COS 210 Worksheet 4 Dylan Kapnias (u18108467)

## Question 1

$A^k = A^0 \cup A^1 \cup A^2 \cup \ldots \cup A^k$, $\forall k \in \mathbb{Z}, k \geq 0$,

$A^2 = AA$
$A^3 = AAA$
$\vdots$
$A^n = \underbrace{A \ldots A}_{n \text{ times}}$

Basis Step: $k = 0$ ; $A^0 = \epsilon$
$\epsilon$ is regular.

Induction Hypothesis: Let $n \in \mathbb{Z}$ exist such that $A^n$ is regular, $n \geq 0$, $n = k$
$A^n = A^0 \cup A^1 \cup A^2 \cup \ldots \cup A^n$ $(*)$

Induction Step: Show $n+1$ is regular:

$A^{n+1} = A^0 \cup A^1 \cup A^2 \cup \ldots \cup A^n \cup A^{n+1}$
$= A^n \cup A$ $(*)$

$\therefore$ This is regular as $A^n$ is regular $(*)$, $A$ is regular (given) and regular languages are closed under concatenation and union.

$\therefore$ By the principle of weak mathematical induction $A^k$ is regular for $k \geq 0$

## Question 2

Create Object: $A^k = A^1 \cup A^2 \cup A^3 \cup \ldots \cup A^k$, $\forall k \in \mathbb{Z}, k \geq 1$
$\therefore$ as $k$ tends towards $\infty$:
$A^+ = \bigcup_{k=1}^{\infty} A^k$

Proof: ① $A$ is regular (given)
② Regular languages are closed under union

$A^m = A$, $m = 1$, regular
$A^m = A \cup A$, $m = 2$, regular
$A^m = \underbrace{A \cup A \cup \ldots \cup A}_{m}$, $m \geq 1$, regular

Tending to $\infty$: $\lim_{k \to \infty} A^k = M$
$\therefore M = \bigcup_{k=1}^{\infty} A^k = A^+$, regular

## Question 3

$(a^* baaa^*) \cup (aa^* baa^*) \cup (aaa^* ba^*)$

## Question 4

$(a \cup b \cup c)^* acb (a \cup b \cup c)^* acb (a \cup b \cup c)^* acb (a \cup b \cup c)^*$

## Question 5

$((c \cup d)((a \cup b)(c \cup d))^*) \cup (((c \cup d)(a \cup b))^*)$