Cellular Automata

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Cellular Automata

Cellular automata CA
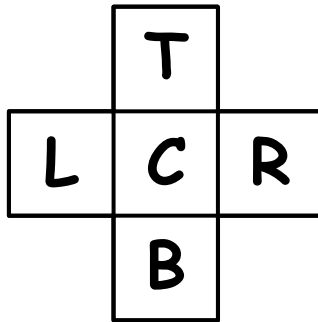
- A regular grid model made of many "automata" whose states are finite and discrete ($\rightarrow$ nonlinearity)

- Their states are simulta-neously updated by a uniform state-transition function that refers to states of their neighbors
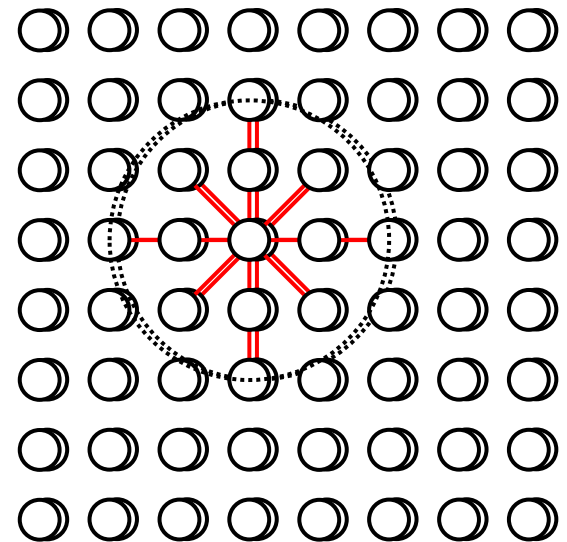
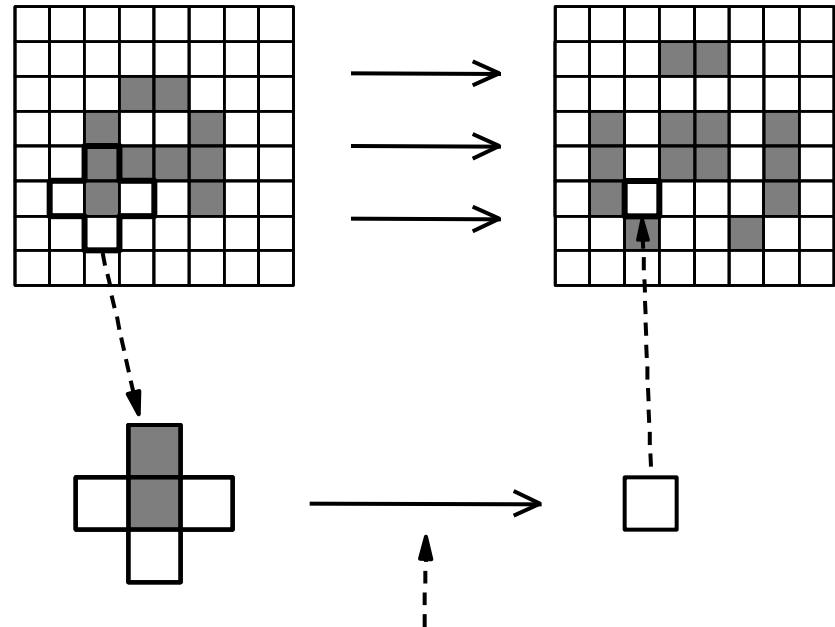$$s_{t+1}(x) = F (s_t(x+x_0), s_t(x+x_1), \ldots , s_t(x+x_{n-1}))$$

How CA works

Neighborhood

State set

$$\{ \square , \blacksquare \}$$

State-transition function

CTR	CTR	CTR	CTR

Some terminologies

- **Configuration**
  - A mapping from spatial coordinates to states ($s_t(x)$); global arrangement of states at time t

- **Local situation**
  - A specific arrangement of states within a local neighborhood, to be given to the state-transition function as an input

- **Quiescent state** ($\square$)
  - A state that represents "empty" space; never changes if surrounded by other quiescent states
    - Some CA have no quiescent state

Boundary conditions

- **Periodic boundary condition**
  - Cells at the edge of the space is connected to the cells at the other edge

- **Cut-off boundary condition**
  - Cells at the edge of the space do not have neighbors outside the space

- **Fixed boundary condition**
  - Cells at the edge of the space are fixed to specific states

Exercise: 1-D majority rule

- Each cell looks at its own and two nearest neighbors' states, and then change to the local majority state

- Periodic boundary conditions assumed

- Complete time evolution of this CA
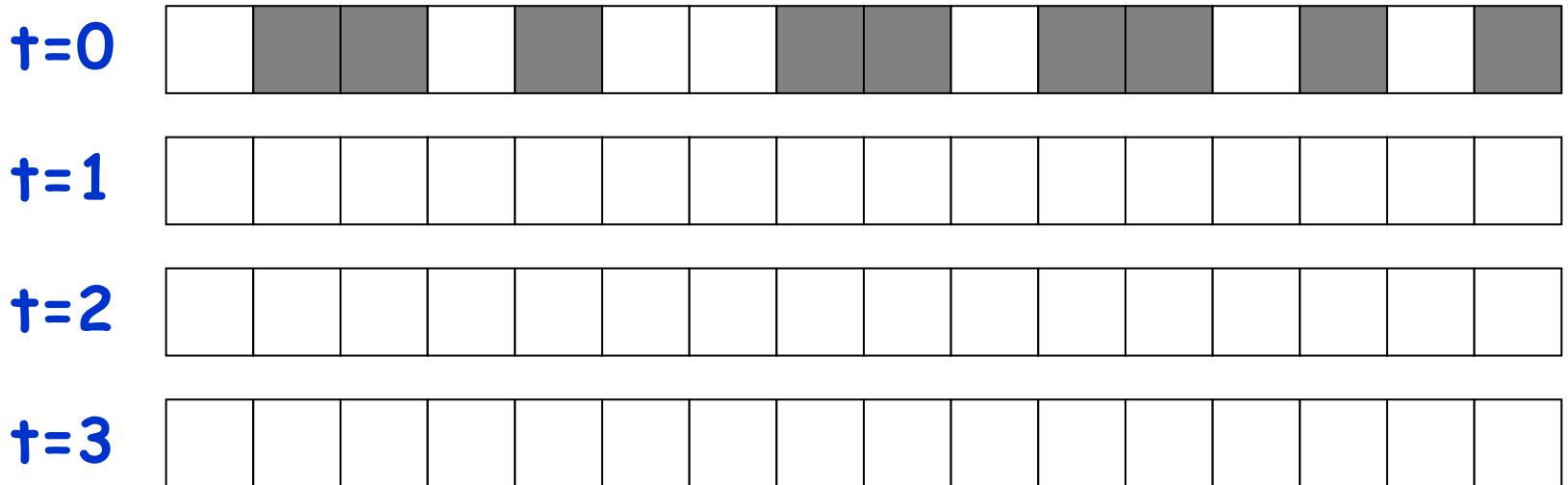
Reason CA so useful

- because value of cell depends on finite neighs, there are only finite # possible values and finite # rules possible so can study all possible sets of dynamics
- easy to program

Wolfram CA

- 1D CA
- Cells can be +1 (black) or 0 (white)
- Value of cell depends on itself and two neighbors

- How many total rules possible?

Wolfram CA

UNIVERSALITY AND COMPLEXITY IN CELLULAR AUTOMATA

Stephen WOLFRAM*
The Institute for Advanced Study, Princeton NJ 08540, USA

Cellular automata are discrete dynamical systems with simple construction but complex self-organizing behaviour. Evidence is presented that all one-dimensional cellular automata fall into four distinct universality classes. Characterizations of the structures generated in these classes are discussed. Three classes exhibit behaviour analogous to limit points, limit cycles and chaotic attractors. The fourth class is probably capable of universal computation, so that properties of its infinite time behaviour are undecidable.

Wolfram CA

$x_-(t)$	$x_0(t)$	$x_+(t)$	$x_0(t+1)$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

Rule

Wolfram CA

- 8 possible states of the three cells
- for each of the 8 local configurations, the cell can take on 2 values
- so total possible rules - 2^8

- GENERAL
  - A possible values
  - A^k possible values of k cells
  - $A\hat{\ }A\hat{\ }k$ possible rules
  - Can get nuts $-A=5, k=3, 10^{600}$ rules !!

Wolfram CA

Rule 254
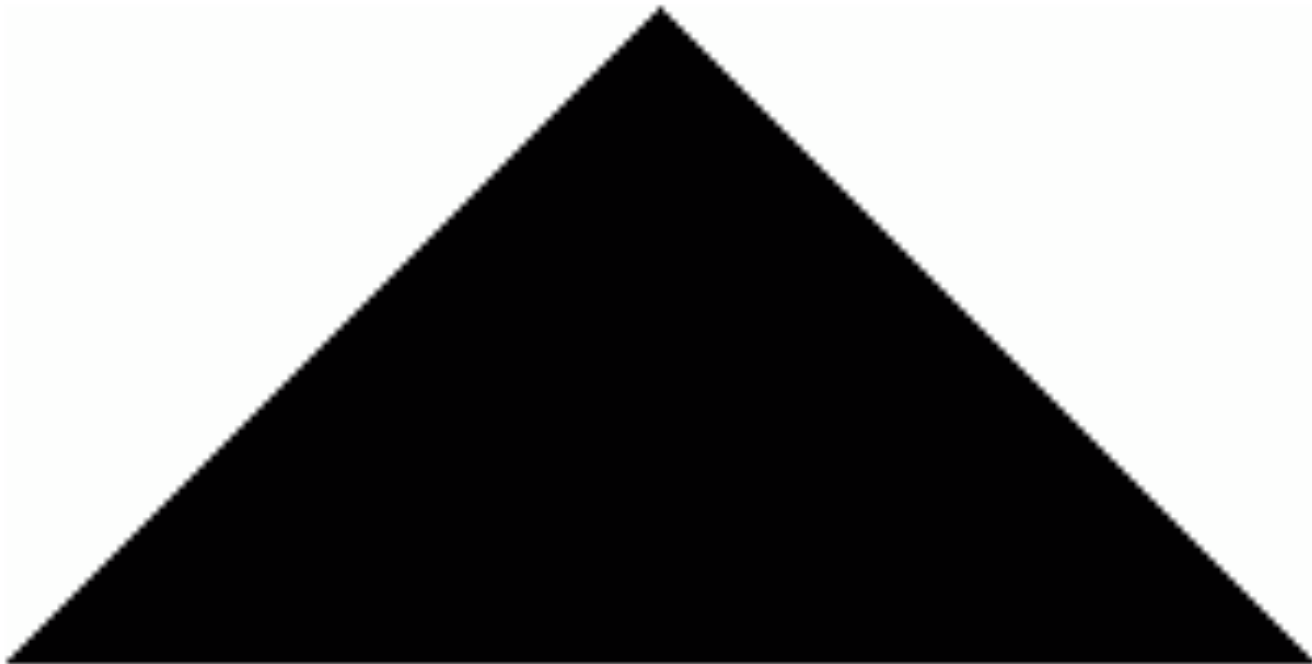
Wolfram CA

Rule 254

Wolfram CA

Rule 250

Wolfram CA

Rule 250
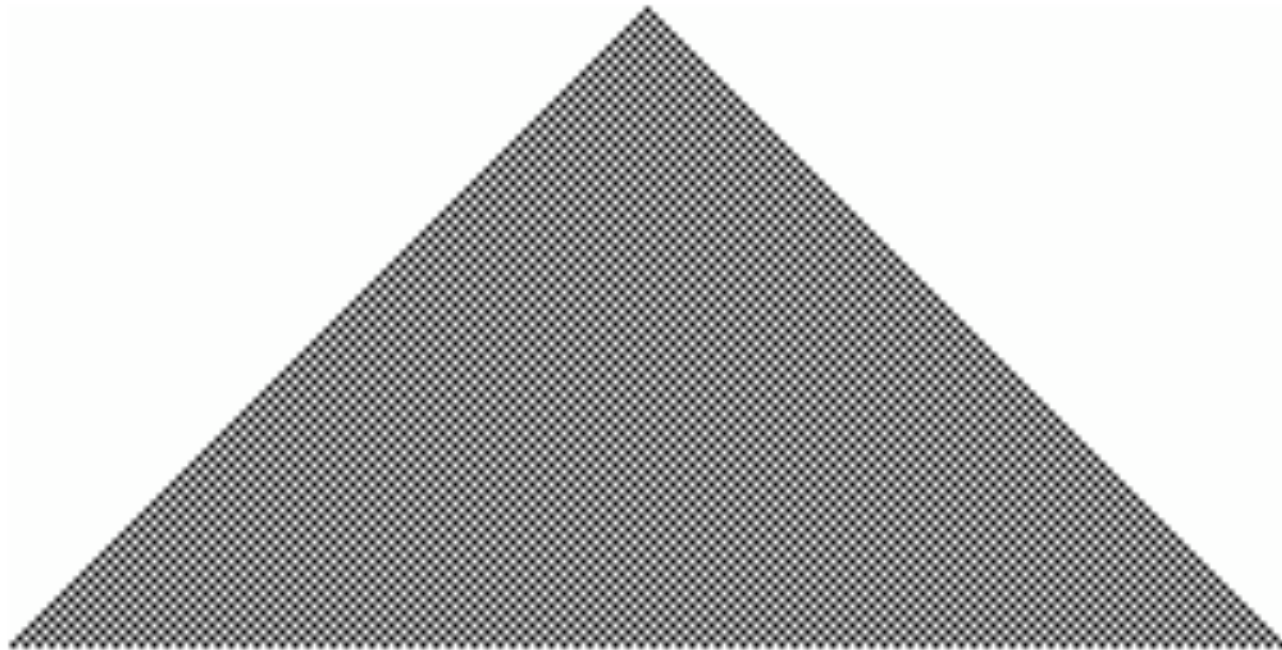
Wolfram CA

Rule 150

Wolfram CA

Rule 150
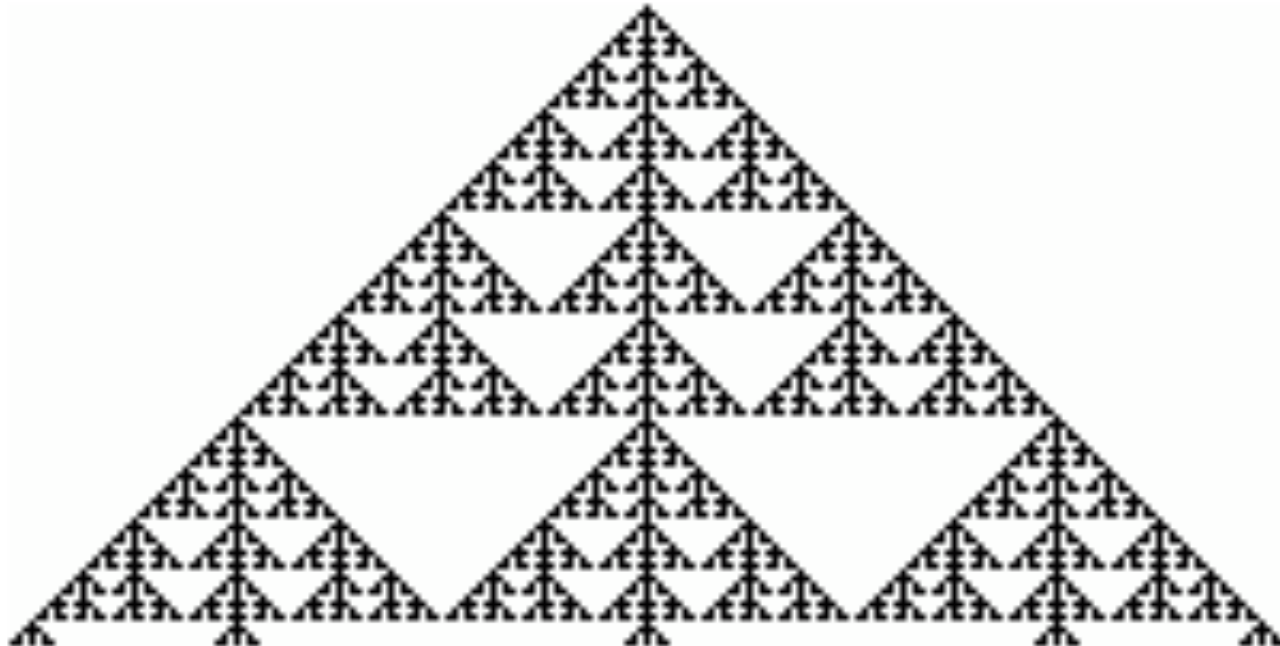
Wolfram CA

Rule 30

Wolfram CA

Rule 30
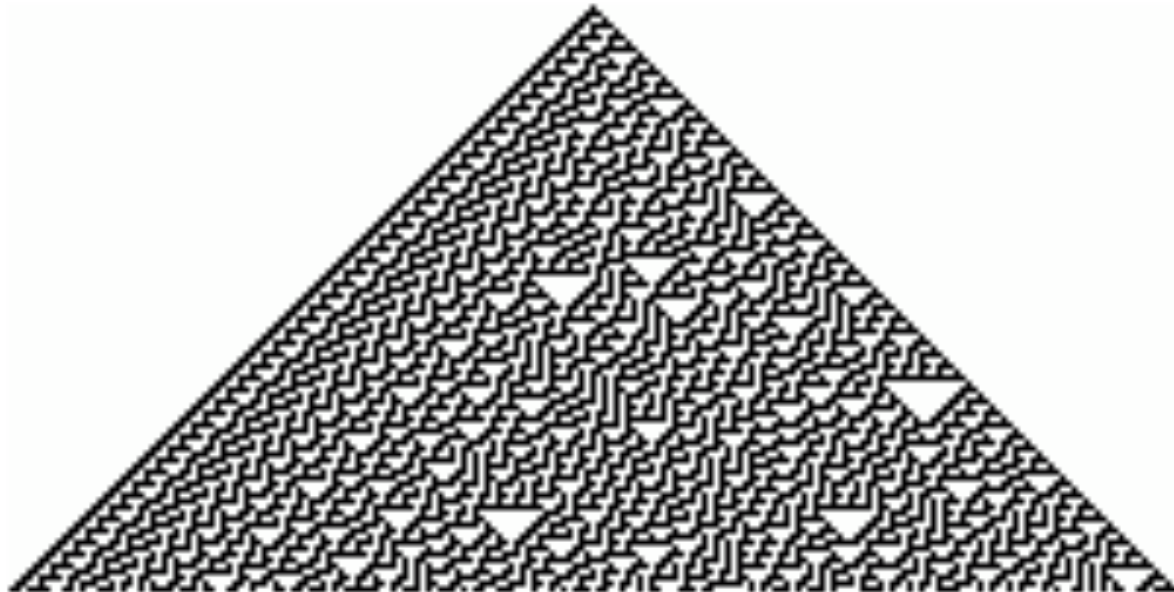
Wolfram's classification

- **Wolfram classified binary-state 1-D CA based on their typical attractors' properties**
  - Fixed point of homogeneous states
  - Fixed point or small cycle involving heterogeneous states
  - Chaotic attractor very long cycle
  - Complex, long-lived localized structures attractor preceded by very long tree-like basins of attraction

Wolfram's classification

Class I (Code 4)

Class II (Code 24)

Class III (Code 10)

Class IV (Code 20)

0	32	72	104	128	160	200	232
4	36	76	108	132	164	204	236
18	50	90	122	146	178	218	250
22	54	94	126	150	182	222	254
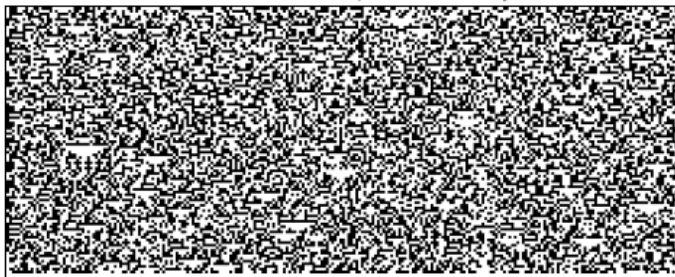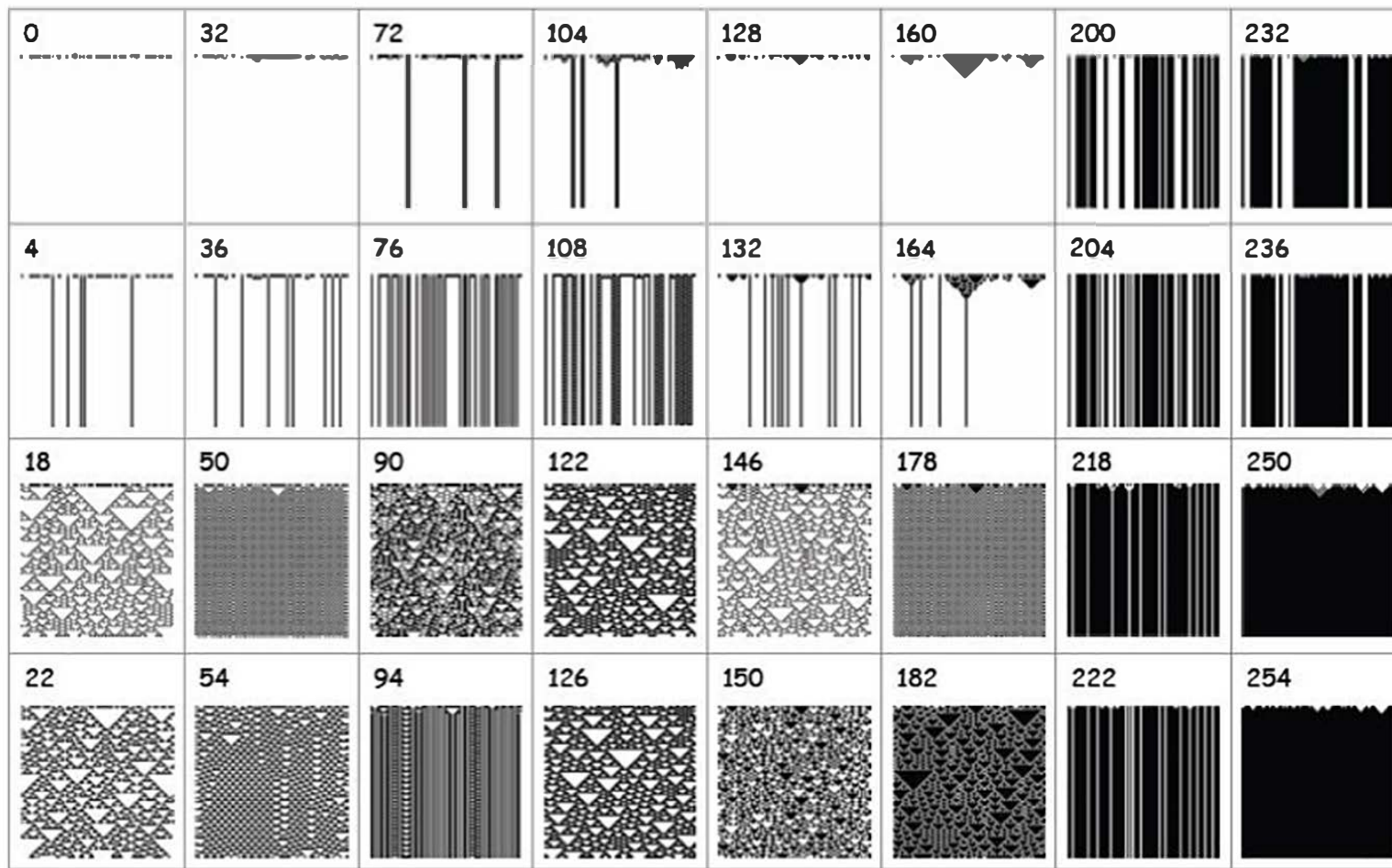
- **Wolrfram found empircally that the fraction of states in different CA settings went like this**

	A=2	A=2	A=2	A=3
	K=3	K=5	K=7	K=3
Class 1	0.50	0.25	0.09	0.12
Class 2	0.25	0.16	0.11	0.19
Class 3	0.25	0.53	0.73	0.60
Class 4	0.00	0.06	0.06	0.07

Wolfram's applications

Rule 180 – good use for what?
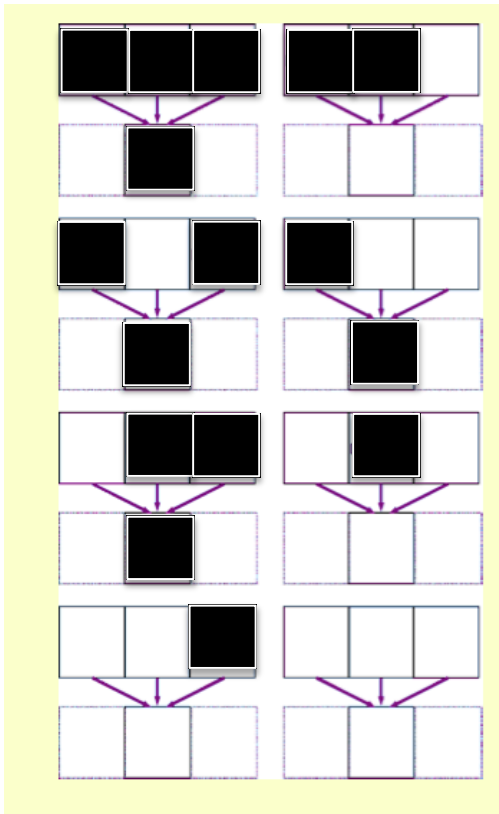
Wolfram's applications

- A very simplistic model of traffic
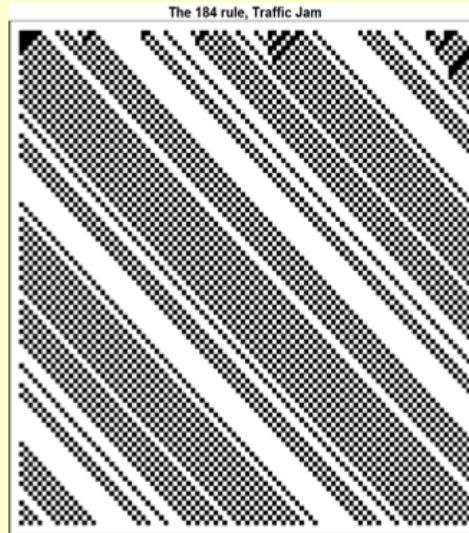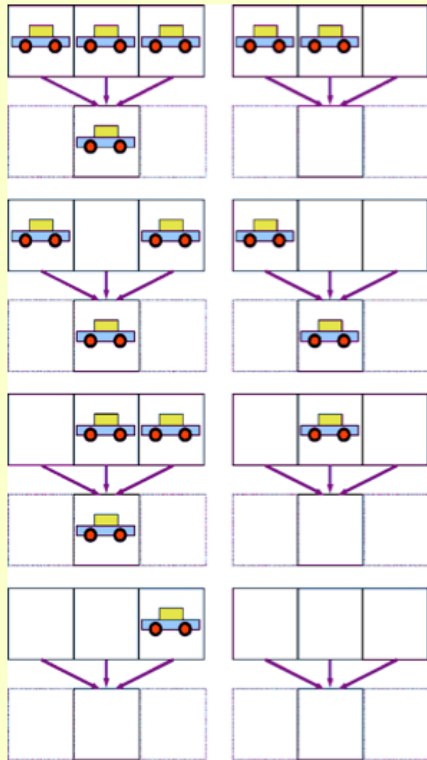
$\rho = 0.3$

$\rho = 0.7$
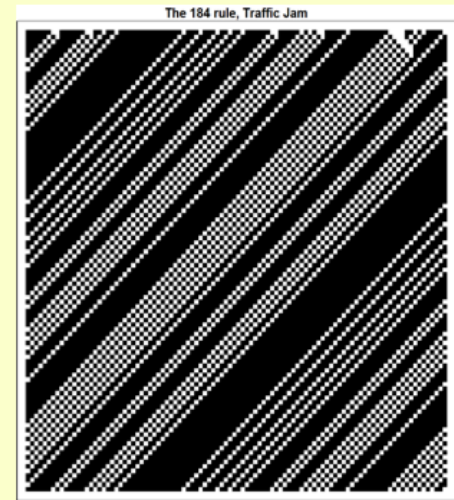
Wolfram's applications
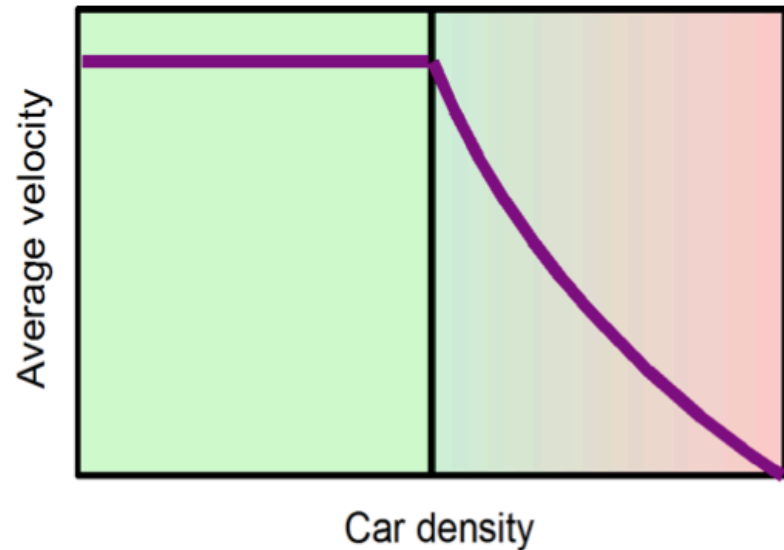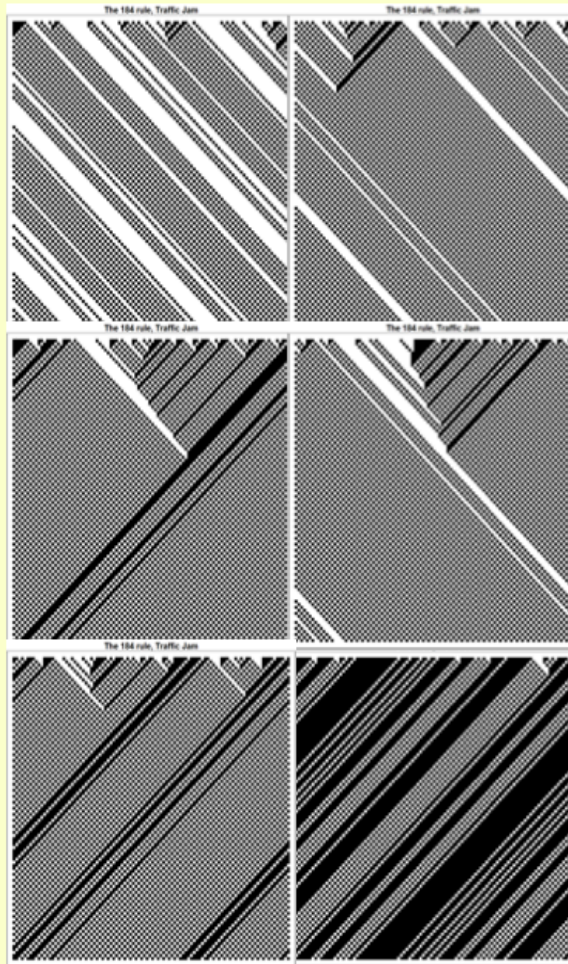
At ρ= 0.5, we observe a transition from flowing traffic to nose-to-tail traffic

Typical   2-D neighborhood shapes
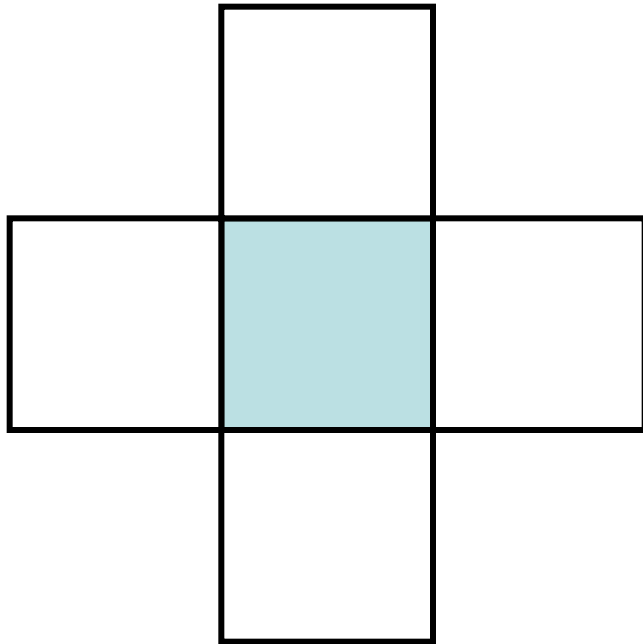
von Neumann neighborhood
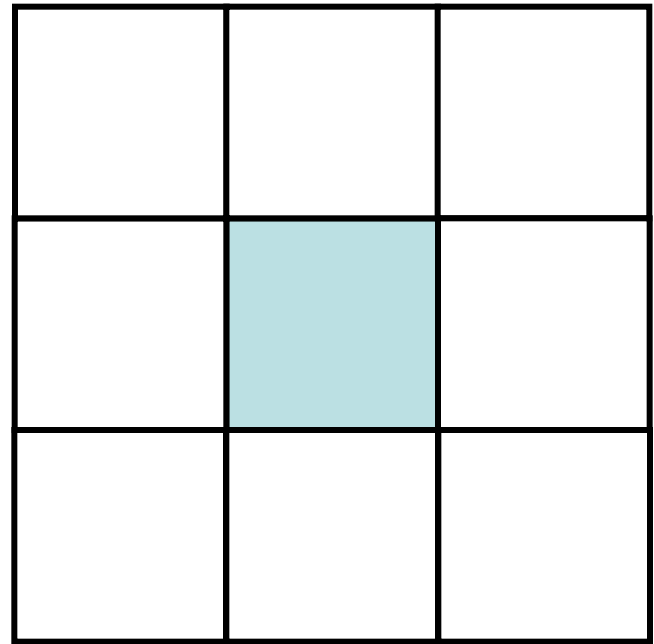
Moore neighborhood

Conway's Game of Life

- Each cell can be alive or dead
- Moore neighborhood
- Dead cell with 3 live neighbors becomes alive
- Living cell with 2 or 3 living neighbors stays alive
- Living cell with less than 2 or more than 3 alive becomes dead
- What happens

Langton's Ant

- **If the ant exits a square the color inverts**
- **If the ant exits a black (white) square it turns right (left)**
- **What happens**
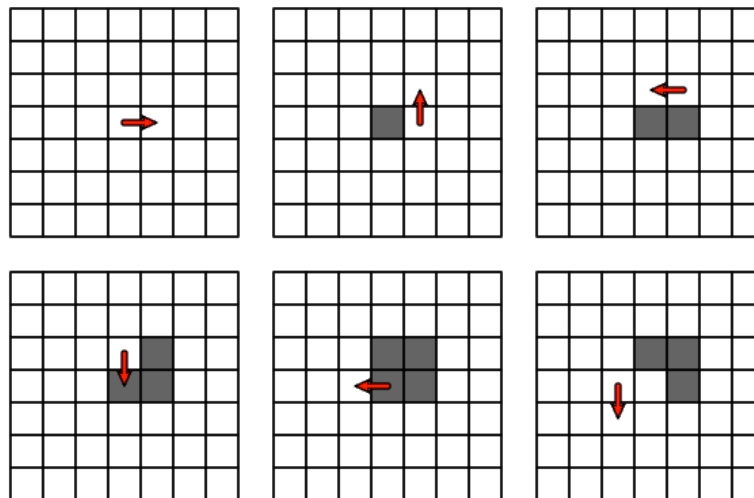
Golly

- Golly: game of life simulator is free software that can simulate very large-scale CA space in a very fast and efficient way
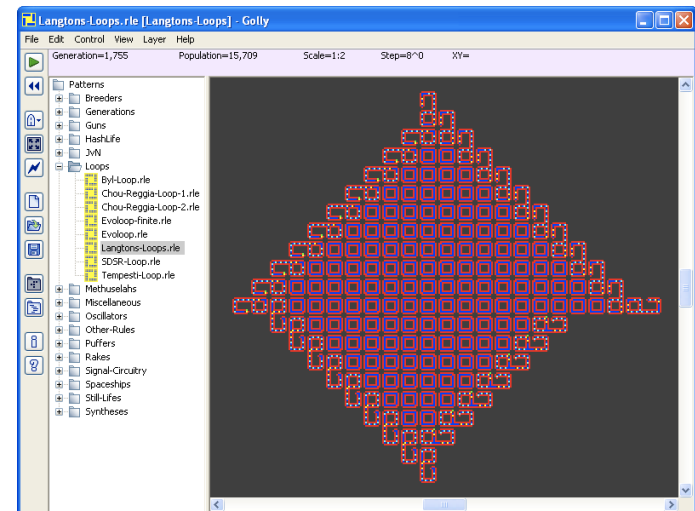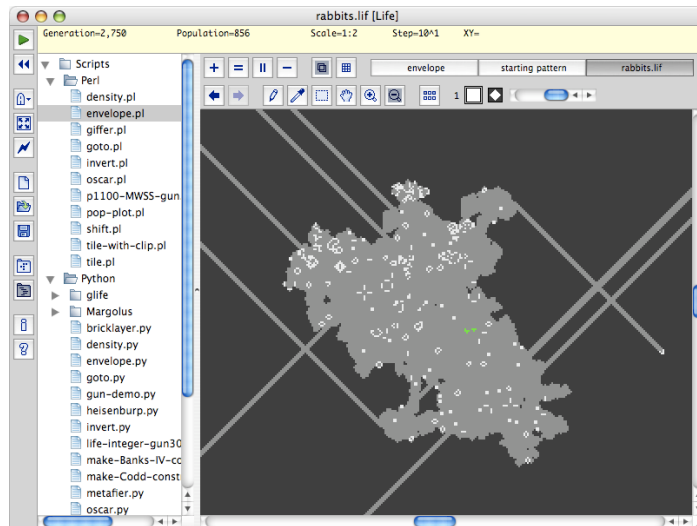
Wolfram

- Educated at Eton
- First pub at 16 in particle physics
- Left Eton for Oxford (boredom)
- 18, published well cited paper (quarks)
- Published 10 more by 19
- CalTech PhD at 20
- Immediately joined faculty
- After disputes about software, left...

Wolfram - A New Kind of Science

- **CA is new way of doing science**
- **Totally different than anything done before**
- **Wolfram single handedly is responsible for this idea**

"Three centuries ago science was transformed by the dramatic new idea that rules based on mathematical equations could be used to describe the natural world. My purpose in this book is to initiate another such transformation…"

"It has taken the better part of twenty years to build the intellectual structure that is needed but I have been amazed by its results"

Wolfram - A New Kind of Science

Simple programs are capable of a remarkable range of behavior.

Another feature of simple programs is that, according to the book, making them more complicated seems to have little effect on their overall complexity.

A New Kind of Science argues that this is evidence that simple programs are enough to capture the essence of almost any complex system.

In order to study simple rules and their often complex behaviour, Wolfram believes it is necessary to systematically explore all of these computational systems and document what they do.

He believes this study should become a new branch of science, like physics or chemistry.
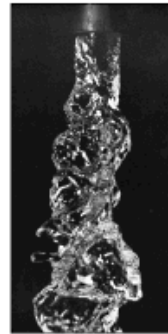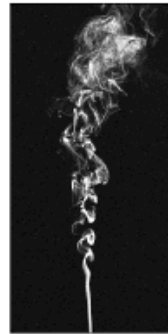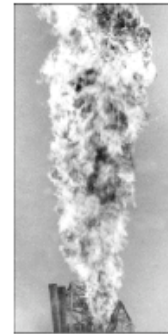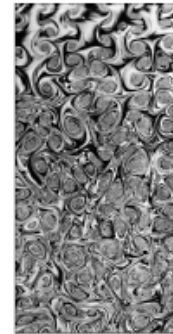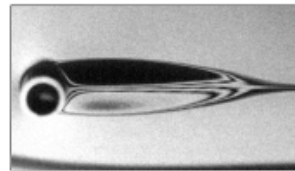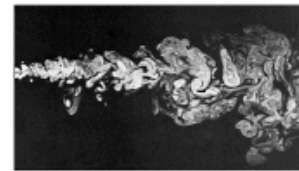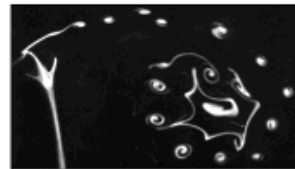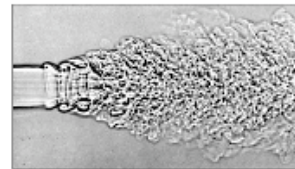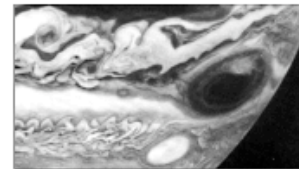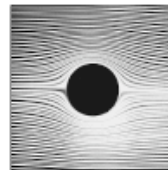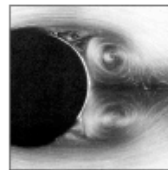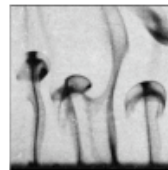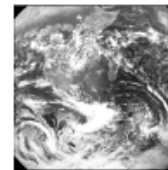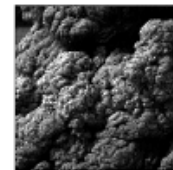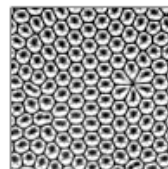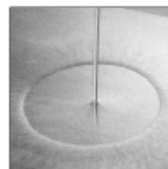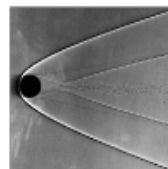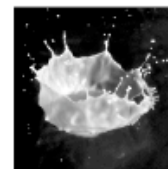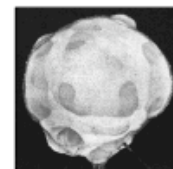
Wolfram - A New Kind of Science

- Fluids

Wolfram - A New Kind of Science

- Fluids

step 1000   step 1001   step 1002   step 1003

(a) individual cells   (b) 5 x 5 averages   (c) 25 x 25 averages

A simple cellular automaton system set up to emulate the microscopic behavior of molecules in a fluid. At each step the configuration of particles is updated according to the simple collision rules shown above. Particles are reflected whenever they hit the plate. A steady stream of particles is inserted in a regular way far to the left, with an average speed 3/10 of the maximum possible. Picture (a) shows the configuration of individual particles; pictures (b) and (c) show total velocities of successively larger blocks of particles. Picture (d) is obtained by transforming to a reference frame in which the fluid is on average at rest.

(d) moving with fluid
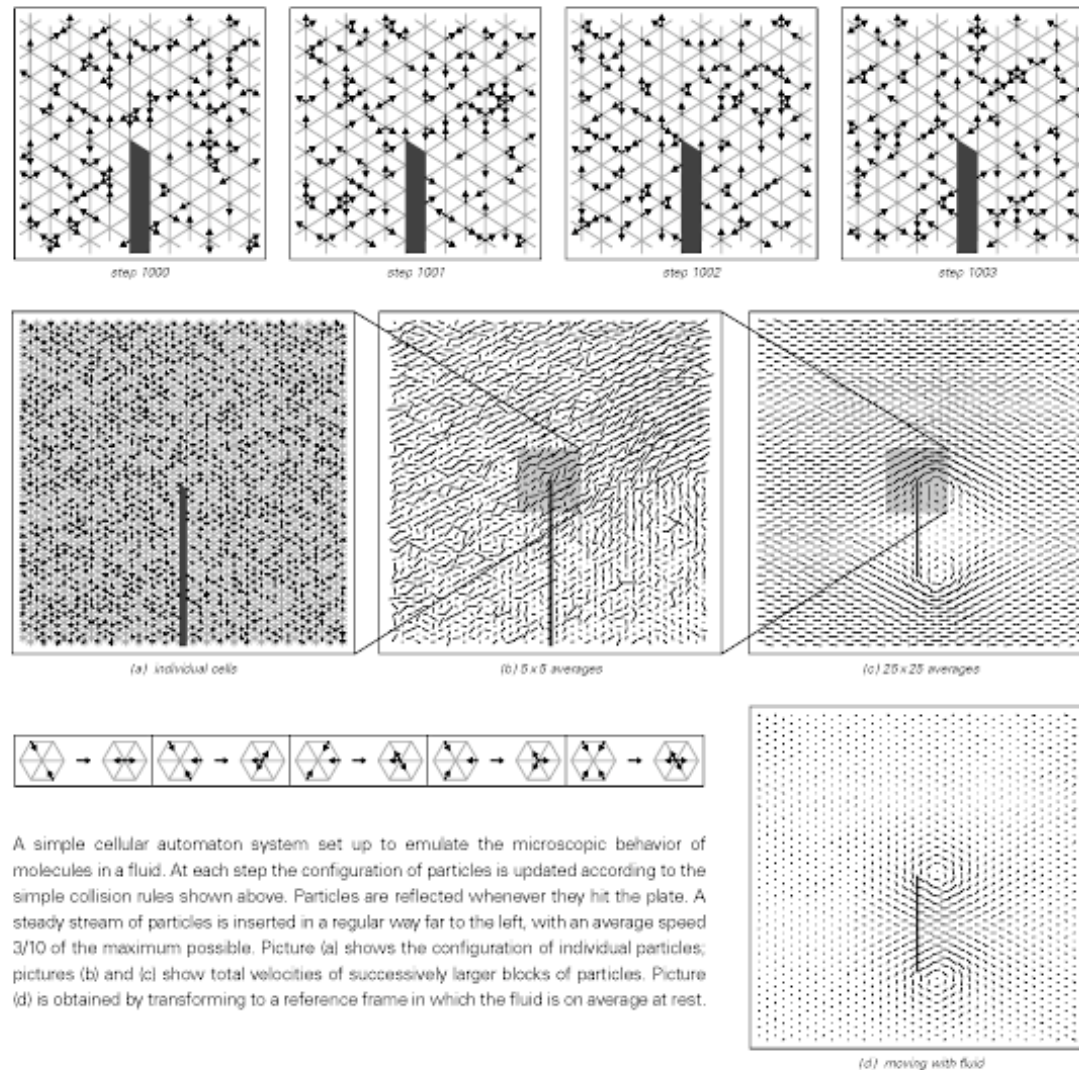
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- **Fluids**

A larger example of the cellular automaton system shown on the previous page. In each picture there are a total of 30 million underlying cells. The individual velocity vectors drawn correspond to averages over 20 × 20 blocks of cells. Particles are inserted in a regular way at the left-hand end so as to maintain an overall flow speed equal to about 0.4 of the maximum possible. To make the patterns of flow easier to see, the velocities shown are transformed so that the fluid is on average at rest, and the plate is moving. The underlying density of particles is approximately 1 per cell, or 1/6 the maximum possible—a density which more or less minimizes the viscosity of the fluid. The Reynolds number of the flow shown is then approximately 100. The agreement with experimental results on actual fluid flows is striking.
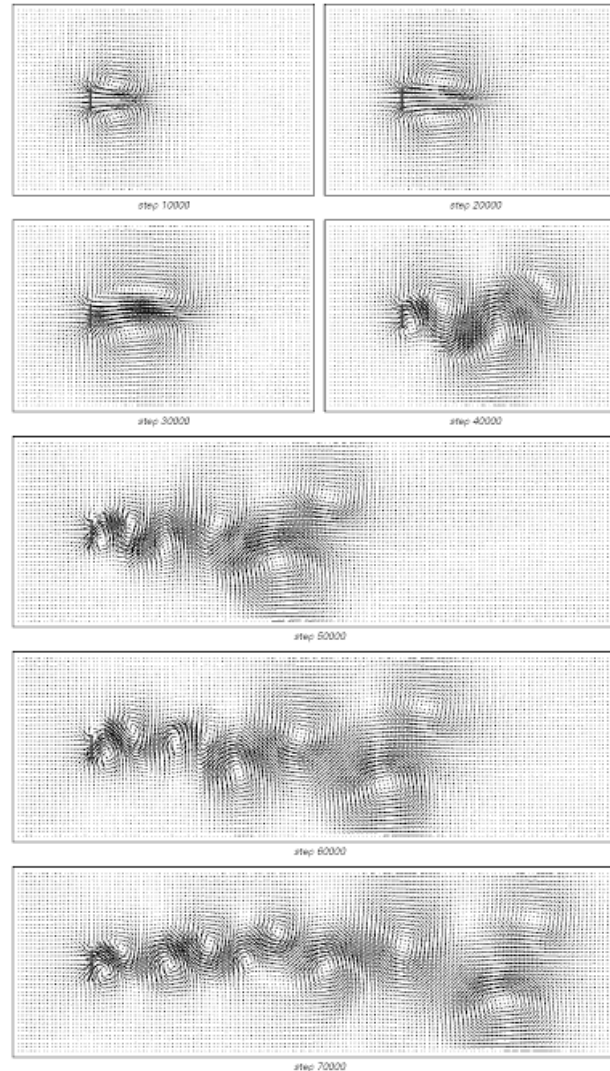
Wolfram - A New Kind of Science

- ## Reviews

An anonymous [customer review at the amazon.com site for ANKOS:](#)
A New Kind of Review
by "a reader"
I can only imagine how fortunate you must feel to be reading my review. This review is the product of my lifetime of experience in meeting important people and thinking deep thoughts. This is a new kind of review, and will no doubt influence the way you think about the world around you and the way you think of yourself.

Bigger than infinity Although my review deserves thousands of pages to articulate, I am limiting many of my deeper thoughts to only single characters. I encourage readers of my review to dedicate the many years required to fully absorb the significance of what I am writing here. Fortunately, we live in exactly the time when my review can be widely disseminated by "internet"
technology and stored on "digital media", allowing current and future scholars to delve more deeply into my original and insightful use of commas, numbers, and letters.

Wolfram - A New Kind of Science

• Reviews

My place in history My review allows, for the first time, a complete and total understanding not only of this but *every single* book ever written. I call this "the principle of book equivalence." Future generations will decide the relative merits of this review compared with, for example, the works of Shakespeare. This effort will open new realms of scholarship.

More about me I first began writing reviews as a small child, where my talent was clearly apparent to those around me, including my mother. She preserved my early writings which, although simpler in structure, portend elements of my current style. I include one of them below (which I call review 30) to indicate the scholarly pedigree of the document now in your hands or on your screen or committed to your memory:

"The guy who wrote the book is also the publisher of the book. I guess he's the only person smart enough to understand what's in it. When I'm older I too will use a vanity press. Then I can write all the pages I want."...

It is staggering to contemplate that all the great works of literature can be derived from the letters I use in writing this review. I am pleased to have shared them with you, and hereby grant you the liberty to use up to twenty (20) of them consecutively without attribution. Any use of additional characters in print must acknowledge this review as source material since it contains, implicitly or explicitly, all future written documents.

Wolfram - A New Kind of Science

- ## Panel Discussion

Although it is clear that Wolfram is no crank, not someone skeptics
would label a pseudoscientist, skeptics will notice that, despite his
flawless credentials, staggering intelligence, and depth of knowledge, Wolfram
possesses many attributes of a pseudoscientist:

(1) he makes grandiose claims,

(2) works in isolation,

(3) did not go through the normal peer-review process,

(4) published his own book,

(5) does not adequately acknowledge his predecessors, and

(6) rejects a well-established theory of at least one famous scientist.

Wolfram - A New Kind of Science

- ## Panel Discussion

Steven Koonin pointed out that a paradigm shift cannot arise simply by asserting something is a paradigm shift. One must convince the scientific community that this description is warranted. To the contrary, according to David Stevenson, Wolfram fails to satisfy rules of what constitutes good science. Creating programs that generate images that look like things found in nature is not sufficient. One needs specific predictions. Wolfram does not offer any laboratory experiments or observations that could verify or falsify his grand claims.

Wolfram - A New Kind of Science

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Wolfram response: his claims have the character of mathematics rather than physics, employing calculus as an analogy.
Newton showed how calculus provides a new way of doing science. Calculus
itself, however, is not tested to determine whether it is true or false. Its justification is that it works.
The panel rebutted that if this analogy is true, then Wolfram is just proposing a new kind of computational method, not a new kind of science.

Wolfram - A New Kind of Science

- ## Review

- Simple Rules/Programs Produce Complex Behavior. There is not much controversy about this assertion, which covers such well-studied behaviors as chaos and self-organization, except in the sense that Wolfram claims that the origin and development of the hypothesis is more or less entirely due to his work.

- These Simple Rules, in the form of CA, underlie all of the Complexity of Nature. To test this hypothesis in any sort of reasonable way, one would have to at the very least examine a very broad range of behaviors in Nature. Despite a lot of expansive talk, Wolfram really only looks at fairly simple, basic behaviors.

- Cellular Automata Can Model very Complicated Systems, Whereas Traditional Approaches can only treat Simple Systems. This is ludicrous. Wolfram treats fairly basic behaviors but ignores a vast range of work dealing with very complicated systems, both from the perspective of complexity and also from traditional approaches. This is what happens when you isolate yourself from active research communities.

- Computation is the Framework for Nature. A well-studied topic, but Wolfram's spin is that his very simple CA models do the trick. Again, this is ridiculous. CA models treat only behaviors of systems that correspond to spatially distributed dynamics, ignoring dynamics that do not have an explicit spatial component - think: THE BRAIN.

Wolfram - A New Kind of Science

- A Final Note From Wolfram

Modeling example: Panic in a gym
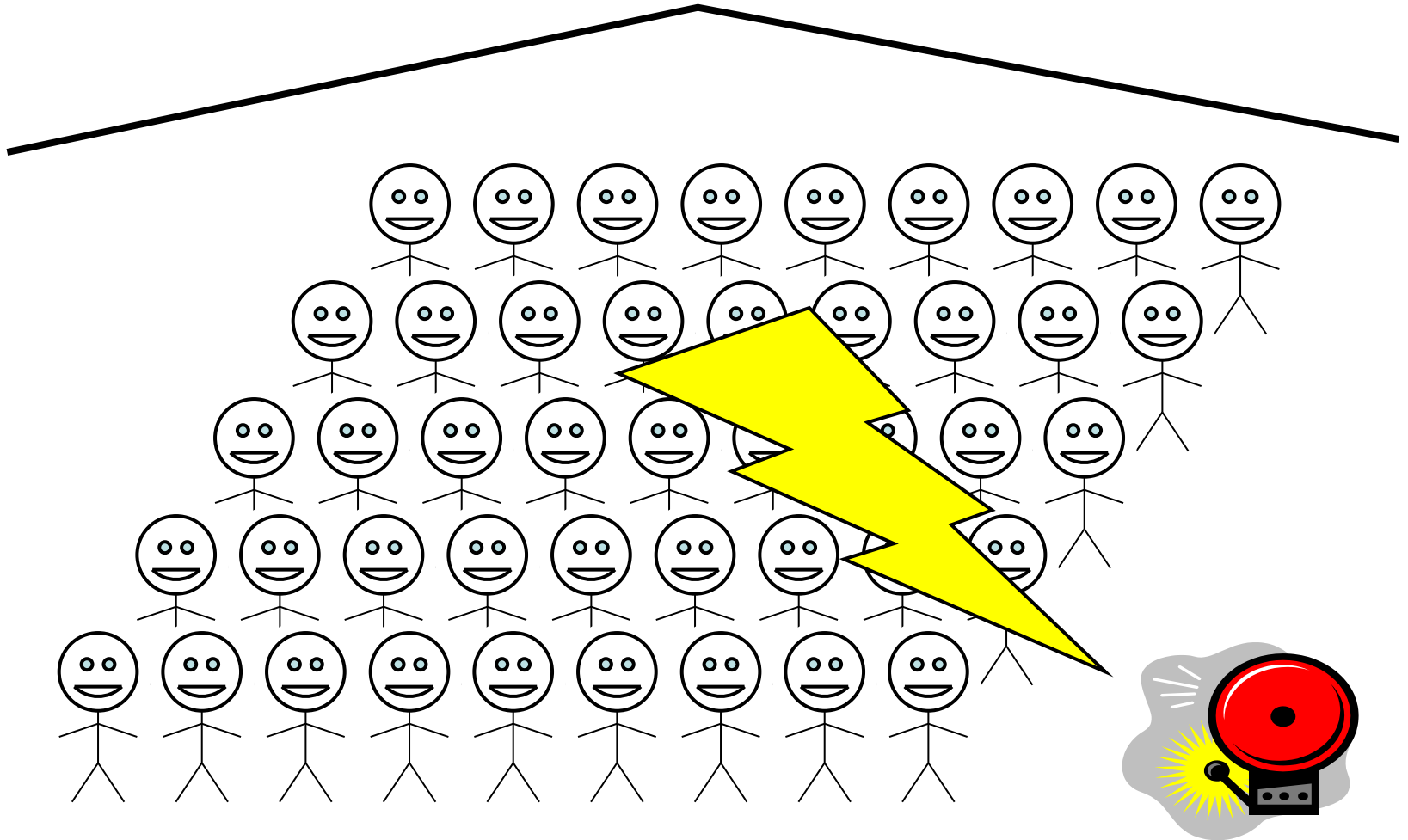
Fire alarm causes initial panics
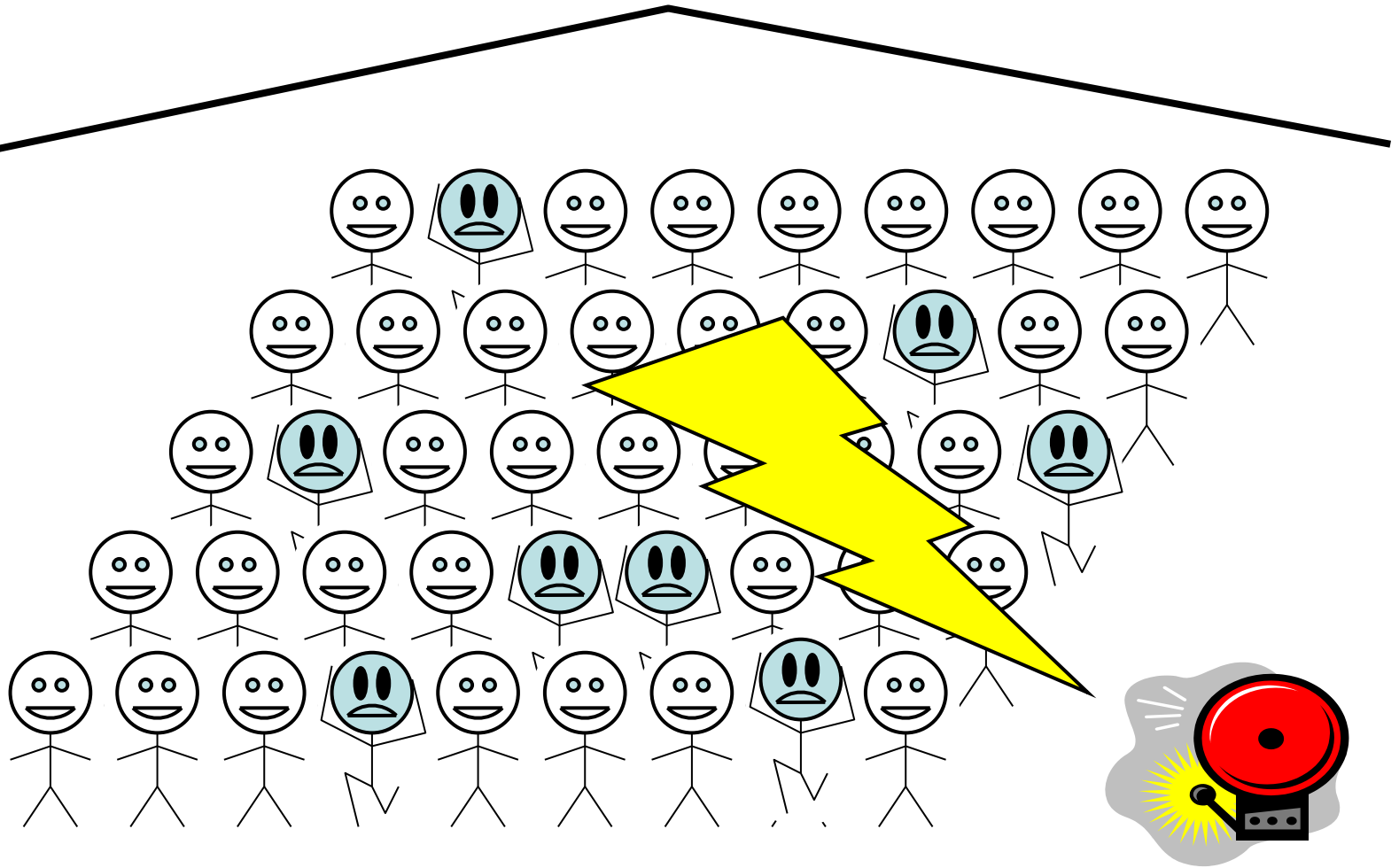
Rules of local interaction
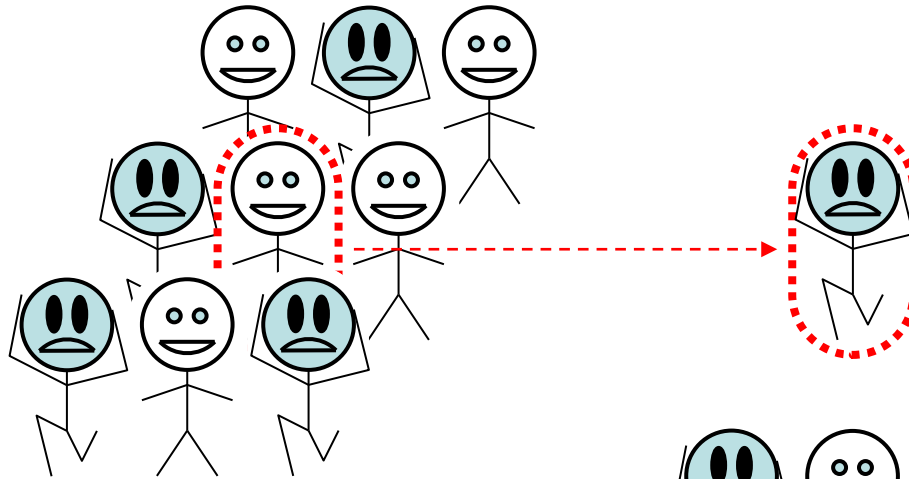
With four or more panicky persons around you
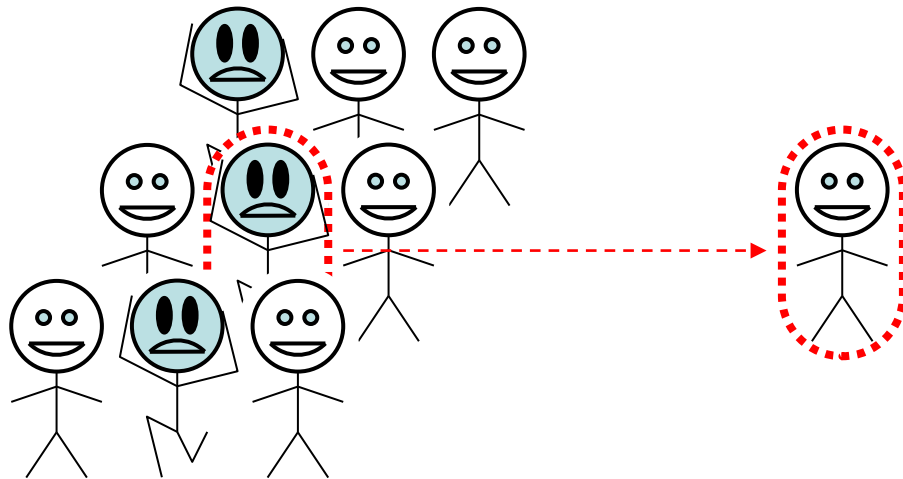
With two or fewer panicky persons around you

Exercise

- **Implement the simulator of "majority" CA (each cell turns into a local majority state) and see what kind of patterns arise**

- **What will happen if:**
  - Number of states are increased
  - Size of neighborhoods is increased
  - "Minority" rule is adopted

Other Extensions of Cellular Automata

Stochastic CA

- CA with stochastic state-transition functions
  - Stochastic growth models
  - Ecological/epidemiological models
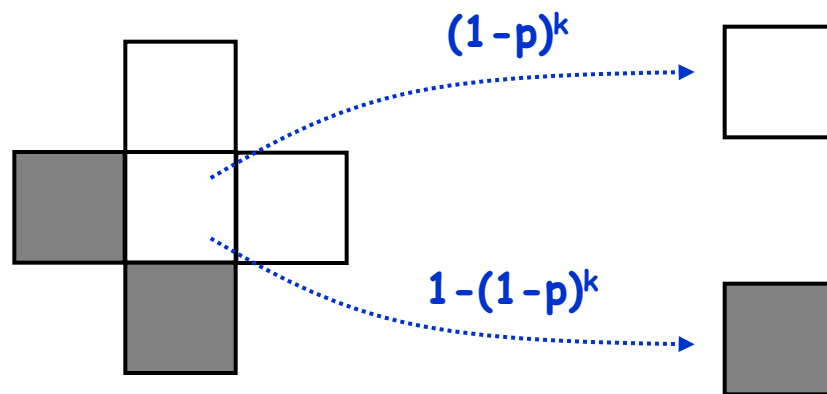
Multi-layer CA

- **CA whose cells have composite states**
  - Multiple "layers" in a state
  - Similar to vector states, but each layer can have non-numerical values as well
  - Represents multiple CA spaces interacting with each other

State for layer 1
State for layer 2
State for layer 3
…
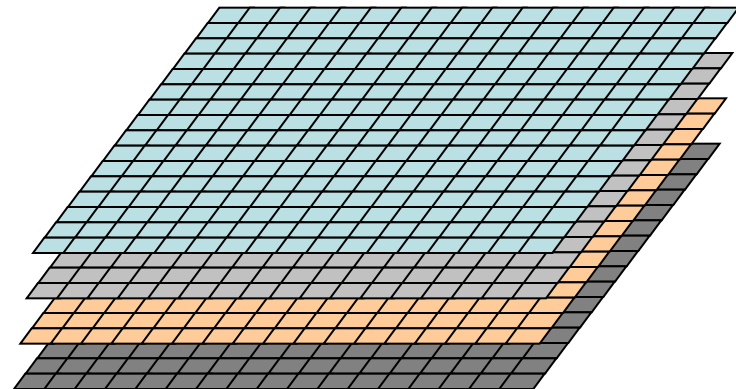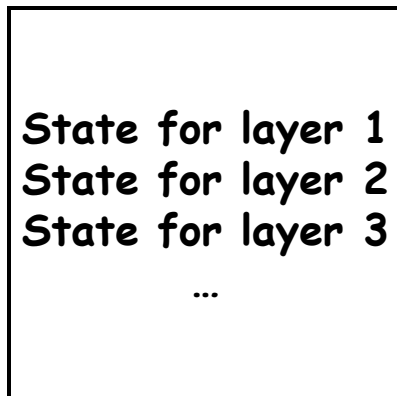
Multi-layer CA

- Examples
  - Models of spatially distributed agents with complex internal states
  - Models of interaction between agents and environment
  - CA that depend on more than one step past configurations

Asynchronous CA

- CA where cells are updated asynchronously
  - Random updating, sequential updating, etc.
  - Can simulate any synchronous CA
    Global synchrony is not strictly required for the dynamics of CA!!
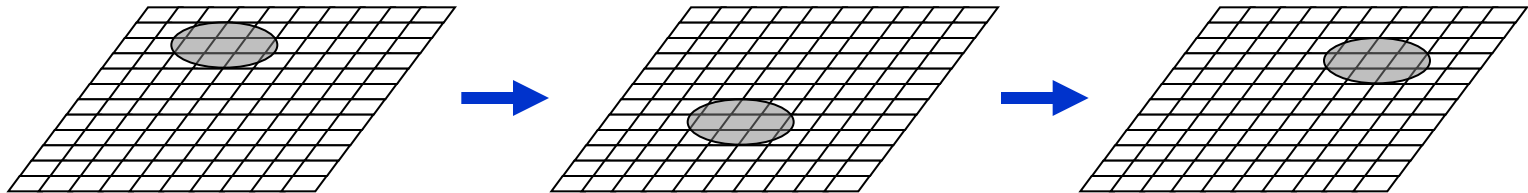
FYI: Emergent evolution on CA

- **Evoloops (Sayama 1998)**
- **Self-replicating worms (Sayama 2000)**
- **Ecology of self-replicating worms (Suzuki et al. 2003)**
- **Genetic evolution of self-replicating loops (Salzberg et al. 2003, 2004)**

  http://bingweb.binghamton. edu/~sayama/
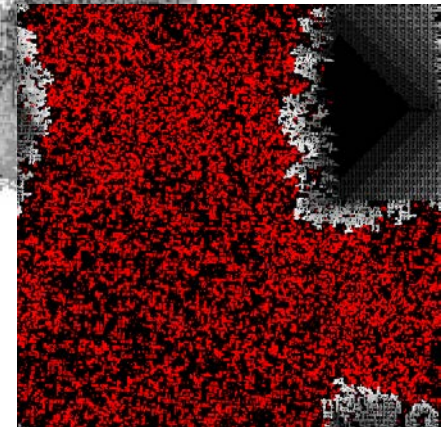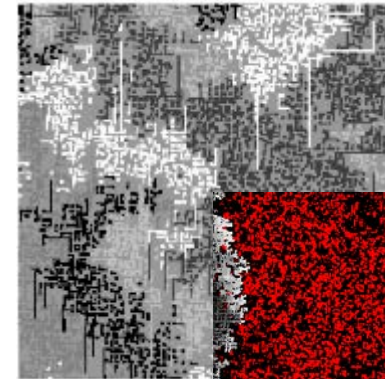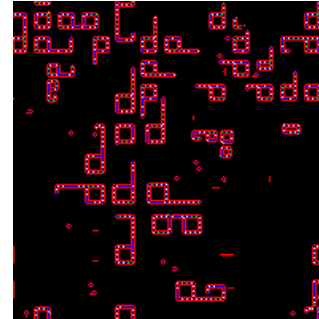
Biological Models on CA

Several biological models on CA

- **Turing patterns**

- **Waves in excitable media**

- Host-pathogen models

- Epidemic / forest fire models

Turing patterns

- **Chemical pattern formation proposed by Alan Turing (original model in PDEs)**
  - **Strong short-range activation**
  - **Relatively weak long-range inhibition**
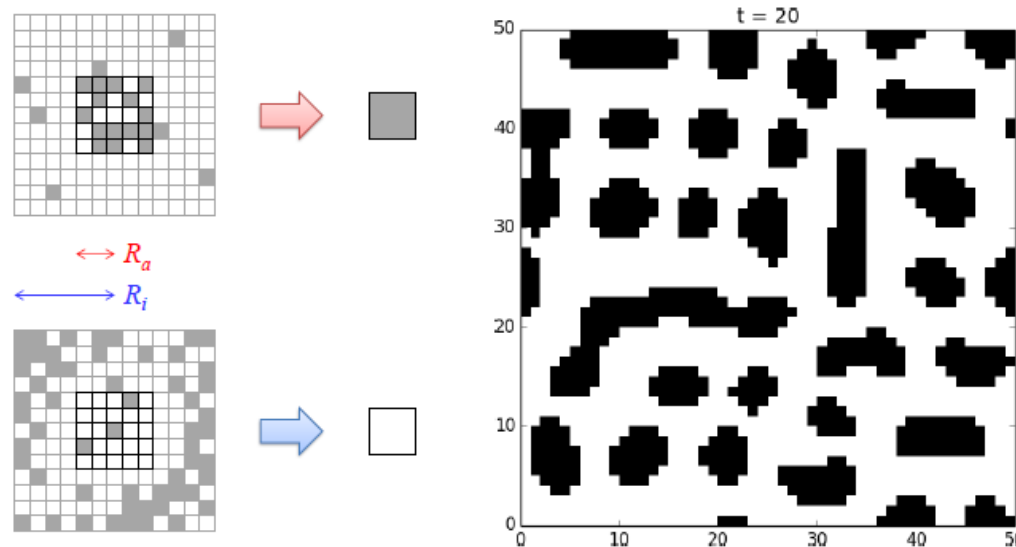
Waves in excitable media

- **Propagation of signals over excitable neural/muscular tissues**
  - Excitation of resting cells by neighbors
  - Excitation followed by refractory states
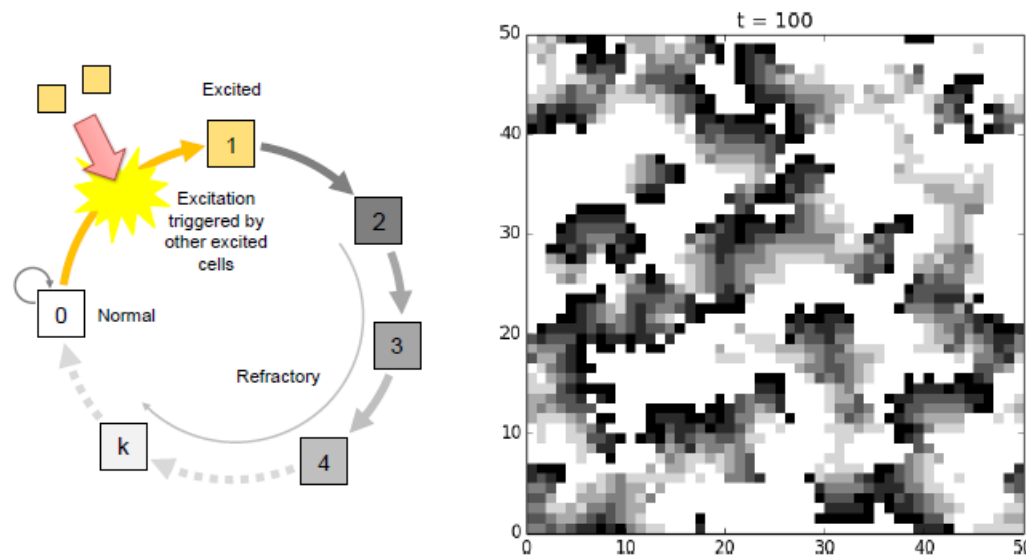
Host-pathogen models

- **Propagation of pathogens over dynamically growing hosts**
  - Spatial growth of hosts
  - Infection of pathogens to nearby hosts
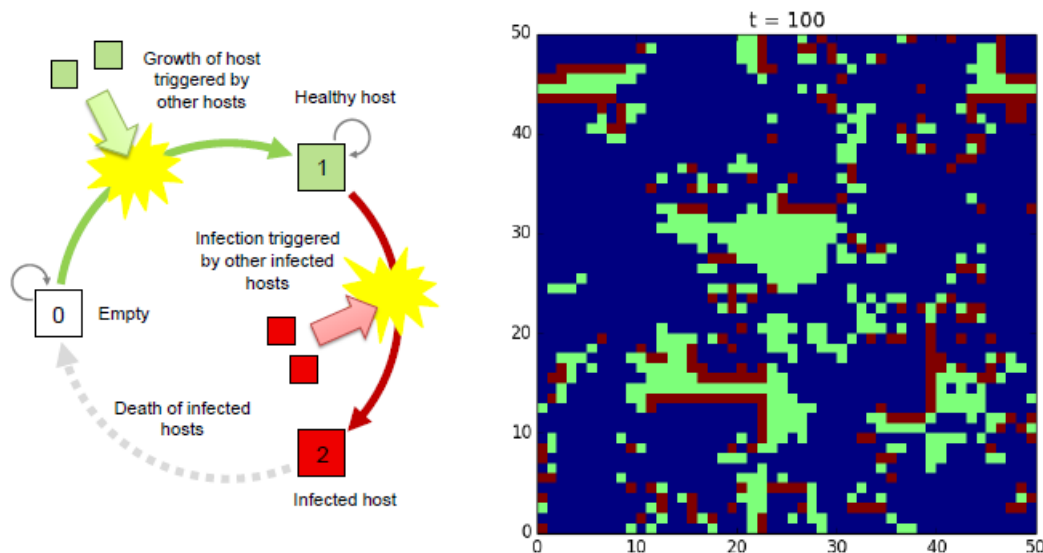
Epidemic/forest fire models

- **Propagation of disease or fire over statically distributed hosts**
  - **Propagation of disease to nearby hosts**
  - **Death of hosts caused by propagation**
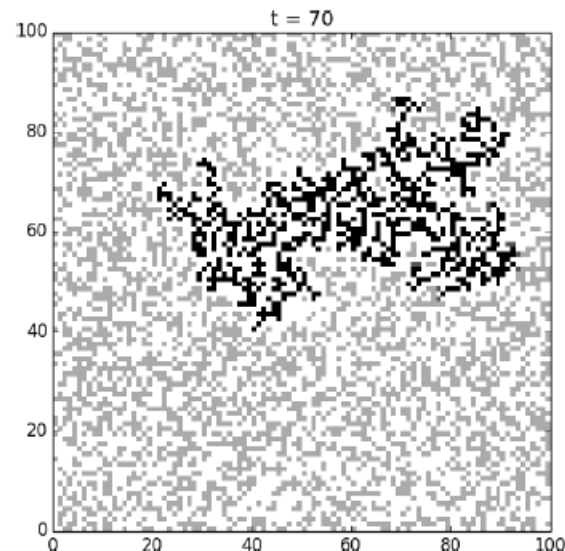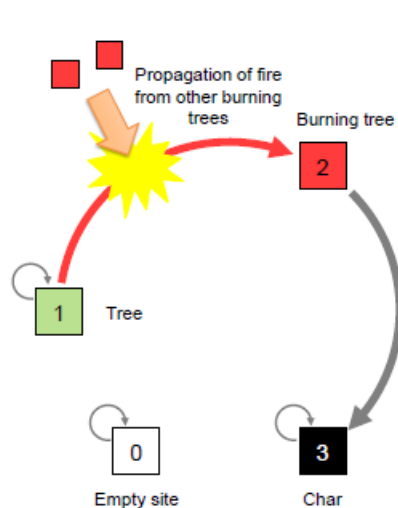
Coral/Algae model

A battle for space on the reef
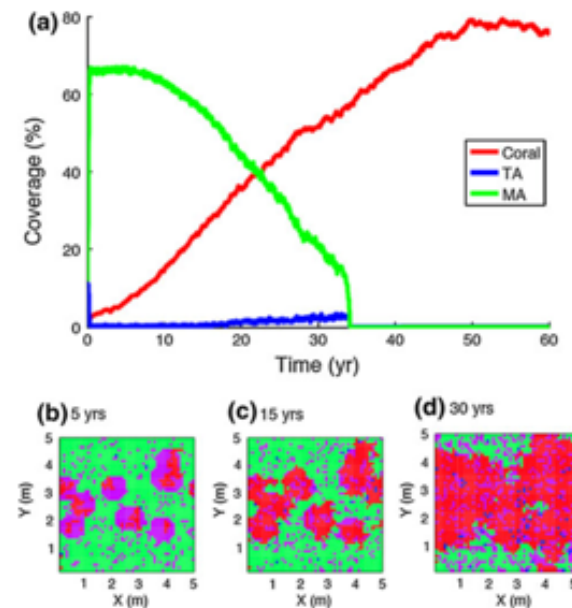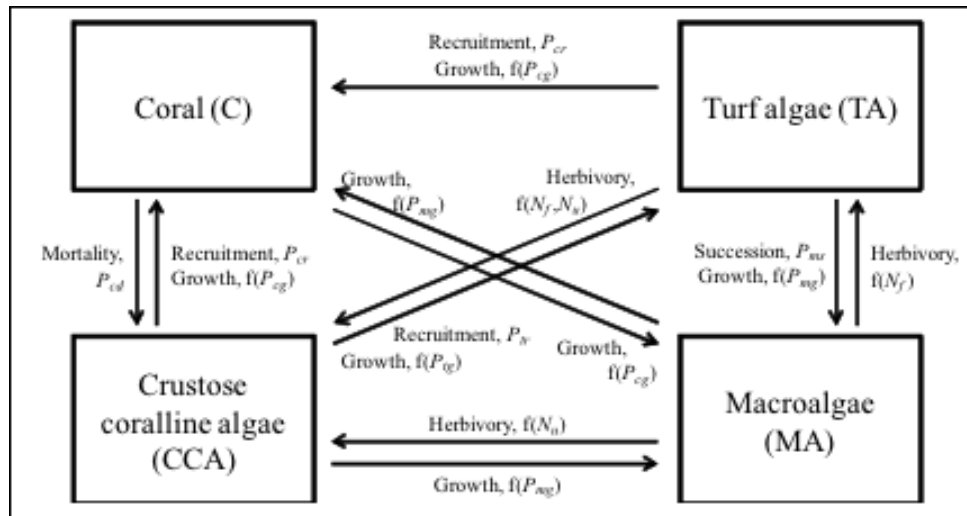
Fig. 2 a Time series of benthic community development over 60 years showing percent coverage for coral, turf algae (TA) and macroalgae (MA) starting from a random initial configuration of 10% of each type (remaining 70% coverage was by crustose coralline algae, CCA) and including both herbivorous fish and sea urchins. Snapshots of benthic landscape at b 5, c 15 and d 30 years showing coverage by coral (red), turf algae (blue), macroalgae (green) and CCA (purple). Note that ephemeral single cells of CCA are largely caused by spatially uncorrelated herbivory by fish while larger circles of CCA are caused by spatially correlated herbivory by sea urchins, consistent with so-called 'urchin halos'

Phase Space of Cellular Automata

Phase space of cellular automata

- **For a fixed number of cells, the total number of possible configurations is also fixed and finite: k^n**
  - **k: Number of states**
  - **n: Number of cells**
- **For deterministic CA, each configuration is always mapped to just one configuration**
  - **You can draw a state-transition diagram**

Counting #s of possibilities

- Dimensions of space: D
- Length of space in each dimension: L
- Radius of neighborhood: r
- Number of states for each cell: k

- # of configurations: $k^{\{L^D\}}$

- # of possible rules: $k^{\{k^{\{(2r+1)^D\}}\}}$

Features in phase space

- **Self-loop** $\rightarrow$ **fixed point**
- **Cycle** $\rightarrow$ **periodic attractor**
- **Configuration with no predecessor**
  - $\rightarrow$ **"Garden of Eden" states**
- **Just one big basin of attraction**
  - $\rightarrow$ **Initial configurations don't matter**
- **Many separate basins of attraction**
  - $\rightarrow$ **Sensitive to initial configurations**

Phase space visualization
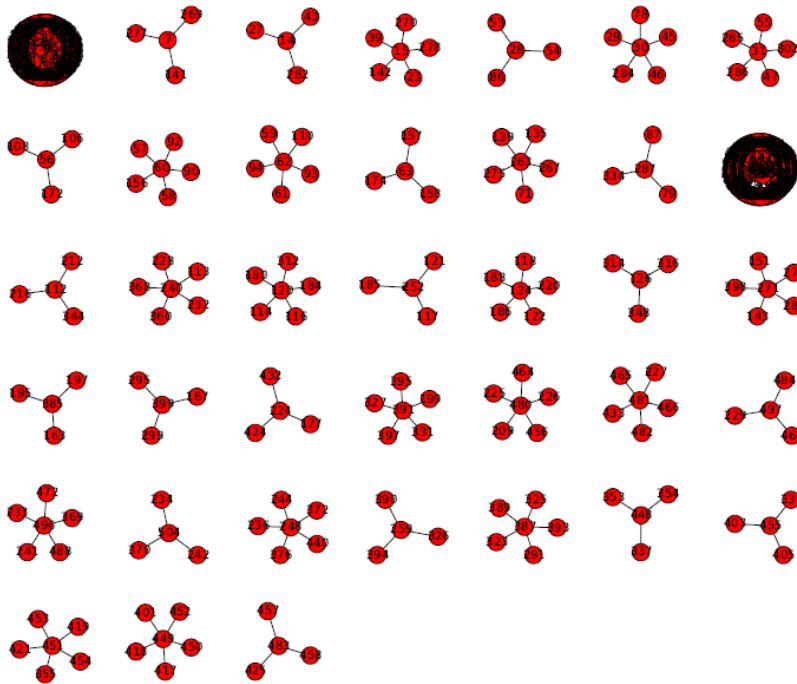
Visualization in Python
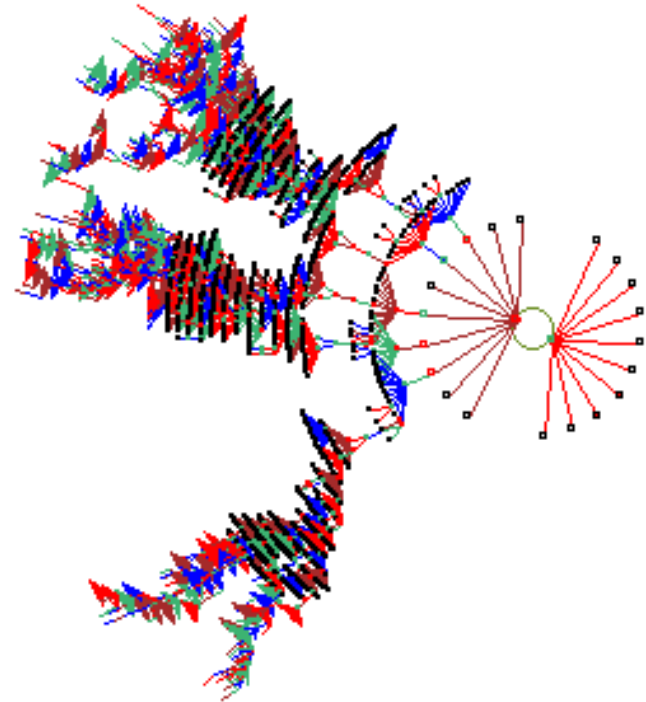
Visualization by DDLab

Example

see detail

garden-of-Eden states

attractor cycle

transient trees and sub-trees
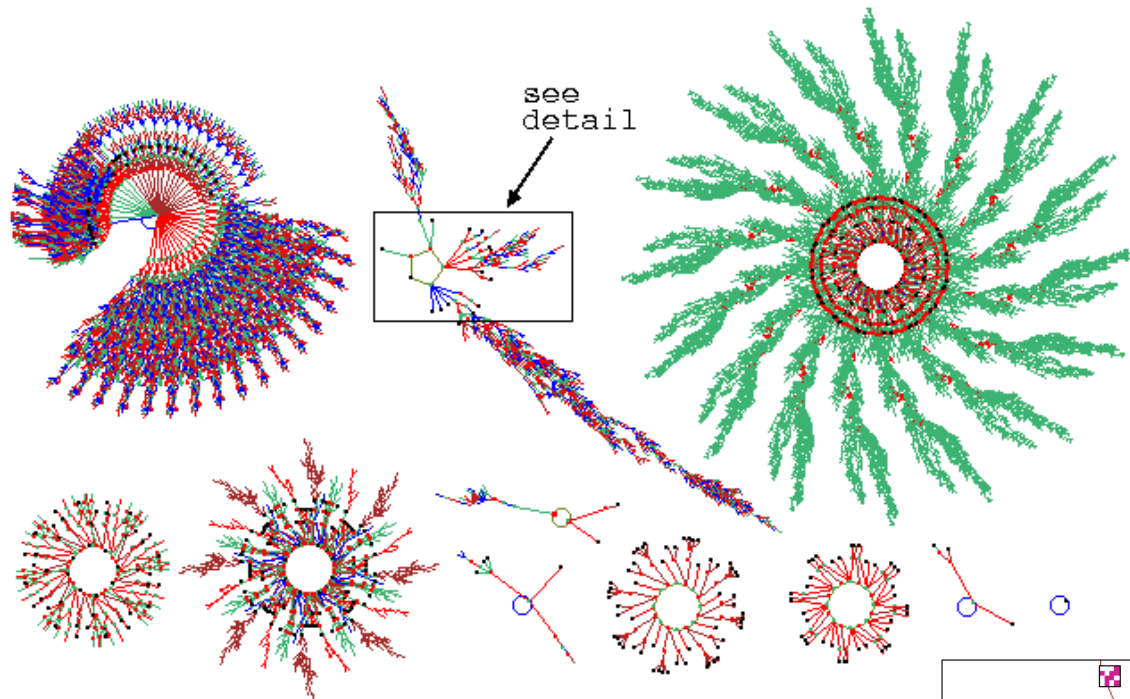
k=2, n=16

© Andy Wuenshe
http://www.ddlab.com/
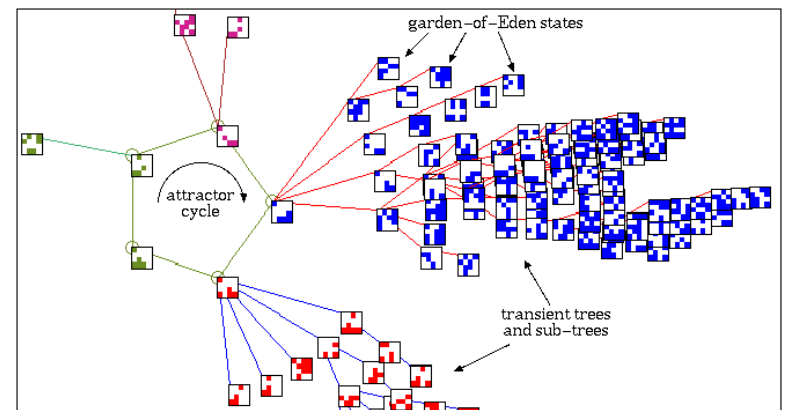
Mean-Field Approximation

Mean-field approximation

- An approximation to drastically reduce the dimensions of the system by reformulating the dynamics in terms of "a state of one part" and "the average of all the rest (= mean field)"

MFA

A cell

0

1
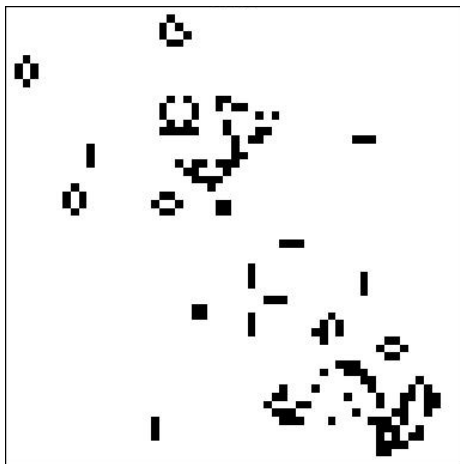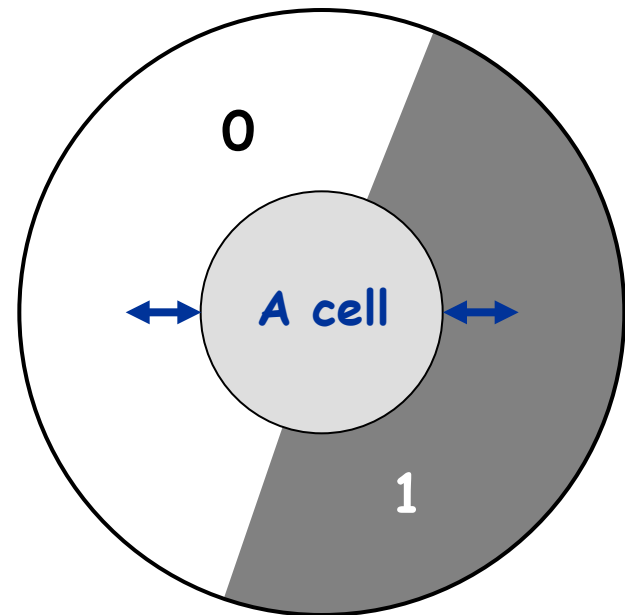
How MFA works
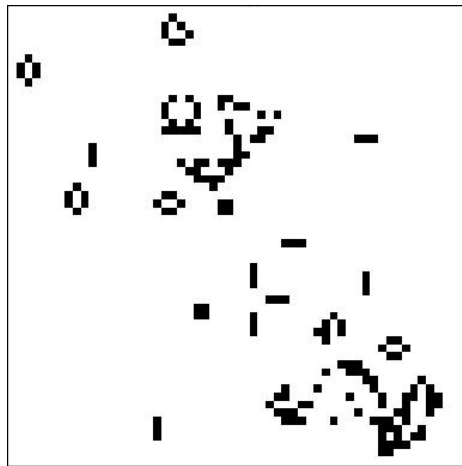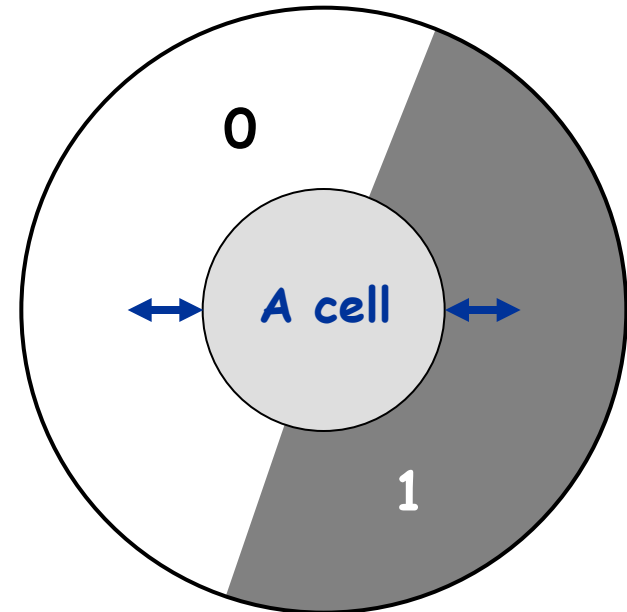
1. Make an approximated description about how one part changes its state through the interaction with the average of all the rest (= mean field)
2. Assume this approximation applies to all the parts, and analyze how the mean field itself behaves

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Mathematical description of MFA (difference equations)

- **Original equations:**

$$x^i_t = F^i(\{ x^i_{t-1} \})$$
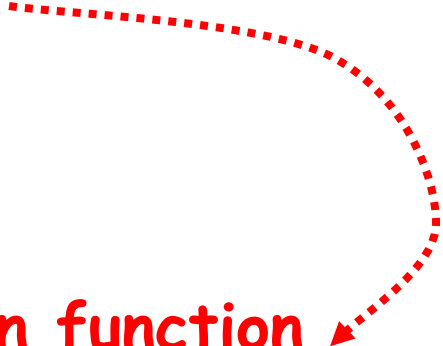
- **Approximate equations with MFA:**

$$x^i_t = F'^i(x^i_{t-1}, <x>_{t-1})$$

$$<x>_t = \Sigma_i \, x^i_{t-1} \, / \, n$$

Each state-transition function takes only two arguments: its own state and the "mean field"

Exercise

- Describe the dynamics of average density of states, p_t, in majority CA

- Consider the following:
  - For each state '0' or '1', what is the probability for the cell to stay in the same state?
  - Based on the results above, how does the "mean field" p_t change over time?

Applicability of MFA

- MFA works quite well for systems whose parts are well-connected or randomly interacting
- MFA does not work (a) if interactions are local or non-homogeneous, and/or (b) if the system has a non-uniform "pattern" as its macroscopic state
- Use MFA as a preliminary approximation, but not to derive a final conclusion

Critical Behavior in CA-Based Models

Critical behavior in CA-based models

- **Contact processes on CA-based models may show critical behavior**
  - Drastic changes of macroscopic properties when some local parameter value is varied
  - Called "phase transition" – bifurcation of macroscopic properties occurring in aggregates
  - Many interesting phenomena occur at a critical parameter value

Example: Epidemic / forest fire models

- **Propagation of disease or fire over statically distributed hosts with initial density p:**

  - Propagation of disease or fire to nearby hosts
 - Death or breakdown of hosts caused by propagation

Analytical estimation of threshold

- **Percolation threshold may be estimated analytically by renormalization group analysis**

- **Key idea:**

  Critical behavior arises when local dynamics and global dynamics are nearly identical

  - Similar percolation process occurs at every scale, from micro to macro

Renormalization group analysis

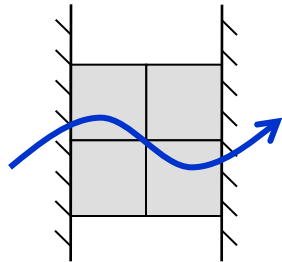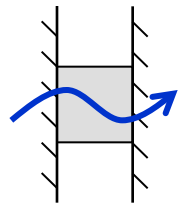1. **Obtain the relationship between percolation probabilities at lowest level (p_1) and at one-step higher level ($p_2 = \Phi(p_1)$)**

Single cell level:
$p_1 = p$

2x2 block level:
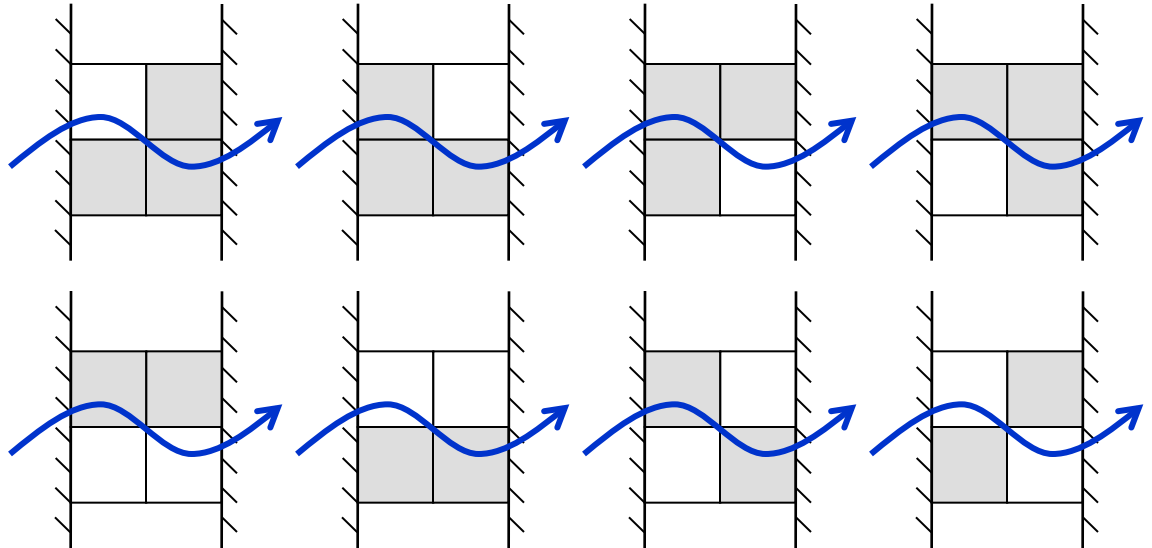$p_2 = \Phi(p_1)$
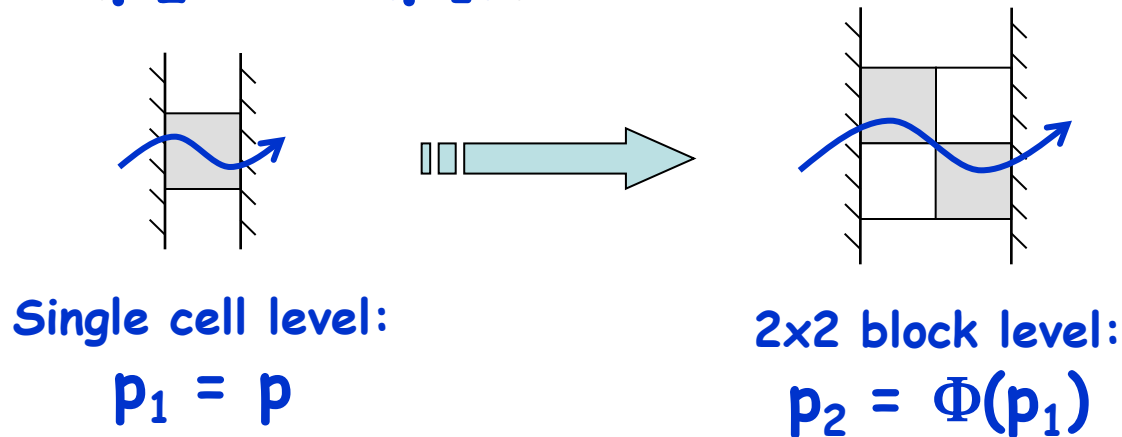
2. **Obtain p that satisfies $p = \Phi(p)$**

Example: Forest fire model with Moore neighborhoods

Single cell level:

$$p_1 = p$$

2x2 block level:

$$p_2 = \Phi(p_1)$$
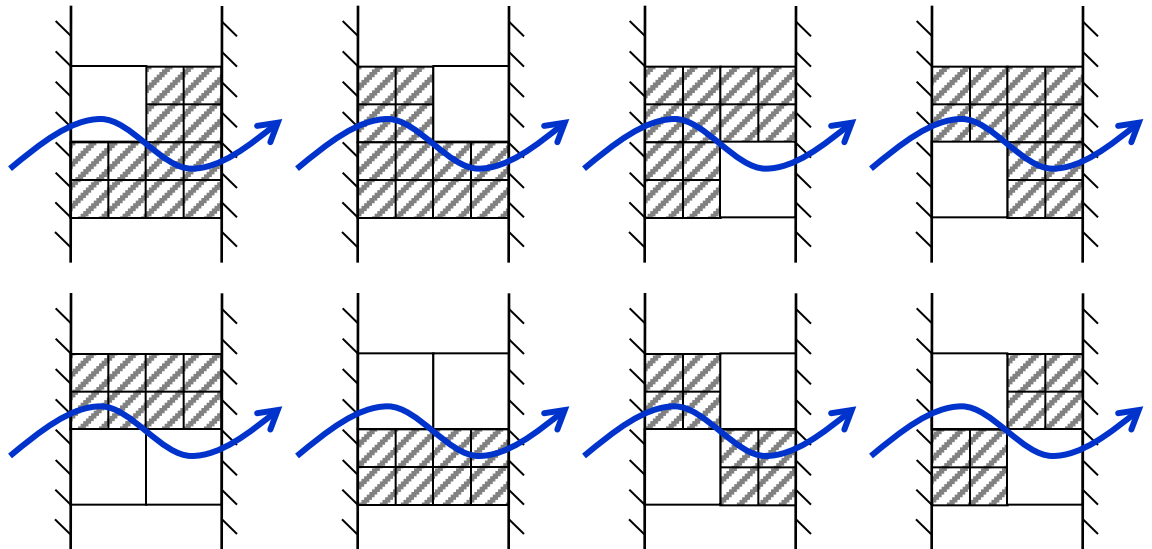$$= p_1^4 + 4p_1^3(1-p_1)$$
$$+ 4p_1^2(1-p_1)^2$$

At a larger scale...

2x2 block level:
$$p_2 = \Phi(p_1)$$
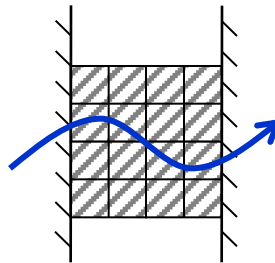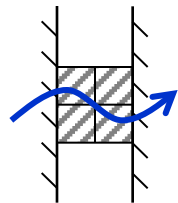
4x4 block level:
$$p_4 = \Phi(p_2)$$
$$= \Phi(\Phi(p_1))$$

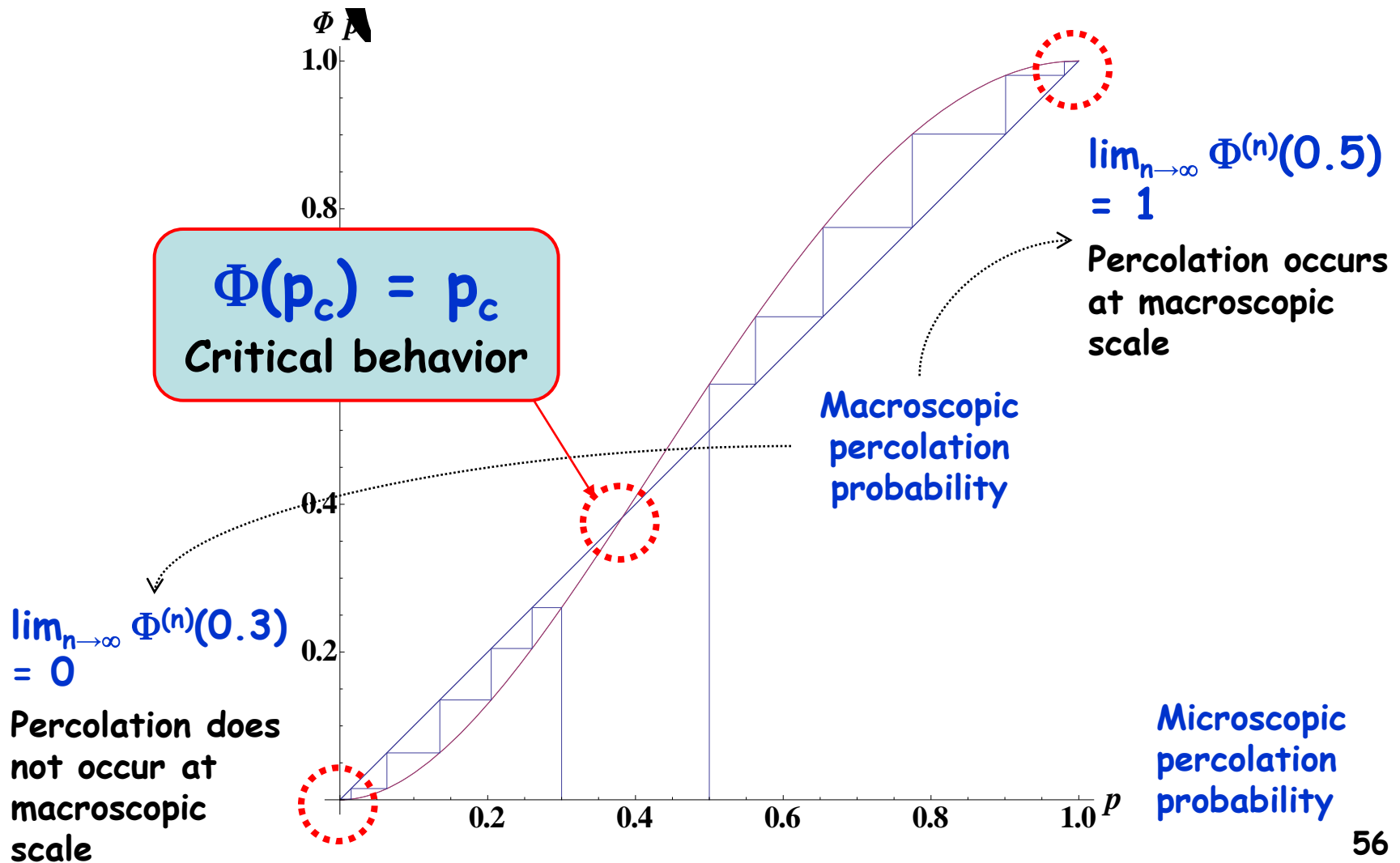**(Note: This is an approximation, not an exact calculation)**

Thinking even larger…

- Percolation probability at $2^n \times 2^n$ block level:

$$p_{2^n} = \Phi^{(n)}(p_1)$$

- Percolation probability at macroscopic level ($n \to \infty$):

$$p_\infty = \lim_{n \to \infty} \Phi^{(n)}(p_1)$$

Fixed point of Φ(p)

$\Phi(p_c) = p_c$
Critical behavior

$\lim_{n \to \infty} \Phi^{(n)}(0.5) = 1$

Percolation occurs at macroscopic scale

Macroscopic percolation probability

$\lim_{n \to \infty} \Phi^{(n)}(0.3) = 0$

Percolation does not occur at macroscopic scale

Microscopic percolation probability

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Therefore…

- Solving $p = \Phi(p)$ gives an analytical estimate of percolation threshold p_c