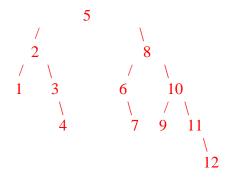
3) (10 pts) ALG (AVL Trees)

There is a unique AVL tree of height 4 with 12 nodes storing the integers 1 through 12, inclusive, such that for every non-leaf node, the left subtree is strictly shorter than the right subtree.

- (a) (7 pts) Draw this AVL Tree.
- (b) (3 pts) Explain how you constructed the tree based on the prompt.



In order to minimize the number of nodes in an AVL tree of a fixed height, one side of the tree must have a height that is one less than the other side of the tree. Based on the directions given in the question, we must always make the left side of a node shorter than the right side. We can build up the size of the desired AVL trees of each height. Height 0 will have 1 node, Height 1 will have 2 nodes. It follows that the AVL tree with the minimal number of nodes of height 3 will have 1 (root) + 1 (left) + 2 (right) = 4 nodes. It follows that our answer must have the root node store the value 5. Using similar logic, we can deduce that the node to the left of the root node must store 2. We can then fill in the appropriate AVL trees on the left and right of the node storing 2. To fill in the right side, we note that the right node of 5 will store an AVL tree with height 3, meaning that its left subtree will have 2 nodes and its right subtree will have 4 nodes. We recursively apply the previous logic to determine that 8 is stored to the right of 5 and fill out the left and right subtrees of 8 accordingly.

Grading: 1 pt if the tree is a valid AVL tree (so must be search tree with valid heights...)

1 pt if the tree has 12 nodes storing the values 1 through 12

1 pts for having 5 at the root of the tree

1 pts total for the left subtree of 5 (give partial as needed)

3 pts for the right subtree of 5 (1 pt for 8, 1 pt for left of 8, 1 pt for right of 8)

Explanation: 3 pts total, no need to be as verbose or thorough as what's above. Give full credit for general logic in the direction of figuring out how many nodes are on the left side. Give partial credit as needed.

Computer Science Foundation Exam

August 24, 2024

Section C

ALGORITHM ANALYSIS

NO books, notes, or calculators may be used, and you must work entirely on your own.

SOLUTION

Question #	Max Pts	Category	Score
1	10	ANL	
2	10	ANL	
3	5	ANL	
TOTAL	25		

You must do all 3 problems in this section of the exam.

Problems will be graded based on the completeness of the solution steps and <u>not</u> graded based on the answer alone. Credit cannot be given unless all work is shown and is readable. Be complete, yet concise, and above all <u>be neat</u>. For each coding question, assume that all of the necessary includes (stdlib, stdio, math, string) for that particular question have been made.