2) (5 pts) ANL (Algorithm Analysis)

An algorithm processing a two dimensional array with R rows and C columns runs in $O(RC^2)$ time. For an array with 100 rows and 200 columns, the algorithm processes the array in 120 ms. How long would it be expected for the algorithm to take when processing an array with 200 rows and 500 columns? Please express your answer in ***seconds.***

Let the algorithm with input size n have a runtime of $T(R,C) = kRC^2$, for some constant k. Using the given information we have:

$$T(100,200) = k(100)(200)^2 = 120ms$$
$$c = \frac{120}{4 \times 10^6} ms$$

Now, we must find T(200,500):

$$T(200,500) = k(200)(500)^2 = \frac{120ms}{4 \times 10^6} \times (200)(500)^2 = \frac{120 \times 50 \times 10^6}{4 \times 10^6} ms = 1500ms = 1.5s$$

Thus, our final answer is **1.5 seconds.**

Grading: 1 pt to set up the initial equation for k, 1 pt to solve for k, 2 pts to get answer in ms, 1 pt to convert to seconds. Give partial credit for the 2 pts if the setup is correct but some algebra issue occurred. Also, give full credit if the ratio method (which is also valid) is used instead of this method. Map points accordingly if some error is made using that method.

FURTHER GRADING NOTE: If the student used an algorithm run time of $O(R^2C^2)$ for any reason and correctly solved the problem for this different run time, 4 points out of 5 were awarded.

3) (10 pts) ANL (Summations and Recurrence Relations)

Determine the following summation in terms of n (assume n is a positive integer 2 or greater), expressing your answer in the form $an^3 + bn^2 + cn$, where a, b and c are rational numbers. (Hint: Try rewriting the summation into an equivalent form that generates less algebra when solving.)

$$\sum_{i=n^2-3}^{n^2+n-4} (i+4)$$

To simplify the algebra, do an index shift. Notice that the terms getting added are actually $n^2 + 1$, $n^2 + 2$, …, $n^2 + n$:

$$\sum_{i=n^2-3}^{n^2+n-4} (i+4) = \sum_{i=1}^{n} (n^2+i)$$

$$= \left(\sum_{i=1}^{n} n^2\right) + \left(\sum_{i=1}^{n} i\right)$$

$$= n(n^2) + \frac{n(n+1)}{2}$$

$$= n^3 + \frac{1}{2}n^2 + \frac{1}{2}n$$

Grading: 4 pts for a correct index shift (give partial as necessary), 1 pt for splitting the sum correctly, 2 pts for the sum of the constant, 3 pts for the sum of i.

If they don't do the index shift, then they are likely to be subtracting two sums. 3 pts for each of the two sums, 4 pts for the algebra of subtracting those sums.

If they try something else, try your best to map points to one of these two schemes.