STAT451 HW2

Russell Miller

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1. A student selected from a class will either be a boy or a girl. If the probability that a boy will be selected is .3, what is the probability that a girl will be selected?

Let the number of boys be 3, and the total class size be 10. The P(boy)=.3 would be satisfied, and there would be 7 girls remaining.

$$\boxed{P(girl) = .7}$$

3. If the probability that student A will fail a certain statistics examination is 0.5, the probability that student B will fail the examination is 0.2, and the probability that both student A and student B will fail the examination is 0.1, what is the probability that at least one of these two students will fail the examination?

$P(A) = .5$, $P(B) = .2$, $P(A \cap B) = .1$

What we're looking for is when A will fail *or* B will fail. This could be represented as $P(A \cup B)$.
By Theorem 6 we know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

$$\boxed{P(A \cup B) = .5 + .2 - .1 = .6}$$

6. Consider two events A and B such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{2}$. Determine the value of $P(B \cap A')$ for each of the following conditions:

a. **A and B are disjoint**
$P(B)$ does not share any outcomes with $P(A)$, so $P(B)$ is a subset of $P(A')$. The intersection of $P(B)$ with $P(A')$ is just $P(B)$.

$$\boxed{P(B \cap A') = P(B) = \frac{1}{2}}$$

b. $\mathbf{A \subset B}$
This is the donut hole problem. If you want all of $P(B)$ except for the $P(A)$ part:

$$\boxed{P(B \cap A') = P(B) - P(A) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}}$$

c. $\mathbf{P(A \cap B) = \frac{1}{8}}$
Now we know they are not disjoint. The overlap section of the venn diagram is the $\frac{1}{8}$ part. So in order to make sure we get the $P(B)$ without the $P(A)$, we have to take:

$$\boxed{P(B \cap A') = P(B) - P(A \cap B) = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}}$$

9. Prove that for any two events A and B, the probability that exactly one of the two events will occur is given by the expression:

$$\mathbf{P(A) + P(B) - 2P(A \cap B)}$$

We want to know the probability that A will occur and not B, or that B will occur and not A. These can be represented as $P(A \cap B')$ and $P(B \cap A')$ respectively.
We also know that $P(A \cap B') = P(A) - P(A \cap B)$ and $P(B \cap A') = P(B) - P(A \cap B)$.
Thus:
$P(A \cap B') + P(B \cap A')$
$P(A) - P(A \cap B) + P(B) - P(A \cap B)$
$P(A) + P(B) - 2P(A \cap B)\blacksquare$

10. A point (x, y) is to be selected from the square S containing all points (x, y) such that $0 \le x \le 1$ and $0 \le y \le 1$. Suppose that the probability that the selected point will belong to any specified subset of S is equal to the area of that subset. Find the probability of each of the following subsets:

a. $(x - \frac{1}{2})^2 + (y - \frac{1}{2})^2 \ge \frac{1}{4}$
This equation is the area above a semicircle within S. We can find this area by:

$$\frac{Area\ of\ square - Area\ of\ circle}{2}$$
$$\boxed{\frac{1 - \pi\frac{1}{2}^2}{2} = .1073}$$

b. $\frac{1}{2} < x + y < \frac{3}{2}$
This is the area between two right triangles, of height $\frac{1}{2}$ and width $\frac{1}{2}$.

$$Area\ of\ square - 2(Area\ of\ triangle)$$
$$\boxed{1 - 2\left(\frac{1}{2}\left(\frac{1}{2}\right)^2\right) = \frac{3}{4}}$$

c. $y \le 1 - x^2$
To get the area under this curve, simply integrate from 0 to 1.

$$\boxed{\int_0^1 (1 - x^2)dx = \frac{2}{3}}$$

d. $x = y$
This is a line connecting one corner to another. If the square has 10000 possible points, that is 100x100. Then for each of the 100 possible values of x, only one matches y. So there would be one for each value of y. Meaning $\frac{100}{10000}$ or $\frac{1}{100}$.
If, however, there were only 100 possible points, that is 10x10. Then for each of the 10 possible values of x, only one will match y. But this is true for each of the 10 values of y, so it is $\frac{10}{100}$ or $\frac{1}{10}$.
So it seems it would depend on the number of possible outcomes.

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