2) (5 pts) ANL (Algorithm Analysis)

A O($\sqrt{n}$) search algorithm took 45 milliseconds to complete a search amongst $\boldsymbol{n}$ = 4 x 10^6 entries. How long would it be expected for this algorithm execute a search amongst a database of 10^8 entries, in milliseconds?

Let $T(n) = c\sqrt{n}$ be the amount of time the algorithm takes to execute on a input of size n. Using the given information we have:

$$T(4 \times 10^6) = c\sqrt{4 \times 10^6} = 45\ ms$$

$$c\sqrt{4 \times 10^6} = 45\ ms$$

$$(2 \times 10^3)c = 45\ ms$$

$$c = \frac{45}{2000} ms$$

Now, we solve for T(10^8):

$$T(10^8) = \frac{45}{2000} ms\sqrt{10^8} = \frac{45 \times 10^4}{2 \times 10^3} ms = 45 \times 5ms = \mathbf{225\ ms}$$

Grading: 1 pt set up equation for c.
1 pt solve for c.
1 pt plug in n = 10^8
2 pts to get to correct final answer simplified as 225 ms. (1 pt for intermediate form)

3) (10 pts) ANL (Recurrence Relations)

Determine the following summation in terms of n, **<u>in factorized form</u>**. (Do NOT multiply the answer out into polynomial form. Note: Your answer should NOT have a fraction in it.)

$$\sum_{i=1}^{2n-1} (i + 3i^2)$$

$$\sum_{i=1}^{2n-1} (i + 3i^2) = \left(\sum_{i=1}^{2n-1} i\right) + (\sum_{i=1}^{2n-1} 3i^2)$$

$$= \frac{(2n-1)(2n)}{2} + \frac{3(2n-1)(2n)(2(2n-1)+1)}{6}$$

$$= n(2n-1) + \frac{3(2n-1)(2n)(4n-2+1)}{6}$$

$$= n(2n-1) + (2n-1)(n)(4n-1)$$

$$= n(2n-1)(1 + 4n - 1)$$

$$= n(2n-1)(4n)$$

$$= \mathbf{4n^2(2n-1)}$$

Grading: 1 pt split sum
2 pts formula sum of i
2 pts formula sum of i^2
2 pts to get to non-fractional form (canceling 2, 6)
2 pts factor out n(2n – 1)
1 pt to simplify to final form
Note: Grade was 7 pts out of 10 for polynomial form.