$$\int \frac{dx}{\sqrt{9x^2 + 4}}$$

$(3x)^2 + (2)^2$ $\quad a = 2$

$X = a\,Tg(\alpha)$

$dX = a\,SEC^2(\alpha)\,d\alpha$

$3X = 2Tg(\alpha) \rightarrow x = \frac{2}{3}Tg(\alpha)$

$3dx = 2SEC^2(\alpha)\,d\alpha \rightarrow dx = \frac{2}{3}SEC^2(\alpha)\,d\alpha$

$$\frac{2}{3}\int \frac{SEC^2(\alpha)}{\sqrt{9\left(\frac{2}{3}Tg(\alpha)\right)^2 + 4}}\,d\alpha = \frac{2}{3}\int \frac{SEC^2(\alpha)}{\sqrt{\cancel{9}\left(\frac{4}{\cancel{9}}Tg^2(\alpha)\right) + 4}}\,d\alpha$$

$$\frac{2}{3}\int \frac{SEC^2(\alpha)}{\sqrt{4Tg^2(\alpha) + 4}}\,d\alpha = \frac{2}{3}\int \frac{SEC^2(\alpha)}{\sqrt{4\cdot\underbrace{(Tg^2(\alpha) + 1)}}}\,d\alpha$$

$Tg^2(\alpha) + 1 = SEC^2(\alpha)$

$$= \frac{2}{3}\int \frac{SEC^2(\alpha)}{\sqrt{4}\cdot\sqrt{SEC^2(\alpha)}}\,d\alpha$$

$$= \frac{2}{3}\int \frac{SEC^2(\alpha)}{2SEC(\alpha)}\,d\alpha = \frac{\cancel{2}}{3}\cdot\frac{1}{\cancel{2}}\int \frac{SEC^2(\alpha)}{SEC(\alpha)}\,d\alpha$$

$$= \frac{1}{3}\int \frac{SEC^2(\alpha)}{SEC(\alpha)}\,d\alpha = \frac{1}{3}\int SEC(\alpha)\,d\alpha$$

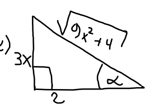
$\ln|Tg(\alpha) + SEC(\alpha)| + C$

$$\frac{1}{3}\ln|Tg(\alpha) + SEC(\alpha)| + C$$

$3X = 2Tg(\alpha) \rightarrow \frac{3X}{2} = Tg(\alpha)$

CAT OP

CAT ADX



$(3x)^2 + (2)^2 = C^2$

$9x^2 + 4 = C^2 / \sqrt{\ }$

$\sqrt{9x^2+4} = C$

$\frac{3X}{2} = Tg(\alpha) \Rightarrow \alpha = ARCTg\left(\frac{3X}{2}\right) = Tg^{-1}\left(\frac{3x}{2}\right)$

$$I_1 = \frac{1}{3}\ln\left|Tg\left(ARCTg\left(\frac{3x}{2}\right)\right) + SEC\left(ARCTg\left(\frac{3x}{2}\right)\right)\right| + C$$

$$I_2 = \frac{1}{3}\ln\left|\frac{3X}{2} + \frac{\sqrt{9x^2+4}}{2}\right| + C$$

$SEC(\alpha) = COS(\alpha)^{-1} = \frac{CAT\ ADY}{hip} = \frac{hip}{CAT\ ADX}$