BALLOONING MODE ANALYSIS OF HIGH-β EQUILIBRIA

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INTRODUCTION AND APPROACH

- For a typical fusion device $Q \sim \mathcal{O}(\beta^2 B^4)$. Given a value of B it implies that high beta operation is favorable.
- The ballooning(or high wave-number) mode is useful for determining an upper limit of β that a tokamak plasma can achieve.
- Conventional methods assume that the least stable modes lie at the outboard mid-plane i.e. $\theta_0 = 0$.
- Our aim is to use $s-\alpha$ model for concentric flux surfaces and compare its results to gyrokinetic simulations of an equilibrium with finite θ_0

The two *equivalent* forms of the Ideal ballooning equation are

$$\underset{\sim}{B}\cdot\nabla\left(\frac{|\nabla S|^2}{B^2}\underset{\sim}{B}\cdot\nabla F\right)+2\frac{dp}{d\psi}\,\frac{(\underset{\sim}{B}\times\nabla S)\cdot\underset{\sim}{K}}{B^2}\,F,\quad \underset{\sim}{K}=\frac{\nabla(p+B^2/2)}{B^2},\quad \underset{\sim}{B}\cdot\nabla S=0 \qquad (1)$$

$$\frac{1}{\mathcal{J}}\frac{\partial}{\partial y}\left[\frac{1}{\mathcal{J}R^2B_\chi^2}\left\{1+\left(\frac{R^2B_\chi^2}{B}\int_{y_0}^{y}\nu' dy\right)^2\right\}\frac{\partial F}{\partial y}\right]+F\left[\frac{2\mathcal{J}p'}{B^2}\frac{\partial}{\partial\psi}\left(p+\frac{B^2}{2}\right)-\frac{I\,p'}{B^4}\left(\int_{y_0}^{y}\nu' dy\right)\frac{\partial B^2}{\partial y}\right]=0 \qquad (2)$$

We apply Mercier-Luc formulation to these equations to obtain their lowest order form in ρ around a flux surface. For a $\mathcal{O}(\beta)\gg 1$, large aspect ratio $\mathcal{O}(\epsilon^{-1})\gg 1$ equilibrium, the orderings of various the figures of merit are as follows

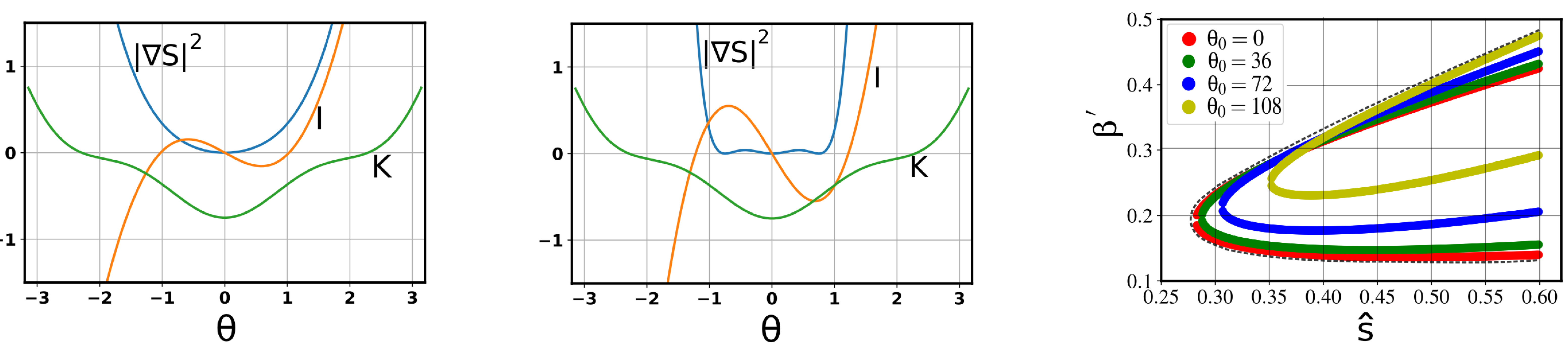
$$\beta_\varphi=\mathcal{O}\left(\frac{p}{B_\varphi^2}\right)\sim\epsilon,\quad \beta_p=\mathcal{O}\left(\frac{p}{B_p^2}\right)\sim\frac{1}{\epsilon},\qquad \frac{B_p}{B_\varphi}\sim\mathcal{O}(\epsilon),\qquad q\sim\mathcal{O}(1),\qquad \frac{\rho\,\psi_1}{\psi_0}\sim\mathcal{O}(\epsilon)$$

Applying these orderings to equation (1), we get the $s-\alpha$ equation

$$\frac{d}{d\theta}\left\{1+\Big(s(\theta-\theta_0)-\alpha\left(\sin(\theta)-\sin(\theta_0)\right)\Big)^2\right\}\frac{dF}{d\theta}$$
$$+\,\alpha\left\{\cos(\theta)+\sin(\theta)\Big(s(\theta-\theta_0)-\alpha(\sin(\theta)-\sin(\theta_0))\Big)\right\}F=0 \qquad (3)$$

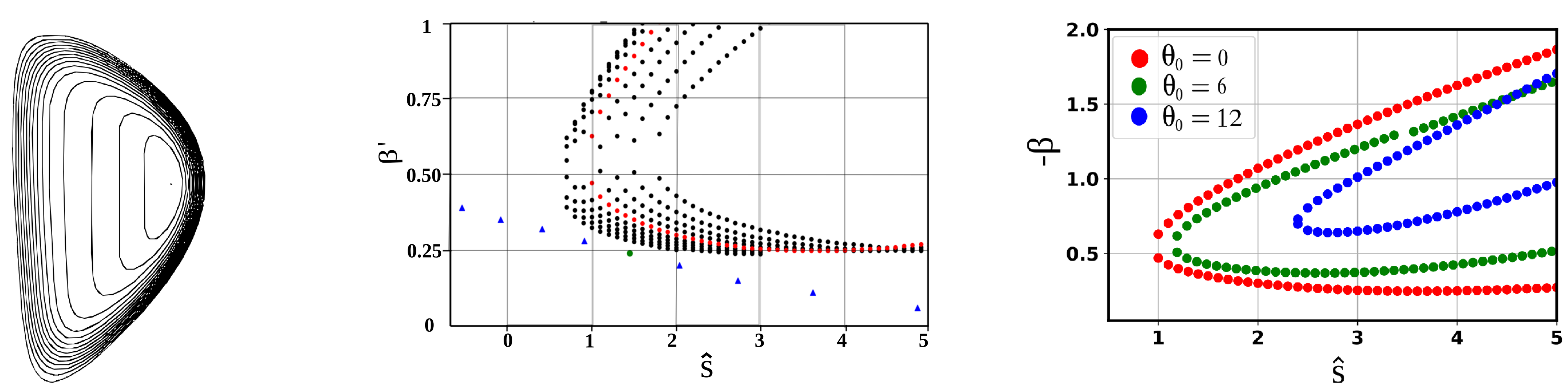
- We integrate the $s-\alpha$ equation for a concentric circle test profile and use Newcomb's criterion to determine the regions of marginal stability.
- Then we take a Grad-Shafranov stable high β equilibrium profile and plot $s-\alpha$ diagram and also use GS2 to perform Greene-Chance analyses to determine marginal stability

RESULTS



(a) $\hat{s}=1, \beta=0.4$ (b) $\hat{s}=1, \beta=1.1$ (c) $s-\alpha,\quad \rho_c=0.05$

Figure 1. a) and b)Qualitative variation of the stretching($|\nabla S|^2$), total shear(I and curvature(K) terms) terms, c) $s-\alpha$ diagrams for various values of θ_0



(d) (e) $\rho_c=[0.90, 0.99]$ (f) $s-\alpha\quad \rho_c=0.95$

Figure 1. d) High beta equilibrium profile and e)$s-\alpha$ diagrams for different surfaces ρ_c near the LCFS and the stability boundary from GS2 f) $s-\alpha$ diagrams for various values of θ_0 for the surface $\rho_c=0.95$

SUMMARY

- We have investigated the effect of finite θ_0 on the boundary of marginal stability to ballooning modes for a test profile and an actual high β equilibrium profile.
- Although the test profile shows significant changes to the $s-\alpha$ envelope, the actual profile doesn't change as much for the tested range of $\hat{s}$.
- Future work would involve checking the effect of large θ_0 and $\hat{s}$ values and a broader range of flux surfaces(ρ_c).

REFERENCES

[1] Chance *et al.* *Plasma Physics and Controlled Nuclear Fusion Research*, 1, 1978.

[2] Mercier C and Luc N. *Technical Report No EUR-5127e*, 1974.

[3] Greene and Chance. *Nuclear Fusion*, 21(4):453, 1981.

[4] Connor, Hastie, and Taylor. *Phys. Rev. Lett.*, 40:396–399, Feb 1978.

[5] Miller *et. al.* *Physics of Plasmas*, 5(4):973–978, 1998.