STAT451 HW5

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3.27 The time to failure in hours of an important piece of electronic equipment used in a manufactured DVD player has the density function

$$f(x) = \begin{cases} \left(\frac{1}{2000}\right) e^{\frac{-x}{2000}}, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

a. Find $F(x)$.

$$F(x) = \int f(x)dx = \begin{cases} 0, & x < 0 \\ -e^{\frac{-x}{2000}}, & x \geq 0 \end{cases}$$

b. Determine the probability that the component (and thus the DVD player) lasts more than 1000 hours before the component needs to be replaced.

$$P(X > 1000) = \int_{1000}^{\infty} \frac{e^{\frac{-x}{2000}}}{2000} dx = .6065$$

c. Determine the probability that the component fails before 2000 hours.

$$P(X < 2000) = \int_{0}^{2000} \frac{e^{\frac{-x}{2000}}}{2000} dx = .6321$$

3.29 An important factor in solid missile fuel is the particle size distribution. Significant problems occur if the particle sizes are too large. From production data in the past, it has been determined that the particle size (in micrometers) distribution is characterized by

$$f(x) = \begin{cases} 3x^{-4}, & x > 1, \\ 0, & \text{elsewhere} \end{cases}$$

a. Verify that this is a valid density function.
From my notes, a Probability Density Function is any function $f(x)$ that satisfies:

- $f(x) \geq 0$ for all real x,
- $\int_{-\infty}^{\infty} f(x)dx = 1$,
- For event A
  $P(x \in A) = \int_{x \in A} f(x)dx$

The first bullet is satisfied, there are no negative values of $f(x)$. The second bullet can be verified by

$$\int_{1}^{\infty} f(x)dx = \frac{-1}{\infty} - \frac{-1}{1^4} = 1$$

Since we're not given any events or probabilities, this is sufficient.

b. Evaluate $F(x)$.

$$F(x) = \begin{cases} 0, & x < 1 \\ -x^{-3}, & x \geq 1 \end{cases}$$

c. What is the probability that a random particle from the manufactured fuel exceeds 4 micrometers?

$$P(X > 4) = \int_4^{\infty} 3x^{-4} dx = \frac{1}{64}$$

3.30 Measurements of scientific systems are always subject to variation, some more than others. There are many structures for measurement error and statisticians spend a great deal of time modeling these errors. Suppose the measurement error X of a certain physical quantity is decided by the density function

$$f(x) = \begin{cases} k(3 - x^2), & -1 \leq x \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

a. Determine k that renders $f(x)$ a valid density function.

In order for $f(x)$ to be a valid density function, the following must hold.

$$\begin{aligned} \int_{-1}^{1} k(3 - x^2) dx &= 1 \\ \int_{-1}^{1} (3k - kx^2) dx &= 1 \\ 3kx - \frac{x^3}{3k} \Big|_{-1}^{1} &= 1 \\ k(3x - \frac{x^3}{3}) \Big|_{-1}^{1} &= 1 \\ \frac{8k}{3} - \frac{-8k}{3} &= 1 \\ \frac{16k}{3} &= 1 \\ k &= \frac{3}{16} \end{aligned}$$

And we'll double check by plugging it in

$$\int_{-1}^{1} \left(\frac{3}{16}\right)(3 - x^2) dx = 1$$

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