



**SCHULICH**  
School of Engineering

# ENGO 419 Lab Assignment 1

Pre-Analysis of Surveying Observations

Instructor: Ivan Detchev

TA: Claire Mah

Assignment Deadline: September 29, 2023

Titus Castillon (UCID 30156837)

Dylan Conley (UCID 30140483)

Nick Kennedy (UCID 30145355)

Group 7

Title of Lab Assignment: Lab 1 - Pre-analysis of Surveying Observations

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Your Name: Dylan Conley Student ID: 30140483

Contribution % 33.33% Hours: 12

[signature]  Date Sept 29, 2023


Your Name: Titus Castillon Student ID: 30156837

Contribution % 33.33% Hours: 14

[signature]  Date Sept 29, 2023

Your Name: Nick Kennedy Student ID: 30145355

Contribution % 33.33% Hours: 10

[signature]  Date Sept 29, 2023

Your Name: \_\_\_\_\_ Student ID: \_\_\_\_\_

Contribution % \_\_\_\_\_ Hours: \_\_\_\_\_

[signature] \_\_\_\_\_ Date \_\_\_\_\_

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## II List of Definitions

### Geomatics network:

A system of nodes or stations connected by geodetic observations. These stations may be connected through survey measurements, for example, through height measurements using a levelling network, or through angle and distance measurements using a total station. [1]

### Correlation coefficient $\rho_{ab}$ :

Scales in a range from -1 to 1 [6]. 0 correlation means that datasets  $a$  and  $b$  are completely uncorrelated [6]. -1 represents complete negative correlation, and 1 represents complete positive correlation.

If  $0 \leq |\rho_{ab}| < 0.35$ , then point  $a$  and  $b$  are weakly correlated (or a strong solution) [6].

If  $0.35 \leq |\rho_{ab}| < 0.75$ , then point  $a$  and  $b$  are significantly correlated [6].

If  $0.75 \leq |\rho_{ab}| < 1$ , then datasets  $a$  and  $b$  are strongly correlated (or a weak solution) [6].

# 1 Introduction

This lab assignment requires performing and presenting a pre-analysis on surveying observation scenarios. Particularly, the pre-analysis specifics of estimating accuracies and screening of observations will be presented in this assignment. A learning goal of this assignment is for the lab members to gain a comprehensive understanding of how to mathematically perform a pre-analysis on a set of observations. Another learning goal is to observe and be aware of the potential implications of how blunder observations affect a dataset in the pre-analysis phase of establishing geomatics networks.

An important objective of creating this lab report is to detect inconsistencies and gaps in the understanding of each lab member concerning the pre-analysis phase of developing geomatics networks. By the end of this lab, each member will be confident in their knowledge of pre-analysis in geomatics networks so that they can retain and apply the knowledge effectively in future academic and professional endeavours.

This lab will be broken into three parts. The first part will look at error propagation and covariances by solving for angles from directions. The impact of independent vs. dependently collected observations will also be considered. The second part and third parts will then look at statistical testing for outliers in a levelling loop and total station distances, respectively. The misclosure of the system and individual blunders will be checked for.

## 2 Procedures

### 2.1 First Scenario - Estimation of Observation Accuracy

#### 2.1a Computing the Variance-Covariance Matrix of the Three observations

First, mean values of the angular observations,  $a$ ,  $b$ , and  $c$  were calculated, then the residuals,  $\hat{v}$  of each angular observation were found (a total of  $n = 11$  repetitive observations per angle) [6]:

$$\hat{v}_{x_i} = \bar{x} - x_i \quad [\text{Equation 2.1a.1}]$$

Where  $x_i$  is the individual observation, and  $\bar{x}$  is the mean of the set of observations of dataset  $x$ .

The next procedure is to calculate the variance-covariance matrix for the three observations. The formula for covariance is the following [6]:

$$\sigma_{ab} = \frac{1}{n-1} \sum_{i=1}^n (\hat{v}_a \cdot \hat{v}_b) \quad [\text{Equation 2.1a.2}]$$

Where  $a$  is the first dataset,  $b$  is the second dataset, and  $n$  is the number of observations. The form of a 3x3 variance-covariance matrix is shown below [6]:

$$C_{abc} = \begin{bmatrix} \sigma_a^2 & \sigma_{ab} & \sigma_{ac} \\ \sigma_{ab} & \sigma_b^2 & \sigma_{bc} \\ \sigma_{ac} & \sigma_{bc} & \sigma_c^2 \end{bmatrix} \quad [\text{Equation 2.1a.3}]$$

Once the variance-covariance matrix is found, correlation can then be determined. The correlation matrix will be 3x3. Correlation is solved by [6]:

$$\rho_{ij} = \rho_{ji} = \frac{\sigma_{ij}}{\sigma_i \sigma_j} \quad [\text{Equation 2.1a.4}]$$

### 2.1c Computing the Angles $\alpha$ , $\beta$ , $\gamma$ , and Their Variance-Covariance Matrix

The specified angles were computed with the following formulas:

$$\alpha = \bar{b} - \bar{a} \quad [\text{Equation 2.1c.1}]$$

$$\beta = \bar{c} - \bar{b} \quad [\text{Equation 2.1c.2}]$$

$$\gamma = \bar{c} - \bar{a} \quad [\text{Equation 2.1c.3}]$$

Where  $\bar{a}$ ,  $\bar{b}$ , and  $\bar{c}$  are the mean directions for  $a$ ,  $b$ , and  $c$ . The variance-covariances were then propagated from the directions to the angles by the formula [3]:

$$C_{3 \times 3, \alpha\beta\gamma} = J_{3 \times 3} C_{3 \times 3, abc} J_{3 \times 3}^T \quad [\text{Equation 2.1c.4}]$$

Where  $J$  represents the Jacobian Matrix as defined by [3]:

$$J = \frac{df}{dobs} = \frac{df}{d(a, b, c)} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \quad [\text{Equation 2.1c.5}]$$

This step was repeated twice, once with the full variance-covariance matrix of the observations and once where the observations were considered to be independent, indicating all covariances are 0 while variances went unchanged. The difference between independent and dependent observations will be looked at in the discussion.

The correlation coefficient matrix of the angles was then solved for each of the two scenarios, uncorrelated observations and correlated observations, by using equation 2.1a.2.

## 2.2 Second Scenario - Observation Screening of Levelling Loop

### 2.2a Testing the Loop Misclosure

The first step in this procedure is to calculate the misclosure of the overall levelling loop:

$$\Delta i = \Delta \bar{h}_{AB} + \Delta \bar{h}_{BC} + \Delta \bar{h}_{CD} + \Delta \bar{h}_{DE} + \Delta \bar{h}_{EF} + \Delta \bar{h}_{FA} \quad [\text{Equation 2.2a.1}]$$

where  $\Delta i$  is the misclosure of the levelling loop, and  $\Delta \bar{h}$  is the average height difference of a levelling loop leg. It will be checked if the misclosure is 0 under a normal distribution[2]:

$$H_0: E\{\Delta\} = 0 \quad H_1: E\{\Delta\} \neq 0 \quad [\text{Equation 2.2a.2}]$$

The following test statistic would be:

$$y = \frac{\Delta i}{\sigma_{\Delta i}} \sim n(0, 1) \quad [\text{Equation 2.2a.3}]$$

where  $\sigma_{\Delta i}$  is the standard deviation of the levelling loop.  $\sigma_{\Delta i}$  is calculated as [4]:

$$\sigma_{\Delta i}^2 = \frac{df}{dl_{obs}}^2 = \sigma_{\Delta \bar{h}_{AB}}^2 + \sigma_{\Delta \bar{h}_{BC}}^2 + \dots + \sigma_{\Delta \bar{h}_{FA}}^2 = \frac{\sigma_{\Delta h_{AB}}^2}{n} + \frac{\sigma_{\Delta h_{BC}}^2}{n} + \dots + \frac{\sigma_{\Delta h_{FA}}^2}{n} = \frac{6}{5} * \sigma_{\Delta h}^2 \quad [\text{Equation 2.2a.4}]$$

where  $\sigma_l = 2.00 \text{ mm}$  (a given value from the lab handout - source [3]). A two-tailed test will then be done to compare the test statistic with a statistical distribution. A Z-distribution with 95% two-tailed confidence will be used. A Z-distribution is used as our interpretation of the standard deviation because it is a known condition of the equipment.

### 2.2b Detecting Specific Blunders in the Repeated Observations in the Dataset

As stated in Section 2.2.1, the standard deviation of each observation is  $\sigma_l = 2 \text{ mm}$ . The null hypothesis for this scenario is that each repeated observation,  $l$ , should follow a normal distribution and can fit into the set of observations collected [2]:

$$H_0: E\{l_i\} = \bar{l} \quad H_1: E\{l_i\} \neq \bar{l} \quad [\text{Equation 2.2b.1}]$$

Since the true mean of each set of repeated observations is unknown (where the approximation of the mean is calculated by averaging the sample dataset), and the standard deviation of each



measurement is given from source [3] (i.e., the standard deviation is known), the form of the test statistic  $y$  is the following [5]:

$$y = \frac{l_i - \bar{l}}{\sigma \sqrt{\frac{n-1}{n}}} \sim n(0, 1) \quad [\text{Equation 2.2b.2}]$$

If the absolute value of  $y$  is less than the retrieved Z-value at a 95% confidence level, then that particular observation is an outlier. Then the final procedure of this section was to repeat Task 2.2a (testing for levelling observation misclosure) and Task 2.2b (testing for individual blunders in the dataset) with the blunder observations removed from the dataset to see if any changes happen in the results.

If blunders are detected, they will be removed and the procedure in Section 2.2 will be repeated with an updated observation list.

## 2.3 Third Scenario - Observation Screening of Triangular Network

### 2.3a Testing the Distance Misclosure

In the third scenario, three sides of a right-angle triangle are known. Therefore, two sides of the triangle should be able to be used to solve the third side. The difference between the measured third side and the calculated third side will be the misclosure represented as:

$$\Delta i = \sqrt{AC^2 + BC^2} - AB \quad [\text{Equation 2.3a.1}]$$

Where  $\Delta i$  is the misclosure and  $AC$ ,  $BC$  and  $AB$  were the direct observations. Each side of the triangle also carried an error of  $\pm (3 + D * 2ppm)mm$ . The variance of each triangle was therefore solved by the formula [7]:

$$\sigma_{XY}^2 = (0.003m)^2 + \left(\frac{2*XY_{(meters)}}{1\,000\,000}\right)^2 \quad [\text{Equation 2.3a.2}]$$

After calculating the variance for each observation, the variance for the mean of each set of observations was calculated. The mean variance is required due to the mean observations being used in the calculation of the misclosure loop.

$$\sigma_{\overline{XY}}^2 = \frac{\sigma_{XY}^2}{n} \quad [\text{Equation 2.3a.3}]$$

This calculated variance was then used for all error propagation. The same hypothesis test as setup in equation 2.2a.2 was used to see if the misclosure can be 0 based on a normal

distribution. The same test statistic in equation 2.2a.3 is also used where the misclosure is divided by the misclosures standard deviation. In this case  $\sigma_{\Delta i}$  will be solved as:

$$\sigma_{\Delta i}^2 = \frac{df}{dl_{obs}}^2 = \frac{\sigma_{AC}^2 AC^2}{AC^2 + BC^2} + \frac{\sigma_{BC}^2 BC^2}{AC^2 + BC^2} + \sigma_{AB}^2 \quad [\text{Equation 2.3a.4}]$$

A two-tailed test will then be used to see if the test statistic (equation 2.2a.3) will fall into a normal Z-distribution with a two-tailed 90% confidence. A Z-distribution was selected as the given standard deviation was based on the manufacturer's specifications [3].

### 2.3b Detecting for Individual Blunders in Distance Observations

The null hypothesis formed to detect individual blunders in distance observations is identical to Equation 2.2b.1. The test statistic formed for each observation is identical to Equation 2.2b.2, since in this scenario the standard deviation values are known (the instrument specifications are provided in Source [3]), and the true value is not known (the true value of the distance of each triangle side is approximated by averaging the repeated observations). If blunders were detected, Task 2.3a (testing for distance observation misclosure) and Task 2.3b (testing for individual blunders in the dataset) were repeated with the blunders removed from the dataset to obtain more accurate results.

## 3 Results

### 3.1 First Scenario

#### 3.1a Variance-Covariance Matrix

Table 3.1a.1: Variance-Covariance Matrix of Initial Observations  $[s^2]$

2.74	-1.55	-0.85
-1.56	5.24	-0.73
-0.85	-0.73	2.65

### 3.1b Correlation Matrix

Table 3.1b.1: Correlation Matrix of Initial Observations [unitless]

1	-0.41	-0.32
-0.41	1	-0.20
-0.32	-0.20	1

### 3.1c Angles and Their Variance-Covariance Matrix

Table 3.1c.1: Unknown Angles [DMS]

$\alpha$	17°52'51.8"
$\beta$	42°32'11.1"
$\gamma$	60°25'02.9"

Table 3.1c.2: Angles Variance-Covariance  $[s^2]$

11.08	-6.67	4.41
-6.67	9.35	2.68
4.41	2.68	7.09

### 3.1d Uncorrelated Angles' Variance-Covariance Matrix

Table 3.1d.1: Uncorrelated Angles Variance-Covariance  $[s^2]$

7.97	-5.24	2.74
-5.24	7.88	2.65
2.74	2.65	5.38

### 3.1e Correlation Matrix for Unknown Angles

Table 3.1e.1: Correlation Matrix of Unknown Angles [unitless]

1	-0.66	0.50
-0.66	1	0.33
0.50	0.33	1

### 3.1f Uncorrelated Angles Correlation Matrix

Table 3.1f.1: Correlation Matrix of Uncorrelated Unknown Angles [unitless]

1	-0.66	0.42
-0.66	1	0.401
0.42	0.41	1

## 3.2 Second Scenario

### 3.2a Misclosure Test for Outliers

After using the averages of the measurements as  $\Delta h$  values, we calculate the misclosure.

Table 3.2a.1: Computed Misclosure of the Levelling Loop

Misclosure [m]	-0.0135
Standard Deviation of Misclosure [m]	0.0022
Test Statistic, y	-6.1765

From the statistical testing for outliers on the misclosure at a 95% confidence level, we calculate a y value of -6.1765. Since this value does not fall between our calculated critical values of -1.96 and 1.96, we see that there is likely an outlier in the data. See below.

$[-1.96 < -6.1765 < 1.96]$  is not true!

### 3.2b Observations Test for Outliers

Because we know that there is likely an outlier, we now test each individual observation at a 95% confidence level to see if it is an outlier. A  $y$  value is calculated for each observation via Equation 2.2b.2.

Table 3.2b.1: Matrix of Y-Values for Outlier Test

0.0000	1.1080	-0.0559	0.000	-0.0671	-0.0559
0.0000	1.1303	0.0559	0.1118	0.1006	0.1677
-0.1118	1.1080	-0.0559	0.1677	-0.0112	-0.0559
0.0559	-4.4710	0.0000	0.0000	-0.0112	-0.1677
0.0559	1.1247	0.0559	-0.2795	-0.0112	0.1118

These values are then checked to see if it falls between the critical values -1.96 and 1.96. We see that the 4th observation for BC is a clear outlier.

### 3.2c Misclosure Test of Levelling Network After Removing Initial Outlier

Table 3.2c.1 shows the misclosure values of the levelling network after the initial detected outlier is removed:

Table 3.2c.1 Misclosures of Levelling Loop Without Initial Detected Outlier

Misclosure [m]	-0.0115
Standard Deviation of Misclosure [m]	0.0022
Test Statistic, $y$	-5.1765

The initial hypothesis states that there is 95% confidence that there is no misclosure in the levelling loop. However, since the test statistic does not fall within the 95% range of the  $z$ -table ( $|-5.1765| > 1.96$ ), the initial hypothesis is rejected and the null hypothesis is accepted that there is still a misclosure in the levelling loop, despite having the initial blunder removed.

### 3.2d Checking for Individual Blunders After Removing Initial Outlier

We now remove the outlier and repeat the check for individual blunders from before.

Table 3.2d.1: Matrix of Y-Values for Outlier Test (Second Trial)

0.0000	-0.0101	-0.0559	0.0000	-0.0671	-0.0559
0.0000	0.0130	0.0559	0.1118	0.1006	0.1677
-0.1118	-0.0101	-0.0559	0.1677	-0.0112	-0.0559
0.0559	NaN	0.0000	0.0000	-0.0112	-0.1677
0.0559	0.0072	0.0559	-0.2795	-0.0112	0.1118

We see that all of the Y-Values fall within the critical values of -1.96 and 1.96, meaning that the data is free of any remaining outliers.

### 3.3 Third Scenario

#### 3.3a Misclosure Test for Outliers

These tests are done much the same as for the second scenario. First, the misclosure is calculated to be:

Table 3.3a.1: Computed Misclosure of the Triangle

Misclosure [m]	-0.0229
Standard Deviation of Misclosure [mm]	0.007
Test Statistic, y	-3.2748

From the statistical testing for outliers on the misclosure at a 90% confidence level, we calculate a y value of -3.2748. Since this value does not fall between our calculated critical values of -1.6449 and 1.6449, we see that there is likely an outlier in the data. See below.  
 $[-1.6449 < -3.2748 < 1.6449]$  is not true!

#### 3.3b Observations Test for Outliers

We know that there is likely an outlier, so we now test each observation at a 90% confidence level to see if it is an outlier. A y value is calculated for each observation via Equation 2.2b.2.

Table 3.3b.1: Matrix of Y-Values for Outlier Test

0.0626	0.4170	0.0553
-0.4594	1.6331	-0.4978
0.0626	0.0695	-0.9587
-0.0418	-0.7992	1.9911
0.3758	-1.3204	-0.5900

These values are then checked to see if it falls between the critical values -1.6449 and 1.6449. We see that the 4th observation for AB is an outlier.

### 3.3c Misclosure Test of Triangular Network After Removing Initial Outlier

Table 3.3c.1 shows the misclosure values of the triangular network after the initial detected outlier is removed:

Table 3.3c.1 Misclosures of Triangular Network Without Initial Detected Outlier

Misclosure [m]	-0.018
Standard Deviation of Misclosure [m]	0.008
Test Statistic, y	-2.3335

The initial hypothesis states that there is 90% confidence that there is no misclosure in the levelling loop. However, since the test statistic does not fall within the 90% range of the Z-table ( $|-2.3335| > 1.6449$ ), the initial hypothesis is rejected and the null hypothesis is accepted that there is still a misclosure in the levelling loop, despite having the initial blunder removed.

### 3.3d Checking for Individual Blunders After Removing Initial Outlier

We now remove the outlier and repeat the check from before.

Table 3.3d.1: Matrix of Y-Values for Outlier Test (Second Trial)

0.0626	0.4170	0.5712
--------	--------	--------

-0.4594	1.6331	0.0000
0.0626	0.0695	-0.4760
-0.0418	-0.7992	NaN
0.3758	-1.3204	-0.0952

We see that all of the Y-Values fall within the critical values of -1.6449 and 1.6449, meaning that the data is free of any remaining outliers.

## 4 Discussion

### 4.1 First Scenario

Based on the initial observations and their variance-covariance matrix, we can see that the direction measurement for C is more precise than the A or B as it has the lowest variation and standard deviation. However, the measurement for A is not that much more imprecise ( $\sim 0.09 \text{ s}^2$  difference).

From the correlation matrix of the measurements, we see that A is more negatively correlated with C than B is with C. This makes sense as B's precision is noticeably lower than A's, meaning it would be less likely to affect the measurements. Additionally, all of the observations are not strongly correlated indicating that there is some independence between the measurements. There was a strong solution between B and C but there was significant correlation between observations A and the rest of the data. This indicates that the observations were not completely independently collected.

After creating two variance-covariance matrices for the unknown angles; one assuming the observations are independent and one assuming they are not; we see that the variance for the matrix that assumes independent observation collection is noticeably more precise. This can be justified from an understanding that if observations are correlated, there is going to be less trust placed in the results due to a correlation of the observations which can mean increased error when solving for the unknowns.

From looking at the correlation matrix that assumed the directions were correlated and dependent on each other, it can be seen that angle alpha is strongly negatively correlated with beta. This makes sense because alpha and beta are supposed to sum to make gamma, meaning that if the sum were to be maintained, one would have to lessen as the other grows. Gamma maintains a positive correlation with both alpha and beta, making sense because gamma is the sum of them



both, so as they grow gamma should also grow. When we assume no correlation, the correlation matrix shows a very similar result, with all correlations slightly increasing in magnitude but keeping the same signs.

## 4.2 Second Scenario

This is a levelling loop, meaning that the sum of all the legs is supposed to be zero. However, after testing the misclosure we see that there is an outlier. The test failed as the test statistic of -6.1765 did not fall in the range [-1.96,1.96]. We can reject the null hypothesis and say that the misclosure does not equal 0 with 95% confidence.

This then brings specific outlier analysis, upon visual inspection, the 0.04635m measurement in the second column appears suspicious when compared to the other values. The outlier test on individual values verified that it was an outlier with a confidence level of 95%. It had a large residual by a magnitude of 4 times and when tested its standardized residual did not fall within the confidence bounds. All other values were also tested and no other blunders were detected. The root cause of this blunder can not be determined with certainty but it appears to be a reading or booking error as the observation appears to be about 1cm off of what it should be.

This value was then removed and the misclosure was tested again. With a y-value of 5.1575, the misclosure test failed again. A second blunder check indicated no blunders. This indicates that there are two possible errors in the system:

- The 2mm accuracy model for the noise in the system was incorrect, and/or,
- There is a bias in the system affecting average height measurements at one or more of the stations

A completely independent set of rounds of observations would need to be completed to identify if there was a systematic error at one of the turning points.

## 4.3 Third Scenario

This scenario is almost identical to the second one, except it is measuring the sides of a right-angle triangle as opposed to a levelling loop. However, a key difference in this scenario is that each measurement had its own standard deviation, while in the second scenario, the standard deviation was constant across all values. Regardless, the same checks are applied with a 90% confidence level.

Firstly, the test on the misclosure indicated that there was likely an outlier in the data. A test statistic of the misclosure of -3.27 did not fall in the normal distribution range of [-1.6449,1.6449]. We can again reject the null hypothesis and say that the misclosure does not equal 0 with 90% confidence in the system.

Upon visual inspection, none of the data appears to contain blunders, but doing the test on individual values at a 90% confidence level reveals that the 5875.893 m measurement for AB is an outlier. It had a test statistic of 1.9911 which fell out of the distribution range. The root cause of this blunder can not be identified without a greater knowledge of the field operations.

Upon removing the blunder the misclosure and outliers were tested for again. No more outliers were detected but the misclosure continued to not equal zero with 90% confidence. As a blunder cannot be detected, it indicates there was either an issue with random noise or systematic errors which can be specified as

- The manufacturer's specifications to model the bias/noise in the system was incorrect based on the conditions of the equipment, and/or,
- There is bias in the collection of some or all of the distances across the whole system

The exact issue of the misclosure cannot be identified without more fieldwork being performed.

## 5 Conclusion

The angles were successfully solved from the bearings in problem 1. The errors were propagated and it was identified that there is more confidence in the results if the observations are considered to be independent. In problem 2, an outlier was identified and removed. The misclosure test continued to fail which indicates that there is likely unconsidered bias in the system or an error in the error model. In problem 3, an outlier was also identified and removed. The misclosure test also failed even after outlier removal which again indicates bias or an error in the error model.

The results show that the test statistic values and residuals of a set of observations significantly decrease when you remove the blunder observation from the dataset. Despite that, this lab is a good reminder that even if no outliers are identified there may still be errors in the system. This finding shows that it is important to remove blunders in pre-analysis before performing a least-squares adjustment on a dataset so that the final adjusted results will be more precise and accurate. The adjustment may also potentially not converge if a blunder is not removed in the least squares adjustment [5], therefore this pre-screening was a necessary step in the analysis of the observations.

This lab was valuable for learning outcomes in group members becoming more familiar with the pre-analysis of a set of observations. Familiarity with statistical testing was reestablished and this lab was excellent practice to become more familiar with C++ and Matlab.

## References

- [1] M.G. Sideris, "Introduction - Network Concepts," ENGO 419 L01 - (Fall 2023) - Geomatics Networks - F2023ENGO419L01, <https://d2l.ucalgary.ca/d2l/le/content/543211/viewContent/6044575/View> (accessed Sep. 24, 2023).
- [2] M. G. Sideris, "Preanalysis - Concepts and Components," ENGO 419 L01 - (Fall 2023) - Geomatics Networks - F2023ENGO419L01, <https://d2l.ucalgary.ca/d2l/le/content/543211/viewContent/6044576/View> (accessed Sep. 24, 2023).
- [3] I. Datchev, C. Mah, "Precision analysis of observations, data pre-processing, and pre-adjustment screening," ENGO 419 L01 - (Fall 2023) - Geomatics Networks - F2023ENGO419L01, <https://d2l.ucalgary.ca/d2l/le/content/543211/viewContent/6077200/View> (accessed Sep. 24, 2023).
- [4] W. NAVIDI, "Confidence Intervals," in *Statistics for engineers and scientists*, MCGRAW-HILL EDUCATION, 2020, pp. 323–326
- [5] I. Datchev "Data Screening", Lecture, ENGO 419 University of Calgary, September 20, 2023
- [6] M. Blois, "ENGO 363 Estimation and Statistical Testing - Multivariate statistics and confidence intervals," ENGO 363 L01 - (Fall 2023) - Estimation and Statistical Testing - W2023ENGO363L01, <https://d2l.ucalgary.ca/d2l/le/content/499544/viewContent/5733291/View>.
- [7] I. Datchev "Slope Distance Measurements Error", Lecture, ENGO 419 University of Calgary, September 15, 2023