## 3) (10 pts) ANL (Recurrence Relations)

Using the iteration technique, determine a closed-form solution for the following recurrence relation in terms of n. Note: Your answer should be **EXACT** and not a Big-Oh bound.

$$T(0) = 1$$

$$T(n) = 4T(n-1) + 2^n, for integers n > 0$$

Using the iteration technique for three iterations, we get:

$$T(n) = 4T(n-1) + 2^{n}$$
 Grading: 1 pt 
$$T(n) = 4(4T(n-2) + 2^{n-1}) + 2^{n}$$
 
$$T(n) = 16T(n-2) + (2^{2})2^{n-1} + 2^{n}$$
 Grading: 2 pts 
$$T(n) = 16(4T(n-3) + 2^{n-2}) + [2^{n+1} + 2^{n}]$$
 Grading: 2 pts 
$$T(n) = 64T(n-3) + (2^{4})2^{n-2} + [2^{n+1} + 2^{n}]$$
 T(n) =  $64T(n-3) + [2^{n+2} + 2^{n+1} + 2^{n}]$  Grading: 2 pts

From here we see that after k iterations, the recurrence is:

$$T(n) = 4^k T(n-k) + \sum_{i=0}^{k-1} 2^{n+i}$$
 Grading: 2 pts

Since we know T(0), let's plug in k = n into the formula above to yield:

$$T(n) = 4^{n}T(n-n) + \sum_{i=0}^{n-1} 2^{n+i}$$
 Grading: 1 pt 
$$T(n) = 4^{n}T(0) + \sum_{i=0}^{n-1} 2^{n}2^{i}$$
 
$$T(n) = 2^{2n} + 2^{n}\sum_{i=0}^{n-1} 2^{i}$$
 
$$T(n) = 2^{n}(2^{n}) + 2^{n}\sum_{i=0}^{n-1} 2^{i}$$
 
$$T(n) = 2^{n}\left[2^{n} + \sum_{i=0}^{n-1} 2^{i}\right]$$
 
$$T(n) = 2^{n}\left[\sum_{i=0}^{n} 2^{i}\right]$$
 
$$T(n) = 2^{n}\left(\frac{2^{n+1}-1}{2^{-1}}\right)$$
 Grading: 2 pts

Grading Note: Please accept any reasonable final form (there are quite a few that are reasonably simplified). For the last two points, award 1 pt if there is some algebra but not quite enough to get to a reasonable final form.