IDENTIDADES TRIG

$\sin^2\alpha + \cos^2\alpha = 1 \rightarrow \frac{1-\sin^2\alpha = \cos^2\alpha}{1-\cos^2\alpha = \sin^2\alpha}$

$(1) \frac{1}{\cos^2\alpha} + \frac{\sin^2\alpha}{\cos^2\alpha} = \frac{\sec^2(\alpha)}{\cos^2\alpha} \rightarrow \tan^2(\alpha) + 1 = \sec^2(\alpha)$

$\cos^2(\alpha) = \frac{1+\cos(2\alpha)}{2}$ $\quad \sin(2\alpha) = 2\sin(\alpha)\cos\alpha$

$\sin^2(\alpha) = \frac{1-\cos(2\alpha)}{2}$

$\sin\alpha = \frac{CAT\ OP}{HIP} = \csc\alpha = \frac{HIP}{CAT\ OP}$

$\cos\alpha = \frac{CAT\ ADY}{HIP} = \sec\alpha = \frac{HIP}{CAT\ ADY}$

$\tan\alpha = \frac{CAT\ OP}{CAT\ ADY} = \cot g\alpha = \frac{1}{\tan\alpha}$

TIPO I:

$\int (a^2 - x^2)\,dx \qquad x = a\sin(\alpha)$
$dx = a\cos(\alpha)\,d\alpha$

$\int \sqrt{4 - x^2}\,dx \qquad \frac{x = 2\sin(\alpha)}{dx = 2\cos(\alpha)\,d\alpha}$

$\sqrt{4^2 - (x)^2}$

$\int \sqrt{4 - (2\sin(\alpha))^2} \cdot 2\cos(\alpha)\,d\alpha$

$\int \sqrt{4 - 4\sin^2(\alpha)} \cdot 2\cos(\alpha)\,d\alpha$

$\int \sqrt{4(1 - \sin^2(\alpha))} \cdot 2\cos(\alpha)\,d\alpha \qquad \sqrt{x^2} = |x| \qquad \sqrt{x^2 + |x|^2}$

$\int \sqrt{4\cos^2(\alpha)} \cdot 2\cos(\alpha)\,d\alpha$

$\int 2\cos(\alpha) \cdot 2\cos(\alpha)\,d\alpha$

$\int 4\cos^2(\alpha)\,d\alpha = 4\int \cos^2(\alpha)\,d\alpha$

$= 4\int \frac{1+\cos(2\alpha)}{2}\,d\alpha = 2\int 1 + \cos(2\alpha)\,d\alpha$

$2\left[\int 1\,d\alpha + \int \cos(2\alpha)\,d\alpha\right] \qquad \int \cos(\alpha) = \sin(\alpha)$

$\int \cos(bx) = \frac{1}{b}\sin(bx)$

$2\left[\alpha + \frac{1}{2}\sin(2\alpha)\right] + C$

$2\alpha + \sin(2\alpha) + C$

$x = 2\sin(\alpha)$

$\frac{x}{2} = \sin(\alpha)$

$\tan(\alpha) = \frac{x}{\sqrt{4-x^2}}$

$a^2 + b^2 = c^2$
$x^2 + b^2 = 2^2$
$x^2 + b^2 = 4$
$b^2 = 4 - x^2 / \sqrt{\ }$
$b = \sqrt{4 - x^2}$

$2\alpha + \sin(2\alpha) + C$

$\frac{x}{2} = \sin(\alpha) \;/\; \arcsin = \sin^{-1}$

$\alpha = \arcsin\left(\frac{x}{2}\right) \quad \alpha = \sin^{-1}(\frac{x}{2})$

$2\alpha + 2\sin(\alpha)\cdot\cos(\alpha) + C$

$2\arcsin\left(\frac{x}{2}\right) + x\frac{\sqrt{4-x^2}}{2} + C$

$2\arcsin\left(\frac{x}{2}\right) + x\frac{\sqrt{4-x^2}}{2} + C$

$2\alpha + \sin(2\alpha) + C$

$\alpha = \arcsin\left(\frac{x}{2}\right)$

$2\arcsin\left(\frac{x}{2}\right) + \sin(2\arcsin\left(\frac{x}{2}\right)) + C$

TIPO II:

$\int (x^2 - a^2)\,dx \qquad x = a\sec(\alpha)$
$dx = a\sec(\alpha)\tan(\alpha)\,d\alpha$

$\int \frac{dx}{\sqrt{x^2 - 16}} \qquad x = 4\sec(\alpha)$
$dx = 4\sec(\alpha)\tan(\alpha)\,d\alpha$

$\sqrt{(x^2 - (4)^2}$

$\int \frac{4\sec(\alpha)\cdot\tan(\alpha)}{\sqrt{(4\sec(\alpha))^2 - 16}}\,d\alpha$

$= \int \frac{4\sec(\alpha)\cdot\tan(\alpha)}{\sqrt{16\sec^2(\alpha) - 16}}\,d\alpha$

$= \int \frac{4\sec(\alpha)\cdot\tan(\alpha)}{\sqrt{16(\sec^2(\alpha) - 1)}}\,d\alpha$

$= \int \frac{4\sec(\alpha)\cdot\tan(\alpha)}{\sqrt{16\tan^2(\alpha)}}\,d\alpha$

$\int \frac{4\sec(\alpha)\cdot\tan(\alpha)}{4\tan(\alpha)}\,d\alpha$

$\int \sec(\alpha)\,d\alpha \rightarrow \ln|\sec(\alpha) + \tan(\alpha)| + C$

$= \ln\left|\sec(\alpha) + \tan(\alpha)\right| + C$

$x = 4\sec(\alpha)$

$\frac{x}{4} = \sec(\alpha)$

$\tan(\alpha) = \frac{CAT\ OP}{CAT\ ADY}$

$a^2 + 4^2 = x^2$
$a^2 = x^2 - 16$
$a = \sqrt{x^2 - 16}$

$= \ln\left|\frac{x}{4} + \frac{\sqrt{x^2-16}}{4}\right| + C$

TIPO III:

$\int (a^2 + x^2)\,dx \;\vee\; (x^2 + a^2) \qquad x = a\tan(\alpha)$
$dx = a\sec^2(\alpha)\,d\alpha$

$\int \frac{1}{\sqrt{x^2+1}}\,dx \qquad x = \tan(\alpha)$
$dx = \sec^2(\alpha)\,d\alpha$

$\sqrt{(x)^2 + (1)^2}$

$\int \frac{1}{\sqrt{(\tan(\alpha))^2 + 1}}\sec^2(\alpha)\,d\alpha$

$\int \frac{\sec^2(\alpha)\,d\alpha}{\sqrt{\sec^2(\alpha)}} = \int \frac{\sec^2(\alpha)}{\sec(\alpha)}\,d\alpha$

$= \int \sec(\alpha)\,d\alpha$

$= \ln\left|\sec(\alpha) + \tan(\alpha)\right| + C$

$= \ln\left|\frac{\sqrt{x^2+1}}{1} + \frac{x}{1}\right| + C$

$x = \tan(\alpha)$

$\alpha = \tan^{-1}\frac{x}{1} = \tan(\alpha)$