Midterm Exam - Solutions

Problem 1

a) N2: $\vec{F} = m\vec{a} \Rightarrow -k\vec{r} = m\vec{a}$

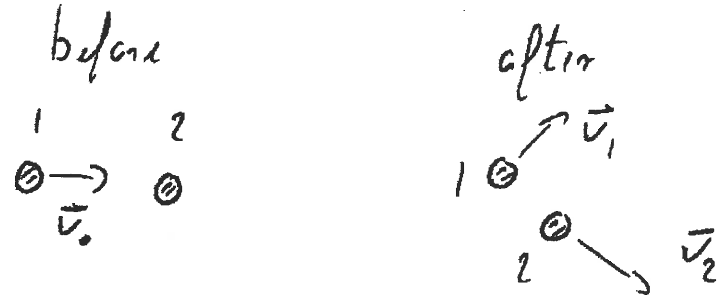
$$\Rightarrow \vec{a} = -\frac{k}{m}\vec{r} \quad (1)$$

b) $$\frac{d}{dt}(\vec{r}\times\vec{v}) = \underbrace{\left(\frac{d}{dt}\vec{r}\right)}_{\vec{v}}\times\vec{v} + \vec{r}\times\underbrace{\left(\frac{d}{dt}\vec{v}\right)}_{\vec{a}}$$

$$= \underbrace{\vec{v}\times\vec{v}}_{=0} + \vec{r}\times\vec{a} \overset{(1)}{=} -\frac{k}{m}\underbrace{\vec{r}\times\vec{r}}_{=0} = 0$$

c) $\vec{r}\times\vec{v} = \frac{\vec{l}}{m}$ conserved $\Rightarrow$ $\vec{r}, \vec{v}$ stay in plane $\perp \vec{l}$

Problem 2



$$m\vec{v}_0 = m\vec{v}_1 + m\vec{v}_2$$

$$\Rightarrow \vec{v}_0 = \vec{v}_1 + \vec{v}_2$$

$$v_0^2 = \vec{v}_0\cdot\vec{v}_0 = (\vec{v}_1+\vec{v}_2)\cdot(\vec{v}_1+\vec{v}_2)$$

$$= v_1^2 + 2\underbrace{\vec{v}_1\cdot\vec{v}_2}_{=0,\ b/c\ \vec{v}_1\perp\vec{v}_2} + v_2^2 = v_1^2 + v_2^2$$

$$= (3^2+4^2)\left(\frac{cm}{s}\right)^2 = 25\left(\frac{cm}{s}\right)^2 \rightarrow v_0 = 5\ \frac{cm}{s}$$

Problem 3)

a) See figure in question

b) $\vec{F}_g = -mg\hat{y}$

$\Rightarrow \vec{F}_n \overset{N3}{=} -\vec{F}_g = mg\hat{y}$

$\Rightarrow F_n = mg$

c) Net force on sliding block: $F = -f$

$F \overset{N2}{=} ma = -f = -\mu F_n \overset{b)}{=} -\mu mg$

$\Rightarrow a = -\mu g$

d) $v(t) = v_0 + at \overset{c)}{=} v_0 - \mu g t$

e) $v(t_f) \overset{d)}{=} v_0 - \mu g t_f = 0$ (rest)

$\Rightarrow t_f = \frac{v_0}{\mu g}$

f) $x(t) = \underbrace{x_0}_{=0} + v_0 t + \frac{1}{2} a t^2 \overset{c)}{=} v_0 t - \frac{1}{2}\mu g t^2$

g) $x_f = x(t_f) \overset{f)}{=} v_0 t_f - \frac{1}{2}\mu g t_f^2 \overset{e)}{=} \frac{1}{2}\frac{v_0^2}{\mu g}$

Problem 4

cp. Figure 1.11b, p. 27

$\vec{r}(t) = R\,\hat{r}(t)$, $R = \text{const}$, $\theta(t) = \frac{1}{2}\alpha t^2$

a) $\vec{v} = \frac{d}{dt}\vec{r} = R\frac{d}{dt}\hat{r} \underset{\uparrow}{=} R\dot{\theta}\hat{\theta} = R\alpha t\,\hat{\theta}$

(1.42), p. 27

b) $v = |\vec{v}| \overset{a)}{=} R\alpha t$

c) $\vec{a} = \frac{d}{dt}\vec{v} \overset{a)}{=} R\alpha\hat{\theta} + R\alpha t \underbrace{\frac{d}{dt}\hat{\theta}}_{-\dot{\theta}\hat{r},\ (1.46)}$

$= R\alpha\hat{\theta} - R(\alpha t)^2\hat{r}$

d) $\vec{F} \overset{N2}{=} m\vec{a} \overset{c)}{=} mR\alpha\hat{\theta} - mR(\alpha t)^2\hat{r}$

e) $a_{\parallel} = \vec{a}\cdot\hat{\theta} \overset{c)}{=} R\alpha$ using $\hat{\theta}\cdot\hat{\theta} = 1$, $\hat{r}\cdot\hat{\theta} = 0$

$\frac{d}{dt}v \overset{b)}{=} R\alpha \implies a_{\parallel} = \frac{d}{dt}v$ speeding up

f) $a_{\perp} = \vec{a}\cdot\hat{r} \overset{c)}{=} -R(\alpha t)^2 = -R\omega^2$

centripetal acceleration (here $\omega = \dot{\theta} = \alpha t$)