STAT451 HW4

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3.2 An overseas shipment of 5 foreign automobiles contains 2 that have slight paint blemishes. If an agency receives 3 of these automobiles at random, list the elements of the sample space S using the letters B and N for blemished and nonblemished, respectively; then to each sample point assign a value x of the random variable X representing the number of automobiles purchased by the agency with paint blemishes.

Out of the 5 automobiles in each shipment, only 2 of them have blemishes. This is every combination of 3 cars from 2 blemished and 3 nonblemished.

S	BBN	BNB	NBB	BNN	NBN	NNB	NNN
x	2	2	2	1	1	1	0

3.5 Determine the value c so that each of the following functions can serve as a probability distribution of the discrete random variable X:

a. $f(x) = c(x^2+4)$, for $x = 0, 1, 2, 3$

The sum of $f(x)$ for all values of x, is 1.

$$\begin{aligned}\sum_{x=0}^{3} c(x^2+4) &= 1\\ c(4+5+8+13) &= 1\end{aligned}$$

$$\boxed{c = \frac{1}{30}}$$

b. $f(x) = c\binom{2}{x}\binom{3}{3-x}$, for $x = 0, 1, 2$

$$\begin{aligned}\sum_{x=0}^{2} c\binom{2}{x}\binom{3}{3-x} &= 1\\ c\binom{2}{0}\binom{3}{3} + \binom{2}{1}\binom{3}{2} + \binom{2}{2}\binom{3}{1} &= 1\\ c(1)(1) + (2)(3) + (2)(3) &= 1\end{aligned}$$

$$\boxed{c = \frac{1}{13}}$$

3.7 The total number of hours, measured in units of 100 hours, that a family runs a vacuum cleaner over a period of one year is a continuous random variable X that has the density function

$$f(x) = \begin{cases} x, & 0 < x < 1, \\ 2 - x, & 1 \le x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the probability that over a period of one year, a family runs their vacuum cleaner

a. **less than 120 hours.**

$$\begin{aligned} P(x < 1.2) &= \int_0^1 x dx + \int_1^{1.2} (2 - x) dx \\ &= .5 + .18 \end{aligned}$$

$$\boxed{P(x < 1.2) = .68}$$

b. **between 50 and 100 hours.**

$$P(.5 < x < 1) = \int_{.5}^1 x dx$$

$$\boxed{P(.5 < x < 1) = .375}$$

3.8 Find the probability distribution of the random variable W in Exercise 3.3, assuming that the coin is biased so that a head is twice as likely to occur as a tail.

The sample space of Example 3.3 was:

S	HHH	HHT	HTH	THH	HTT	TTH	THT	TTT
w	3	1	1	1	-1	-1	-1	-3

The probability distribution was:

w	-3	-1	1	3
f(w)	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

But that's when all of those combinations were equally likely. If heads is twice as likely as tails, then the probability of getting a HHH is $\frac{2}{3}^3 = \frac{8}{27}$. The probability of getting a HHT is $\left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{1}{3}\right) = \frac{4}{27}$, and so on.

The new probability distribution looks like:

w	-3	-1	1	3
f(w)	$\frac{1}{27}$	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{8}{27}$

3.12 An investment firm offers its customers municipal bonds that mature after varying numbers of years. Given that the cumulative distribution function of T, the number of years to maturity for a randomly selected bond, is,

$$F(t) = \begin{cases} 0, & t < 1, \\ \frac{1}{4}, & 1 \le t < 3, \\ \frac{1}{2}, & 3 \le t < 5, \\ \frac{3}{4}, & 5 \le t < 7, \\ 1, & t \ge 7, \end{cases}$$

find

a. $P(T = 5)$

$$\begin{aligned} P(T = 5) &= F(5) - F(3) \\ &= \frac{3}{4} - \frac{1}{2} \end{aligned}$$

$$\boxed{P(T = 5) = \frac{1}{4}}$$

b. $P(T > 3)$

$$\begin{aligned} P(T > 3) &= 1 - P(T < 3) \\ P(T < 3) &= \frac{1}{4} \end{aligned}$$

$$\boxed{P(T > 3) = \frac{3}{4}}$$

c. $P(1.4 < T < 6)$

$$\begin{aligned} P(1.4 < T < 6) &= P(T < 6) - P(T < 1.4) \\ &= \frac{3}{4} - \frac{1}{4} \end{aligned}$$

$$\boxed{P(1.4 < T < 6) = \frac{1}{2}}$$

3.23 Find the cumulative distribution function of the random variable W in Exercise 3.8. Using $F(w)$, find

The cumulative distribution function of W is

$$F(w) = \begin{cases} 0, & w < -3, \\ \frac{1}{27}, & -3 \le w < -1, \\ \frac{7}{27}, & -1 \le w < 1, \\ \frac{19}{27}, & 1 \le w < 3, \\ 1, & w \ge 3. \end{cases}$$

a. $P(W > 0)$

$$\begin{aligned} P(W > 0) &= F(3) - F(-1) \\ &= 1 - \frac{7}{27} \end{aligned}$$

$$\boxed{P(W > 0) = \frac{20}{27}}$$

b. $P(-1 \leq W < 3)$

$$
\begin{aligned}
P(-1 \leq W < 3) &= P(1) - P(-3) \\
&= \frac{19}{27} - \frac{1}{27}
\end{aligned}
$$

$$\boxed{P(-1 \leq W < 3) = \frac{2}{3}}$$

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