2) (5 pts) DSN (Sorting)

Quick Sort is not a stable sort. This means that if there are two elements in the input array, a[i] and a[j], are considered equal before the sort is executed with i < j, the element originally stored in a[j] may appear ***before*** the element originally stored in a[i]. Namely, the relative order of equivalent elements may not be maintained in the sort. (For example, if we are sorting only by last name and in the original list "Doug Adams" appeared before "Sandy Adams", then after the sort, it's possible that "Sandy Adams" could appear before "Doug Adams".)

The reason this is the case is due to the partition function used in Quick Sort. Explain why the partition function doesn't maintain the stability property mentioned above and provide a specific example where stability isn't maintained. In your example, you can use letters with subscripts and sort the letters in alphabetical order.

In the partition function, there is both a low pointer and a high pointer for which the contents potentially get swapped. This means that without inspecting other items, an element in the lowest part of the array can be swapped into the highest part of the array, potentially skipping over an equivalent item.

Consider the partition of the following subarray, from index x to index y, inclusive, using index x as the partition element.

D_1	A_1	F_1	B_1	A_2	F_2	Q_1	B_2	G_1
x		low					high	y

The typical in-place implementation of partition would move index low upto the letter F, moving past A, since F comes after D and would move index high down to B since B comes before D. After identifying these two items that are out of place (F on the left side of the array and B on the right side of the array), these items are swapped, so the new subarray looks like this:

D_1	A_1	B_2	B_1	A_2	F_2	Q_1	F_1	G_1
x		low					high	y

as the partition concludes, no further changes are made to the array from indexes x to low, or from indexes high to y, as shown in the diagram above.

Notice that with this one swap, what has happened is that F_1, which used to appear before F_2 in the array now appears after it. Similarly, B_2, which used to appear after B_1, got swapped into a position such that it now appears before B_1.

Grading: 3 pts for a general explanation of how partition swaps elements far away, and 2 pts for the specific example. The example need not use letters and subscripts, but it must illustrate the idea clearly. Also, there are many different examples, even examples where the instability deals with the pivot element. Give full credit to all responses that have a trace through that includes instability.