

1) (10 pts) ANL (Algorithm Analysis)

What is the worst case Big-Oh runtime for the function **f**, in terms of its input parameter **n**? You may assume that the array pointed to by **arr** is of length **n**. (Grading note: 2 pts will be awarded for the answer, 8 pts for the proof of the answer. Your proof must include either summations or recurrence relations related to the code below.)

```
int f(int* arr, int n, int minVal) {
    return fHelp(arr, 0, n-1, minVal);
}

int fHelp(int* arr, int low, int high, int minVal) {
    if (low > high) return 0;
    if (low == high) return arr[low] >= minVal;

    int mid = (low+high)/2;
    int left = fHelp(arr, low, mid, minVal);
    int right = fHelp(arr, mid+1, high, minVal);
    int res = left;
    if (right > left)
        res = right;

    int alt = 0, i;
    for (i=mid; i>=low; i--) {
        if (arr[i] < minVal) break;
        alt++;
    }
    for (i=mid+1; i<=high; i++) {
        if (arr[i] < minVal) break;
        alt++;
    }

    if (alt > res) res = alt;
    return res;
}
```

There are two recursive calls in **fHelp**, both to arrays of half the size of the original array. Let $T(n)$ be the run time of function **f**. Effectively, $T(n)$ breaks down into two function calls, each of which take time $T(n/2)$, plus the work after the recursive calls. There are two loops, each which run $n/2$ times at most, so in total the loops run n times with constant time operations inside the loops. Thus, the total amount of work beyond the recursive calls is $O(n)$. It follows that $T(n)$ satisfies the following recurrence relation:

$$T(n) = 2T(n/2) + O(n).$$

We can solve this recurrence relation via the Master Theorem, getting a solution of **$O(n \lg n)$** . ($A = 2$, $B = 2$ and $k = 1$. Since $B^k = 2$ and $A = 2$, the solution follows.)

Grading: 2 pts for correct answer (give pts even if no work)

2 pts for ANY recurrence relation

2 pts (additional) if recurrence has $T(n/2)$ on RHS

2 pt (additional) if term is $2T(n/2)$

2 pt for $O(n)$ added extra work