$\int f(g(x)) * g'(x)dx = \int f(u)du$

$\int x\sqrt{x^2+1}$ dx

$\int x^3\sqrt{x^2-9}$ dx

$\int \underline{f(g(x)) \cdot g'(x) = f(u)\,du}$

$\int f(x)\,dx = \int 2x \quad \int \frac{x^3}{3}+1$

$\int x\sqrt{x^2+1}\,dx$ $\sqrt{x^2+1}$ $\sqrt{x}$ x^2+1

$2x$

$u^2 = x^2+1 \; ( )'$ $\sqrt{x^2} = |x|$ x $\cancel{-x}$

$2u\,du = 2x\,dx$

$\frac{\cancel{2}u\,du}{\cancel{2}} = x\,dx$

$u\,du = x\,dx$

$\int x\,dx\sqrt{x^2+1} = \int u\,du \cdot \sqrt{u^2}$

$= \int u \cdot du \cdot u = \int$

$\int a^n = \frac{a^{n+1}}{n+1} + C$

$\int u^2\,du = \frac{u^3}{3} + C$ $u^2 = x^2+1 = u = \sqrt{x^2+1}$

$= \boxed{\frac{(\sqrt{x^2+1})^3}{3} + C}$

$\int x^3\underbrace{\sqrt{x^2-9}}_{2x}$ dx

$x^3 = x^2 \cdot x$

$\int x^2\, x\,dx\sqrt{x^2-9}$

$\boxed{u^2 = x^2-9 \; ( )'} \Rightarrow u^2+9 = x^2$

$2u\,du = 2x\,dx$

$\frac{\cancel{2}u\,du}{\cancel{2}} = x\,dx$

$\boxed{u\,du = x\,dx}$ $u^1 \cdot u^1 = u^{1+1} = u^2$

$\int (u^2+9)\,u\,du \cdot \sqrt{u^2} = \int (u^2+9)\,u\,du\,u$

$\int u^2 (u^2+9)\,du = \int (u^4 + 9u^2)\,du$

$= \int u^4\,du + \int 9u^2\,du$ $\frac{a^{n+1}}{n+1}$

$\frac{u^5}{5} + 9\int u^2\,du = \frac{u^5}{5} + \cancel{9}\cdot\frac{u^3}{\cancel{3}} + C$ $\frac{a^{n+1}}{n+1}$

$= \frac{u^5}{5} + 3u^3 + C \rightarrow u^2 = x^2-9 \,/\, \sqrt{\;}$

$\sqrt{u^2} = \sqrt{x^2-9}$

$u = \sqrt{x^2-9}$

$\boxed{\frac{(\sqrt{x^2-9})^5}{5} + 3\cdot(\sqrt{x^2-9})^3 + C}$