

## 3) (10 pts) ANL (Summations)

Recall that  $\sum_{i=0}^{n-1} 2^i = 2^n - 1$ .

Use the iteration technique to find a Big-Oh bound for the recurrence relation below. Note: you may find the following mathematical results helpful:  $2^{\log_3 n} = n^{\log_3 2}$ , and  $\sum_{i=0}^{\infty} (\frac{2}{3})^i = 3$ . You may use these without proof in your work below.

$$T(n) = 2T\left(\frac{n}{3}\right) + O(n), \text{ for } n > 1$$

$$T(1) = O(1)$$

$$T(n) = 2T\left(\frac{n}{3}\right) + cn$$

$$T(n) = 2(2T\left(\frac{n}{9}\right) + c\left(\frac{n}{3}\right)) + cn$$

$$T(n) = 4T\left(\frac{n}{9}\right) + c\left(\frac{2n}{3}\right) + n$$

$$T(n) = 4(2T\left(\frac{n}{27}\right) + c\left(\frac{n}{9}\right)) + c\left(\frac{2n}{3}\right) + n$$

$$T(n) = 8T\left(\frac{n}{27}\right) + c\left(\frac{4n}{9}\right) + \left(\frac{2n}{3}\right) + n$$

Now that we've done three iterations, we can guess the form of the recurrence after k iterations:

$$T(n) = 2^k T\left(\frac{n}{3^k}\right) + cn \left( \sum_{i=0}^{k-1} \left(\frac{2}{3}\right)^i \right)$$

We want to plug in a value of k to this formula such that  $\frac{n}{3^k} = 1$ , which occurs when  $n = 3^k$ . By definition of log, we have that  $k = \log_3 n$ . We will bound the summation by taking it to infinity instead of k-1:

$$T(n) \leq 2^{\log_3 n} T(1) + cn \left( \sum_{i=0}^{\infty} \left(\frac{2}{3}\right)^i \right)$$

Now, we can use both given hints to arrive at:

$$T(n) \leq n^{\log_3 2} + 3cn = O(n)$$

Note that  $\log_3 3 = 1$ , so it follows that  $\log_3 2 < 1$ . Thus, the dominant term is  $3cn$ , which is  $O(n)$ .

**Grading: Part A - 1 pt for restating original recurrence, 1 pt for getting to second iteration, 2 pts for getting to third iteration, 2 pts for the correct guess of the general form after k iterations, 1 pt for getting the appropriate value of k to plug in, 2 pts to properly simplify both terms, 1 pt to decide which of the two terms is dominant and give the final answer.**

# Computer Science Foundation Exam

May 18, 2019

## Section II B

### ALGORITHMS AND ANALYSIS TOOLS

### **SOLUTION**

**NO books, notes, or calculators may be used,  
and you must work entirely on your own.**

Question #	Max Pts	Category	Score
1	10	DSN	
2	5	ALG	
3	10	DSN	
TOTAL	25		

**You must do all 3 problems in this section of the exam.**

**Problems will be graded based on the completeness of the solution steps and not graded based on the answer alone. Credit cannot be given unless all work is shown and is readable. Be complete, yet concise, and above all be neat. For each coding question, assume that all of the necessary includes (stdlib, stdio, math, string) for that particular question have been made.**