STAT451 HW2

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1. A student selected from a class will either be a boy or a girl. If the probability that a boy will be selected is .3, what is the probability that a girl will be selected?

Let the number of boys be 3, and the total class size be 10. The P(boy)=.3 would be satisfied, and there would be 7 girls remaining.

$$\boxed{P(girl) = .7}$$

3. If the probability that student A will fail a certain statistics examination is 0.5, the probability that student B will fail the examination is 0.2, and the probability that both student A and student B will fail the examination is 0.1, what is the probability that at least one of these two students will fail the examination?

$P(A) = .5$, $P(B) = .2$, $P(A \cap B) = .1$
What we're looking for is when A will fail *or* B will fail. This could be represented as $P(A \cup B)$.
By Theorem 6 we know that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

$$\boxed{P(A \cup B) = .5 + .2 - .1 = .6}$$

6. Consider two events A and B such that $P(A) = \frac{1}{3}$ and $P(B) = \frac{1}{2}$. Determine the value of $P(B \cap A')$ for each of the following conditions:

a. **A and B are disjoint**
$P(B)$ does not share any outcomes with $P(A)$, so $P(B)$ is a subset of $P(A')$. The intersection of $P(B)$ with $P(A')$ is just $P(B)$.

$$\boxed{P(B \cap A') = P(B) = \frac{1}{2}}$$

b. $\mathbf{A \subset B}$
This is the donut hole problem. If you want all of $P(B)$ except for the $P(A)$ part:

$$\boxed{P(B \cap A') = P(B) - P(A) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}}$$

c. $\mathbf{P(A \cap B) = \frac{1}{8}}$
Now we know they are not disjoint. The overlap section of the venn diagram is the $\frac{1}{8}$ part. So in order to make sure we get the $P(B)$ without the $P(A)$, we have to take:

$$\boxed{P(B \cap A') = P(B) - P(A \cap B) = \frac{1}{2} - \frac{1}{8} = \frac{3}{8}}$$

9. Prove that for any two events A and B, the probability that exactly one of the two events will occur is given by the expression:

$$\mathbf{P(A) + P(B) - 2P(A \cap B)}$$

We want to know the probability that A will occur and not B, or that B will occur and not A. These can be represented as $P(A \cap B')$ and $P(B \cap A')$ respectively.
We also know that $P(A \cap B') = P(A) - P(A \cap B)$ and $P(B \cap A') = P(B) - P(A \cap B)$.
Thus:
$P(A \cap B') + P(B \cap A')$
$P(A) - P(A \cap B) + P(B) - P(A \cap B)$
$P(A) + P(B) - 2P(A \cap B)\blacksquare$

10. A point (x, y) is to be selected from the square S containing all points (x, y) such that $0 \leq x \leq 1$ and $0 \leq y \leq 1$. Suppose that the probability that the selected point will belong to any specified subset of S is equal to the area of that subset. Find the probability of each of the following subsets:

a. $(x - \frac{1}{2})^2 + (y - \frac{1}{2})^2 \geq \frac{1}{4}$
This equation is the area above a semicircle within S. We can find this area by:

$$\frac{Area\ of\ square - Area\ of\ circle}{2}$$

$$\boxed{\frac{1 - \pi\frac{1}{2}^2}{2} = .1073}$$

b. $\frac{1}{2} < x + y < \frac{3}{2}$
This is the area between two right triangles, of height $\frac{1}{2}$ and width $\frac{1}{2}$.

$$Area\ of\ square - 2(Area\ of\ triangle)$$

$$\boxed{1 - 2\left(\frac{1}{2}\left(\frac{1}{2}\right)^2\right) = \frac{3}{4}}$$

c. $y \leq 1 - x^2$
To get the area under this curve, simply integrate from 0 to 1.

$$\boxed{\int_0^1 (1 - x^2)dx = \frac{2}{3}}$$

d. $x = y$
This is a line connecting one corner to another. If the square has 10000 possible points, that is 100x100. Then for each of the 100 possible values of x, only one matches y. So there would be one for each value of y. Meaning $\frac{100}{10000}$ or $\frac{1}{100}$.
If, however, there were only 100 possible points, that is 10x10. Then for each of the 10 possible values of x, only one will match y. But this is true for each of the 10 values of y, so it is $\frac{10}{100}$ or $\frac{1}{10}$.
So it seems it would depend on the number of possible outcomes.

σ_σ

Text 2.79 A random sample of 200 adults are classified by sex and their level of education attained. If a person is picked at random from this group, find the probability that:

a. the person is a male, given that the person has a secondary education.
   78 people have a secondary education, from that there are 28 males.

$$\boxed{\frac{28}{200} = .14}$$

b. the person does not have a college degree, given that the person is a female.
   There are 112 females. Of them 95 do not have a degree.

$$\boxed{\frac{95}{200} = .475}$$

Text 2.86 For married couples living in a certain suburb the probability that the husband will vote on a bond referendum is 0.21, the probability that his wife will vote on the referendum is 0.28, and the probability that both the husband and the wife will vote is 0.15. What is the probability that:

a. at least one member of a married couple will vote
   Let $P(h) = 0.21$ be the probability the husband will vote. Let $P(w) = 0.28$ be the probability the wife will vote. Then, $P(h \cap w) = 0.15$ is the probability that both will vote. $P(h \cup w)$ is the probability that at least one will vote.

$$\boxed{P(h \cup w) = P(h) + P(w) - P(h \cap w) = 0.21 + 0.28 - 0.15 = 0.34}$$

b. a wife will vote, given that her husband will vote
   By the division rule we know that $P(w \mid h) = \frac{P(w \cap h)}{P(h)}$.

$$\boxed{P(w \mid h) = \frac{0.15}{0.21} = 0.7143...}$$

c. a husband will vote, given that his wife will not vote
   We can see that $P(w') = 1 - P(w) = 1 - 0.28 = 0.72$
   Also, $P(h \cap w') = P(h) - P(h \cap w) = 0.21 - 0.15 = 0.06$ Thus:

$$\boxed{P(h \mid w') = \frac{P(h \cap w')}{P(w')} = \frac{0.06}{0.72} = 0.08\bar{3}}$$

Text 2.87 The probability that a vehicle entering the Luray Caverns has Canadian license plates is 0.12; the probability that it is a camper is 0.28; and the probability that it is a camper with Canadian license plates is 0.09. What is the probability that:

a. a camper entering the Luray Caverns has Canadian license plates?

$$\boxed{P(Canadian \mid camper) = \frac{P(Canadian \cap camper)}{P(camper)} = \frac{0.09}{0.28} = 0.32143...}$$

b. a vehicle with Canadian license plates entering the Luray Caverns is a camper?

$$\boxed{P(camper \mid Canadian) = \frac{P(camper \cap Canadian)}{P(Canadian)} = \frac{0.09}{0.12} = 0.75}$$

c. a vehicle entering the Luray Caverns does not have Canadian plates or is not a camper?
Using DeMorgan's Law:

$$\boxed{P(Canadian' \cup camper') = P((Canadian \cap camper)') = 1 - P(Canadian \cap camper) = 1 - 0.09 = 0.91}$$

Text 2.93 A town has 2 fire engines operating independently. The probability that a specific engine is available when needed is 0.96.

a. What is the probability that neither is available when needed?
Let $P(A)$ represent the probability that engine 1 is available, and let $P(B)$ represent the probability that engine 2 is available. We're given that $P(A) = P(B) = 0.96$.

$$\boxed{(1 - P(A))(1 - P(B)) = 0.0016}$$

b. What is the probability that a fire engine is available when needed?

$$\boxed{1 - (1 - P(A))(1 - P(B)) = 0.9984}$$

Text 2.94 The probability that Tom will be alive in 20 years is 0.7, and the probability that Nancy will be alive in 20 years is 0.9. If we assume independence for both, what is the probability that neither will be alive in 20 years?

Let $P(T)$ represent the probability that Tom will be alive in 20 years, and let $P(N)$ represent the probability that Nancy will be alive in 20 years.. We're given that $P(T) = 0.7$ and $P(N) = 0.9$.

$$\boxed{(1 - P(T))(1 - P(N)) = 0.03}$$