

1. In the notes the Wolfe dual is used to find a constrained minimum of $(x - 2)^2$ subject to $x \geq 4$, and later to find a minimum of $(x - 2)^2$ subject to $x \geq -2$. But the Wolfe dual approach works for any convex objective, e.g., $x \log x$.
 - (a) Show that $x \log x$ is strictly convex for $x > 0$.
 - (b) Find a minimum of $x \log x$ subject to $x \geq 0.5$.
 - (c) Find a minimum of $x \log x$ subject to $x \geq 0.25$.

2. (a) Show that the dual form of the Lagrangian for a hard-margin, linear SVM solving the 2-input AND problem may be represented as

$$W(\lambda_2, \lambda_3, \lambda_4) = 2\lambda_4 + \lambda_2\lambda_4 + \lambda_3\lambda_4 - 0.5\lambda_2^2 - 0.5\lambda_3^2 - \lambda_4^2.$$

State clearly all assumptions you make.

- (b) To find an optimum solution for λ_i , W must be maximized subject to certain constraints. What are these constraints?
 - (c) Find an appropriate solution for W , stating clearly which variables are slack, and why.
 - (d) Find the equation of the maximum margin hyperplane for this problem.
3. (a) Can a linear SVM solve the 2-input XOR problem? Explain.
 - (b) Does the introduction of the *nonlinear kernel*

$$K(\vec{v}, \vec{w}) = v_1w_1 + v_2w_2 + v_1v_2w_1w_2$$

allow the problem to be solved? (Note that $\vec{v} = [v_1, v_2]^T$, etc.) Discuss.

4. In the notes it was claimed that the 2-input XOR problem may be solved using SVM methods by first performing the nonlinear mapping:

$$(x, y) \longrightarrow (x, y, (x + y)^2).$$

Show this to be the case by finding the Lagrange multipliers and associated weights and bias for the mapped problem. Demonstrate that the mapped problem with these weights and bias does actually solve the 2-input XOR problem.