Homework 8

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Problem 1.

(a) Construct a twice differentiable function $v(x,t)$ such that

$$v(0,t) = g(t), \quad v_x(\pi,t) = h(t).$$

(b) Construct a twice differentiable function $v(x,t)$ such that

$$v_x(0,t) = g(t), \quad v_x(\pi,t) = h(t).$$

For both parts, show some computations to justify that the functions you construct do satisfy those boundary conditions.

Solution. □

Problem 2. *Solve the heat equation with source:*

$$\begin{cases} u_t - u_{xx} = e^{-t}\sin(3x), & 0 < x < \pi, \quad t > 0, \\ u(0,t) = u(\pi,t) = 0, & t > 0, \\ u(x,0) = 1, & 0 < x < \pi. \end{cases}$$

Solution. □

Problem 3. *Solve the heat equation with inhomogeneous Dirichlet boundary condition:*

$$\begin{cases} u_t - u_{xx} = 0, & 0 < x < \pi, \quad t > 0, \\ u(0,t) = 0, & t > 0, \\ u(\pi,t) = t, & t > 0, \\ u(x,0) = 0, & 0 < x < \pi. \end{cases}$$

Solution. □

Problem 4. *Solve the wave equation with a constant gravitational force:*

$$\begin{cases} u_{tt} - u_{xx} = -1, & 0 < x < \pi, \quad t > 0, \\ u(0,t) = u(\pi,t) = 0, & t > 0, \\ u(x,0) = u_t(x,0) = 0, & 0 < x < \pi. \end{cases}$$

Solution. □