

2) (10 pts) ANL (Algorithm Analysis)

A program takes $O(n \lg m)$ time to process n data sets, each which have m values. For 100,000 data sets, each with 2^{16} values, the program takes 500 ms (milliseconds) to complete. Given this information, how many milliseconds would we expect the program to take to process 60,000 data sets, each with 2^{20} values?

For some constant c , let $T(n, m) = cn \lg m$ be the run time of the algorithm for a given input size of n data sets, each with m values. Using the given information, we have:

$$\begin{aligned} T(100000, 2^{16}) &= c(100000)(\lg 2^{16}) = 500ms \\ c(100000)(16 \lg 2) &= 500ms \end{aligned}$$

$$c = \frac{500ms}{(100000)(16 \lg 2)} = \frac{1ms}{3200 \lg 2}$$

Now, using this value of c , solve for the desired information:

$$\begin{aligned} T(60000, 2^{20}) &= \frac{1ms}{3200 \lg 2} \times 60000 \times (\lg 2^{20}) \\ &= \frac{1ms \times 60000 \times 20 \times \lg 2}{3200 \times \lg 2} \\ &= \frac{1ms \times 60000}{160} = \frac{3000ms}{8} = 375ms \end{aligned}$$

Grading: 2 pts - Setting up equation for constant

2 pts – Solving for c (no simplification necessary)

2 pts – Plugging in appropriate new values to solve problem

4 pts – arriving at correct answer in ms (give partial as necessary)