Part 1: An electromagnetic velocity sensor

A) Explain the Biot-Savart law.

The Biot-Savart law gives the magnetic field of a steady line current and is analogous to Coulomb's law in electrostatics:

$$\mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{\mathbf{I}\times\hat{\mathbf{r}'}}{r'^2}dl = \frac{\mu_0}{4\pi}I\int \frac{d\mathbf{l}\times\hat{\mathbf{r}'}}{r'^2}dl$$

Where $\mathbf{I}$ is the electric current, $\mathbf{r}'$ is the vector from the source to the point $\mathbf{r}$, dl is an element along the wire in the direction of the current and $\mu_0 = 4\pi \times 10^{-7} N/A^2$ is the permeability of free space. The units for the magnetic field are newtons per ampere-meter (N/ (A m)) or teslas (T) and the integral is along the current path in the same direction as the flow.

Source: Griffiths, David. Introduction to Electrodynamics. Fourth Edition

B) Explain Onsager's reciprocal theorem

Onsager's reciprocal theorem involves the reciprocity of coupled electrical and thermal systems. Lars Onsager's 1931 paper uses the following notation:

X_1 and X_2 are driving electrical and thermodynamic forces, respectively and, if they were independent, could be written as:

$$X_1 = R_1 J_1$$
$$X_2 = R_2 J_2$$

where R_1 and R_2 are the electrical resistivity and thermal resistance, respectively. J_1 and J_2 are electrical and thermal current.

However, these systems are coupled since electrical current is not independent of the temperature. Therefore we can add in this dependency using cross coefficients R_{12} and R_{21} so that:

$$X_1 = R_{11}J_1 + R_{12}J_2,$$

and

$$X_2 = R_{21}J_1 + R_{22}J_2.$$

My understanding of this is that the electrical force is equal to electrical resistance times current density plus the current scaled by a coeffecient that represents the coupling of the thermal heat on the electrical system.

Similarly, the thermodynamic force is equal to the thermal resistance times thermal current density plus the electrical current density scaled by a coeffecient representing the coupling of the electrical system on the thermal. The reciprocity theorem states that these two coefficients are equal,

$R_{12} = R_{21}$

Sources:

Onsager,Lars. Reciprocal Relations in Irreversible Processes. I. Physical Review, Vol 37. 1931

http://www.iue.tuwien.ac.at/phd/holzer/node24.html

C)How would you compute l given a coil with radius r and number of coils n?

$l = 2\pi rn$

D) Derive equation 12.49

First list the forces by looking at Figure 12.15.

1. Force of gravity on the mass
2. Force from the spring
3. Magnetic force

So using Newton, we have: $F = F_{gravity} - F_{spring} - F_{mag}$

We saw in class that the force from the spring ($K(z - l_0)$) and the force of gravity ($Mg = k(z - l_0)$) combine and can be written as kz(t). We also saw that there are two sources of acceleration, from the seismometer and from the ground, so that we have:

$$z'' + u'' = -\omega_0^2 z - \frac{1}{M} Fmag. \tag{1}$$

Rearranging and dividing by M:

$$z'' + u'' = -\omega_0^2 z - \frac{1}{M} Fmag, \tag{2}$$

where $w_o^2 = \frac{k}{m}$.

Aki and Richards give the magnetic force (from the Lorentz force law) as:

$F = IlB$,

where I is the current, l is the length of the wire and B is the flux density. The mechanical power is the rate that work is done so they multiply both sides by the velocity of the moving mass, or z'. so that :

$Fz' = IlBz'$

Then they say that the mechanical power must be consumed by the resistance and since electrical power is equal to $P = VI$ (by Ohm's law) they obtain

$V = lBz'$.

Now they define lB as G so $V = Gz'$. Using Ohm's law and the total resistance equal to $R_0 + R$, $I(R_0 + R) = Gz'$ and simple division yields

$$I = \frac{Gz'}{R_0 + R}. \tag{3}$$

Now $F = GI$ and we can substitute in equation (3) to get

$F = \frac{G^2 z'}{R_0 + R}$,

and in turn substitute this back into equation (2) so that:

$$z'' + u'' = -\omega_0^2 z - \frac{G^2}{R_0 + R}\frac{z'}{M} \tag{4}$$

or

$$z'' + \omega_0^2 z = -u'' - \frac{G^2}{R_0 + R}\frac{z'}{M}. \tag{5}$$

In class we saw that for a similar system with a dashpot instead of a magnet we get:

$$z'' + \omega_0^2 z = -u'' - 2\epsilon z'. \tag{6}$$

and comparing the two equations:

$$2\epsilon = \frac{G^2}{R_0 + R}\frac{1}{M} \tag{7}$$

This equation relates the two dampening terms in the equation of motion for both systems. If there is also mechanical attenuation, ϵ_0 then

$$\epsilon = \epsilon_0 + \frac{G^2}{2(R_0 + R)}\frac{1}{M}. \tag{8}$$

E) When was the electromagnetic sensor introduced in seismology and who introduced it?

The electromagnetic sensor was introduced in 1914 by Russian scientist Galitzin.

Table of Contents

Part 2: Creating the frequency response

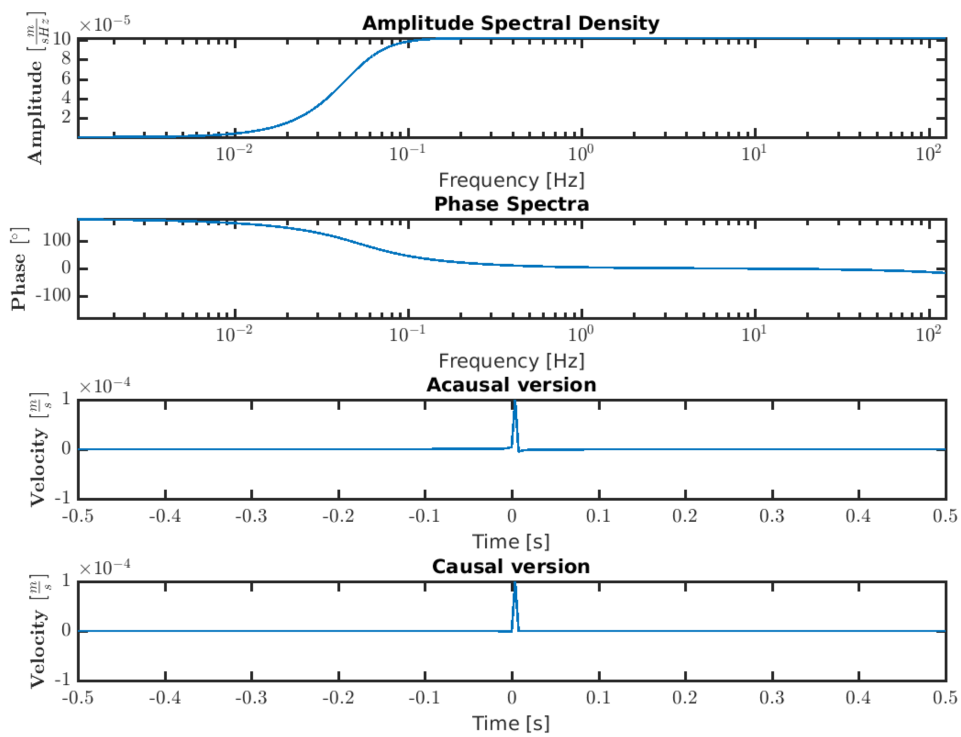
```
addpath '/home/rflee/Documents/MATLAB/mseed/'
f = '../../Data/XX.BSM7.HHZ_MC-PH1_0426_20170112_180000.miniseed';
Meta = rdmseed(f);

sf= Meta.SampleRate; %sampling frequeny in Hz
ny= .5*sf; % nyquist frequency
n = 10e4+1; % number of points for frequency vector
dt = 1/sf;

% create frequency array
frequencies= linspace(0,ny,n);
w = frequencies*2*pi;

A0=4.344928e17; % conversion from counts to m/s for this instrument

zeros = [ % the zeros in 1/sec
    0.000000E+00;
    0.000000E+00;
    -3.920000E+02;
    -1.960000E+03;
    -1.490000E+03 + 1i*1.740000E+03;
    -1.490000E+03 + 1i*-1.740000E+03
    ];

poles = [ % the poles in 1/sec
    -3.691000E-02 + 1i* 3.702000E-02;
    -3.691000E-02 + 1i*-3.702000E-02;
    -3.430000E+02 + 1i* 0.000000E+00;
    -3.700000E+02 + 1i* 4.670000E+02;
    -3.700000E+02 + 1i*-4.670000E+02;
    -8.360000E+02 + 1i* 1.522000E+03;
    -8.360000E+02 + 1i*-1.522000E+03;
    -4.900000E+03 + 1i* 4.700000E+03;
    -4.900000E+03 + 1i*-4.700000E+03;
    -6.900000E+03 + 1i* 0.000000E+00;
    -1.500000E+04 + 1i* 0.000000E+00
```

```
    ];

% get coefficients for polynomials
num = poly(zeros*2*pi); %convert poles and zeros to rad/s for function
denom= poly(poles*2*pi);

%get the frequency response
fResponse=A0*freqs(num,denom,w);

% convert frequency response to polar (magnitude and phase)
mag =abs(fResponse);
phase = angle(fResponse);
phase = rad2deg(phase);

% get time series amplitudes
Vacausal = ifft(fResponse);
Vacausal = ifftshift(Vacausal);

% create times
t1 = 0:dt:dt*((n-1)/2); % positive times
t2= -dt*((n-1)/2):dt:-dt; % negative times
t = [t2, t1];

% get times
df = frequencies(2)-frequencies(1);

% Apply Haney method to enforce causality
Vcausal = real(fResponse) -1i*(hilbert(real(fResponse)));
Vcausal = real(ifftshift(ifft(Vcausal)));
```

Plots

```
fsize=16;

scrsz = get(0,'ScreenSize');
HH=figure('Position',[scrsz(3) scrsz(4) scrsz(3) scrsz(4)]);
subplot(4,1,1)
semilogx(frequencies,mag,'LineWidth',2);
axis tight
xlabel('Frequency [Hz]')
ylabel('{\bf Amplitude [$\frac{m}{s Hz}$]}','Interpreter','latex')
title('Amplitude Spectral Density')


subplot(4,1,2)
semilogx(frequencies,phase,'Linewidth',2)
axis tight
ylim([-180 180])
xlabel('Frequency [Hz]')
ylabel('{\bf Phase [$^{\circ}$]}','Interpreter','latex')
title('Phase Spectra')
```

```
subplot(4,1,3)
plot(t,real(Vacausal),'Linewidth',2)
ylim([-1e-4 1e-4])
xlim([-.5 .5])
ylabel('{\bf Velocity [$\frac{m}{s}$]}','Interpreter','latex')
xlabel('Time [s]')
title('Acausal version')


subplot(4,1,4)
plot(t,Vcausal,'Linewidth',2)
ylim([-1e-4 1e-4])
xlim([-.5 .5])
ylabel('{\bf Velocity [$\frac{m}{s}$]}','Interpreter','latex')
xlabel('Time [s]')
title('Causal version')

figproperties
```



Discussion

For this instrument, the response starts to fall off below about .1 Hz The phase spectra shows the response is about zero from nyquist down to .1 hz. Below .1 Hz the phase slowly increases until about pi. I interpret this to mean there can be change in the phase up to pi at low frequencies so that by .01 Hz the polarity is reversed. The time domain of the frequency response shows a near delta function, so the response should reflect the motion since the time series does not show any resonance.

Part 3: Instrument Deconvolution

Import Data

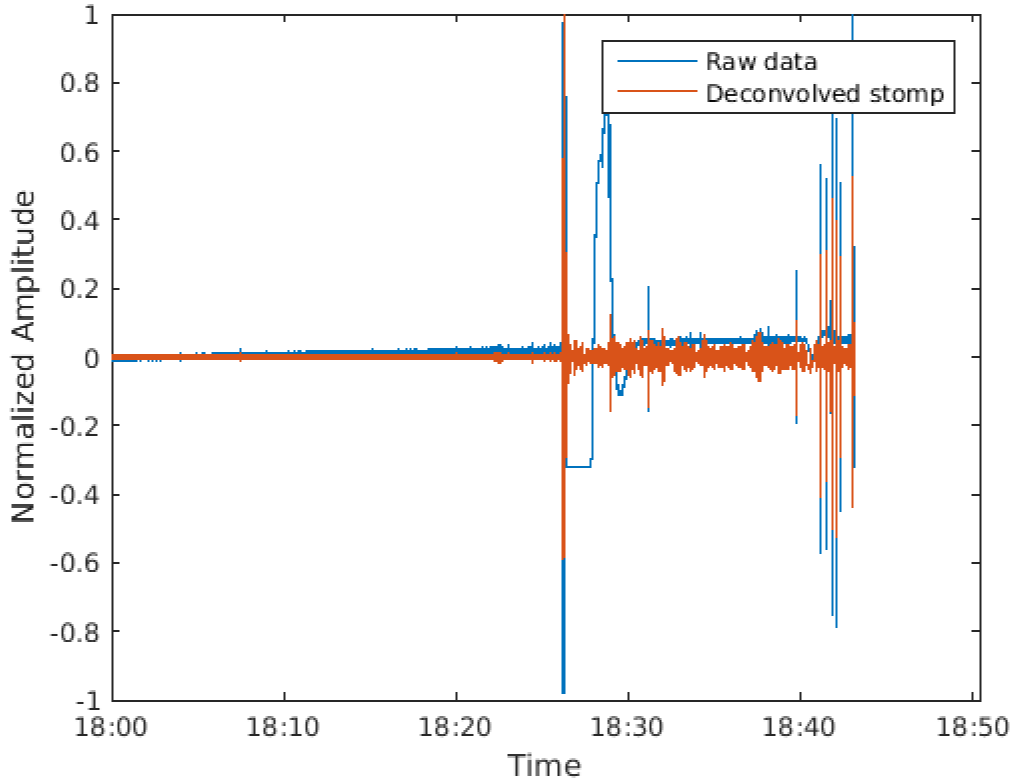
```
t = cat(1,Meta.t);
rawdata = cat(1,Meta.d);

% %plot initial data
% Xlim = [min(cat(1,Meta.t)),max(cat(1,Meta.t))];
% figure;
% plot(t,rawdata)
% set(gca,'XLim',Xlim)
% datetick('x','keeplimits')
```

Deconvolve the data

```
Sensitivity = 4.000000E+05;
% ordl Butterworth order at flo, the low cutoff (between 2 and 4)
ordl = 2;
% ordh Butterworth order at fhi, the high cutoff (between 3 and 7)
ordh = 3;
% digout: inverse gain (m/s/count)
digout = Sensitivity/A0;
% digoutf: frequency of normalization (Hz)
digoutf = 1;
% ovrsampl: over-sampling factor for digital filter accuracy (e.g., 5)
ovrsampl = 10;
% idelay: intrinsic delay in the acquisition system
idelay = 0;

flo= 10e-2;
fhi = 10e2;
badvals= -2e31;

decondata =
 rm_instrum_resp(rawdata,badvals,sf,poles,zeros,flo,fhi,ordl,ordh,digout,digoutf,o
```

plot all data

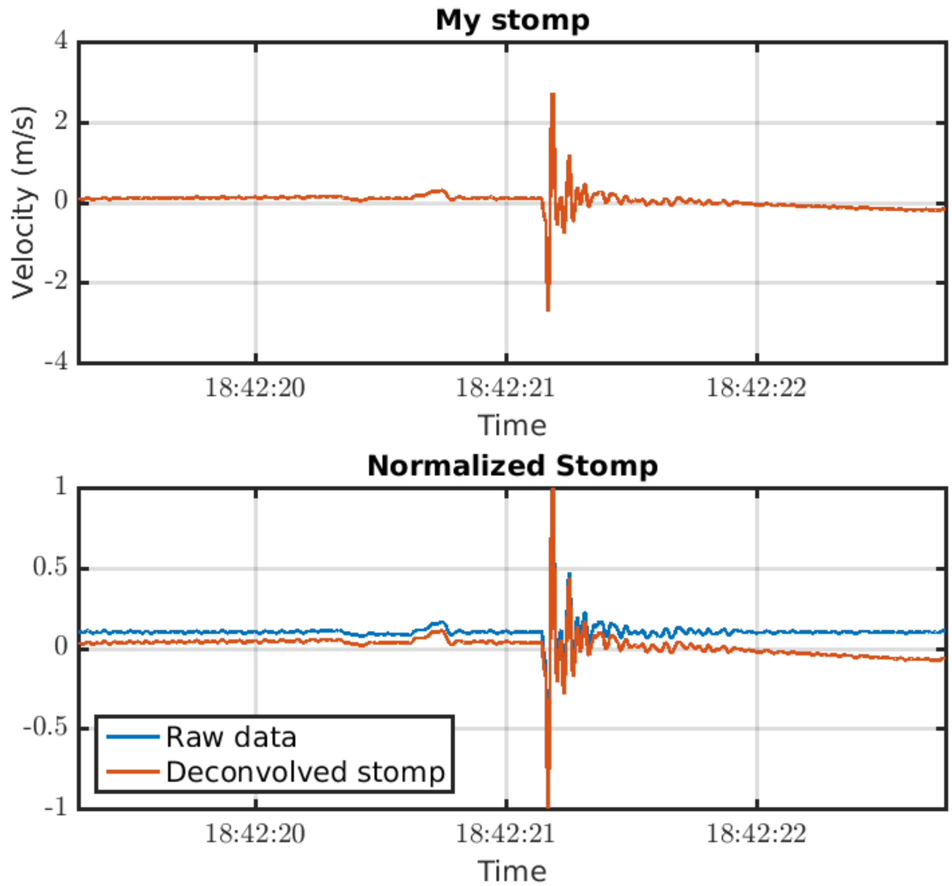
```
normraw = rawdata./max(abs(rawdata));
normdecon = decondata./max(abs(decondata));
figure;
plot(t,normraw)
hold on
plot(t,normdecon)
ylabel('Normalized Amplitude')
xlabel('Time')
datetick('x','keeplimits')
legend('Raw data','Deconvolved stomp')
```



Plot my stomp

```
mylim = [7.3670777939 7.3670777943]*1e5; % I zoomed into mystop and
 got xlim to get these numbers

% cut the data to my stomp

%find index numbers
[~,idxstart] = min(abs(t-mylim(1)));
[~,idxstop] = min(abs(t-mylim(2)));

% cut data and time vectors
mystompraw = rawdata(idxstart:idxstop);
mystompraw = mystompraw./max(abs(mystompraw));

mystompdecon = decondata(idxstart:idxstop);
mytim = t(idxstart:idxstop);
% plot my stomp-----------------------
figure;
subplot(2,1,1)
plot(mytim,mystompdecon);hold on;
plot(mytim,mystompdecon)
xlim(mylim)
datetick('x','keeplimits')
title('My stomp')
```

```
xlabel('Time')
ylabel('Velocity (m/s)')

figproperties;
grid on
%--------------------------------
mystompdecon = mystompdecon./(max(abs(mystompdecon)));




subplot(2,1,2)
plot(mytim,mystompraw)
hold on
plot(mytim,mystompdecon);
xlim(mylim)
datetick('x','keeplimits')
legend('Raw data','Deconvolved stomp','Location','SouthWest')
hold off
title('Normalized Stomp')
xlabel('Time')

figproperties;
grid on

set(gcf,'Units','Inches')
set(gcf,'Position',[1 1.5 8.7 7.3])
```



Discussion

Compared to the deconvolved data, the raw data shows the same shape as the deconvolved data. The main difference is that there seems to be a D.C. offset that is nearly corrected to zero for the deconvolved data. Where there is no event, however, the deconvolved data fluctuates around zero.

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