UT Rio Grande Valley | October 2, 2021
PHYS-3305 Classical Mechanics | Andreas Hanke

Midterm Exam – due Friday, October 8 by 12:00 PM midnight

The exam has **4 problems** on **3 pages**. The maximum credit of the exam is **320 points**.
Please submit your exam as a single file (not multiple files) using blackboard,
Course Materials / Midterm Exam.

Problem 1) Three-dimensional harmonic oscillator [60 points]

Consider a mass m with three-dimensional position vector $\vec{\mathbf{r}}(t)$ and velocity vector $\vec{\mathbf{v}}(t) = \frac{d}{dt}\vec{\mathbf{r}}(t)$. The mass is connected by a spring with spring constant k to the origin $\vec{\mathbf{r}} = 0$, resulting in the harmonic restoring force $\vec{\mathbf{F}} = -k\,\vec{\mathbf{r}}$ on the mass m.

a) Write down Newton's 2nd law for the mass m.

b) Show: $\frac{d}{dt}(\vec{\mathbf{r}} \times \vec{\mathbf{v}}) = 0$.

c) Argue why the trajectory of the mass is confined to a plane.

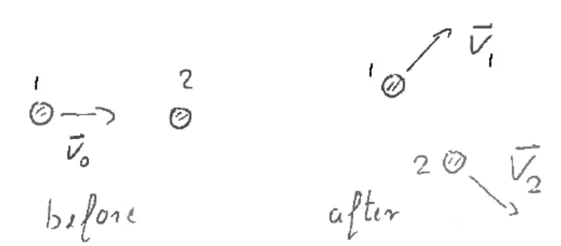

Figure for Problem 2

Problem 2) Billiard balls [60 points]

Two billiard balls of equal mass m are subject to no external forces. Ball 1 is traveling with velocity vector $\vec{\mathbf{v}}_0$ when it collides with the <u>stationary</u> ball 2. After the collision the two balls move off with velocity vectors $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$, respectively (see figure). It is found that $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$ are mutually perpendicular, i.e., $\vec{\mathbf{v}}_1 \perp \vec{\mathbf{v}}_2$, with speeds $v_1 = 3\frac{\text{cm}}{\text{s}}$ and $v_2 = 4\frac{\text{cm}}{\text{s}}$, respectively.

Question: Find the speed v_0 of ball 1 before the collision.

Hint: Use conservation of the total momentum $\vec{\mathbf{P}} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2$ to express $\vec{\mathbf{v}}_0$ in terms of $\vec{\mathbf{v}}_1$ and $\vec{\mathbf{v}}_2$.
Then use $v_0^2 = \vec{\mathbf{v}}_0 \cdot \vec{\mathbf{v}}_0$.

Problem 3) Block sliding on a horizontal plane with friction [100 points]

A student kicks a block of mass m with initial speed v_0 so that it slides on a <u>horizontal</u> plane. The magnitude of the friction force on the sliding block is $f = \mu F_n$ where μ is the coefficient of friction and F_n is the magnitude of the normal force of the plane on the block (see figure).

a) Make a diagram of the sliding block indicating all forces acting on the block, assuming the block is sliding in positive x- direction.

b) Use Newton's 3rd law to find F_n in terms of m and the acceleration of gravity g.

c) Use Newton's 2nd law to find the acceleration a of the block in terms of μ and g.

Hint: Keep in mind that a is negative because the friction force is slowing the block down.

d) Find the velocity $v(t)$ of the block as a function of time t for the initial condition $v(0) = v_0$.

e) Find the time t_f at which the block comes to rest in terms of v_0, μ, and g.

f) Find the position $x(t)$ of the block as a function of time t for the initial conditions $x(0) = 0$ and $v(0) = v_0$.

g) Find the distance x_f the block slides before it comes to rest, in terms of v_0, μ, and g.

(x_f is the horizontal range of the block.)

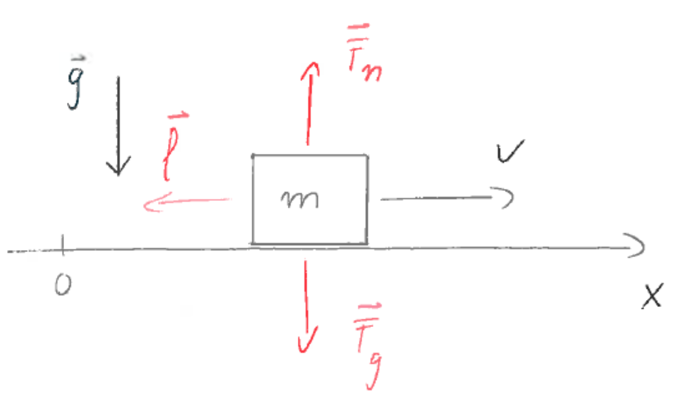

Figure for Problem 3

Problem 4) Accelerating circular motion [100 points]

A mass m is moving on a circular orbit of radius R in the xy - plane with position vector $\vec{\mathbf{r}}(t) = R\hat{\mathbf{r}}(t)$. The unit vector in radial direction is given by $\hat{\mathbf{r}}(t) = \cos[\phi(t)]\,\hat{\mathbf{x}} + \sin[\phi(t)]\,\hat{\mathbf{y}}$ with $\phi(t) = \frac{1}{2}\alpha t^2$, where $\alpha > 0$ is the <u>constant</u> angular acceleration.

a) Find the velocity vector $\vec{\mathbf{v}}(t) = \frac{d}{dt}\vec{\mathbf{r}}(t)$ in terms of the unit vector $\hat{\phi}(t)$ (planar polar coordinates).

b) Find the speed $v(t) = |\vec{\mathbf{v}}(t)|$.

c) Find the acceleration vector $\vec{\mathbf{a}}(t) = \frac{d}{dt}\vec{\mathbf{v}}(t)$ in terms of the unit vectors $\hat{\mathbf{r}}(t)$, $\hat{\phi}(t)$ (planar polar coordinates).

Result: $\vec{\mathbf{a}} = R\alpha\hat{\phi} - R(\alpha t)^2\,\hat{\mathbf{r}}$

d) Find the total force $\vec{\mathbf{F}}$ on the mass m in terms of the unit vectors $\hat{\mathbf{r}}(t)$, $\hat{\phi}(t)$.

Hint: Use Newton's 2nd law and the result from c).

e) Show: $\vec{\mathbf{a}}(t)\cdot\hat{\phi}(t) = \frac{d}{dt}v(t)$ (tangential acceleration). Hint: Use the results of c) and b).

f) Show: $\vec{\mathbf{a}}(t)\cdot\hat{\mathbf{r}}(t) = -R(\alpha t)^2$ (centripetal acceleration).