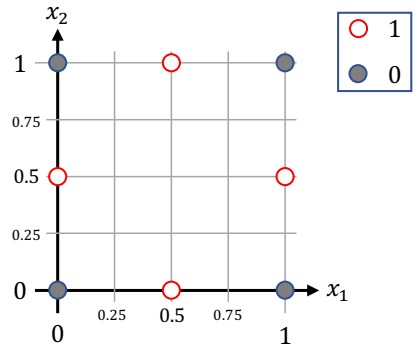
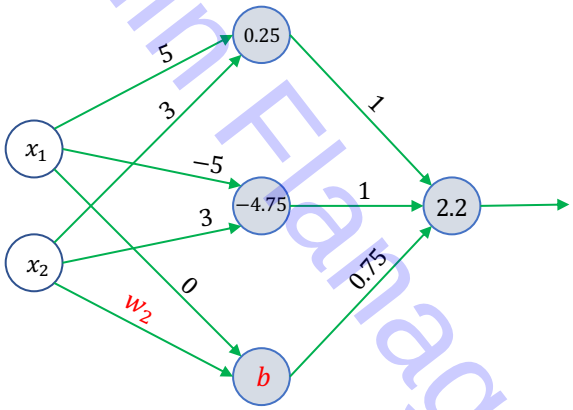
1. Consider the following classification problem.



(a) Design a neural network with a single hidden layer to solve the problem. Your network should employ hard nonlinearities. It should be a *maximum margin classifier*, i.e., the decision boundary should be as far away from the training points as possible.

(b) If sigmoids instead of hard nonlinearities are employed, the following network can solve the problem, given appropriate values for w_2 and b.



Find appropriate values and justify your choices. You should approach the problem in a structured fashion.

(c) Comment on the solution in (b). In particular, is it a maximum margin solution? Would it work using hard nonlinearities?

2. (a) Consider a 2-input AdaBoost system learning the mapping

$$\begin{aligned}(-1,-1) &\rightarrow -1,\\ (-1,+1) &\rightarrow +1,\\ (+1,-1) &\rightarrow +1,\\ (+1,+1) &\rightarrow -1\end{aligned}$$

using weighted weak linear base classifiers.

The first base classifier generates the discriminator function

$$h_1(x,y) = \text{sgn}(1-x-y)$$

(note that $\text{sgn}(z) = -1$ if $z \leq 0$, otherwise $\text{sgn}(z) = 1$).

What is α_1? What is the weight distribution used to generate the second base classifier? What are ϵ_2, h_2 and α_2?

(b) Consider a general AdaBoost classifier.

i. Show that a weak learner with $\epsilon = 0.5$ makes *no contribution* to the behaviour of the overall strong classifier.

ii. If weak learner i returns $\epsilon_i = 0.5$ show that learning has *stopped* and that there is no point in generating any more weak learners.