Explanation of algorithm for sampling from $s_k(x) = u_k(x)/\int_D u_k(x')dx' = c\ u_k(x)$. First we need to find the normalizing constant $c := 1/\int_D u_k(x')dx'$:

$$\int_D u_k(x')dx' = \sum_{j=1}^{k} \int_{z_{j-1}}^{z_j} \exp(h(x_j) + (x' - x_j)h'(x_j))dx'$$

$$= \sum_{j=1}^{k} \frac{1}{h'(x_j)} \exp(h(x_j) + (x' - x_j)h'(x_j))\Big|_{x'=z_{j-1}}^{z_j}$$

$$= \sum_{j=1}^{k} \frac{\exp u_k(z_j) - \exp u_k(z_{j-1})}{h'(x_j)}$$

Next, in order to use the inverse CDF method for sampling, we must find the CDF for $s_k(x)$, $S_k(x) = c\int_{z_0}^{x} u_k(x')dx'$:

$$S_k(x) = c\left(\sum_{j=1}^{t-1} \int_{z_{j-1}}^{z_j} \exp(h(x_j) + (x' - x_j)h'(x_j))dx' + \int_{z_{t-1}}^{x} \exp(h(x_t) + (x' - x_t)h'(x_t))dx'\right)$$

$$= c\left(\sum_{j=1}^{t-1} \frac{\exp u_k(z_j) - \exp u_k(z_{j-1})}{h'(x_j)} + \frac{\exp u_k(x) - \exp u_k(z_{t-1})}{h'(x_t)}\right)$$

where t is the index of which interval of z's that x lies in. Formally, it is $t(x) = i : x \in (z_{i-1}, z_i)$. For convenience, we call the sum term above `partialSums[t-1]`. Note that our normalizing constant $c =$ `1/partialSums[k]`. Thus,

$$S_k(x) = c\left(\texttt{partialSums[t-1]} + \frac{\exp u_k(x) - \exp u_k(z_{t-1})}{h'(x_t)}\right)$$

Now we can determine the inverse transform $S_k^{-1}(U)$, where $U \sim$ Uniform[0,1]. Because t is actually a function of x, it too must be inverted. Intuitively, we want to pick the biggest t such that $S_k(z_{t-1}) < U$. Formally, $t(U) = i : U \in (c\ \texttt{partialSums[i - 1]}, c\ \texttt{partialSums[i]})$. Solving for the inverse, we have:

$$S_k^{-1}(U) = \frac{\log\left(h'(x_t)(\frac{U}{c} - \texttt{partialSums[t-1]}) + \exp u_k(z_{t-1})\right) - h(x_t)}{h'(x_t)} + x_t$$