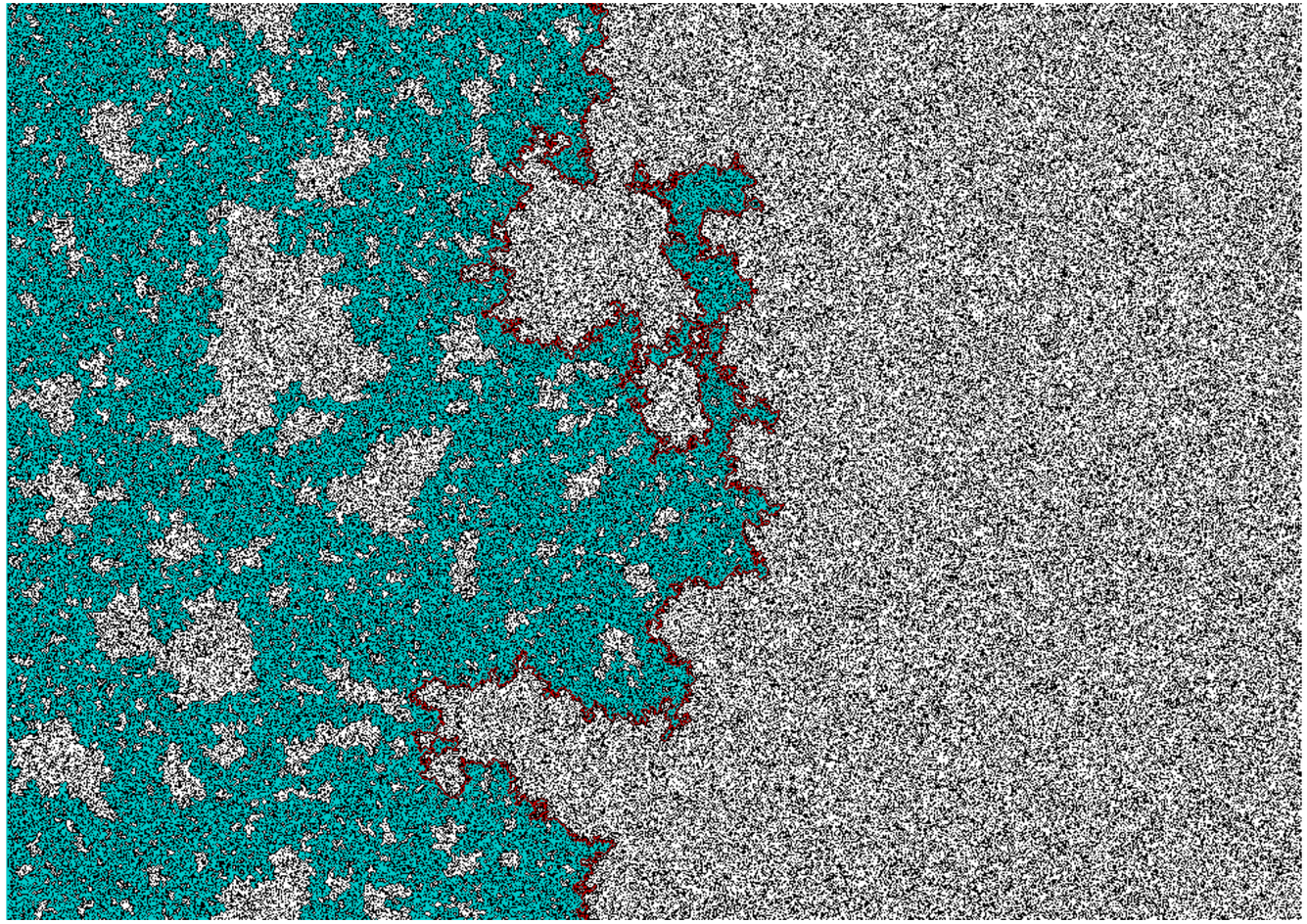
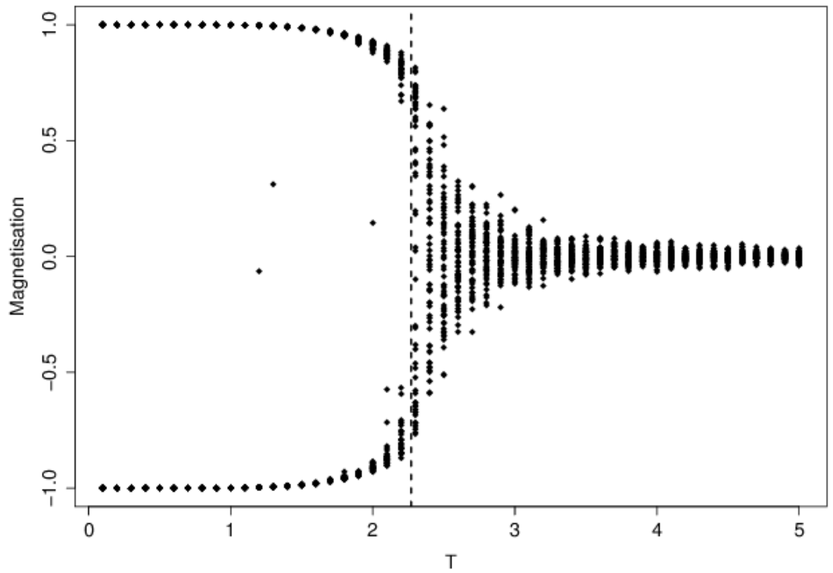
Criticality - Percolation and Sandpiles

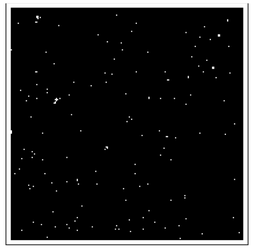


Dr. Dylan McNamara
people.uncw.edu/mcnamarad

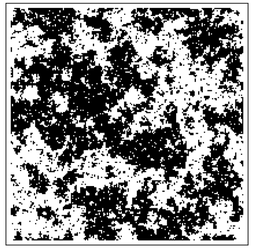
Criticality

- Asteroids vs Green Sahara
- Phase changes
- Ising model

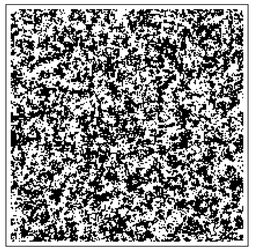




$T < T_c$



$T \sim T_c$



$T > T_c$

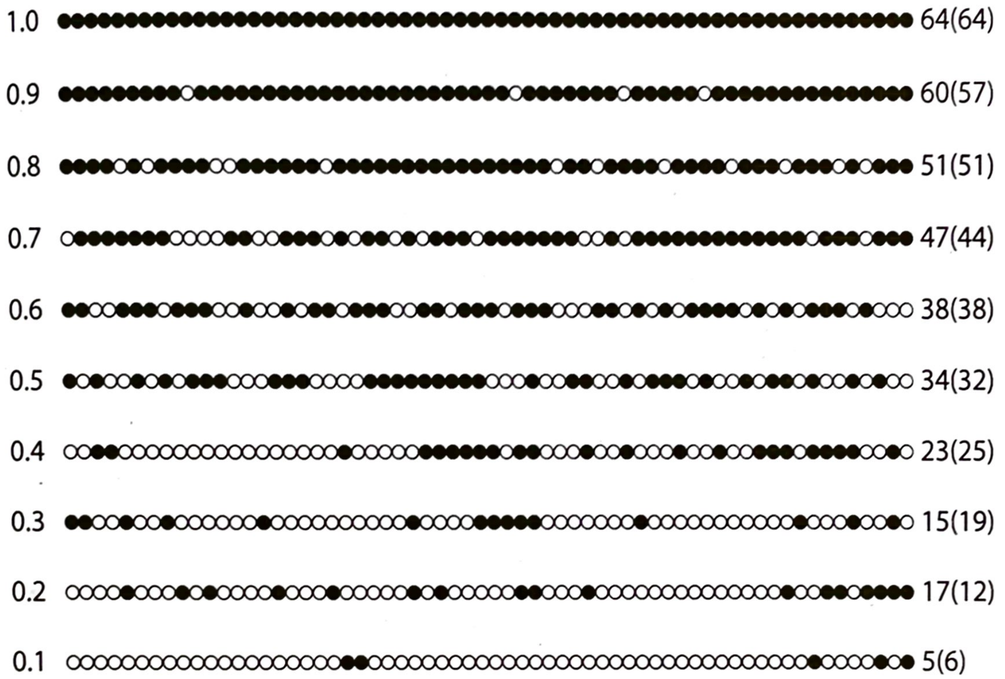
Percolation

- Percolation - passage of liquid through a porous media
- Can serve as canonical example of criticality - phase transitions
- What we will explore is an abstraction isn't percolation and isn't a phase transition

Percolation (1D)

N = 64

p - occupation probability



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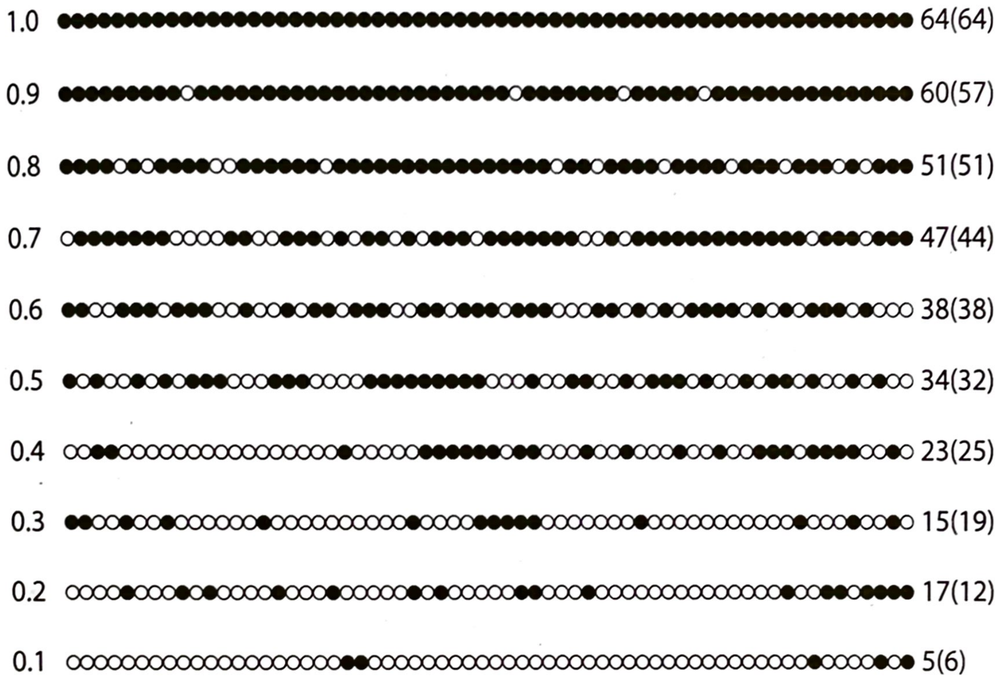
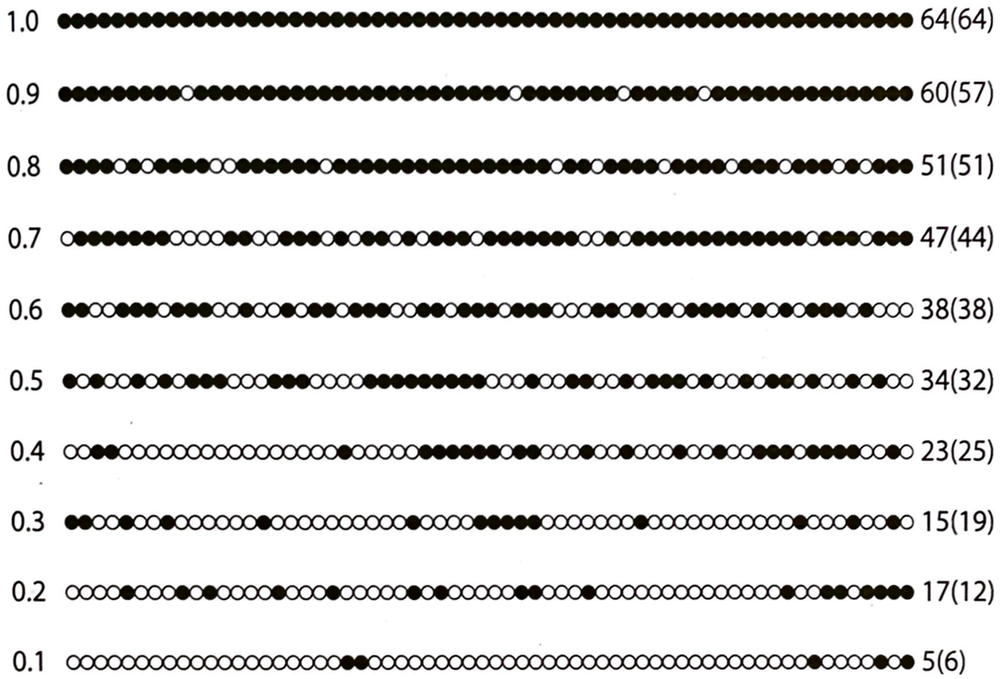


- What is the probability of having a cluster of length s?

Percolation (1D)

N = 64

p - occupation probability

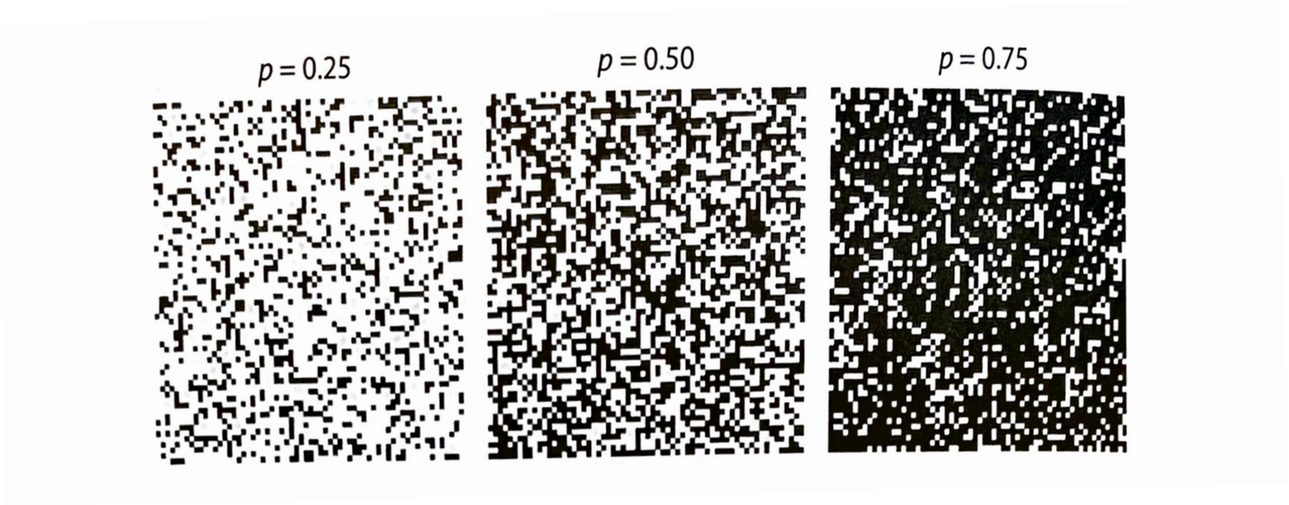


- What is the probability of having a cluster of length s? (1-p) x p x p x p …x (1-p) = $p^s(1-p)^2$

Percolation (1D)

- Size of largest cluster S(k)= max(s_k)? Linear at first…but then pieces connect
- Can be shown - lim N-> infinity S = (1+p) / (1-p)
- In other words, largest clusters reach system size as p-> 1
- p_c = 1 is percolation threshold

Percolation (2D)



$N = 64 \times 64$

- Size of largest cluster $S(k) = \max(s_k)$? When does connect edge to edge?

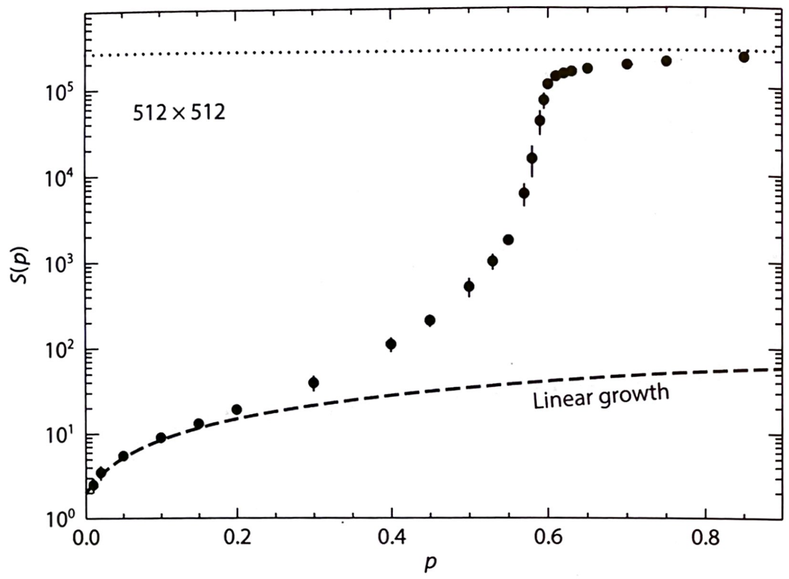
- $p_c = 1$ is not the percolation threshold!

Percolation (2D)

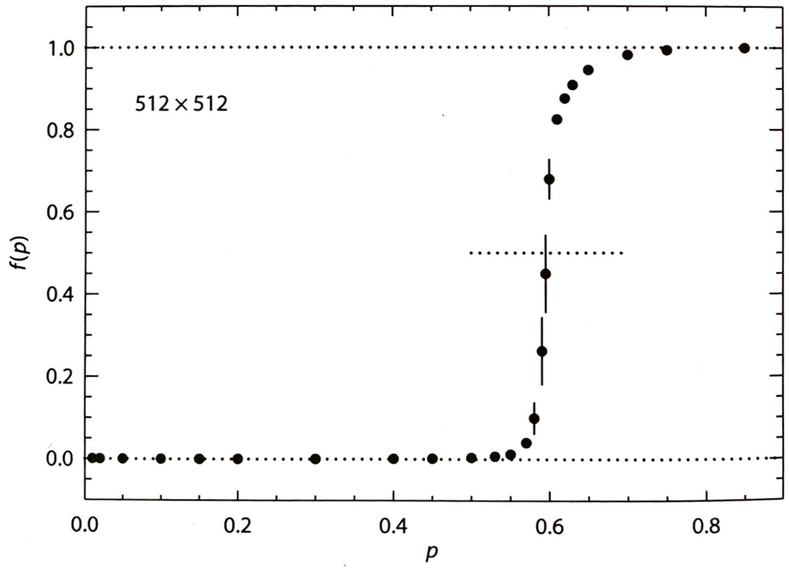
p = 0.59

N = 512 x 512

Percolation (2D)

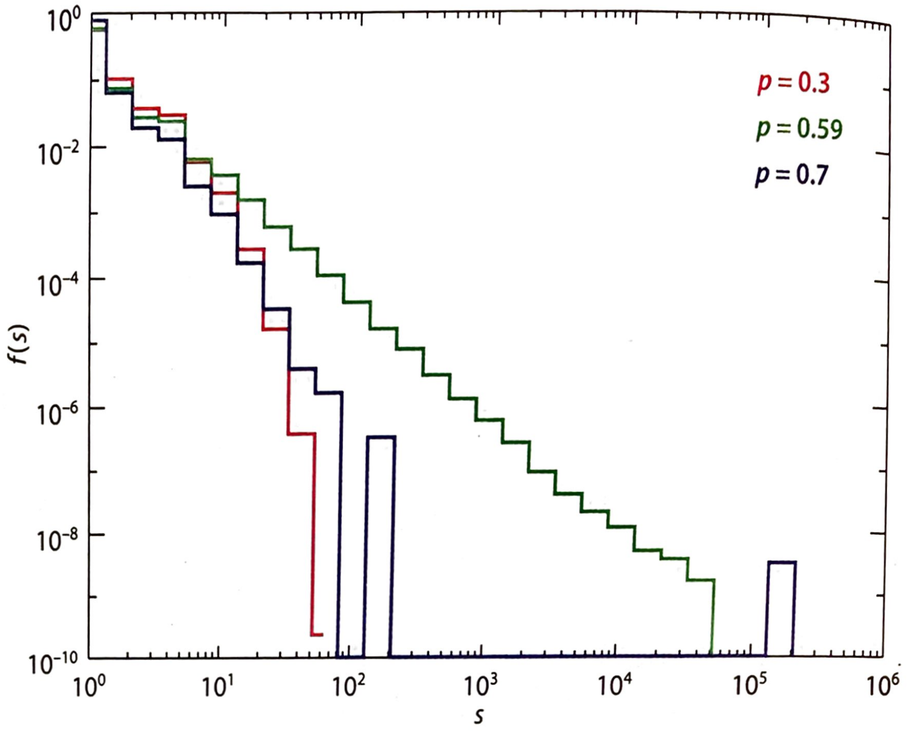


$f(p) = S(p) / (pN^2)$



Percolation (2D)

f(s) = PDF
probability of
finding cluster
between s and s+ds
is f(s)*ds



Percolation (2D)

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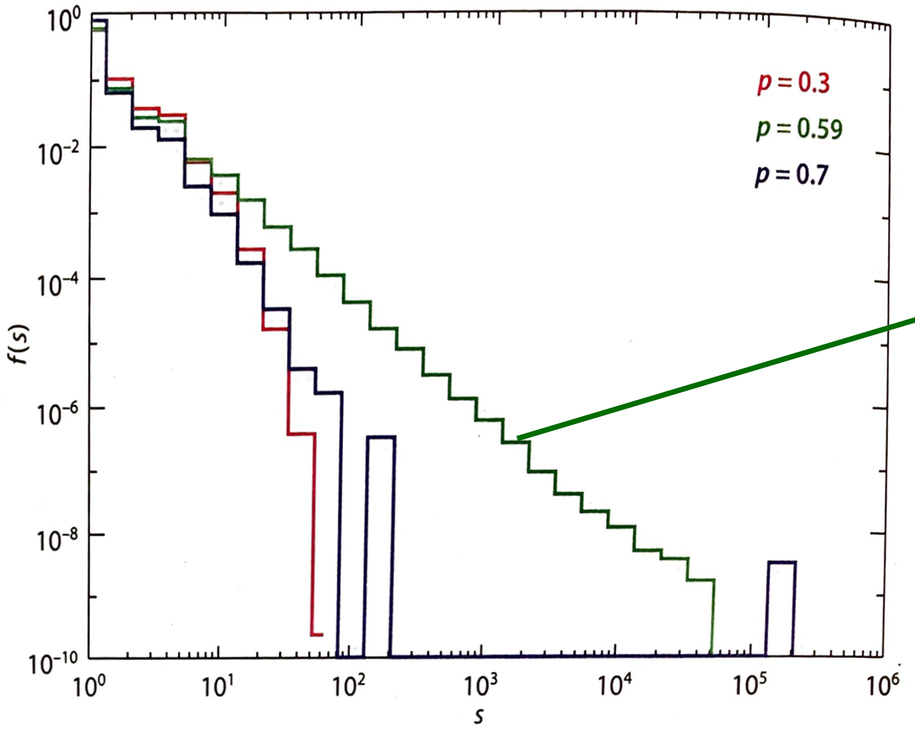
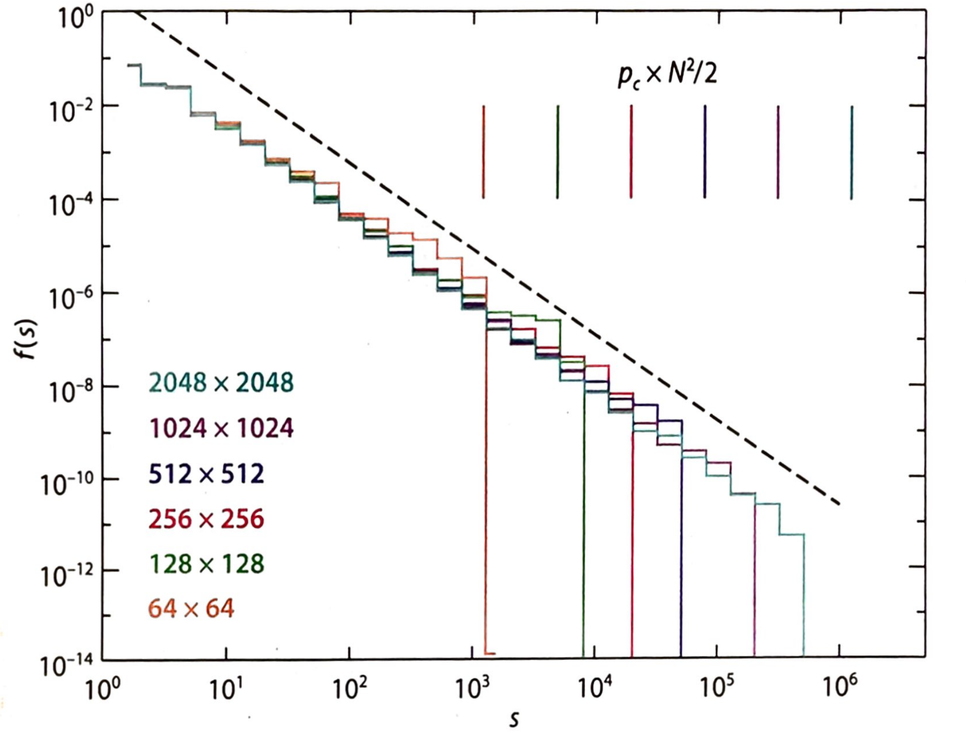
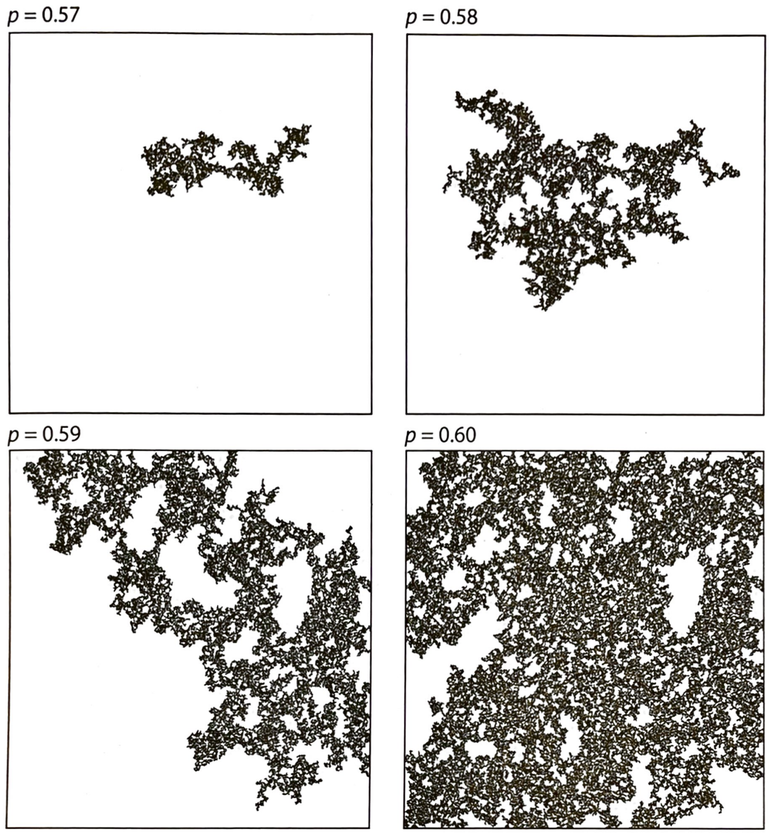


$f(s) = f_o s^{-a}$
$a \sim 1.85$

Percolation (2D)



Percolation (2D)



Percolation - criticality

- AT PERCOLATION THRESHOLD:
  - The sizes of clusters is power law distributed
  - Linear dimension of largest cluster ~ N
  - Largest cluster collects F = 0.5 of occupied nodes
  - The growth rate of S diverges (dS/dp -> infinity) in limit p -> pc

Percolation - criticality

Criticality was achieved as a control parameter was varied. Always needed?

Sandpiles - self-organized criticality

- CA model for sandpiles

- Rules:

$$S_j^o = 0,\ j = 0, ..., N-1$$

j = node on lattice (1D), superscript is iteration

$$S_r^{n+1} = S_r^n + s,\ r \in [0, N-1],\ s \in [0, \epsilon]$$

drop "sand grain" each iteration

$$z_j^n = |S_{j+1}^n - S_j^n|,\ j = 0, ..., N-2$$

slope grows

Sandpiles - self-organized criticality

- CA model for sandpiles

- Rules:

if slope exceeds
critical value (Z_c)

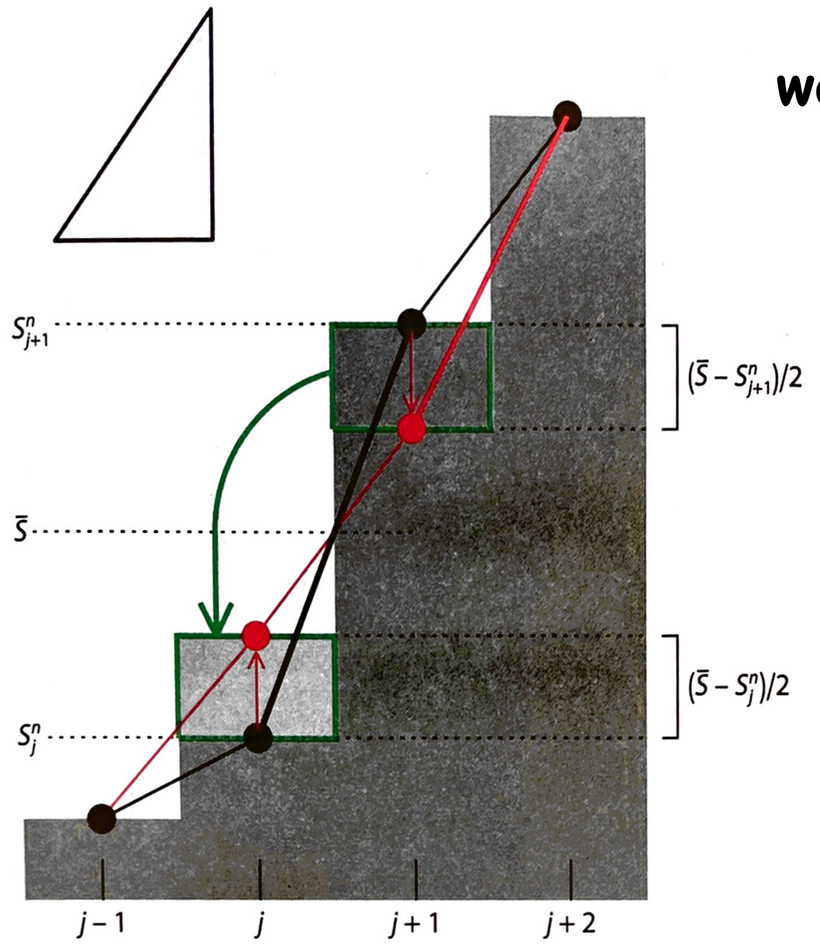
$$S_j^{n+1} = S_j^n + \frac{1}{2}(\bar{S} - S_j^n)$$

$$S_{j+1}^{n+1} = S_{j+1}^n + \frac{1}{2}(\bar{S} - S_{j+1}^n)$$

$$\bar{S} = \frac{1}{2}(S_{j+1}^n + S_j^n)$$

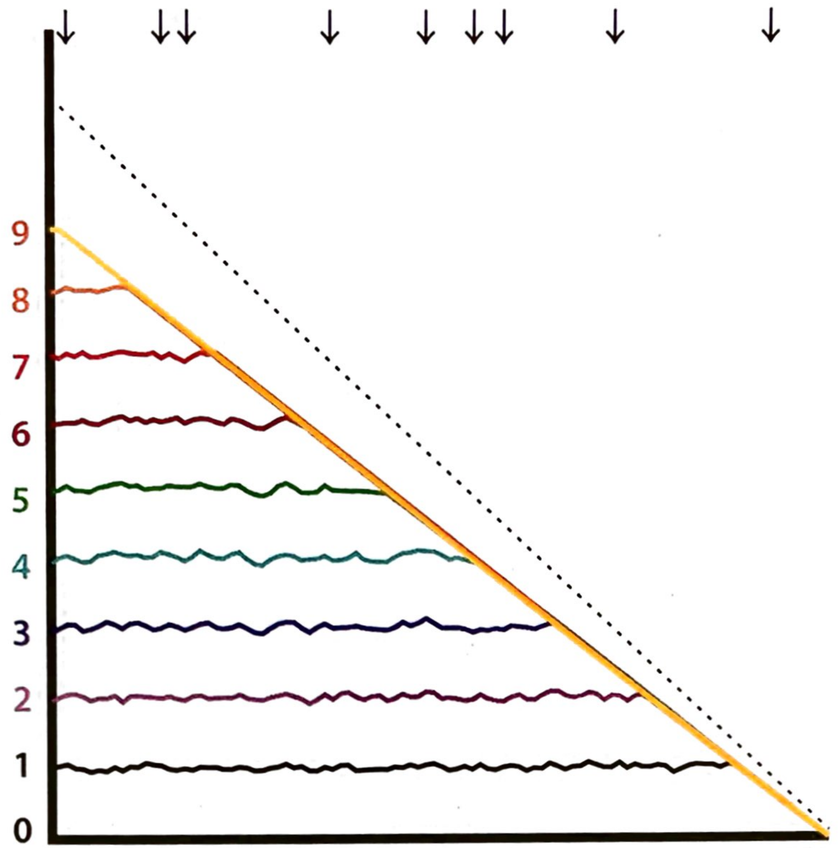
Sandpiles - self-organized criticality

- CA model for sandpiles
- Rules:

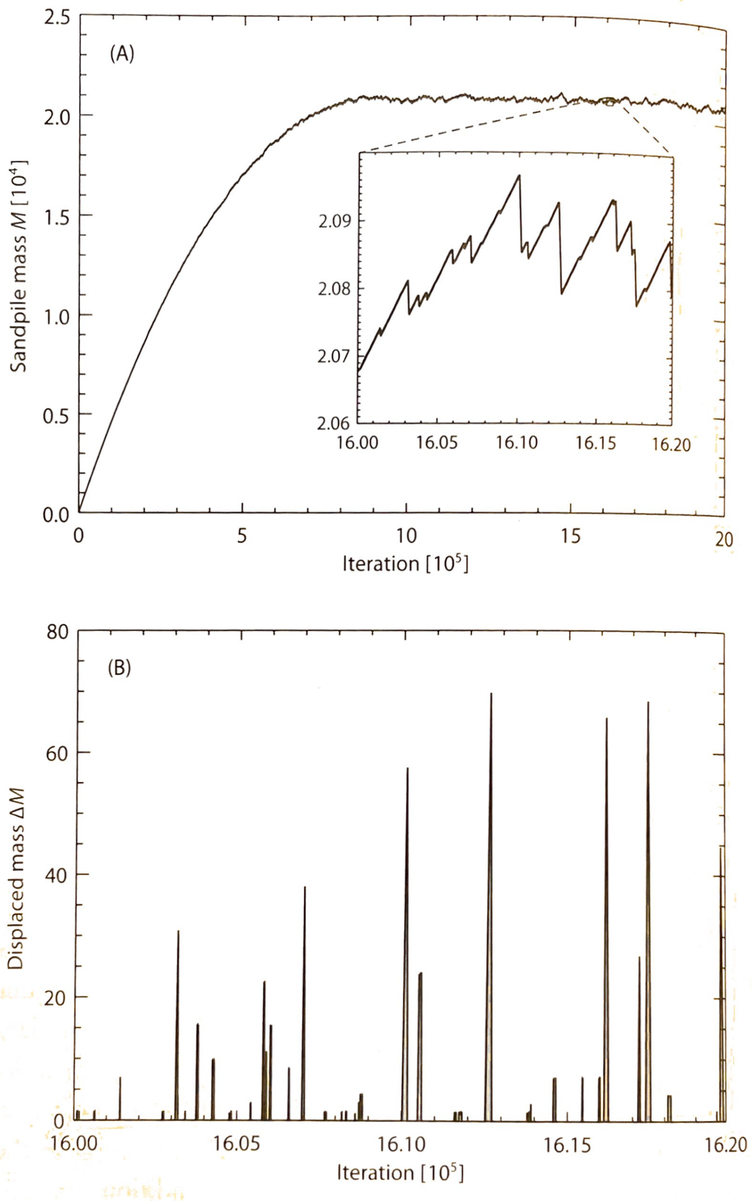


Sandpiles - self-organized criticality

$Z_c = 5$
epsilon = 0.1

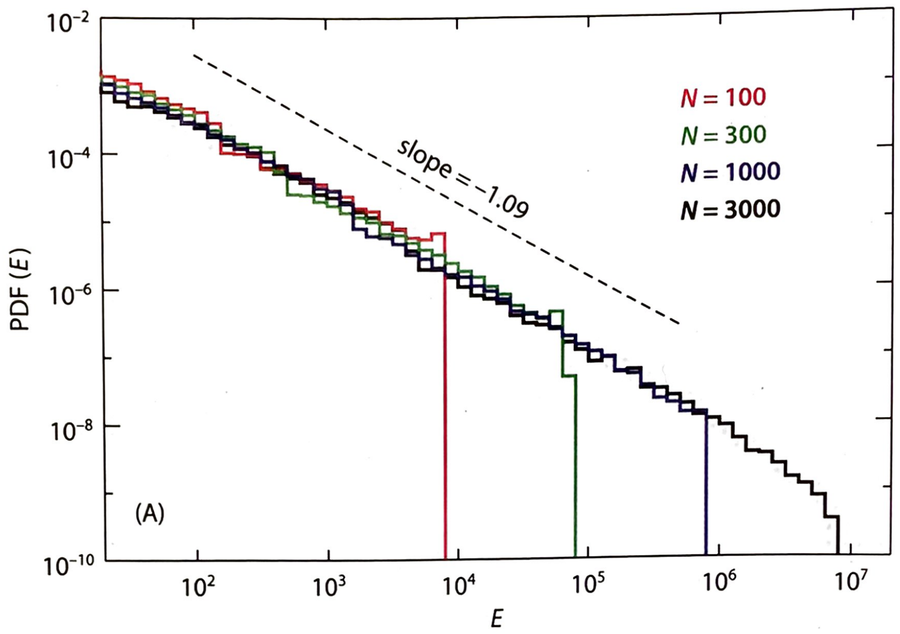


Sandpiles - self-organized criticality



Sandpiles - self-organized criticality

"Energy"
sum of an avalanche



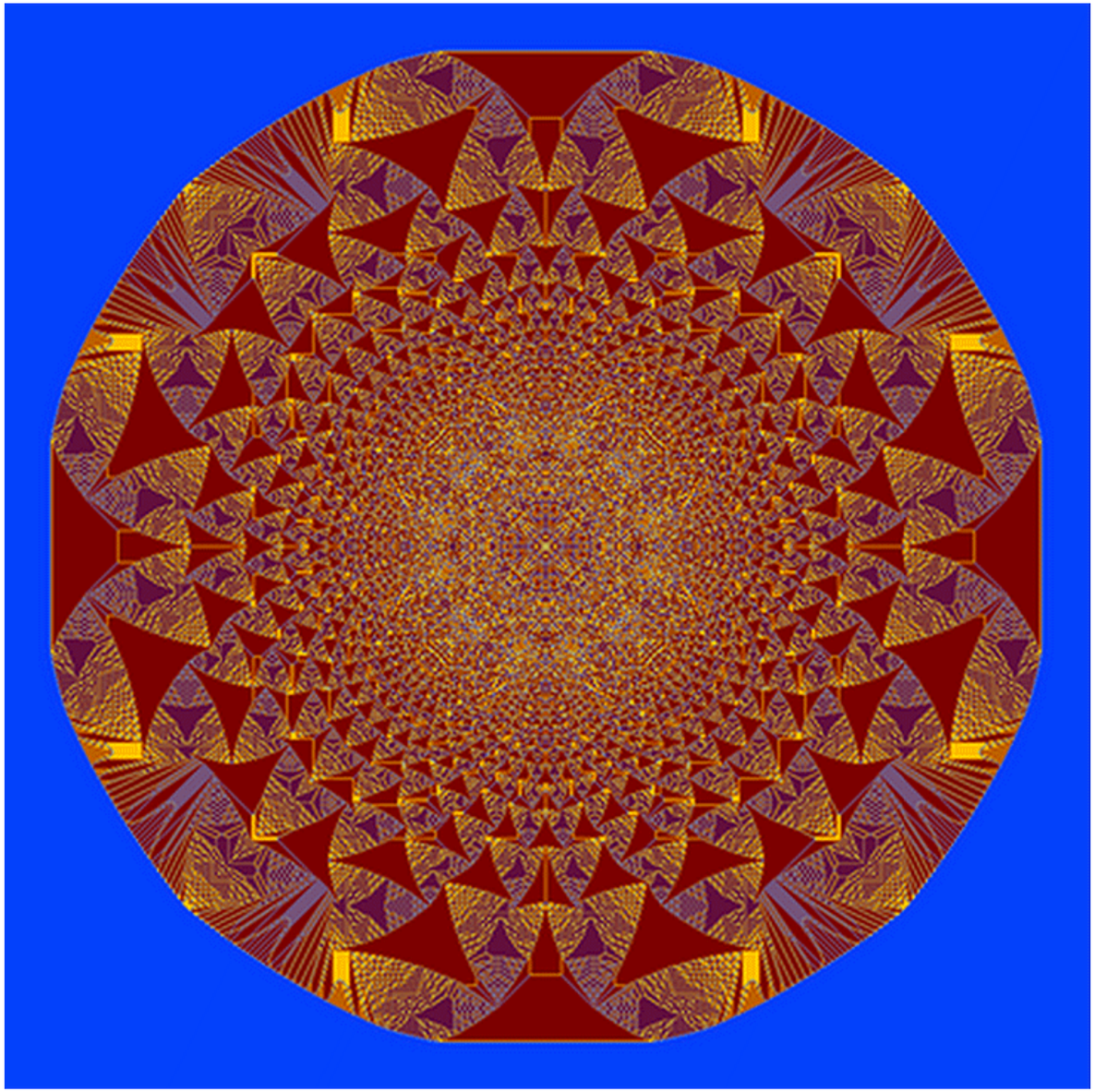
Sandpiles - self-organized criticality

2D

4 grains most allowed

piled million grains on center dot

blue empty
light blue 1
yellow 2
maroon 3



Sandpiles - self-organized criticality

2D

4 grains most allowed

piled BILLION grains
on center dot

blue empty
light blue 1
yellow 2
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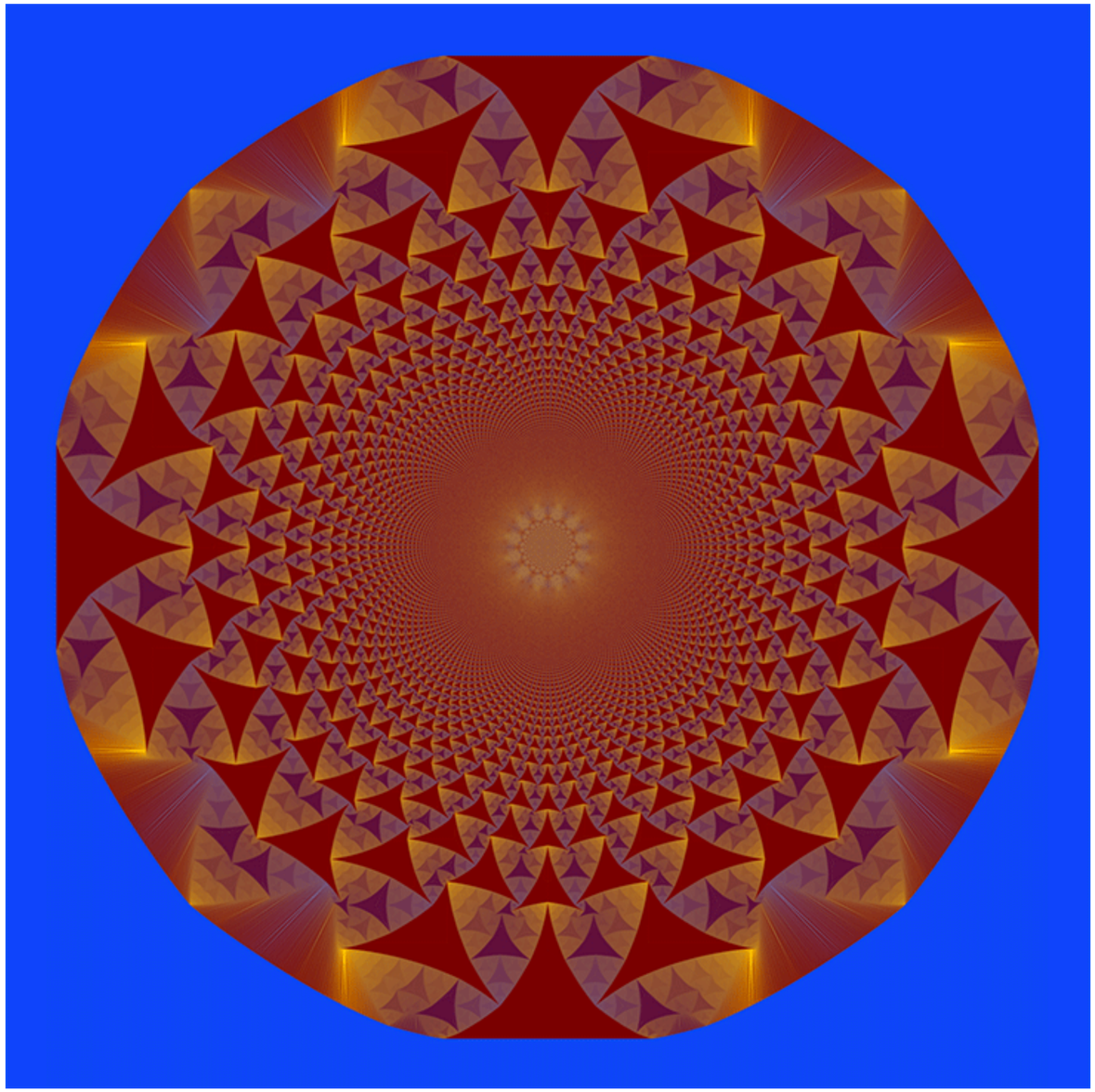

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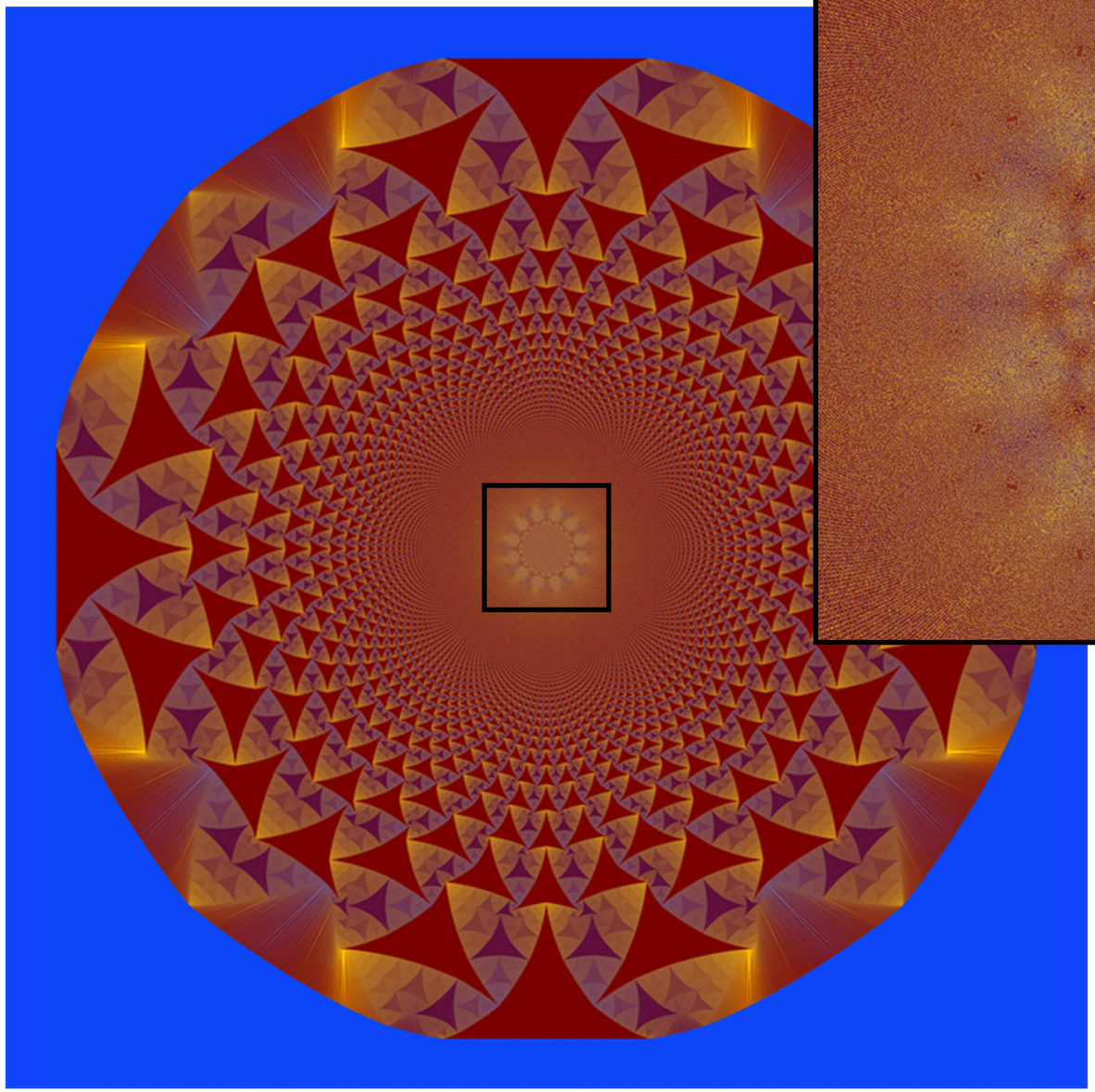

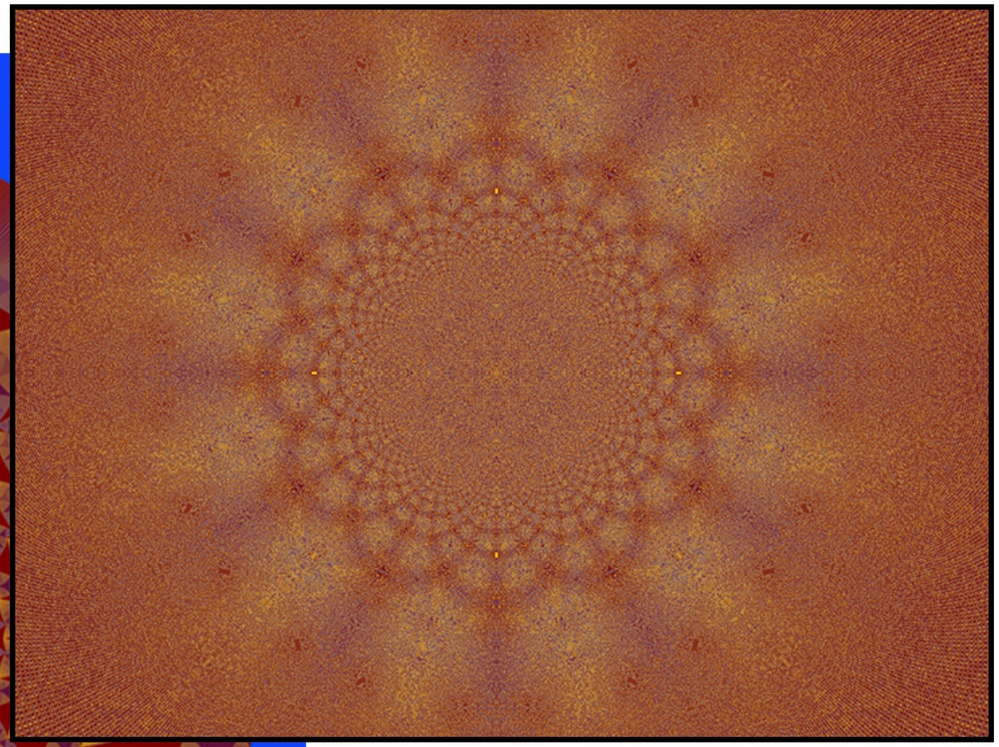

Sandpiles - self-organized criticality

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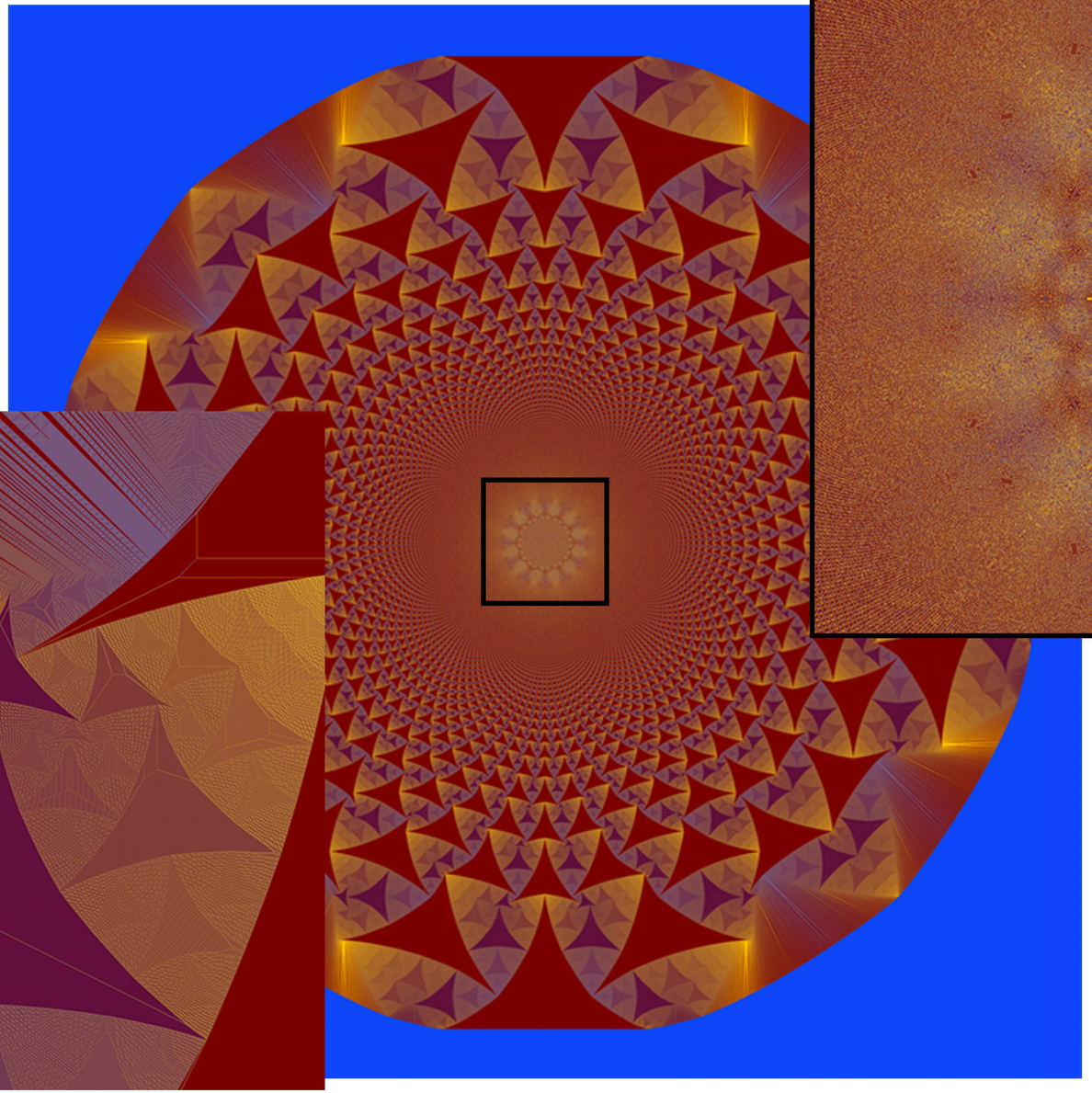
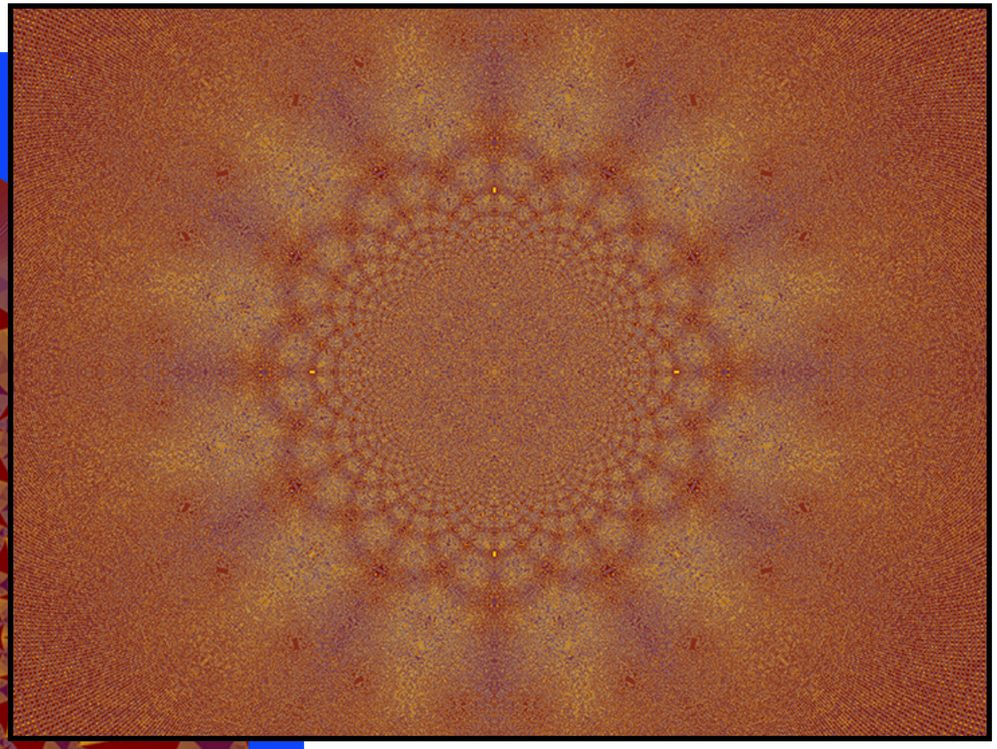
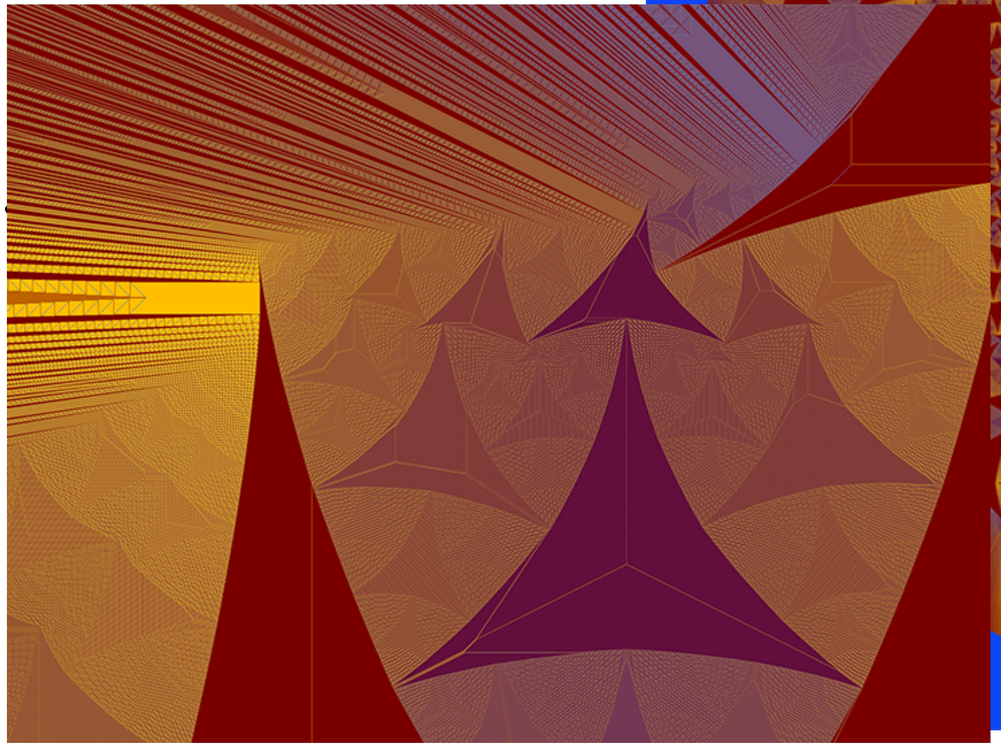




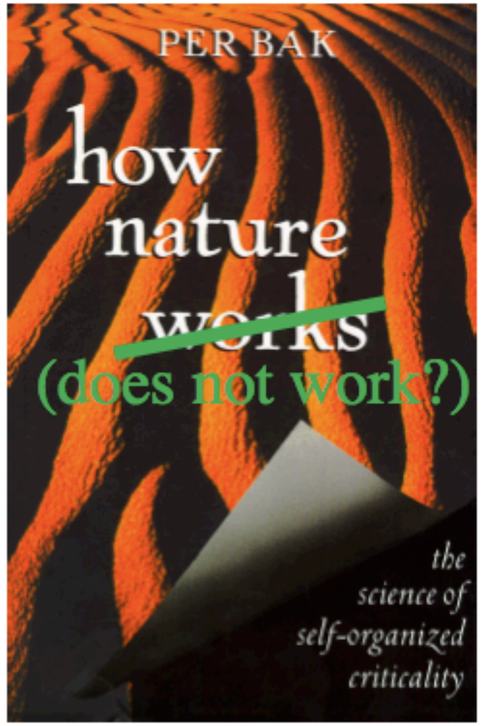


Sandpiles - self-organized criticality

- Criticality:
  - impact of small thing can hit whole system (occupying one more node in percolation)
  - system suddenly changes state (permeable)
  - here percolation threshold is angle of repose
  - here was found "naturally" - attractor!
  - Other possible examples: earthquakes, forest fires, stocks, war

Sandpiles - self-organized criticality

- **Per Bak (another Wolfram)**
  - Fashionable -> Unfashionable
  - $$$$$
  - Dogmatic
    - "Excuse me but what is actually non-trivial about what you did?"
  - A real sandpile though…



Sandpiles - self-organized criticality

Avalanche dynamics in a pile of rice

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THE idea of self-organized criticality[1] (SOC) is commonly illustrated conceptually with avalanches in a pile of sand grains. The grains are dropped onto a pile one by one, and the pile ultimately reaches a stationary 'critical' state in which its slope fluctuates about a constant angle of repose, with each new grain being capable of inducing an avalanche on any of the relevant size scales. Some numerical models of sand-pile dynamics do show SOC[1-8], but the behaviour of real sand piles remains ambiguous[9-18]. Here we describe experiments on a granular system—a pile of rice—in which the dynamics exhibit self-organized critical behaviour in one case (for grains with a large aspect ratio) but not in another (for less elongated grains). These results show that SOC is not as 'universal' and insensitive to the details of a system

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