2) (10 pts) ANL (Algorithm Analysis)

A program processing an array of size 100 took 50 ms to finish and on an array of size 1000 it took 75 ms to finish. What Big Oh runtime would be most reasonable for the program? (Hint: make a couple guesses to the function and see if those guesses are consistent with the run-times listed.)

A quick analysis shows that the function must be sub-linear since multiplying the input size by 10 resulted in a multiplicative factor of only 1.5 (less than 10) to the run-time.

So, two candidates we might try for the run-time are f(n) = $\sqrt{n}$ or f(n) = lg n.

Let's try the first one: Let T(n) be the run time of the function for an array of input size n, then we have

T(n) = $c\sqrt{n}$, for some constant c. Thus:

T(100) = $c\sqrt{100} = 50\ ms$ → c = 5 ms

T(1000) = = $c\sqrt{1000} = 75\ ms \rightarrow c = \frac{75}{10\sqrt{10}} = \frac{7.5}{\sqrt{10}} ms$. Note that the square root of 10 is greater than 3, so this second equation indicates that c is in between 2 and 3 ms and isn't very consistent with the previous result.

Let's try the second guess: : Let T(n) be the run time of the function for an array of input size n, then we have

T(n) = $clg\ n$, for some constant c. Thus:

T(100) = $clg(100) = 50\ ms$ → c = $\frac{50\ ms}{\lg(100)} = \frac{50\ ms}{2\lg(10)} = \frac{25ms}{\lg 10}$

T(1000) = $clg(1000) = 75\ ms$ → c = $\frac{75\ ms}{\lg(1000)} = \frac{75\ ms}{3\lg(10)} = \frac{25ms}{\lg 10}$

Notice that both pieces of information result in the same value of c. Thus, the given data points are consistent with the hypothesis that the run-time of the program is **O(lg n).**

Grading: **5 pts for the correct answer without any proof**
5 pts for the logic behind the answer. There are many ways to quantitatively make the argument.
3 pts for trying to verify any answer at all.
3 pts for any sublinear guess without any verification or proof.