3) (10 pts) ANL (Recurrence Relations)

What is the closed form for the following recurrence relation, T(N)? For full credit your work must be shown. Note: Your answer should be **EXACT** and not a Big-Oh bound.

$$T(N) = 2T(N-1) + N \text{ (for N} \geq 1)$$
$$T(0) = 0$$

Note: this problem does not fit the form for using master's theorem. Use the iteration technique:

$T(N) = 2T(N-1) + N$ **Grading: 1pt**

$$T(N-1) = 2T(N-1-1) + N - 1$$
$$T(N-1) = 2T(N-2) + N - 1$$
$$T(N) = 2(2T(N-2) + N - 1) + N$$

$T(N) = 4T(N-2) + 2(N-1) + N$ **Grading: 1 pt**

$$T(N-2) = 2T(N-2-1) + N - 2$$
$$T(N-2) = 2T(N-3) + N - 2$$
$$T(N) = 4(2T(N-3) + N - 2) + 2(N-1) + N$$

$T(N) = 8T(N-3) + 4(N-2) + 2(N-1) + N$ **Grading: 1 pt**

General Form after *k* iterations

$T(N) = 2^k T(N-k) + \sum_{i=0}^{k-1} 2^i (N-i)$ **Grading: 2pts**

We want *k* to be large enough to terminate the recursion i.e. *T* (*N* - *k*) = *T* (0). Thus *N* - *k* = 0, or *N* = *k*

Plugging in *N* for *k* we get

$T(N) = 2^N T(0) + \sum_{i=0}^{N-1} 2^i (N-i)$ **Grading: 2 pts**

$$T(N) = 2^N 0 + (2^{N-1}(1) + 2^{N-2}(2) + 2^{N-3}(3) + \ldots + 2^0(N))$$
$$T(N) = (2^{N-1}(1) + 2^{N-2}(2) + 2^{N-3}(3) + \ldots + 2^0(N))$$

Multiply the equation above through by 2 to obtain: **Grading: 1 pt**

$$2T(N) = (2^N(1) + 2^{N-1}(2) + 2^{N-2}(3) + \ldots + 2^1(N))$$

Take the difference of 2T(N) and T(N)

$$2T(N) - T(N) = (2^N(1) + 2^{N-1}(2) + 2^{N-2}(3) + \ldots \qquad + 2^1(N)) -$$
$$(2^{N-1}(1) + 2^{N-2}(2) + 2^{N-3}(3) + \ldots + 2^1(N-1) + 2^0(N))$$
$$T(N) = (2^N(1) + 2^{N-1}(1) + 2^{N-2}(1) + \ldots + 2^1(1) - 1(N))$$

$$T(N) = (\mathbf{2^{N+1}} - \mathbf{2}) - \boldsymbol{N}$$

Grading: 2pts to subtract and get to final answer.

Grading Notes: Give 6/10 if wrong general guess but the rest of it is correct after the general guess. (So 4 pts for upto the general part, 2 pts for solving the incorrect recurrence.)

Computer Science Foundation Exam

January 14, 2023

Section D

ALGORITHMS

NO books, notes, or calculators may be used,
and you must work entirely on your own.

SOLUTION

Question #	Max Pts	Category	Score
1	10	DSN	
2	10	DSN	
3	5	ALG	
TOTAL	25		

You must do all 3 problems in this section of the exam.

Problems will be graded based on the completeness of the solution steps and not graded based on the answer alone. Credit cannot be given unless all work is shown and is readable. Be complete, yet concise, and above all be neat. For each coding question, assume that all of the necessary includes (stdlib, stdio, math, string) for that particular question have been made.