3) (10 pts) ANL (Summations)

Using the fact that if $x \neq 1$, then $\sum_{i=0}^{n} x^i = \frac{x^{n+1}-1}{x-1}$, for positive integers n, determine the following summation, in terms of n (assume n is a positive integer):

$$\sum_{i=2n+1}^{3n} 4^i$$

First, notice that we can factor out 4^{2n+1} from each term of our summation. Next, we can re-index the summation by noticing that inside the new sum, the terms are $4^0 + 4^1 + \ldots + 4^{n-1}$. Formally, we set j = i - (2n+1).

$$\sum_{i=2n+1}^{3n} 4^i = 4^{2n+1} \sum_{i=2n+1}^{3n} 4^{i-(2n+1)}$$

$$= 4^{2n+1} \sum_{j=0}^{n-1} 4^j$$

$$= 4^{2n+1}\left(\frac{4^n - 1}{4 - 1}\right)$$

$$= \frac{4^{3n+1} - 4^{2n+1}}{3}$$

Another way to solve the sum is to take the sum from i=1 to 3n, and subtract from it the same sum form i=1 to 2n. If we proceed in this way, we'll get $\frac{4^{3n+1}-1}{4-1} - \frac{4^{2n+1}-1}{4-1}$, after evaluating both sums. Notice that both terms equal to one-third (first -, second +) cancel out and that we arrive at the same answer as above.

Grading Method #1: 3 pts factor out, 3 pts rewrite sum, 3 pts apply formula, 1 pt final answer (note, final answer can be factored form instead.)

Grading Method #2: 2 pts split sum, 3 pts apply formula first sum, 3 pts apply formula second sum, 2 pts algebra to get to final answer

Computer Science Foundation Exam

August 8, 2020

Section II B

ALGORITHMS AND ANALYSIS TOOLS

SOLUTION

Directions: You may either directly edit this document, or write out your answers in a .txt file, or scan your answers to .pdf and submit them in the COT 3960 Webcourses for the Assignment "Section II B". Please put your *name, UCFID and NID* on the top left hand corner of each document you submit. Please aim to submit 1 document, but if it's necessary, you may submit 2. Clearly mark for which question your work is associated with. If you choose to edit this document, please remove this cover page from the file you submit and make sure your *name, UCFID and NID* are on the top left hand corner of the next page (first page of your submission).

Question #	Max Pts	Category	Score
1	5	DSN	
2	10	ALG	
3	10	DSN	
TOTAL	25		

You must do all 3 problems in this section of the exam.

Problems will be graded based on the completeness of the solution steps and not graded based on the answer alone. Credit cannot be given unless all work is shown and is readable. Be complete, yet concise, and above all be neat. For each coding question, assume that all of the necessary includes (stdlib, stdio, math, string) for that particular question have been made.