2) (10 pts) ANL (Algorithm Analysis)

You are using an algorithm that can multiply 2 N-digit integers in $O(N^{1.5})$ time. It takes $(10/13)^3$ seconds to multiply 2 numbers that have 100,000 digits. What is the expected number of digits of 2 numbers we could multiply together that would take exactly 1 second? **Please show all your work, including algebraic simplification, which is part of what is being tested with this question.**

Let T(N) be the run time for the algorithm to multiply two N-digit integers. Using the given information, there exists a constant c such that $T(N) = cN^{1.5}$

Let's plug in N = 100,000 into this equation to find c:

$(\frac{10}{13})^3 = c(100{,}000)^{3/2}$ **Grading: 2 pts**

$\frac{10^3}{13^3} = c(10)^{5\times(\frac{3}{2})}$ **Grading: 1 pt**

$\frac{10^3}{13^3} = c(10)^{\frac{15}{2}}$ **Grading: 1 pt**

$c = \frac{10^3}{13^3 \times 10^{\frac{15}{2}}} = \frac{1}{13^3 \times 10^{\frac{9}{2}}}$ **Grading: 1 pt (5 pts total to find c)**

Now, let N be the answer to the question, plugging in 1 second for the time and the value of c found above:

$1s = \frac{1}{13^3 \times 10^{\frac{9}{2}}} \times N^{1.5}$ **Grading: 2 pts**

$N^{1.5} = 13^3 \times 10^{4.5}$ **Grading: 1 pts**

$N = 13^{\frac{3}{1.5}} \times 10^{\frac{4.5}{1.5}} = 13^2 \times 10^3 = \mathbf{169{,}000}$ **Grading: 2 pts**