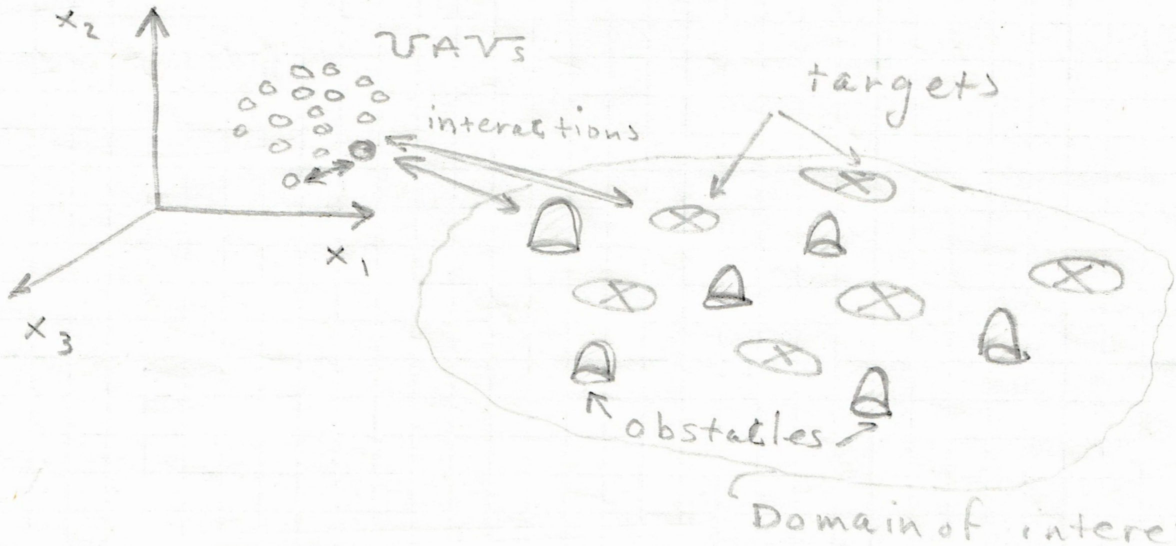
Modeling and Simulation of Swarm of UAVs

- For historical perspectives – typed notes

Model Problem:



Governing Equation:

member–target   member member

$$m_i \dot{\underline{v}}_i = m_i \ddot{\underline{r}}_i = \underline{T}_i^{total} = \underline{\mathcal{F}}\left(\underline{N}_i^{mt}, \underline{N}_i^{mo}, \underline{N}_i^{mm}\right)$$

mass of the "i"th swarmmember   velocity   position   total interaction   member obstacle

position: $\underline{r}_i = r_{i1}\underline{e}_1 + r_{i2}\underline{e}_2 + r_{i3}\underline{e}_3$

velocity $\underline{v}_i = \dot{\underline{r}}_i = \dot{r}_{i1}\underline{e}_1 + \dot{r}_{i2}\underline{e}_2 + \dot{r}_{i3}\underline{e}_3$

acceleration $\underline{a}_i = \dot{\underline{v}}_i = \ddot{\underline{r}}_i = \ddot{r}_{i1}\underline{e}_1 + \ddot{r}_{i2}\underline{e}_2 + \ddot{r}_{i3}\underline{e}_3$

- N_m = # of swarm-members
- N_t = # of targets
- N_o = # of obstacles

Characterization of interactions:

Member-target interaction

$$\|\underline{r}_i - \underline{T}_j\| = \left((r_{i1}-T_{j1})^2 + (r_{i2}-T_{j2})^2 + (r_{i3}-T_{j3})^2\right)^{1/2}$$

$$\stackrel{def}{=} d^{mt}_{ij}$$

Direction vector: $\underline{n}_{i\to j} = \dfrac{\underline{T}_j - \underline{r}_i}{\|\underline{T}_j - \underline{r}_i\|}$ = unit vector

since $\underline{r}_i + \underline{r}_{i\to j} = \underline{T}_j$

Weighted direction vector:

$$\hat{\underline{n}}_{i\to j} = \left(w_{t1}\, e^{-a_1 d^{mt}_{ij}} - w_{t2}\, e^{-a_2 d^{mt}_{ij}}\right) \underline{n}_{i\to j}$$

decay

weight that reflects importance

Sum all member-target interaction:

$$\underline{N}^{mt}_i = \sum_{j=1}^{N_t} \hat{\underline{n}}_{i\to j}$$

Member-obstacle interaction

$$\|\underline{r}_i - \underline{O}_j\| = \left((r_{i1}-O_{j1})^2 + (r_{i2}-O_{j2})^2 + (r_{i3}-O_{j3})^2\right)^{1/2}$$

$$\stackrel{def}{=} d^{mo}_{ij}$$

Direction vector: $\underline{n}_{i\to j} = \dfrac{\underline{O}_j - \underline{r}_i}{\|\underline{O}_j - \underline{r}_i\|}$

Weighted direction vector:

$$\hat{n}_{i\to j} = \left(w_{o1}\, e^{-b_1 d^{mo}_{ij}} - w_{o2}\, e^{-b_2 d^{mo}_{ij}}\right) \underline{n}_{i\to j}$$

Sum of <u>all</u> member-obstacle interactions:

$$\underline{N}_i^{mo} = \sum_{j=1}^{N_o} \hat{\underline{n}}_{i\to j} \qquad (j \neq i)$$

member-member interaction

$$\|\underline{r}_i - \underline{r}_j\| = \left((r_{i1} - r_{j1})^2 + (r_{i2} - r_{j2})^2 + (r_{i3} - r_{j3})^2\right)^{1/2} \overset{\text{def}}{=} d_{ij}^{mm}$$

Direction vector: $\underline{n}_{i\to j} = \dfrac{(\underline{r}_j - \underline{r}_i)}{\|\underline{r}_i - \underline{r}_j\|}$

Weighted direction vector:

$$\hat{\underline{n}}_{i\to j} = \left(w_{m1} e^{-c_1 d_{ij}^{mm}} - W_{m2} e^{-c_2 d_{ij}^{mm}}\right) \underline{n}_{i\to j}$$

Sum all the member-member interactions:

$$\underline{N}_i^{mm} = \sum_{j=1}^{N_m} \hat{\underline{n}}_{i\to j}$$

Summation of all interactions:

$$\underline{N}_i^{tot} = W_{mt} \underline{N}_i^{mt} + W_{mo} \underline{N}_i^{mo} + W_{mt} \underline{N}_i^{mm}$$

normalized $\quad \underline{n}_i^* = \dfrac{\underline{N}_i^{tot}}{\|\underline{N}_i^{tot}\|}$

Forces are $\underline{\Upsilon}_i^{tot} = F_i \underline{n}_i^*$

$\uparrow$ thrust

yielding $m_i \dot{\underline{v}}_i = m_i \ddot{\underline{r}}_i = \underline{\Upsilon}_i^{tot}, i = 1, \ldots, N_m$

Time discretization:

velocity update: $\underline{v}_i(t+\Delta t) = \underline{v}_i(t) + \frac{\Delta t}{m_i} \underline{f}_i^{tot}(t)$

position update: $\underline{r}_i(t+\Delta t) = \underline{r}_i(t) + \Delta t\ \underline{v}_i(t)$

"Speed Limit": maximum speed $= V_{max}$

Thus, if $\|\underline{v}_i(t+\Delta t)\| > V_{i\,max}$

then (a) $\underline{V}_i^{old}(t+\Delta t) \stackrel{def}{=} \underline{V}_i(t+\Delta t)$

(b) $\underline{V}_i^{New}(t+\Delta t) = V_{i\,max} \dfrac{\underline{V}_i^{old}(t+\Delta t)}{\|\underline{v}_i^{old}(t+\Delta t)\|}$

(c) $\underline{v}_i(t+\Delta t) = \underline{v}_i^{New}(t+\Delta t)$

Mapped criteria:

If $\|\underline{r}_i - \underline{T}_j\| \leq tol \Rightarrow$ take $\underline{T}_j$ out of the system

Obstacle Criteria:

If $\|\underline{r}_i - \underline{O}_j\| \leq tol \Rightarrow$ Immobilize the VAV

Algorithm: at time $= t$

(1) Initialize target Locations: $\underline{T}_j = (T_{j1}, T_{j2}, T_{j3})$
$j = 1, \ldots N_t$

(2) Initialize obstacle Locations:

$\underline{O}_j = (O_{j1}, O_{j2}, O_{j3})$
$j = 1, 2, \ldots N_o$

(3) Initialize swarm Locations

$\underline{r}_i = (r_{i1}, r_{i2}, r_{i3}),\ i = 1, 2, \ldots N_m$

4) Determine relative distances and directions $\underline{n}$, $\underline{d}$

5) For each swarm member determine $\underline{N}_i^{mt}$, $\underline{N}_i^{mo}$, $\underline{N}_i^{mm}$

6) For each swarm member determine $\underline{f}_i^{tot} = F_i \underline{n}_i^*$

7) For each swarm member integrate ODE to yield

(with speed) Limits

$$\underline{v}_i(t+\Delta t) = \underline{v}_i(t) + \frac{\Delta t}{m_i} \underline{f}_i^{tot}(t)$$

$$\underline{r}_i(t+\Delta t) = \underline{r}_i(t) + \Delta t \, \underline{v}_i(t)$$

8) check mapping criteria $\|\underline{r}_i - \underline{T}_j\| \leq tol$

check obstacle criteria $\|\underline{r}_i - \underline{O}_j\| \leq tol$

9) Repeat for next time step

- See example in Reader.
- Design vector:

$$\underline{\Lambda} = \{\Lambda_1, \ldots \Lambda_N\}$$

$$= \{W_{mt}, W_{mo}, W_{mm}, w_{t1}, w_{t2}, w_{o1}, w_{o2}, w_{m1}, w_{m2}, a_1, a_2, b_1, b_2, c_1, c_2\} = 15 \text{ variables}$$

- These parameters are hard to estimate, thus necessitating optimization.

Machine Learning

Objective: Map all targets as fast as possible.

For example: $\Pi(\underline{\Lambda}) = (w_1 A + w_2) T_m$

big

percentage of targets remaining

small

percentage of time used to achieve the result

Nondimensional

Apply MLA / Genetic algorithm:

(1) Generate Population: 5 of them
(2) Evaluate Performance: $\Pi(\underline{\Lambda}^{(i)})$
(3) Rank Performances
(4) "Mate" the top performers => "children"
(5) Eliminate the poor performers
(6) Add new population members
(7) Repeat process until tolerance met
(8) Apply gradient based methods if needed