

2) (10 pts) ANL (Algorithm Analysis)

An algorithm that processes a grid of R rows and C columns runs in $O(R \lg C)$ time. It turns out that for any R by C grid, we can transpose the grid so that it has C rows and R columns instead and solve the problem on that grid to get the same answer. Fred ran the code with $R = 10^6$ and $C = 10^2$ and it took **3 hours** to run. Shanille transposed the grid and reran the code with $R = 10^2$ and $C = 10^6$ to prove to Fred how inefficient his technique was. How long, **in seconds**, would be expect Shanille's execution of the code to take? **Please answer as a decimal to two places.**

Let $T(R, C) = kR \lg C$ be the run-time of the algorithm on a grid with R rows and C columns, where k is a constant.

$$T(10^6, 10^2) = k10^6 \lg(10^2) = 3 \text{ hours}$$

$$k(2 \times 10^6) \lg(10) = 3 \text{ hours} \quad \rightarrow k = \frac{3 \text{ hours}}{2 \times 10^6 \lg(10)}$$

Now, we must solve for $T(10^2, 10^6)$:

$$\begin{aligned} T(10^2, 10^6) &= \left(\frac{3 \text{ hours}}{2 \times 10^6 \lg(10)} \right) 10^2 \lg(10^6) \\ &= \left(\frac{3 \text{ hours}}{2 \times 10^6 \lg(10)} \right) 10^2 (6) \lg 10 \\ &= \left(\frac{18 \text{ hours}}{2 \times 10^4} \right) \times \frac{60 \text{ min}}{1 \text{ hour}} \times \frac{60 \text{ sec}}{1 \text{ min}} \\ &= \left(\frac{9 \text{ hours}}{10^4} \right) \times 3600 \text{ sec/hr} \\ &= (9)(.3600) \text{ sec} \\ &= 3.24 \text{ seconds} \end{aligned}$$

Grading:

- 1 pt setting up equation for k
- 2 pts solving for k no simplification
- 2 pts setting up equation for Shanille
- 2 pts correctly converting from hours to seconds
- 1 pt log simplification
- 2 pts rest of the simplification to the correct final form