, Chapter 10, Problem 10.22 (page 411) "A rigid body comprises 8 equal ... "

Use the definition of the components of the inertia tensor I in Taylor, Eqs. (10.37), (10.38) on page 380. Thus, the diagonal elements are given by:

$$I_{xx} = \sum_{\alpha} m_\alpha \left(y_\alpha^2 + z_\alpha^2 \right) \tag{10.37}$$

and similarly for I_{yy} , I_{zz} . The off-diagonal elements are given by:

$$I_{xy} = -\sum_{\alpha} m_\alpha x_\alpha y_\alpha \;, \quad \text{and so on.} \tag{10.38}$$

In the present problem, the sum over α runs over the 8 masses at the corners of the cube and all masses are equal, i.e., $m_\alpha = m$ for all α .

For part a), consider Taylor, Figure 10.5 (page 382), and consider a rotation of the cube about the z - axis through the origin O. (Rotations about the x - axis and y - axis give the same result due to symmetry). Find the diagonal elements $I_{xx} = \sum_{\alpha} m_\alpha \left(y_\alpha^2 + z_\alpha^2 \right) = m \sum_{\alpha} y_\alpha^2 + m \sum_{\alpha} z_\alpha^2$ and similarly I_{yy} , I_{zz} . You will find that all diagonal elements are equal. Then find the off-diagonal elements $I_{xy} = -\sum_{\alpha} m_\alpha x_\alpha y_\alpha = -m \sum_{\alpha} x_\alpha y_\alpha$, and so on. Again, all off-diagonal elements are equal.

For part b), consider a rotation of the cube parallel to the z - axis through the center of the cube. Again, you will find that all diagonal elements of I are equal. Use a symmetry argument to argue why all off-diagonal elements vanish.

Problem 3) Principal axis transformation of the inertia tensor

For part a), use the procedure in the document “Matrices”, Sections C.2 and C.3 (pages 32-41 of the PDF file posted in Course Materials / Homework 11) to find the three eigenvalues λ_α and eigenvectors $\vec{\mathbf{v}}^{(\alpha)}$ of the matrix I . Then normalize the eigenvectors using $\hat{\mathbf{v}}^{(\alpha)} = \vec{\mathbf{v}}^{(\alpha)} \Big/ \left| \vec{\mathbf{v}}^{(\alpha)} \right|$.