Problem 1

Maximin Optimization Problems

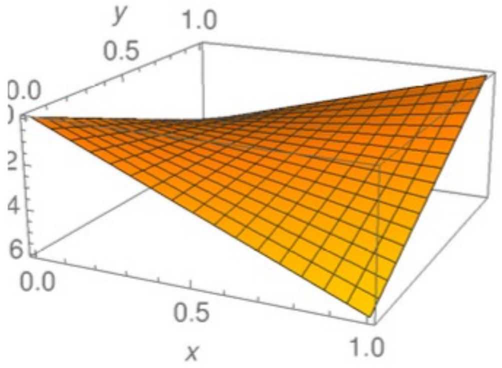
$$f(x) := \min_{y \in \mathcal{Y}} g(x, y).$$

求函数 f(x) 的最大值点

The Bilinear Problem

$$\textsc{Bilinear}(x, y) := |y|(|x| - \beta n) - \alpha n|x|,$$

对于特殊的 Bilinear 函数，求其最大值点



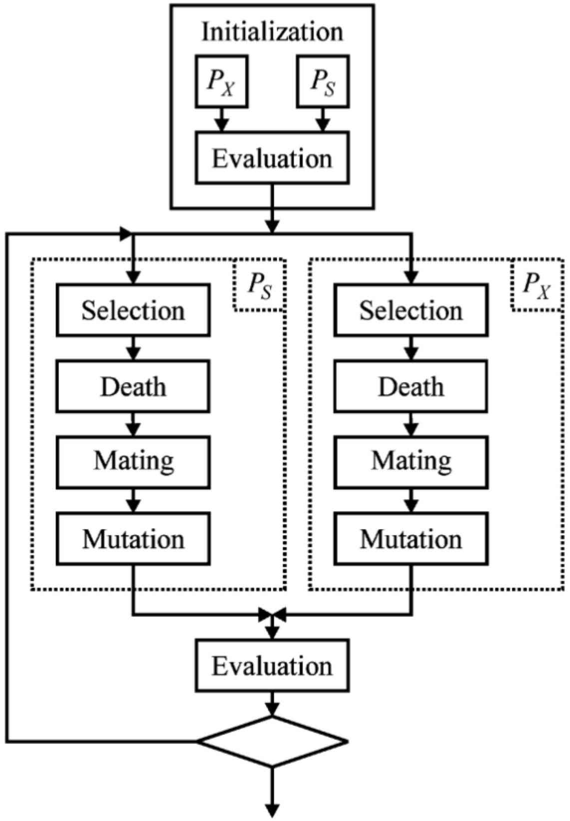
Bilinear 函数图像

Algorithm 1

J. W. Herrmann. A Genetic Algorithm for Minimax Optimization Problems. In Angeline et. al., editors, Proc. 1999 Congress on Evolutionary Computation, pages 1099-1103, 1997.

Parallel Coevolutionary Algorithm

1. 随机生成初始种群 P_1 P_2
2. 根据 P_2 种群对 P_1 中的个体进行估值
3. 依据 P_1 种群对 P_2 中的个体进行估值
4. 对 P_1 进行变异和交换操作，生成新的 P_1 种群
5. 对 P_2 进行变异和交换操作，生成新的 P_2 种群
6. 回到步骤 2



Algorithm 1

Parallel Coevolutionary Algorithm

Algorithm 1: Parallel Coevolutionary Algorithm

Input: Object function $g : \{0,1\}^n \times \{0,1\}^n \to \mathbb{R}$
Input: Population size $\lambda \in \mathbb{N}$
Output: Max value of function $f(x) = \min_{y \in \mathcal{Y}} g(x, y)$

for $i \in [\lambda]$ **do**
    Sample $P_0(i) \sim Unif(\{0,1\}^n)$;
    Sample $Q_0(i) \sim Unif(\{0,1\}^n)$;
end
for $t \in \mathbb{N}$ **do**
    for *individual* $x \in P_t$ **do**
        caculate $h(x) = \min_{y \in Q_t} g(x, y)$;
    end
    for *individual* $y \in Q_t$ **do**
        caculate $v(x) = \max_{x \in P_t} g(x, y)$;
    end
    sort$(P_t,\ h)$;
    sort$(Q_t,\ v)$;
    $x^{'} \leftarrow$ mutation$(P_t(\lambda))$;
    $y^{'} \leftarrow$ mutation$(Q_t(1))$;
    if $\min_{y \in Q_t} g(x^{'}, y) > h(P_t(1))$ **then**
        $P_t(1) \leftarrow x^{'}$;
    end
    if $\max_{x \in P_t} g(x, y^{'}) > v(Q_t(\lambda))$ **then**
        $Q_t(\lambda) \leftarrow y^{'}$;
    end
    $P_{t+1} \leftarrow P_t$;
    $Q_{t+1} \leftarrow Q_t$;
end

Algorithm 1

Parallel Coevolutionary Algorithm 能使用 Theorem 1 中的框架，得到与原文章相同的复杂度和结论

Parallel Coevolutionary Algorithm 满足 Theorem 1 中的条件

Theorem 1

Theorem 3. *Given subsets* $A_j \subseteq \mathcal{X}$, $B_j \subseteq \mathcal{Y}$ *for* $j \in [m]$ *where* $A_1 := \mathcal{X}$ *and* $B_1 := \mathcal{Y}$, *define* $T := \min\{t\lambda \mid (P_t \times Q_t) \cap (A_m \times B_m) \neq \emptyset\}$, *where for all* $t \in \mathbb{N}$, $P_t \in \mathcal{X}^\lambda$ *and* $Q_t \in \mathcal{Y}^\lambda$ *are the populations of Algorithm* $\boxed{1}$ *in generation* t. *If there exist* $z_1, \ldots, z_{m-1}, \delta \in (0,1]$, *and* $\gamma_0 \in (0,1)$ *such that for any populations* $P \in \mathcal{X}^\lambda$ *and* $Q \in \mathcal{Y}^\lambda$ *with "current level"* $j := \max\{i \in [m-1] \mid |(P \times Q) \cap (A_i \times B_i)| \geq \gamma_0 \lambda^2\}$

(G1) *for* $(x, y) \sim \mathcal{D}(P, Q)$

$$\Pr(x \in A_{j+1}) \Pr(y \in B_{j+1}) \geq z_j,$$

(G2a) *for all* $\gamma \in (0, \gamma_0)$ *if* $|(P \times Q) \cap (A_{j+1} \times B_{j+1})| \geq \gamma\lambda^2$, *then for* $(x, y) \sim \mathcal{D}(P, Q)$,

$$\Pr(x \in A_{j+1}) \Pr(y \in B_{j+1}) \geq (1 + \delta)\gamma,$$

(G2b) *for* $(x, y) \sim \mathcal{D}(P, Q)$,

$$\Pr(x \in A_j) \Pr(y \in B_j) \geq (1 + \delta)\gamma_0,$$

(G3) *and the population size* $\lambda \in \mathbb{N}$ *satisfies for a sufficiently large constant* c', *where* $z_* := \min_{i \in [m-1]} z_i$,

$$\lambda \geq c' \log(m/z_*),$$

then for a constant $c'' > 0$, $\mathbb{E}[T] \leq c''\lambda \left(\lambda^2 m + \sum_{i=1}^{m-1} 1/z_i\right)$.

Definition 2

对搜索空间进行划分，定义 $S_0, S_1(k), S_2(k)$

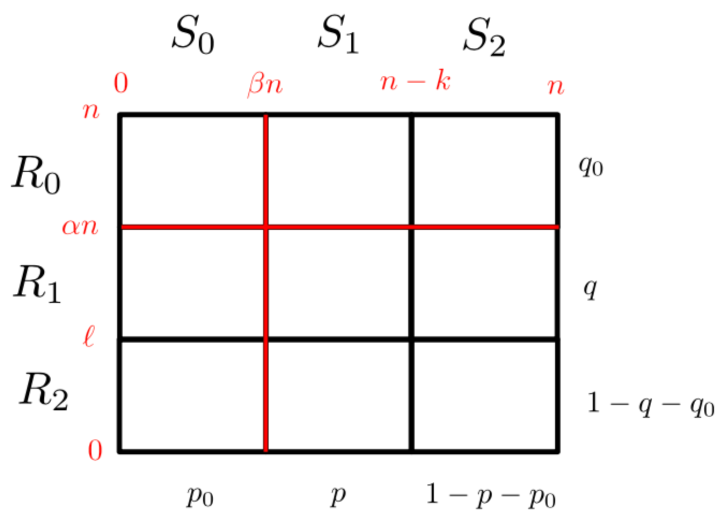
$$
\begin{aligned}
S_0 &:= \{x \in \mathcal{X} \mid 0 \le |x| < \beta n\} \\
S_1(k) &:= \{x \in \mathcal{X} \mid \beta n \le |x| < n-k\}, \\
S_2(k) &:= \{x \in \mathcal{X} \mid n-k \le |x| \le n\}.
\end{aligned}
$$

定义 $R_0, R_1(k), R_2(k)$

$$
\begin{aligned}
R_0 &:= \{y \in \mathcal{Y} \mid \alpha n \le |y| \le n\} \\
R_1(\ell) &:= \{y \in \mathcal{Y} \mid \ell \le |y| < \alpha n\} \\
R_2(\ell) &:= \{y \in \mathcal{Y} \mid 0 \le |y| < \ell\}.
\end{aligned}
$$

同时定义概率 $\mathrm{p}(C)$ $p_{sel}(C)$，其中 $C \subseteq \mathcal{X} \times \mathcal{Y}$,

$$
\begin{aligned}
p(C) &:= \Pr_{(x,y)\sim \mathrm{Unif}(P\times Q)}((x,y) \in C) \\
p_{\mathrm{sel}}(C) &:= \Pr_{(x,y)\sim \mathtt{select}(P\times Q)}((x,y) \in C).
\end{aligned}
$$



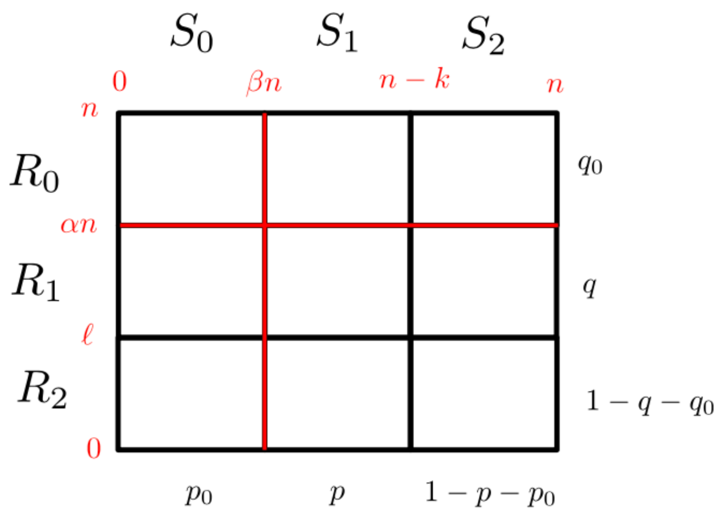
搜索空间的划分

Theorem 1

Theorem 14. *Assume that for a sufficiently large constant c, it holds* $c\log(n) \leq \lambda \in \mathrm{poly}(n)$. *Let* $\alpha, \beta, \varepsilon \in (0,1)$ *be three constants where* $\alpha - \varepsilon \geq 4/5$. *Define* $T := \min\{\lambda t \mid (P_t \times Q_t) \cap S_0 \times R_1((\alpha-\varepsilon))n\}$ *where* P_t *and* Q_t *are the populations of Algorithm 2 applied to* BILINEAR $_{\alpha,\beta}$. *Then if the mutation rate* χ *is a sufficiently small constant, and at most* $1/(1-\alpha+\varepsilon)$, *there exists a constant* c_0 *such that for all* $r \in \mathrm{poly}(n)$, *it holds* $\Pr\left(T > c_0 r \lambda^3 n\right) \leq (1/r)(1+o(1))$.

1. 规定了 Bilinear Problem 中 α 的范围和算法中突变概率 χ 的范围

2. 应用框架的结论得出期望时间和时间的概率分布



搜索空间的划分

Algorithm 1

Parallel Coevolutionary Algorithm 与原文章算法的区别：

1. 产生新种群的方式不同，不会发生“退化”的情况
2. 因此没有种群大小 λ 的限制
3. 实践结果明显更优，种群变化趋势明显

Algorithm 1

Parallel Coevolutionary Algorithm 单个个体研究：
也就是 (1+1)EA 算法

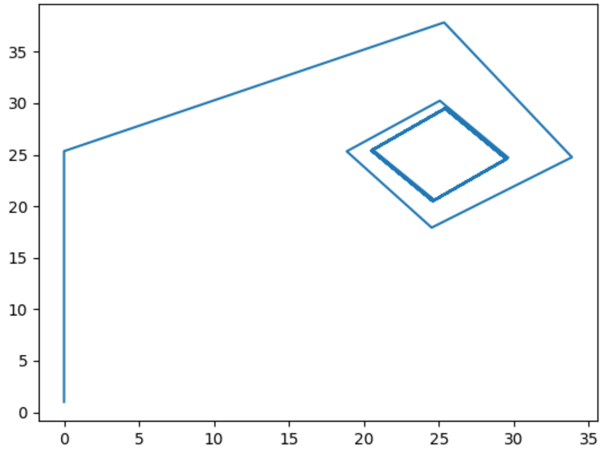
对于位置为 (x,y) 的个体，计算下一次的期望位置 (E(x), E(y))

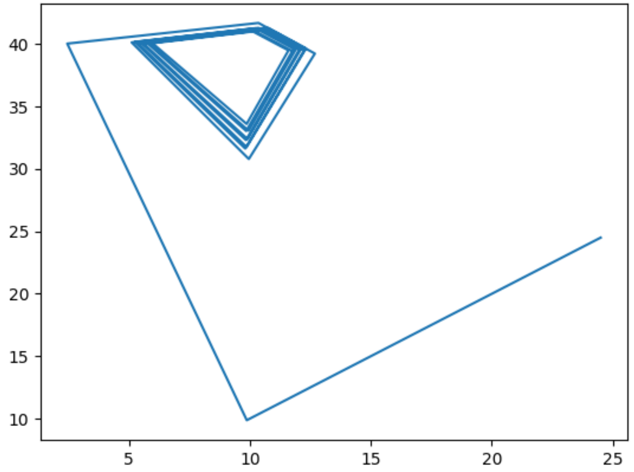
如果 $x < \beta n$：E(x)=x+1-x/n
否则：E(x)=x-x/n

如果 $y < \alpha n$：E(y)=y-y/n
否则：E(y)=y+1-y/n

画出期望曲线，发现结果不收敛

实验也有相同的结果



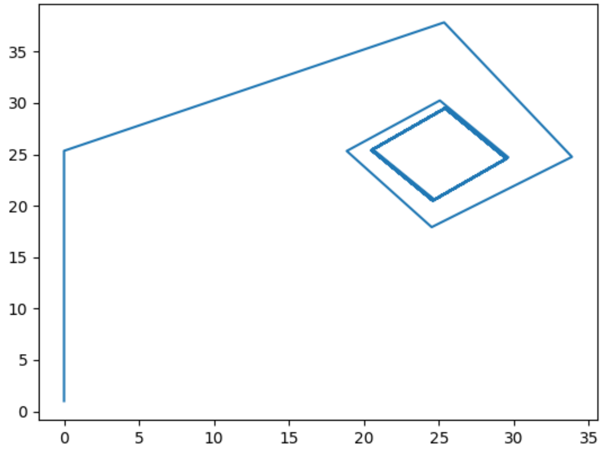


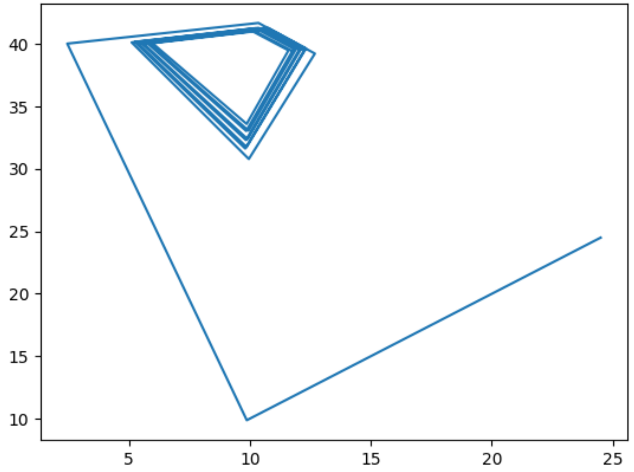
Algorithm 1

方向1：研究种群数量大于1的算法

实验说明不会出现不收敛的情况

在 $x = \beta n$ 或 $y = \alpha n$ 时，由于在直线附近存在多个个体，有向最优解进化趋势，不会存在之前的情况





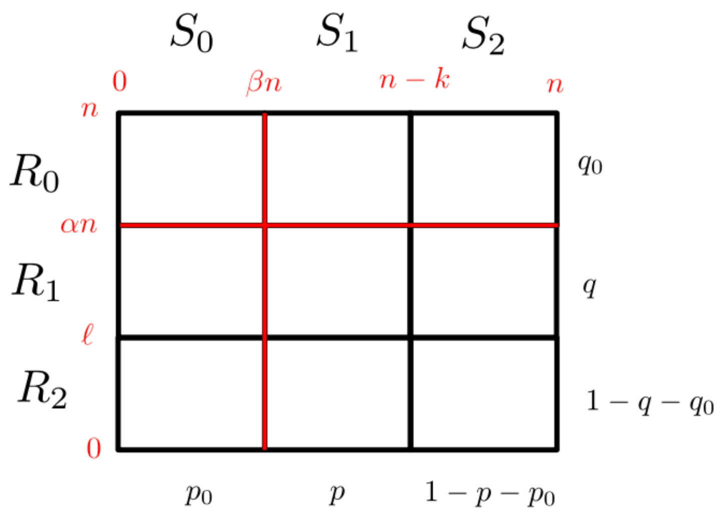
Algorithm 1

方向2：基于原问题的设定 $\alpha > 0.8$， 并且认为没有落在区域 R_0 中的个体

考虑 (1+1)EA 达到最优解的的期望时间 ：

类似于原问题，将进化过程分为两个阶段

每个阶段时间复杂度 $\mathrm{T} = \frac{n}{1} + \frac{n}{2} + \frac{n}{3} + \cdots + \frac{n}{n} = O\big(nln(n)\big)$



搜索空间的划分