$$\int x^3 \operatorname{arctan}(x^2)\, dx$$

ILATE

$u = \operatorname{arctan}(x^2)$ $dv = x^3\,dx$

$du = \frac{1}{(x^2)^2+1} \cdot 2x\,dx$ $v = \frac{x^4}{4}$

$du = \frac{2x}{x^4+1}\,dx$ $v = \frac{x^4}{4}$

$uv - \int v\,du$

$\operatorname{arctg}(x^2) \cdot \frac{x^4}{4} - \int \frac{x^4}{4} \cdot \frac{2x}{x^4+1}\,dx$

$\operatorname{arctg}(x^2) \cdot \frac{x^4}{4} - \frac{1}{2}\int \frac{x^4 \cdot x}{x^4+1}\,dx$ $(x^2)^2$

$\operatorname{arctg}(x^2) \cdot \frac{x^4}{4} - \frac{1}{2}\int \frac{x^4 \cdot x}{(x^2)^2+1}\,dx$

$z = x^2 \rightarrow ()^2$ $z^2 = x^4$

$dz = 2x\,dx$

$\frac{dz}{2} = x\,dx$

$\operatorname{arctg}(x^2) \cdot \frac{x^4}{4} - \frac{1}{2}\int \frac{dz}{2} \cdot \frac{z^2}{z^2+1}$

$\operatorname{arctg}(x^2) \cdot \frac{x^4}{4} - \frac{1}{4}\int \frac{z^2+1-1}{z^2+1}\,dz$ $1-1=0$

$\operatorname{arctg}(x^2) \cdot \frac{x^4}{4} - \frac{1}{4}\left(\int \frac{z^2+1}{z^2+1}\,dz - \int \frac{1}{z^2+1}\,dz\right)$

$\operatorname{arctg}(x^2) \cdot \frac{x^4}{4} - \frac{1}{4}\left(z - \operatorname{arctg}(z)\right) + C$

$$\operatorname{arctg}(x^2) \cdot \frac{x^4}{4} - \frac{1}{4}\left(x^2 - \operatorname{arctg}(x^2)\right) + C$$

$\frac{1}{4}x^4 \arctan(x^2) - \frac{1}{2}\left(\frac{x^2}{2} - \frac{1}{2}\arctan(x^2)\right) + C$

$u = \operatorname{arctan}(x^2)$ $dv = x^3\,dx$

$du = \frac{1}{(x^2)^2+1} \cdot 2x\,dx$ $v = \frac{x^4}{4} + \frac{1}{4}$ $c = \frac{1}{4}$

$\int f(x)\,dx = \dots + C$

$du = \frac{2x}{x^4+1}\,dx$ $v = \frac{x^4}{4} + \frac{1}{4}$ $\boxed{v = \frac{x^4+1}{4}}$

$uv - \int v\,du$

$\operatorname{arctan}(x^2) \cdot \left(\frac{x^4+1}{4}\right) - \int \frac{x^4+1}{4} \cdot \frac{2x}{x^4+1}\,dx$

$\operatorname{arctan}(x^2) \cdot \left(\frac{x^4+1}{4}\right) - \frac{1}{2}\int x^4+1 \cdot \frac{x}{x^4+1}\,dx$

$\operatorname{arctan}(x^2) \cdot \left(\frac{x^4+1}{4}\right) - \frac{1}{2}\int x\,dx$

$\operatorname{arctan}(x^2) \cdot \left(\frac{x^4+1}{4}\right) - \frac{1}{2} \cdot \frac{x^2}{2} + C$

$$\operatorname{arctan}(x^2) \cdot \left(\frac{x^4+1}{4}\right) - \frac{x^2}{4} + C$$