(c) Step 1. Understand Dilated kernel

Dilation rate = 2 means the kernel is applied with 1 element skipped between values.

So, instead of a standard 3×3 sliding window, the receptive field spans a 5×5 area

Step 2: Effective o/p size.

With

i/p size : 7×7

Receptive field size (from dilation): 5×5.

Stride : 1.

No padding.

The o/p dimension is.

$$\left\lfloor \frac{7-(3-1)\times d-1}{s} + 1 \right\rfloor = \left\lfloor \frac{7-4-1}{1} + 1 \right\rfloor = 3.$$

So o/p is 3×3.

Step 3: Compute Convolution.

$$k = \begin{bmatrix} -1 & 0 & 1 \\ -1 & 1 & -1 \\ 1 & 1 & -1 \end{bmatrix}$$

We'll apply the kernel using the dilated receptive field at each valid position.

~~Apply Dilated conv.~~

~~Posn (0,0)~~

Example top left corner of output

Extracting 5×5 region from I starting at (0,0):

5 4 5 4 5

1 4 4 5 1

3 2 4 1 5

3 2 4 2 1

0 2 1 2 1

selecting values with dilation:

5 5 5

1 4 1

0 1 1

Apply kernel element-wise:

$(-1)(5) + (0)(5) + (-1)(5) + (-1)(1) + (1)(4) + (-1)(1)$
$+ (1)(0) + (1)(1) + (-1)(1)$
$= -5 + 0 + 5 - 1 + 4 - 1 + 0 + 1 - 1 = 2$

Continuing similarly for all results, we get

$$\begin{bmatrix} 2 & 6 & 0 \\ -2 & 3 & 1 \\ 2 & 4 & 4 \end{bmatrix}$$

Step 1: Transpose convolution.

A transpose convolution with stride $s = 2$ can be thought of as upsampling the input X by inserting zeros between its elements and then performing a regular convolution with the kernel k. The output size of the transpose convolution is calculated as:

$$O = (X_{size} - 1) \times S + k_{size}.$$

for $X_{size} = 2$, $s = 2$, and $k_{size} = 3$:

$$O = (2 - 1) \times 2 + 3 = 5$$

So, the output Y will be a 5×5 matrix

Step 2: Upsample the i/p X.

Insert zeros between the elements of X and pad it with zeros, to match the size ~~of~~ required. for the direct convolution. The upsampled X (denoted as X_{up}) is.

$$X_{up} = \begin{bmatrix} 3 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 3: Flip the kernel.

For transpose conv., the kernel k is flipped both horizontally and vertically

The flipped kernel K is flipped both horizontally and vertically. The flipped kernel K flipped is:

$$K_{flipped} = \begin{bmatrix} -2 & 3 & 2 \\ 1 & 1 & 2 \\ -1 & -1 & 2 \end{bmatrix}$$

Step 4: Perform Direct Convolution.

Convolve the upsampled X_{up} with flipped kernel $K_{flipped}$ using a stride of 1 and no padding.

The output Y is calculated as follows:

o/p at posn. (1,1):

$$3 \times (-2) = -6.$$

o/p at posn (1,2)

$$3 \times 3 + 1 \times (-2) = 9 - 2 = 7$$

o/p at posn (1,3)

$$3 \times 2 + 1 \times 3 + 1 \times 3 = 6 + 3 = 9.$$

o/p at posn (1,4)

$$1 \times 2 = 2.$$

O/p at pos^n (2,1):

$$3 \times 1 + 4 \times (-2) = 3 - 8 = -5$$

O/p at pos^n (2,2):

$$3\times1+1\times1+4\times3+1\times(-2) = 3+1+12-2 = 14$$

O/p at pos^n (2,3):

$$3\times2+1\times1+4\times2+1\times3 = 6+1+8+3 = 18.$$

O/p at pos^n (2,4):

$$1\times2+1\times1 = 2+1 = 3.$$

O/p at pos^n (3,1):

$$4\times1+3\times(-1) = 4-3 = 1.$$

O/p at pos^n (3,2):

$$4\times1+1\times1+3\times(-1)+1\times3 = 4+1-3+3 = 5$$

O/p at pos^n (3,3):

$$4\times2+1\times(-1)+3\times2+1\times(-1)$$

$$= 8-1+6-1 = 12.$$

O/p at pos^n (3,4)

$$1\times2+1\times(-1) = 2-1 = 1.$$

O/p at pos^n (4,1).

$$4\times(-1) = -4.$$

O/p at pos^n (4,2)

$$4\times(-1)+1\times1 = -4+1 = -3.$$

O/p at pos^n (4,3)

$$4\times2+1\times(-8) = 8-1 = 7$$

o/p at pos^n (4,4)

$1 \times 2 = 2$.

o/p at pos^n (5,1); (5,2), (5,3), (5,4), (5,5):

0 (no verlap)

final o/p matrix $Y = \begin{bmatrix} -6 & 7 & 9 & 2 & 0 \\ -5 & 14 & 18 & 3 & 0 \\ 1 & 5 & 12 & 1 & 0 \\ -4 & -3 & 7 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$