Explanation of algorithm for sampling from $s_k(x) = u_k(x)/\int_D u_k(x')dx' = c\, u_k(x)$. First we need to find the normalizing constant $c := 1/\int_D u_k(x')dx'$:

$$\int_D u_k(x')dx' = \sum_{j=0}^{k+1} I_j,$$

where

$$I_j := \int_{z_{j-1}}^{z_j} \exp(h(x_j) + (x' - x_j)h'(x_j))dx'.$$

Notice that the explicit form of I_j depends on whether $h'(x_j) = 0$ or not. Therefore we have,

$$I_j = \begin{cases} (z_j - z_{j-1}) \exp(h(x_j)) & \text{if} \quad h'(x_j) = 0, \\ \frac{\exp u_k(z_j) - \exp u_k(z_{j-1})}{h'(x_j)} & \text{otherwise.} \end{cases}$$

Next, in order to use the inverse CDF method for sampling, we must find the CDF for $s_k(x)$, $S_k(x) = c\int_{z_0}^{x} u_k(x')dx'$:

$$S_k(x) = c\left(\sum_{j=0}^{t-1} I_j + \int_{z_{t-1}}^{x} \exp(h(x_t) + (x' - x_t)h'(x_t))dx'\right),$$

where t is the index of which interval of z's that x lies in. Formally, it is $t(x) = \{1 \le i \le k+1 : x \in (z_{i-1}, z_i)\}$. For convenience, let

$$\texttt{partialSums[t-1]} := \sum_{j=1}^{t-1} I_j$$

and notice that our normalizing constant can be expressed as $c = \texttt{1/partialSums[k]}$. Moreover, let us define

$$J_{t-1}(x) := \int_{z_{t-1}}^{x} \exp(h(x_t) + (x' - x_t)h'(x_t))dx').$$

Then we have,

$$S_k(x) = c\left(\texttt{partialSums[t-1]} + J_{t-1}(x)\right),$$

where

$$J_{t-1}(x) = \begin{cases} \exp\big(h(x_t)\big)(x - z_{t-1}) & \text{if} \quad h'(x_t) = 0, \\ \frac{\exp u_k(x) - \exp u_k(z_{t-1})}{h'(x_t)} & \text{otherwise.} \end{cases}$$

Now, we can determine the inverse transform $S_k^{-1}(U)$, where $U \sim$ Uniform[0,1]. Because t is actually a function of x, it too must be inverted. Intuitively, we want to pick the biggest t such that $S_k(z_{t-1}) < U$. Formally,

$t(U) = \{1 \leq i \leq k+1 : U \in (c \times \texttt{partialSums[i - 1]}, c \times \texttt{partialSums[i]})\}$. Solving for the inverse, we have:

$$S_k^{-1}(U) = \begin{cases} \frac{\frac{U}{c} - \texttt{partialSums[t-1]}}{\exp\left(h(x_t)\right)} + z_{t-1} & \text{if} \quad h'(x_t) = 0, \\ \frac{\log\left(h'(x_t)(\frac{U}{c} - \texttt{partialSums[t-1]}) + \exp u_k(z_{t-1})\right) - h(x_t)}{h'(x_t)} + x_t & \text{otherwise.} \end{cases}$$