2) (5 pts) ANL (Algorithm Analysis)

A sorting algorithm takes O($n\sqrt{n}$) time to sort n values. The algorithm took .2 milliseconds to sort an array of 1000 values. How many seconds would it take to sort an array of size 900,000?

Let the run time of the algorithm be T(n). It follows that $T(n) = cn\sqrt{n}$, for some constant c. Use the given information to solve for c:

$T(1000) = c1000\sqrt{1000} = .2\ ms$,
$c = \frac{.2}{1000\times\sqrt{1000}}$

Now, let us find T(900000):

$$T(900000) = \frac{.2}{1000 \times \sqrt{1000}} \times (900000) \times \sqrt{900000} = 180\sqrt{900}ms = 180 \times 30ms = 5400ms$$

Converting 5400 ms to seconds, we get **5.4 seconds** as the final answer.

Grading: 1 pt to set up equation for c, 1 pt to solve for c, 1 pt to set up final equation, 1 pt to solve for the answer in milliseconds, 1 pt to cover to seconds.

3) (10 pts) ANL (Recurrence Relations)

What is the closed form solution to the following recurrence relation? Please use the iteration technique, show all of your work and provide your final answer in Big-Oh notation.

T(1) = 1
T(n) = 2T(n/4) + 1

Iterate the recurrence three times:

$T(n) = 2T\left(\frac{n}{4}\right) + 1$ (one iteration)

$T(n) = 2(2T\left(\frac{n}{16}\right) + 1) + 1$

$T(n) = 4T\left(\frac{n}{16}\right) + 3$ (two iterations)

$T(n) = 4(2T\left(\frac{n}{64}\right) + 1) + 3$

$T(n) = 8T\left(\frac{n}{64}\right) + 7$ (three iterations)

Now, let's make a guess as to the form of the recurrence after iterating k times based on the first three iterations:

$$T(n) = 2^k T\left(\frac{n}{4^k}\right) + (2^k - 1)$$

Since we know T(1), we want to plug in the value of k for which $\frac{n}{4^k} = 1$, in for k. Solving, we find that $n = 4^k$. Taking the square root of both sides, we find $\sqrt{n} = \sqrt{4^k} = \sqrt{2^{2k}} = (2^{2k})^{\frac{1}{2}} = 2^k$. Substituting for both 4^k and 2^k, in the right hand of the recurrence, we get:

$$T(n) = \sqrt{n}T\left(\frac{4^k}{4^k}\right) + \left(\sqrt{n} - 1\right) = \sqrt{n}T(1) + \left(\sqrt{n} - 1\right) = \sqrt{n} + \sqrt{n} - 1 \in \boldsymbol{O}(\sqrt{\boldsymbol{n}})$$

Grading: 1 pt for first iteration
1 pt for second iteration
2 pts for third iteration
2 pts for general form guess
2 pts to plug in n = 4^k into general form (or equivalent)
2 pts to substitute and get to the final answer.