

3) (10 pts) ANL (Recurrence Relations)

Use the iteration technique to determine an **exact closed-form solution** for the recurrence relation, $T(N)$, described below. (Note: Be very careful with what occurs towards the end of the iteration, in the general case.)

$$T(N) = (N + 1)T(N - 1) \quad (\text{for } N > 1)$$

$$T(1) = 1$$

$$T(N) = (N + 1)T(N - 1) \quad \text{Grading: 1pt}$$

$$T(N - 1) = (N - 1 + 1)T(N - 1 - 1)$$

$$T(N - 1) = NT(N - 2)$$

$$T(N) = (N + 1)(N)T(N - 2) \quad \text{Grading: 2pts}$$

$$T(N - 2) = (N - 2 + 1)T(N - 2 - 1)$$

$$T(N - 2) = (N - 1)T(N - 3)$$

$$T(N) = (N + 1)(N)(N - 1)T(N - 3) \quad \text{Grading: 2pts}$$

General Form after k iterations

$$T(N) = T(N - k) \prod_{i=1}^k (N + 2 - i) \quad \text{Grading: 2pts}$$

The recursion stops when $N - k = 1$; $k = N - 1$

Plugging in N for k we get

$$T(N) = T(1) \prod_{i=1}^{N-1} (N + 2 - i) \quad \text{Grading: 1pt}$$

$$T(N) = 1 (N + 1)(N)(N - 1) \dots (3)$$

$$T(N) = (N + 1)!/2 \quad \text{Grading: 2 pts}$$

Computer Science Foundation Exam

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Section D

ALGORITHMS

**NO books, notes, or calculators may be used,
and you must work entirely on your own.**

SOLUTION

Question #	Max Pts	Category	Score
1	5	DSN	
2	10	ALG	
3	10	DSN	
TOTAL	25		

You must do all 3 problems in this section of the exam.

Problems will be graded based on the completeness of the solution steps and not graded based on the answer alone. Credit cannot be given unless all work is shown and is readable. Be complete, yet concise, and above all be neat. For each coding question, assume that all of the necessary includes (stdlib, stdio, math, string) for that particular question have been made.