2) (10 pts) ANL (Algorithm Analysis)

Querying a user in our data base of 10^4 users take about 10 milliseconds. The runtime of the query is logarithmic with respect to the number of users. Namely, if there are n users, a query takes O(lg n) time. About how many users can we support while taking no more than 20 milliseconds per query?

Let t(n) represent the run time of a single query. Then, for some constant c, we have

t(n) = c(lg n)

Let n be the answer to the question. Using the given information, we can set up a ratio between the two run-times as follows:

$$\frac{c(\lg n)}{c(\lg 10^4)} = \frac{20\ ms}{10\ ms}$$

$$\frac{\lg n}{4lg10} = 2$$

$$\lg n = 8lg10$$

$$\lg n = lg10^8$$

$$\boldsymbol{n = 10^8}$$

Note: There are quite a few other ways to arrive at the result.

Grading: 2 pts for setting up the run time as a constant times log.
2 pts for setting up a variable to store the answer to the question.
2 pts for writing down the appropriate equations.
4 pts for solving the equations for the correct value of n. (Give partial as needed.)