Problem Set 10 – Solutions 11/20/2021

Problem 9.2 (page 360) "A donut-shaped space station ... "

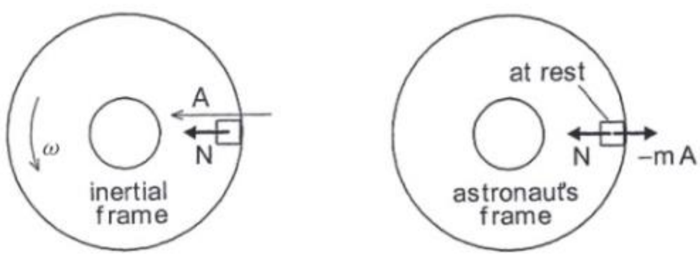
a) Refers to the inertial system C. Choose C such that the space station is in the xy - plane and centered at the origin. Sketch the normal force and centripetal acceleration acting on the astronaut as seen from C.

b) Refers to the accelerated (rotating) system C' in which the rotating space station and the astronaut are at rest. Choose C' such that the space station is in the $x'y'$ - plane and centered at the origin. Sketch the normal force and the inertial force (centrifugal force) acting on the astronaut as seen from C'. Find the centrifugal acceleration $\mathbf{a}'_{\text{cf}} = (\boldsymbol{\omega}\times\mathbf{r}')\times\boldsymbol{\omega}$ on the astronaut in C' (magnitude and direction) using $\boldsymbol{\omega} = \omega\,\hat{\mathbf{e}}'_z$ and $\mathbf{r}' = R\,\hat{\mathbf{e}}'_r$ in C' (cylindrical coordinates).

c) Use the different values of R to find the corresponding values of the perceived $g = |\mathbf{a}'_{\text{cf}}|$ in C'.

Solution:

9.2 ⋆ **(a)** As seen by inertial observers outside the station, the (square) astronaut has a centripetal acceleration $A = \omega^2 R$ which is supplied by the normal force $\mathbf{N}$.



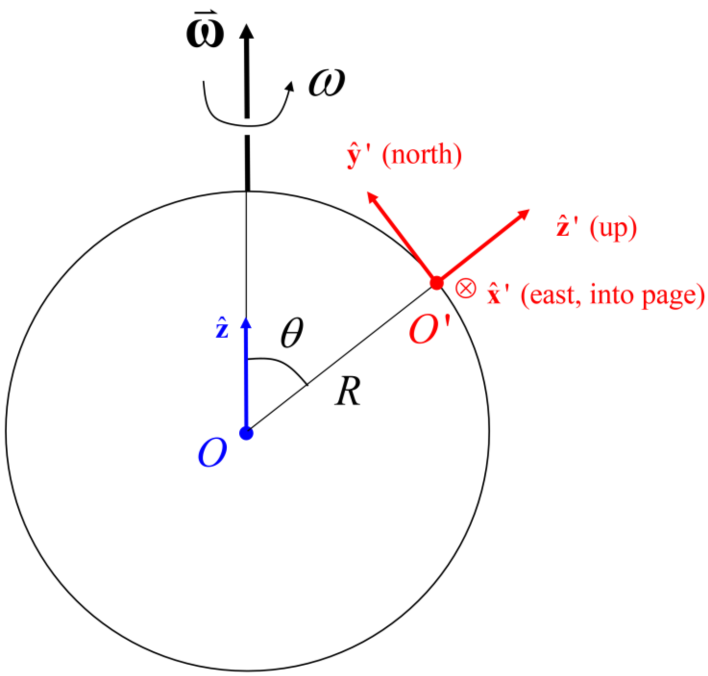
(b) As seen by the crew inside the station, the astronaut is at rest under the action of two forces, the normal force $\mathbf{N}$ and the inertial force $-m\mathbf{A}$. To simulate normal gravity, we must have $A = \omega^2 R = g$ or $\omega = \sqrt{g/R} = 0.5$ rad/s $= 4.8$ rpm.

(c) The apparent gravity $g_{\text{app}} = \omega^2 R$ is proportional to R. Thus if we decrease R from 40 m to 38 m, the fractional change in g_{app} is $\delta g_{\text{app}}/g_{\text{app}} = \delta R/R = -5\%$.

Problem 9.9 (page 361) "A bullet of mass m is fired ... "

Consider the inertial system C (blue, unprimed) and accelerated system C' (red, primed) as shown in the figure below. The system C is centered at the center of the Earth with the z - axis pointing to the north pole, so that $\boldsymbol{\omega} = \omega\hat{\mathbf{z}}$ in C. The system C' is centered at the location on Earth (fixed at Earth's surface) where the bullet is fired. R is the radius of the Earth and the colatitude θ corresponds to the polar angle θ in spherical polar coordinates as shown.

The figure corresponds to Taylor, Figure 9.15 (page 352). However, we here use our notation, where unprimed/blue coordinates refer to C and primed/red coordinates refer to C'. We also write $\boldsymbol{\omega}$ instead of $\boldsymbol{\Omega}$ (and ω instead of Ω for the magnitudes).



Using our notation, the Coriolis force on the bullet in C' is given by $\mathbf{F}'_{cor} = 2m\mathbf{v}'\times\boldsymbol{\omega}$ where $\mathbf{v}' = v_0\hat{\mathbf{y}}'$ and $\boldsymbol{\omega} = \omega\hat{\mathbf{z}}$. Find the direction and magnitude of $\mathbf{F}'_{cor}$. For the comparison with the bullet's weight, consider the ratio $|\mathbf{F}'_{cor}|/(mg)$ and use $\omega = 2\pi/(24\text{h})$.

Solution:

9.9 ⋆ $\mathbf{F}'_{\text{cor}} = 2m\mathbf{v}'\times\boldsymbol{\Omega} = 2mv_o\Omega\cos\theta$ due east, and

$$\frac{F'_{\text{cor}}}{mg} = \frac{2v_o\Omega\cos\theta}{g} = \frac{2\times(1000\text{ m/s})\times(7.3\times10^{-5}\text{ rad/s})\times(\cos 40°)}{9.8\text{ m/s}^2} = 0.0114.$$

Problem 9.14 (page 362) "I am spinning a bucket of water ... "

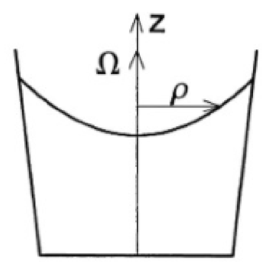
In the rotating frame C' of the bucket, the water is in mechanical equilibrium and its surface is an equipotential surface for the combined gravitational force $-mg\hat{\mathbf{z}}'$ (downward) and centrifugal force $m\omega^2\rho'\hat{\boldsymbol{\rho}}'$ in C' (radially outward, away from the rotation axis) where ρ' is the distance from the rotation axis (cylindrical coordinates). The corresponding potential energies are mgz' and $-m\omega^2\rho'^2/2$, respectively. For an equipotential surface, the sum of these potential energies is constant, U_0. Find z' as a function of ρ' for the equipotential surface.

Solution (note: in our notation, we would write z' instead of z and ρ' instead of ρ)

9.14 ⋆⋆ In the rotating frame of the bucket, the water is in equilibrium and its surface is an equipotential surface for the combined gravitational force (PE $= mgz$) and centrifugal force (force $= m\Omega^2\rho$ and hence PE $= -m\Omega^2\rho^2/2$). Therefore, the surface is given by $mgz - m\Omega^2\rho^2/2 =$ const, or

$$z = \frac{\Omega^2\rho^2}{2g} + \text{const},$$

which is a parabola, as claimed.

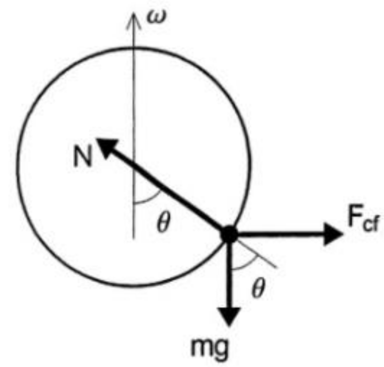


Problem 9.17 (page 362) "Consider the bead threaded on a circular hoop ... "

Consider the rotating frame C' of the hoop in which the hoop is fixed in the $x'y'$- plane and centered at the origin. The only degree of freedom is the angle θ shown in Taylor, Figure 7.9 (page 261). Indicate the centrifugal force $\mathbf{F}'_{\text{cf}}$ and gravitational force $\mathbf{F}_{\text{grav}}$ on the bead in C'. These are the only two forces that affect the equation of motion for θ in C'. Find the equation of motion $ma_{\text{tang}} = F_{\text{tang}}$ where $a_{\text{tang}} = R\ddot{\theta}$ and F_{tang} is the component of the net force $\mathbf{F}'_{\text{cf}} + \mathbf{F}_{\text{grav}}$ tangential to the hoop in C'. Show that the equation of motion for $\ddot{\theta}$ corresponds to Eq. (7.69) on page 261.

Solution (note: in our notation, we would write $\mathbf{F}'_{cf}$ instead of $\mathbf{F}_{cf}$)

9.17 ⋆ As seen in a frame rotating with the hoop, there are five forces on the bead. The first three, all of which act in the plane of the hoop, are the bead's weight $m\mathbf{g}$, the centrifugal force $\mathbf{F}_{cf} = m\omega^2 R \sin\theta \hat{\boldsymbol{\rho}}$, and the normal force $\mathbf{N}$ (actually the component of the normal force in the plane of the hoop). The other two are the Coriolis force $\mathbf{F}_{cor}$ and the component of the normal force normal to the hoop (neither of which is shown in the picture). Since these last two both act normal to the hoop, they cancel one another and need not concern us further. The bead can move only in the tangential direction, and its equation of motion is $ma_{tang} = F_{tang}$ or



$$mR\ddot{\theta} = F_{cf}\cos\theta - mg\sin\theta = (m\omega^2 R\sin\theta)\cos\theta - mg\sin\theta,$$

whence $\ddot{\theta} = (\omega^2\cos\theta - g/R)\sin\theta$, in agreement with Eq.(7.69).

If the bead is in equilibrium, the tangential components of the centrifugal force and the weight must balance. That is, $F_{cf}\cos\theta = mg\sin\theta$, or $\cos\theta = g/(\omega^2 R)$, since $F_{cf} = m\omega^2 R\sin\theta$. This is the same as Eq.(7.71).