

3) (10 pts) ANL (Recurrence Relations)

Using the iteration technique, determine the Big-Oh solution to the recurrence relation below, in terms of n .

$$T(n) = 2T\left(\frac{n}{2}\right) + n^3, \text{ for } n > 1$$

$$T(1) = 1$$

Let's work out three iterations:

$$T(n) = 2T\left(\frac{n}{2}\right) + n^3 \quad (\text{Iteration 1})$$

$$T(n) = 2\left(2T\left(\frac{n}{4}\right) + \left(\frac{n}{2}\right)^3\right) + n^3$$

$$T(n) = 4T\left(\frac{n}{4}\right) + n^3 + \frac{n^3}{4} \quad (\text{Iteration 2 – some students might take more steps to get here})$$

$$T(n) = 4\left(2T\left(\frac{n}{8}\right) + \left(\frac{n}{4}\right)^3\right) + n^3 + \frac{n^3}{4}$$

$$T(n) = 8T\left(\frac{n}{8}\right) + \left(n^3 + \frac{n^3}{4} + \frac{n^3}{16}\right) \quad (\text{Iteration 3})$$

After k iterations, we have the following:

$$T(n) = 2^k T\left(\frac{n}{2^k}\right) + \sum_{i=0}^{k-1} \frac{n^3}{4^i}$$

Plug in $\frac{n}{2^k} = 1$, $n = 2^k$ and substitute:

$$T(n) \leq nT(1) + n^3 \sum_{i=0}^{\infty} \left(\frac{1}{4}\right)^i = n + n^3 \left(\frac{1}{1 - \frac{1}{4}}\right) = n + \frac{4}{3}n^3 = O(n^3)$$

Grading:

- 1 pt first iteration
- 2 pts second iteration
- 2 pts third iteration
- 2 pts guess
- 1 pt substitute $n = 2^k$
- 2 pts work to final answer

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Section D

ALGORITHMS

**NO books, notes, or calculators may be used,
and you must work entirely on your own.**

SOLUTION

Question #	Max Pts	Category	Score
1	10	DSN	
2	10	DSN	
3	5	DSN	
TOTAL	25	----	

You must do all 3 problems in this section of the exam.

Problems will be graded based on the completeness of the solution steps and not graded based on the answer alone. Credit cannot be given unless all work is shown and is readable. Be complete, yet concise, and above all be neat. For each coding question, assume that all of the necessary includes (stdlib, stdio, math, string) for that particular question have been made.