Project 5 Report

Dylan Callaway

Engineering 150
Fall 2019

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0.1 Introduction

This project models the development of a new material comprised of two other constituent materials. The goal is to use the model in order to select constituent materials, as well as the ratio at which they are mixed, such that desired final material properties are achieved.

The model of the new material is constructed by using the bounds proposed by Hashin and Shtrikman (HS) in their 1963 paper.[1] These bounds are the upper and lower limit on the overall properties of the new material, which is assumed to be isotropic, as a function of the properties of constituent isotropic materials.

The properties of interest in this case are the bulk modulus, shear modulus, electrical conductivity, and thermal conductivity; denoted κ, μ, σ_e, and K, respectively. The effective properties of the new material are denoted with the * superscript, and the properties of the constituent materials are denoted with either the $_1$ or $_2$ subscripts.

This paper will develop the necessary physical background for the modeling of the materials, outline the specific methods used in the implementation of the governing physics equations, and finally discuss the resulting material properties achieved and how various algorithmic changes affected the outcomes.

0.2 Background and Theory

Using the HS bounds for the properties of interest in this project produces the following equations for the bulk modulus, shear modulus, electrical conductivity, and thermal conductivity as functions of the volume fraction of each constituent material, v_2.

$$\kappa_1 + \frac{v_2}{\frac{1}{\kappa_2-\kappa_1} + \frac{3(1-v_2)}{3\kappa_1+4\mu_1}} \leq \kappa^* \leq \kappa_2 + \frac{1-v_2}{\frac{1}{\kappa_1-\kappa_2} + \frac{3v_2}{3\kappa_2+4\mu_2}} \tag{1}$$

$$\mu_1 + \frac{v_2}{\frac{1}{\mu_2-\mu_1} + \frac{6(1-v_2)(\kappa_1+2\mu_1)}{5\mu_1(3\kappa_1+4\mu_1)}} \leq \mu^* \leq \mu_2 + \frac{1-v_2}{\frac{1}{\mu_1-\mu_2} + \frac{6v_2(\kappa_2+2\mu_2)}{5\mu_2(3\kappa_2+4\mu_2)}} \tag{2}$$

$$\sigma_{e,1} + \frac{v_2}{\frac{1}{\sigma_{e,2}-\sigma_{e,1}} + \frac{1-v_2}{3\sigma_{e,1}}} \leq \sigma_e^* \leq \sigma_{e,2} + \frac{1-v_2}{\frac{1}{\sigma_{e,1}-\sigma_{e,2}} + \frac{v_2}{3\sigma_{e,2}}} \tag{3}$$

[1]Hashin, Z, and Shtrikman, S, 1963, A variational approach to the elastic behavior of multiphase minerals. Journal of the Mechanics and Physics of Solids

$$K_1 + \frac{v_2}{\frac{1}{K_2-K_1} + \frac{1-v_2}{3K_1}} \leq K^* \leq K_2 + \frac{1-v_2}{\frac{1}{K_2-K_2} + \frac{v_2}{3K_2}} \tag{4}$$

When used to compare with the desired material properties, the average of the upper and lower HS bound is taken for each property according to:

$$y^* \approx \phi y^{*,-} + (1-\phi) y^{*,+} \tag{5}$$

where $\phi = 0.5$, y is some material property, and the $^-$ and $^+$ superscripts denote the lower and upper bound, respectively.

0.3 Procedure and Methods

0.4 Results and Discussion

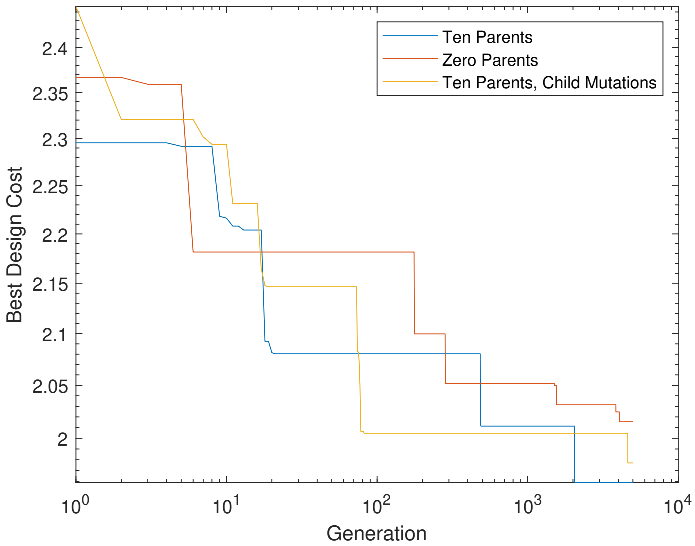

Figure 1: Cost of best design vs. generation for various GA types.

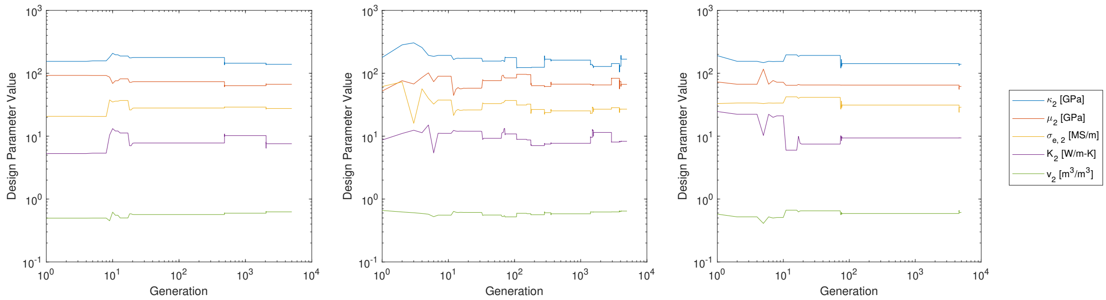

Figure 2: Design parameter value of best design vs. generation for various GA types. From left to right: ten parents, zero parents, ten parents with child mutations

Table 1: Fractional difference between achieved and desired design parameter for various GA types.

GA Variation	κ_2	μ_2	$\sigma_{e,2}$	K_2
Ten Parents	-0.249	-0.419	-0.375	-0.223
Zero Parents	-0.511	-0.417	-0.345	-0.335
Ten Parents, Child Mutations	-0.252	-0.313	-0.439	-0.512

Table 2: Sum of absolute fractional differences for various GA types.

GA Variation	Sum of Frational Differences
Ten Parents	1.266
Zero Parents	1.607
Ten Parents, Child Mutations	1.516

Table 3: Best four design strings for GA with ten parents.

Design	κ_2	μ_2	$\sigma_{e,2}$	K_2	v_2	ν	E^y
1	138.651	66.708	27.504	7.581	0.625	0.293	450.550
2	138.651	66.708	27.504	7.581	0.625	0.293	450.550
3	138.651	66.708	27.504	7.581	0.625	0.293	450.550
4	138.651	66.708	27.504	7.581	0.625	0.293	450.550

Table 4: Best four design strings for GA with zero parents.

Design	κ_2	μ_2	$\sigma_{e,2}$	K_2	v_2	ν	E^y
1	167.687	66.585	26.892	8.279	0.644	0.325	551.208
2	167.687	66.585	26.892	8.279	0.644	0.325	551.208
3	167.687	66.585	26.892	8.279	0.644	0.325	551.208
4	167.687	66.585	26.892	8.279	0.644	0.325	551.208

Table 5: Best four design strings for GA with ten parents and child mutations.

Design	κ_2	μ_2	$\sigma_{e,2}$	K_2	v_2	ν	E^y
1	138.982	61.697	28.771	9.377	0.613	0.307	448.375
2	138.982	61.697	28.771	9.377	0.613	0.307	448.375
3	138.982	61.697	28.771	9.377	0.613	0.307	448.375
4	138.982	61.697	28.771	9.377	0.613	0.307	448.375

0.5 Conclusion

0.6 Appendix

Appendix is empty.