COS 210 Worksheet 6 Dylan Kapnias (u18108467)

Question 1

Assume L_1 is regular:

$\Rightarrow$ There exists a pumping length $p \geq 1$.

Consider $w = a^p b^q a^p \in L_1$, $p, q \in \mathbb{Z}$ $p \geq 0$ $q \geq 0$

- $|w| = p + q + p = 2p + q \geq p$

$\Rightarrow$ w can be written as $w = xyz$, where

$y \neq \epsilon$, $|xy| \leq p$ $\ni$ $xy^kz \in L_1$ $\forall k \geq 0$

- $|xy| \leq p \Rightarrow xy = a \ldots a$ (a's only)
- $y \neq \epsilon \Rightarrow y$ consists of a non-zero amount of a's.

$\Rightarrow$ $xz = a^r b^q a^p$, $r < p$

$\Rightarrow$ xz has less amount of a's before the sequence of b's then after.

$\Rightarrow$ $xz \in L_1$ (contradiction)

$\Rightarrow$ L_1 is not regular.

Question 2

Assume L_2 is regular:

$\Rightarrow$ $\overline{L_2}$ is regular where L_2: $\{0^n 1^m : n \neq m, n \geq 0, m \geq 0\}$ (closure of compliment)

$\Rightarrow$ L_3: $\{0^n 1^n : n \geq 0\}$, $\overline{L_2} = L_3$

$\Rightarrow$ L_3 is not regular (proved in Lecture 12), thus $\overline{L_2}$ is not regular. (contradiction of compliment closure)

$\Rightarrow$ L_2 is not regular.

Question 3

Yes.