



## COS 210 Worksheet 4

- This worksheet consists of **5 questions** for a total of **12 marks**.

### Question 1 ..... (3 marks)

Prove that, if  $A$  is a regular language, then for all natural numbers  $k \geq 0$  the language  $A^k$  is also regular, using induction. (The definition of  $A^k$  can be found in Lecture 5, Slide 10.)

(You may make use of the fact that regular languages are closed under concatenation.)

### Question 2 ..... (3 marks)

Prove by construction that, if  $A$  is a regular language, then  $A^+$  is a regular language. Where  $A^+$  is defined as

$$A^+ = \bigcup_{k=1}^{\infty} A^k$$

( $A^+$  is  $A^*$  without  $k = 0$ .)

### Question 3 ..... (2 marks)

For the alphabet  $\Sigma = \{a, b\}$  we define the language  $A$  as

$$A = \{w : w \text{ contains exactly one } b \text{ and at least two } a\text{'s}\}.$$

Give the regular expression describing this language.

### Question 4 ..... (2 marks)

For the alphabet  $\{a, b, c\}$  we define the language  $A$  as

$$A = \{w : w \text{ contains the substring } acb \text{ at least three times}\}.$$

Give the regular expression describing this language.

### Question 5 ..... (2 marks)

For the alphabet  $\{a, b, c, d\}$  we define the language  $A$  as

$$A = \{w : \text{every even position is } a \text{ or } b, \text{ every odd position is } c \text{ or } d\}.$$

Give the regular expression describing this language.

(Take into account that a string  $w$  may have an even or an odd length.)