

1) (10 pts) ANL (Algorithm Analysis)

Consider a n -bit binary counter, which starts with the binary representation of 0 and increments by 1 until it reaches the binary value of $2^n - 1$. For $n = 3$, the counter would start at 000, and then change as follows:

$$000 \rightarrow 00\underline{1} \rightarrow 0\underline{1}0 \rightarrow 0\underline{1}1 \rightarrow \underline{1}00 \rightarrow 1\underline{0}1 \rightarrow 1\underline{1}0 \rightarrow 1\underline{1}1.$$

The underlined bits represent the ones that had to be changed. In particular, for this example, $1 + 2 + 1 + 3 + 1 + 2 + 1 = 11$ bits were changed as the counter progressed from 0 to $2^n - 1$. Let $f(n)$ equal the number of bits that are changed for an n -bit binary counter counting from 0 to $2^n - 1$. Find a closed-form formula for $f(n)$. (For example, something like $f(n) = 2^{n-1} + 2$. A formula in terms of n without any sort of recursive function definition.) Show all of your work and put a box around your final answer.